Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/20-1.1.2.3-a+b-x^2-^p-c+d-x^2-^q

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [349]. This is test number [20].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (349)	0.00 (0)
Mathematica	100.00 (349)	0.00 (0)
Fricas	79.37 (277)	20.63 (72)
Maple	75.64 (264)	24.36 (85)
Giac	30.37 (106)	69.63 (243)
Sympy	29.51 (103)	70.49 (246)
Maxima	22.64 (79)	77.36 (270)
Mupad	18.91 (66)	81.09 (283)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

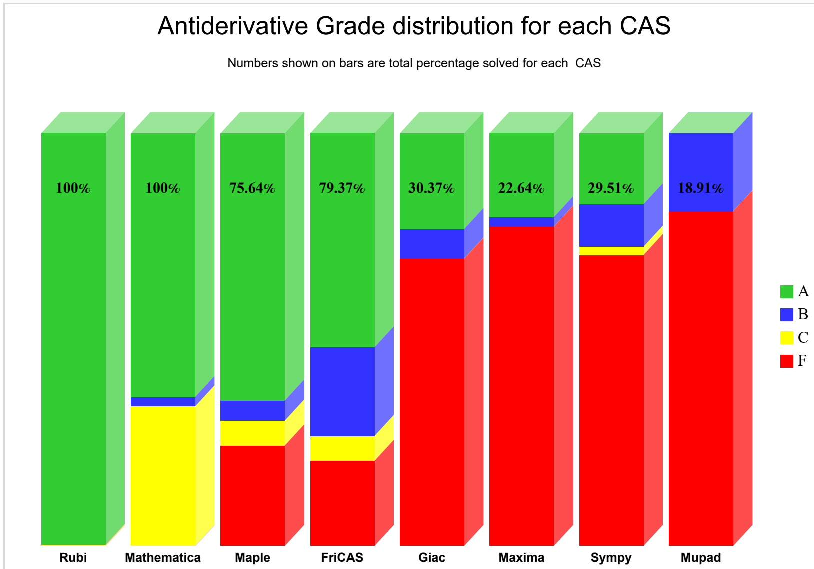
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

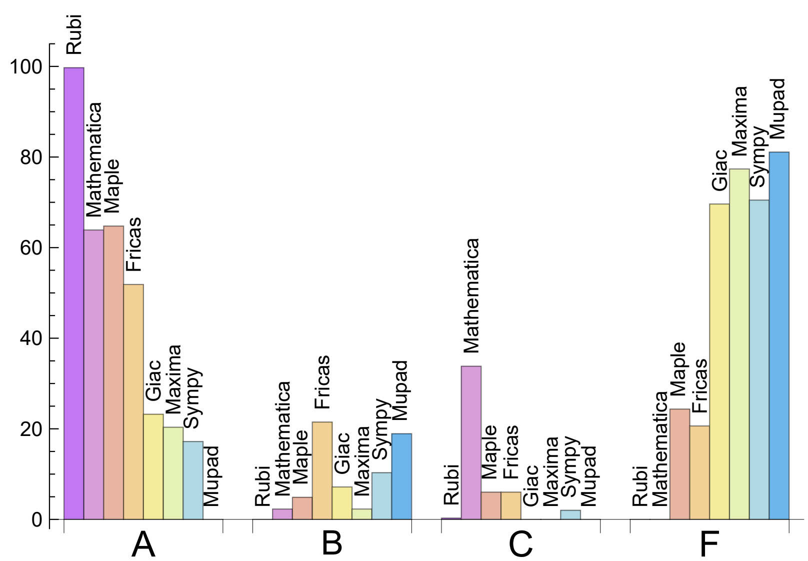
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.713	0.000	0.287	0.000
Maple	64.756	4.871	6.017	24.355
Mathematica	63.897	2.292	33.811	0.000
Fricas	51.862	21.490	6.017	20.630
Giac	23.209	7.163	0.000	69.628
Maxima	20.344	2.292	0.000	77.364
Sympy	17.192	10.315	2.006	70.487
Mupad	0.000	18.911	0.000	81.089

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	72	36.11	63.89	0.00
Maple	85	100.00	0.00	0.00
Giac	243	97.53	0.00	2.47
Sympy	246	91.46	8.54	0.00
Maxima	270	100.00	0.00	0.00
Mupad	283	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.24
Rubi	0.28
Giac	0.49
Sympy	1.91
Mathematica	2.70
Mupad	3.51
Fricas	3.55
Maple	4.19

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	127.63	0.98	93.00	0.99
Sympy	149.86	1.83	94.00	1.63
Maxima	157.72	1.24	116.00	1.16
Maple	165.37	1.44	104.00	1.00
Rubi	184.96	1.01	120.00	1.00
Giac	229.62	1.64	129.50	1.14
Fricas	348.17	2.75	133.00	1.70
Mupad	912.58	5.52	87.50	1.12

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

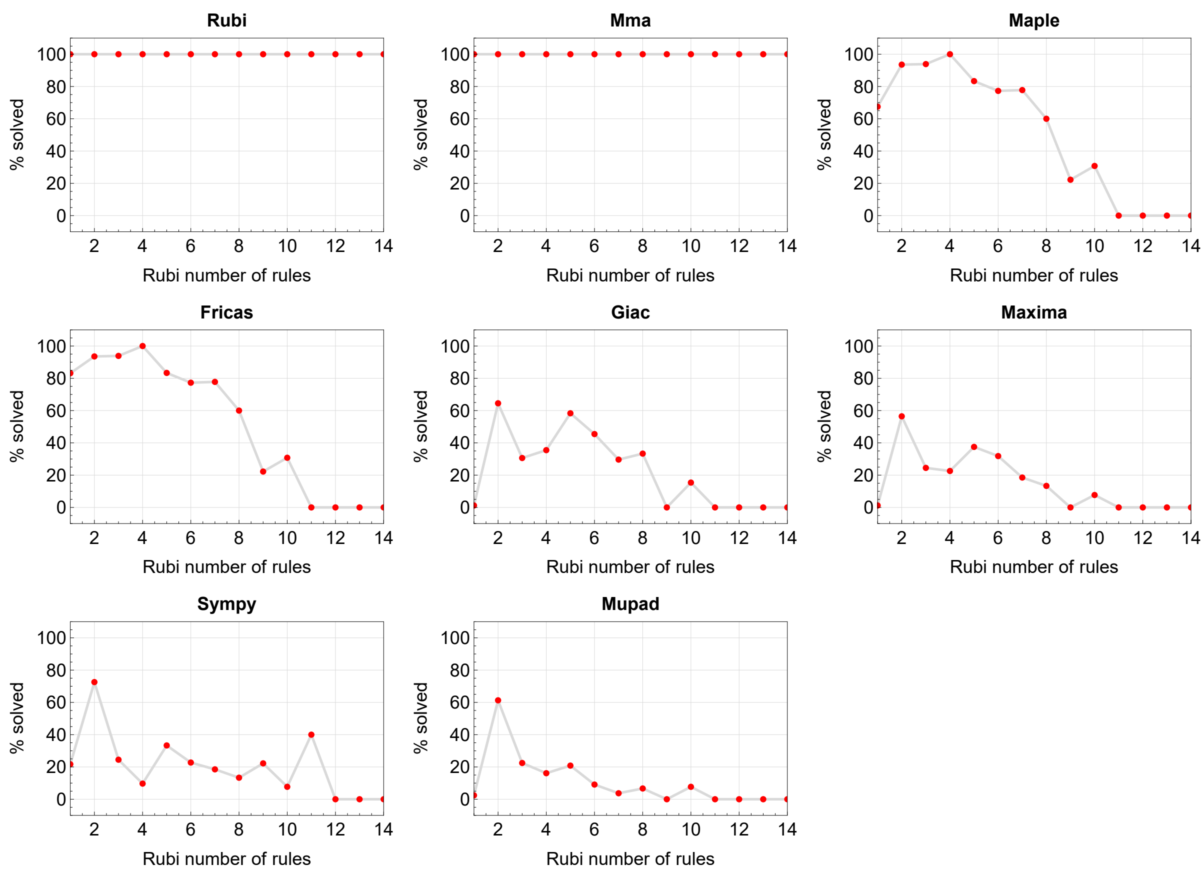


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

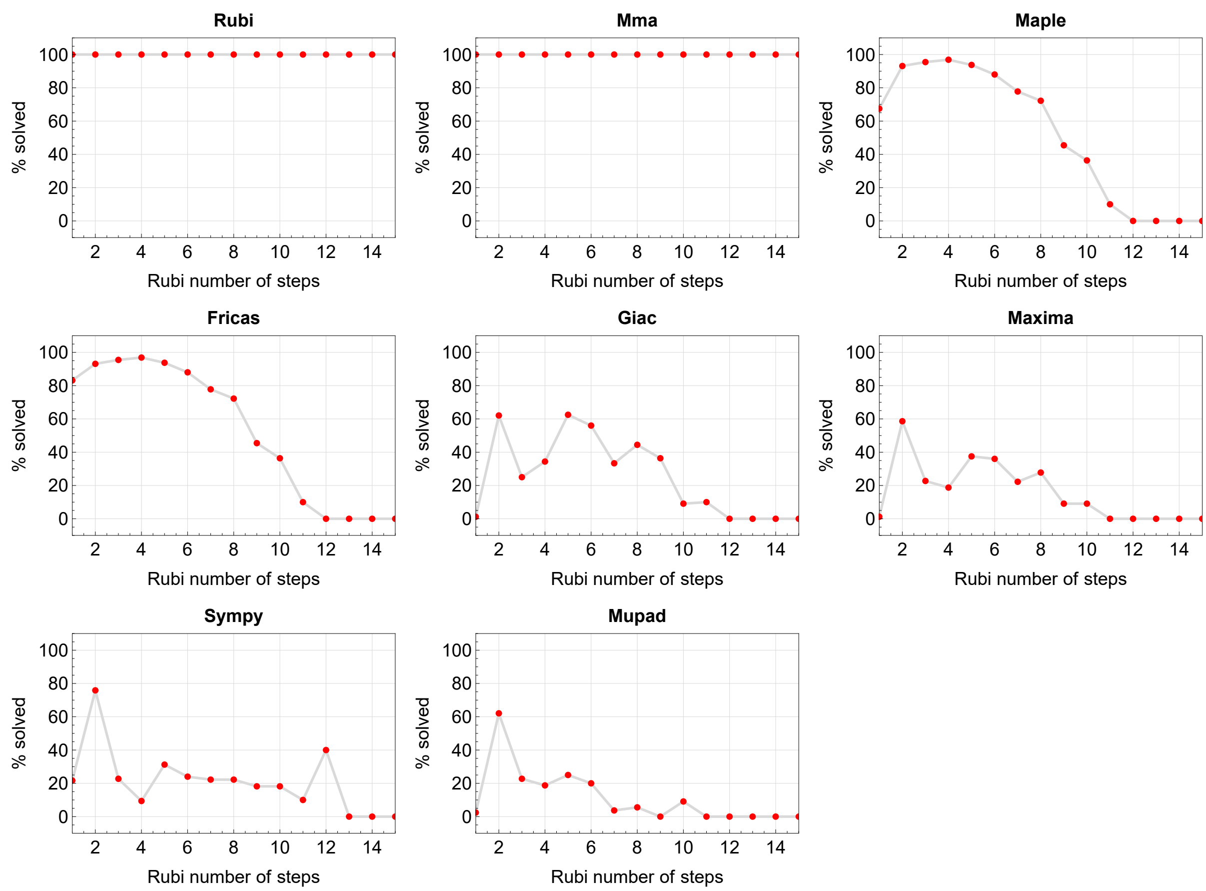


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

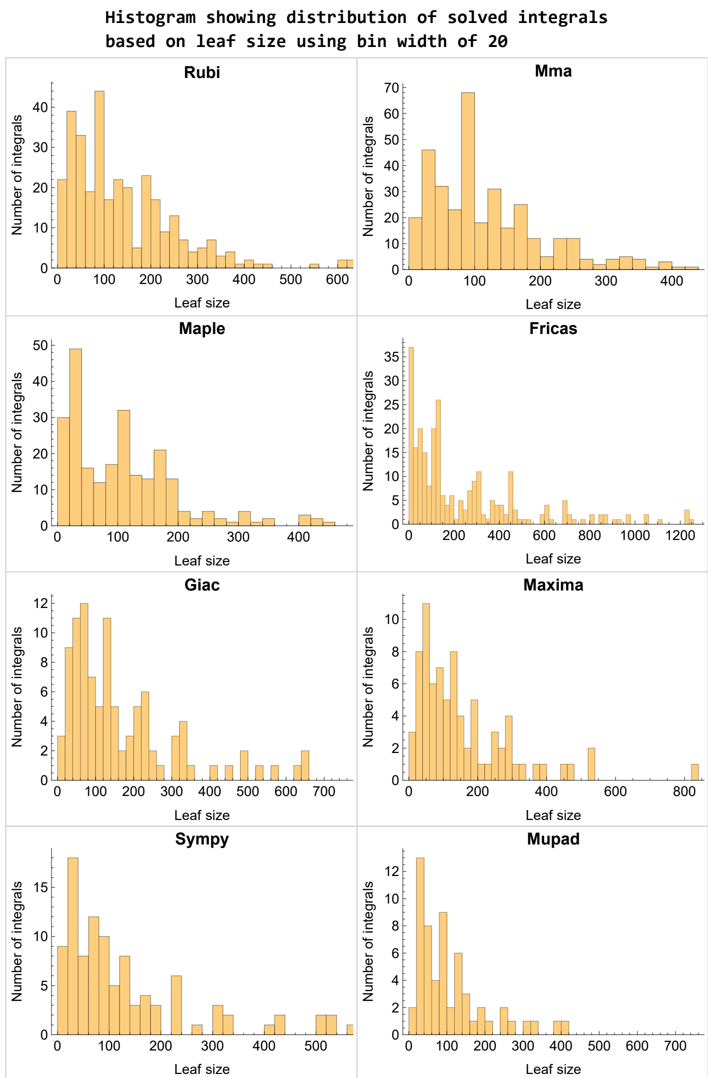


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

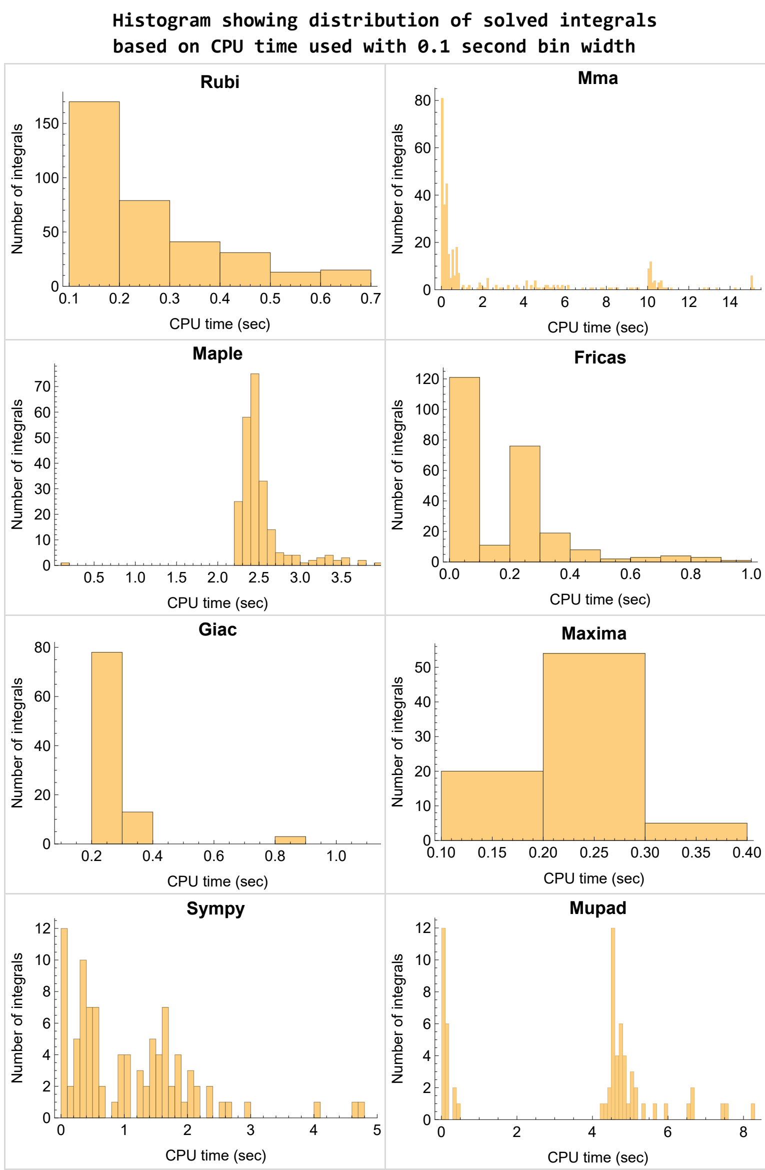


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

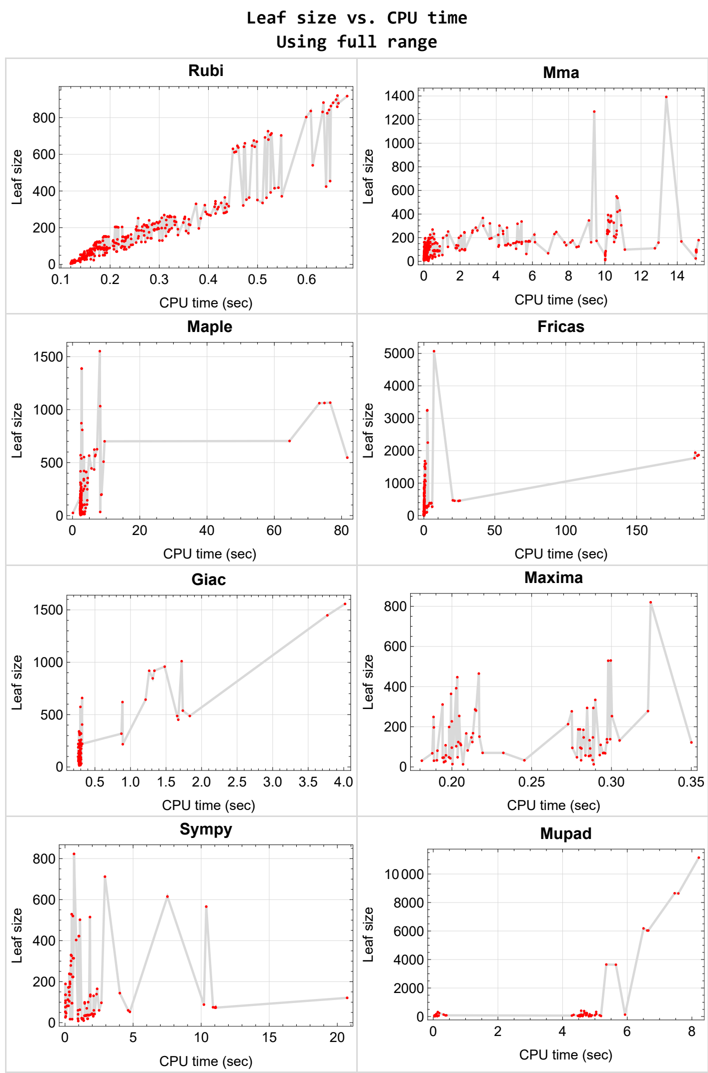


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {114, 115, 119, 120, 121, 127, 128, 132, 133, 134, 139, 140}

Mathematica {59, 69, 110, 112, 114, 115, 119, 120, 121, 124, 126, 127, 128, 130, 132, 133, 134, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 301, 303, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 346, 347, 348}

Maple {148, 156, 157, 158, 159, 305}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

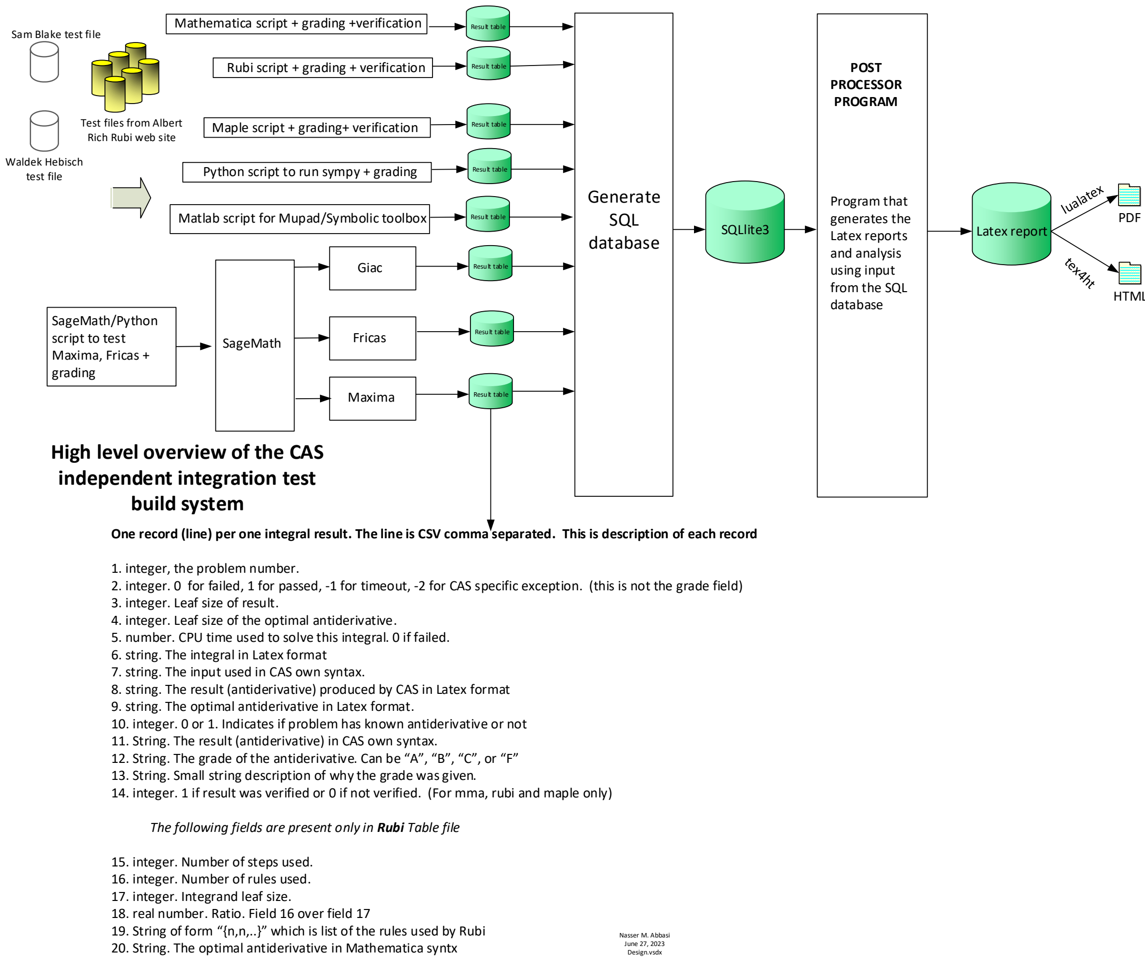
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

B grade { }

C grade { 301 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 137, 148, 149, 163, 164, 165, 168, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 198, 202, 203, 204, 211, 214, 215, 216, 217, 218, 220, 221, 223, 225, 226, 227, 231, 234, 235, 236, 237, 238, 239, 240, 242, 244, 245, 246, 247, 251, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 297, 298, 299, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 326, 342, 343, 344, 345, 349 }

B grade { 51, 224, 243, 293, 341, 346, 347, 348 }

C grade { 88, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 208, 209, 210, 212, 213, 219, 222, 228, 229, 230, 232, 233, 241, 248, 249, 250, 252, 253, 254, 292, 301, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 137, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253, 254, 255, 256, 257, 258, 261, 262, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 277, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 349 }

B grade { 72, 73, 179, 259, 260, 263, 264, 275, 276, 278, 281, 297, 298, 299, 300, 302, 303 }

C grade { 146, 148, 149, 156, 157, 158, 159, 160, 162, 180, 197, 232, 241, 251, 252, 294, 304, 305, 312, 313, 320 }

F normal fail { 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 147, 150, 151, 152, 153, 154, 155, 161, 301, 306, 307, 308, 309, 310, 311, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 30, 31, 32, 39, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 66, 67, 71, 74, 75, 76, 77, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 97, 98, 99, 100, 105, 106, 137, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 178, 179, 181, 182, 183, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 295, 299, 300, 303, 349 }

B grade { 12, 13, 18, 19, 26, 27, 28, 29, 33, 34, 35, 36, 37, 38, 40, 41, 42, 50, 51, 52, 59, 60, 61, 68, 69, 70, 72, 73, 78, 79, 80, 86, 87, 88, 94, 95, 96, 101, 102, 103, 104, 107, 108, 147, 148, 149, 156, 157, 158, 159, 162, 171, 177, 180, 184, 185, 186, 187, 188, 189, 190, 191, 206, 213, 224, 231, 251, 294, 297, 298, 302, 312, 314, 315, 320 }

C grade { 146, 160, 197, 232, 252, 292, 293, 296, 304, 305, 306, 307, 308, 309, 310, 311, 313, 316, 317, 318, 319 }

F normal fail { 109, 110, 111, 113, 116, 117, 118, 122, 123, 124, 125, 129, 130, 131, 135, 136, 138, 301, 341, 342, 343, 344, 345, 346, 347, 348 }

F(-1) timedout fail { 112, 114, 115, 119, 120, 121, 126, 127, 128, 132, 133, 134, 139, 140, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 155, 161, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 62, 63, 64, 65, 74, 75, 76, 77, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 98, 99, 100, 108, 137, 163 }

B grade { 26, 33, 34, 41, 42, 72, 73, 97 }

C grade { }

F normal fail { 49, 50, 51, 52, 57, 58, 59, 60, 61, 66, 67, 68, 69, 70, 71, 78, 79, 80, 86, 87, 88, 94, 95, 96, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 62, 63, 64, 65, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 105, 107, 108, 180 }

B grade { 50, 51, 52, 58, 59, 60, 61, 67, 68, 69, 70, 71, 72, 73, 79, 80, 87, 88, 94, 95, 96, 102, 103, 104, 106 }

C grade { }

F normal fail { 49, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

F(-1) timedout fail { }

F(-2) exception fail { 57, 66, 297, 298, 299, 300 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 48, 56, 65, 71, 72, 73, 76, 77, 84, 85, 92, 93, 97, 98, 99, 100, 105, 106, 108, 137, 345, 349 }

C grade { }

F normal fail { }

F(-1) timedout fail { 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 74, 75, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 94, 95, 96, 101, 102, 103, 104, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 14, 15, 16, 39, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 65, 74, 75, 76, 77, 84, 85, 109, 110, 111, 116, 117, 118, 122, 123, 124, 125, 131, 138, 180, 184, 185, 186, 188, 189, 190, 218, 220, 222, 226, 231, 235, 236, 237, 246, 254, 256, 291 }

B grade { 5, 6, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 35, 36, 37, 38, 62, 63, 64, 92, 93, 99, 100, 108, 187, 221, 224, 225, 227, 234 }

C grade { 241, 245, 247, 342, 343, 344, 345 }

F normal fail { 49, 50, 51, 57, 58, 59, 66, 67, 68, 70, 71, 72, 73, 78, 79, 81, 82, 83, 86, 87, 89, 90, 91, 94, 95, 97, 98, 101, 102, 103, 105, 106, 107, 112, 113, 114, 115, 119, 120, 121, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 223, 228, 229, 230, 232, 233, 238, 239, 240, 242, 243, 244, 248, 249, 250, 251, 252, 253, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

F(-1) timedout fail { 25, 26, 32, 33, 34, 40, 41, 42, 52, 60, 61, 69, 80, 88, 96, 104, 341, 346, 347, 348, 349 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	95	96	96	107	98	88
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.04	0.94
time (sec)	N/A	0.243	0.024	2.974	0.200	0.242	0.035	0.262	0.053

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	71	70	70	76	73	65
N.S.	1	1.00	1.00	1.01	1.00	1.00	1.09	1.04	0.93
time (sec)	N/A	0.228	0.015	2.292	0.219	0.234	0.032	0.305	4.288

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.197	0.012	2.297	0.204	0.232	0.027	0.275	0.048

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.170	0.008	0.113	0.195	0.245	0.029	0.287	0.036

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	34	99	82	34	31
N.S.	1	1.00	1.00	0.85	0.85	2.48	2.05	0.85	0.78
time (sec)	N/A	0.161	0.023	8.221	0.288	0.260	0.150	0.277	0.061

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	182	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.89	1.78	0.90	0.81
time (sec)	N/A	0.170	0.040	2.273	0.287	0.255	0.214	0.266	4.473

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	82	77	92	300	150	78	82
N.S.	1	1.00	0.89	0.84	1.00	3.26	1.63	0.85	0.89
time (sec)	N/A	0.189	0.044	2.291	0.287	0.261	0.311	0.269	4.578

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	122	124	124	136	131	116
N.S.	1	1.00	1.00	1.00	1.02	1.02	1.11	1.07	0.95
time (sec)	N/A	0.284	0.020	2.283	0.204	0.241	0.035	0.275	4.342

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	86	82	82	97	91	75
N.S.	1	1.00	1.00	1.05	1.00	1.00	1.18	1.11	0.91
time (sec)	N/A	0.234	0.015	2.265	0.210	0.234	0.028	0.278	0.046

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.194	0.007	2.270	0.198	0.244	0.024	0.289	0.047

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	64	68	179	172	72	90
N.S.	1	1.00	0.94	1.02	1.08	2.84	2.73	1.14	1.43
time (sec)	N/A	0.207	0.035	2.275	0.296	0.266	0.245	0.289	4.513

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	89	92	96	302	236	95	124
N.S.	1	1.00	1.09	1.12	1.17	3.68	2.88	1.16	1.51
time (sec)	N/A	0.248	0.048	2.277	0.280	0.259	0.404	0.286	4.573

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	129	106	124	138	449	223	126	130
N.S.	1	1.11	0.91	1.07	1.19	3.87	1.92	1.09	1.12
time (sec)	N/A	0.249	0.060	2.287	0.299	0.275	0.594	0.290	4.576

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	161	171	167	167	189	187	152
N.S.	1	1.00	1.05	1.11	1.08	1.08	1.23	1.21	0.99
time (sec)	N/A	0.336	0.019	2.257	0.209	0.260	0.034	0.265	4.841

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	122	124	124	136	131	116
N.S.	1	1.00	1.00	1.00	1.02	1.02	1.11	1.07	0.95
time (sec)	N/A	0.279	0.017	2.284	0.213	0.239	0.034	0.274	4.735

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	71	70	70	76	73	65
N.S.	1	1.00	1.00	1.01	1.00	1.00	1.09	1.04	0.93
time (sec)	N/A	0.224	0.011	2.270	0.295	0.242	0.032	0.276	0.033

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	93	116	122	290	238	130	145
N.S.	1	1.00	0.95	1.18	1.24	2.96	2.43	1.33	1.48
time (sec)	N/A	0.249	0.045	2.304	0.350	0.270	0.319	0.268	4.747

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	107	138	147	444	314	152	181
N.S.	1	1.00	1.00	1.29	1.37	4.15	2.93	1.42	1.69
time (sec)	N/A	0.277	0.047	2.332	0.283	0.250	0.637	0.285	0.102

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	141	167	187	618	422	180	240
N.S.	1	1.00	1.08	1.28	1.44	4.75	3.25	1.38	1.85
time (sec)	N/A	0.347	0.053	2.293	0.280	0.256	1.027	0.291	0.185

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	136	196	187	428	326	198	216
N.S.	1	1.00	0.96	1.38	1.32	3.01	2.30	1.39	1.52
time (sec)	N/A	0.302	0.061	2.326	0.279	0.261	0.482	0.276	4.749

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	92	116	122	292	238	129	146
N.S.	1	1.00	0.94	1.18	1.24	2.98	2.43	1.32	1.49
time (sec)	N/A	0.247	0.045	2.261	0.297	0.260	0.328	0.277	0.083

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	64	69	181	172	72	90
N.S.	1	1.00	0.94	1.02	1.10	2.87	2.73	1.14	1.43
time (sec)	N/A	0.205	0.035	2.275	0.295	0.252	0.268	0.270	4.723

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	34	33	98	82	33	32
N.S.	1	1.00	1.03	0.87	0.85	2.51	2.10	0.85	0.82
time (sec)	N/A	0.157	0.019	2.272	0.281	0.262	0.164	0.309	0.063

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	54	292	712	54	135
N.S.	1	1.00	0.87	0.79	0.77	4.17	10.17	0.77	1.93
time (sec)	N/A	0.169	0.030	2.335	0.286	0.266	2.933	0.279	0.321

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	127	95	93	133	711	0	122	3637
N.S.	1	1.17	0.87	0.85	1.22	6.52	0.00	1.12	33.37
time (sec)	N/A	0.235	0.116	2.377	0.286	0.352	0.000	0.279	5.649

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	198	151	158	277	1585	0	217	6033
N.S.	1	1.24	0.94	0.99	1.73	9.91	0.00	1.36	37.71
time (sec)	N/A	0.325	0.207	2.427	0.275	0.746	0.000	0.297	6.652

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	192	290	294	810	502	306	386
N.S.	1	1.00	1.00	1.51	1.53	4.22	2.61	1.59	2.01
time (sec)	N/A	0.375	0.065	2.406	0.285	0.291	1.097	0.285	4.666

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	142	206	213	612	403	220	261
N.S.	1	1.00	1.00	1.45	1.50	4.31	2.84	1.55	1.84
time (sec)	N/A	0.299	0.060	2.331	0.273	0.259	0.818	0.265	4.683

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	139	147	442	314	152	182
N.S.	1	1.00	1.00	1.31	1.39	4.17	2.96	1.43	1.72
time (sec)	N/A	0.268	0.042	2.318	0.288	0.279	0.583	0.273	0.103

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	88	94	95	297	236	94	124
N.S.	1	1.00	1.07	1.15	1.16	3.62	2.88	1.15	1.51
time (sec)	N/A	0.260	0.043	2.307	0.275	0.280	0.440	0.286	4.645

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	181	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.87	1.78	0.90	0.81
time (sec)	N/A	0.176	0.032	2.284	0.284	0.259	0.214	0.277	4.592

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	125	109	95	132	699	0	121	3649
N.S.	1	1.16	1.01	0.88	1.22	6.47	0.00	1.12	33.79
time (sec)	N/A	0.242	0.088	2.357	0.305	0.359	0.000	0.279	5.352

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	190	136	133	294	1681	0	232	6183
N.S.	1	1.14	0.81	0.80	1.76	10.07	0.00	1.39	37.02
time (sec)	N/A	0.339	0.207	2.419	0.289	0.723	0.000	0.296	6.504

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	277	197	198	529	3239	0	332	8649
N.S.	1	1.20	0.86	0.86	2.30	14.08	0.00	1.44	37.60
time (sec)	N/A	0.436	0.311	2.540	0.298	2.340	0.000	0.281	7.467

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	196	311	334	1044	615	340	409
N.S.	1	1.00	1.00	1.59	1.70	5.33	3.14	1.73	2.09
time (sec)	N/A	0.405	0.078	2.325	0.290	0.262	7.521	0.276	4.575

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	160	231	253	817	515	254	318
N.S.	1	1.00	1.00	1.44	1.58	5.11	3.22	1.59	1.99
time (sec)	N/A	0.366	0.064	2.323	0.300	0.257	1.833	0.278	0.141

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	139	170	185	606	422	178	240
N.S.	1	1.00	1.07	1.31	1.42	4.66	3.25	1.37	1.85
time (sec)	N/A	0.331	0.060	2.322	0.282	0.282	1.013	0.289	4.750

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	129	106	124	138	449	223	126	130
N.S.	1	1.11	0.91	1.07	1.19	3.87	1.92	1.09	1.12
time (sec)	N/A	0.244	0.058	2.307	0.297	0.257	0.545	0.275	4.733

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	76	92	301	150	78	81
N.S.	1	1.00	0.91	0.83	1.00	3.27	1.63	0.85	0.88
time (sec)	N/A	0.190	0.046	2.293	0.281	0.252	0.313	0.287	4.524

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	197	139	158	278	1587	0	218	6033
N.S.	1	1.22	0.86	0.98	1.73	9.86	0.00	1.35	37.47
time (sec)	N/A	0.346	0.234	2.443	0.323	0.802	0.000	0.279	6.618

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	280	197	196	530	3251	0	333	8635
N.S.	1	1.19	0.83	0.83	2.25	13.78	0.00	1.41	36.59
time (sec)	N/A	0.460	0.260	8.528	0.300	2.414	0.000	0.281	7.581

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	371	233	257	820	5070	0	574	11150
N.S.	1	1.18	0.74	0.82	2.60	16.10	0.00	1.82	35.40
time (sec)	N/A	0.589	0.560	2.534	0.325	7.150	0.000	0.296	8.214

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	23	33	33	31	20	31
N.S.	1	1.00	0.71	0.68	0.97	0.97	0.91	0.59	0.91
time (sec)	N/A	0.157	0.009	2.304	0.191	0.244	0.057	0.280	4.622

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	52	41	33	48	67	46	54	47
N.S.	1	1.11	0.87	0.70	1.02	1.43	0.98	1.15	1.00
time (sec)	N/A	0.173	0.011	2.348	0.278	0.249	0.082	0.296	0.046

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	215	180	159	281	398	301	201	0
N.S.	1	0.93	0.78	0.69	1.22	1.72	1.30	0.87	0.00
time (sec)	N/A	0.345	0.252	2.447	0.215	0.319	0.444	0.308	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	140	122	110	168	264	189	129	0
N.S.	1	0.94	0.82	0.74	1.13	1.77	1.27	0.87	0.00
time (sec)	N/A	0.247	0.157	2.360	0.213	0.290	0.369	0.305	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	83	74	62	81	158	104	70	0
N.S.	1	0.95	0.85	0.71	0.93	1.82	1.20	0.80	0.00
time (sec)	N/A	0.188	0.085	2.324	0.191	0.264	0.314	0.285	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	48	36	28	94	41	37	35
N.S.	1	1.00	1.04	0.78	0.61	2.04	0.89	0.80	0.76
time (sec)	N/A	0.155	0.009	2.276	0.196	0.289	0.958	0.279	4.492

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	99	112	0	596	0	0	0
N.S.	1	1.00	1.21	1.37	0.00	7.27	0.00	0.00	0.00
time (sec)	N/A	0.220	0.230	2.517	0.000	0.292	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	99	69	0	369	0	217	0
N.S.	1	1.00	1.21	0.84	0.00	4.50	0.00	2.65	0.00
time (sec)	N/A	0.190	0.301	2.380	0.000	0.322	0.000	0.891	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	148	1268	120	0	698	0	487	0
N.S.	1	0.99	8.51	0.81	0.00	4.68	0.00	3.27	0.00
time (sec)	N/A	0.249	9.417	2.429	0.000	0.355	0.000	1.834	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	234	227	199	0	1220	0	958	0
N.S.	1	1.12	1.09	0.96	0.00	5.87	0.00	4.61	0.00
time (sec)	N/A	0.340	10.643	8.636	0.000	0.639	0.000	1.481	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	231	225	208	364	502	529	260	0
N.S.	1	0.85	0.83	0.76	1.34	1.85	1.94	0.96	0.00
time (sec)	N/A	0.349	0.348	2.485	0.199	0.422	0.489	0.306	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	163	158	142	227	344	330	175	0
N.S.	1	0.83	0.81	0.72	1.16	1.76	1.68	0.89	0.00
time (sec)	N/A	0.251	0.226	2.399	0.200	0.298	0.441	0.307	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	105	99	87	116	210	175	103	0
N.S.	1	0.89	0.84	0.74	0.98	1.78	1.48	0.87	0.00
time (sec)	N/A	0.195	0.137	2.351	0.205	0.283	0.340	0.282	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	68	60	48	43	124	70	49	37
N.S.	1	1.05	0.92	0.74	0.66	1.91	1.08	0.75	0.57
time (sec)	N/A	0.171	0.013	2.293	0.199	0.286	1.597	0.278	4.552

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	119	126	116	0	721	0	0	0
N.S.	1	1.05	1.12	1.03	0.00	6.38	0.00	0.00	0.00
time (sec)	N/A	0.269	0.288	2.446	0.000	0.375	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	142	145	147	0	907	0	317	0
N.S.	1	1.08	1.11	1.12	0.00	6.92	0.00	2.42	0.00
time (sec)	N/A	0.272	0.474	2.428	0.000	0.355	0.000	0.304	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	113	119	163	94	0	526	0	451	0
N.S.	1	1.05	1.44	0.83	0.00	4.65	0.00	3.99	0.00
time (sec)	N/A	0.216	10.514	2.496	0.000	0.327	0.000	1.672	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	185	212	179	0	972	0	919	0
N.S.	1	0.93	1.07	0.90	0.00	4.88	0.00	4.62	0.00
time (sec)	N/A	0.268	10.509	2.585	0.000	0.440	0.000	1.336	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	329	305	270	0	1604	0	1557	0
N.S.	1	1.10	1.02	0.90	0.00	5.35	0.00	5.19	0.00
time (sec)	N/A	0.472	10.908	2.600	0.000	1.374	0.000	4.018	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	259	269	247	447	608	823	321	0
N.S.	1	0.74	0.77	0.71	1.28	1.74	2.36	0.92	0.00
time (sec)	N/A	0.372	0.481	2.538	0.203	0.520	0.660	0.303	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	185	190	172	286	420	520	221	0
N.S.	1	0.77	0.79	0.71	1.19	1.74	2.16	0.92	0.00
time (sec)	N/A	0.273	0.312	2.443	0.215	0.363	0.579	0.321	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	127	123	108	151	260	279	135	0
N.S.	1	0.85	0.83	0.72	1.01	1.74	1.87	0.91	0.00
time (sec)	N/A	0.213	0.189	2.388	0.217	0.320	0.423	0.293	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	90	71	59	58	146	97	63	37
N.S.	1	1.07	0.85	0.70	0.69	1.74	1.15	0.75	0.44
time (sec)	N/A	0.177	0.023	2.322	0.196	0.263	2.672	0.288	4.525

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	170	157	136	0	935	0	0	0
N.S.	1	1.08	1.00	0.87	0.00	5.96	0.00	0.00	0.00
time (sec)	N/A	0.350	0.348	2.454	0.000	0.843	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	182	187	180	0	1236	0	405	0
N.S.	1	1.04	1.07	1.03	0.00	7.06	0.00	2.31	0.00
time (sec)	N/A	0.369	0.575	2.586	0.000	0.659	0.000	0.320	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	218	252	194	0	1517	0	659	0
N.S.	1	1.12	1.30	1.00	0.00	7.82	0.00	3.40	0.00
time (sec)	N/A	0.381	1.340	2.649	0.000	0.498	0.000	0.319	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	156	201	144	0	706	0	846	0
N.S.	1	1.08	1.40	1.00	0.00	4.90	0.00	5.88	0.00
time (sec)	N/A	0.251	10.597	2.674	0.000	0.371	0.000	1.313	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	222	258	234	0	1258	0	1448	0
N.S.	1	0.89	1.04	0.94	0.00	5.05	0.00	5.82	0.00
time (sec)	N/A	0.307	10.655	3.119	0.000	0.786	0.000	3.770	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	33	0	42	0	95	83
N.S.	1	1.00	1.53	1.10	0.00	1.40	0.00	3.17	2.77
time (sec)	N/A	0.163	0.079	2.615	0.000	0.246	0.000	0.277	0.407

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	49	57	59	67	0	70	59
N.S.	1	1.00	1.81	2.11	2.19	2.48	0.00	2.59	2.19
time (sec)	N/A	0.162	0.090	2.431	0.293	0.254	0.000	0.287	0.169

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	37	56	110	74	0	118	85
N.S.	1	1.00	1.48	2.24	4.40	2.96	0.00	4.72	3.40
time (sec)	N/A	0.156	0.046	2.348	0.293	0.243	0.000	0.280	5.001

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	185	138	118	199	300	199	150	0
N.S.	1	1.09	0.82	0.70	1.18	1.78	1.18	0.89	0.00
time (sec)	N/A	0.310	0.164	2.418	0.198	0.285	0.358	0.292	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	116	90	78	109	192	134	90	0
N.S.	1	1.07	0.83	0.72	1.01	1.78	1.24	0.83	0.00
time (sec)	N/A	0.223	0.094	2.351	0.206	0.280	0.329	0.287	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	65	47	47	113	82	49	86
N.S.	1	1.00	1.12	0.81	0.81	1.95	1.41	0.84	1.48
time (sec)	N/A	0.170	0.155	2.292	0.195	0.258	0.255	0.285	5.149

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	37	20
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	1.48	0.80
time (sec)	N/A	0.144	0.002	2.279	0.207	0.265	0.522	0.282	0.124

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	67	42	0	241	0	70	0
N.S.	1	1.00	1.37	0.86	0.00	4.92	0.00	1.43	0.00
time (sec)	N/A	0.162	0.103	2.329	0.000	0.298	0.000	0.278	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	118	87	0	463	0	242	0
N.S.	1	1.00	1.17	0.86	0.00	4.58	0.00	2.40	0.00
time (sec)	N/A	0.216	0.267	2.388	0.000	0.364	0.000	0.294	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	181	160	149	0	864	0	538	0
N.S.	1	1.11	0.98	0.91	0.00	5.30	0.00	3.30	0.00
time (sec)	N/A	0.281	0.656	2.557	0.000	0.487	0.000	1.736	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	270	198	192	311	584	0	235	0
N.S.	1	1.05	0.77	0.75	1.21	2.27	0.00	0.91	0.00
time (sec)	N/A	0.408	0.395	2.476	0.194	0.352	0.000	0.296	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	178	139	137	197	416	0	157	0
N.S.	1	1.05	0.82	0.81	1.17	2.46	0.00	0.93	0.00
time (sec)	N/A	0.333	0.250	2.429	0.189	0.287	0.000	0.291	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	110	95	93	108	276	0	92	0
N.S.	1	1.22	1.06	1.03	1.20	3.07	0.00	1.02	0.00
time (sec)	N/A	0.230	0.164	2.394	0.196	0.267	0.000	0.302	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	58	55	46	167	60	50	53
N.S.	1	1.00	1.07	1.02	0.85	3.09	1.11	0.93	0.98
time (sec)	N/A	0.171	0.081	2.306	0.194	0.267	2.117	0.274	4.818

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	23	17	14	14
N.S.	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	0.88
time (sec)	N/A	0.136	0.001	2.286	0.200	0.251	0.370	0.278	0.042

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	96	92	0	441	0	107	0
N.S.	1	1.00	1.22	1.16	0.00	5.58	0.00	1.35	0.00
time (sec)	N/A	0.197	0.231	2.358	0.000	0.324	0.000	0.293	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	153	151	150	0	864	0	318	0
N.S.	1	1.07	1.06	1.05	0.00	6.04	0.00	2.22	0.00
time (sec)	N/A	0.271	0.590	2.447	0.000	0.491	0.000	0.872	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	250	1392	214	0	1482	0	643	0
N.S.	1	1.11	6.19	0.95	0.00	6.59	0.00	2.86	0.00
time (sec)	N/A	0.370	13.395	2.661	0.000	0.878	0.000	1.213	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	274	202	198	392	684	0	237	0
N.S.	1	1.07	0.79	0.78	1.54	2.68	0.00	0.93	0.00
time (sec)	N/A	0.427	0.476	2.547	0.203	0.398	0.000	0.296	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	187	143	142	254	486	0	158	0
N.S.	1	1.09	0.83	0.83	1.48	2.83	0.00	0.92	0.00
time (sec)	N/A	0.343	0.270	2.479	0.205	0.298	0.000	0.293	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	115	91	105	147	318	0	103	0
N.S.	1	1.10	0.87	1.00	1.40	3.03	0.00	0.98	0.00
time (sec)	N/A	0.226	0.172	2.363	0.212	0.274	0.000	0.282	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	68	54	144	40	33
N.S.	1	1.00	0.79	0.72	1.45	1.15	3.06	0.85	0.70
time (sec)	N/A	0.157	0.077	2.323	0.188	0.253	4.024	0.284	4.550

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	31	47	95	27	28
N.S.	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	0.72
time (sec)	N/A	0.151	0.002	2.293	0.181	0.290	0.502	0.279	4.517

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	139	130	124	0	764	0	320	0
N.S.	1	1.14	1.07	1.02	0.00	6.26	0.00	2.62	0.00
time (sec)	N/A	0.257	0.362	2.398	0.000	0.483	0.000	0.283	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	228	219	208	0	1440	0	620	0
N.S.	1	1.13	1.08	1.03	0.00	7.13	0.00	3.07	0.00
time (sec)	N/A	0.365	1.262	2.628	0.000	0.952	0.000	0.888	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	353	367	302	0	2250	0	1010	0
N.S.	1	1.13	1.17	0.96	0.00	7.19	0.00	3.23	0.00
time (sec)	N/A	0.502	3.262	2.567	0.000	2.764	0.000	1.720	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	187	163	150	465	229	0	218	326
N.S.	1	0.83	0.73	0.67	2.08	1.02	0.00	0.97	1.46
time (sec)	N/A	0.263	0.354	2.415	0.217	0.416	0.000	0.299	5.039

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	150	107	96	249	151	0	138	176
N.S.	1	0.86	0.61	0.55	1.43	0.87	0.00	0.79	1.01
time (sec)	N/A	0.241	0.194	2.382	0.188	0.292	0.000	0.300	5.040

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	88	60	52	103	87	566	72	87
N.S.	1	0.97	0.66	0.57	1.13	0.96	6.22	0.79	0.96
time (sec)	N/A	0.190	0.115	2.418	0.203	0.278	10.373	0.288	4.904

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	31	47	95	27	28
N.S.	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	0.72
time (sec)	N/A	0.150	0.053	2.312	0.189	0.245	0.518	0.285	4.547

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	99	92	0	442	0	107	0
N.S.	1	1.00	1.25	1.16	0.00	5.59	0.00	1.35	0.00
time (sec)	N/A	0.203	0.239	2.389	0.000	0.324	0.000	0.281	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	122	88	0	459	0	225	0
N.S.	1	1.00	1.22	0.88	0.00	4.59	0.00	2.25	0.00
time (sec)	N/A	0.217	0.287	2.395	0.000	0.350	0.000	0.283	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	148	151	121	0	698	0	487	0
N.S.	1	0.99	1.01	0.81	0.00	4.68	0.00	3.27	0.00
time (sec)	N/A	0.239	0.742	2.527	0.000	0.380	0.000	1.657	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	185	179	179	0	972	0	919	0
N.S.	1	0.93	0.90	0.90	0.00	4.88	0.00	4.62	0.00
time (sec)	N/A	0.284	15.182	2.482	0.000	0.483	0.000	1.263	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	27	0	18	18
N.S.	1	1.00	1.00	0.95	0.00	1.35	0.00	0.90	0.90
time (sec)	N/A	0.146	0.034	2.306	0.000	0.265	0.000	0.297	4.812

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	0	23	0	51	79
N.S.	1	1.00	1.00	0.96	0.00	0.92	0.00	2.04	3.16
time (sec)	N/A	0.147	0.054	2.347	0.000	0.267	0.000	0.283	0.380

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	70	41	0	241	0	70	0
N.S.	1	1.00	1.43	0.84	0.00	4.92	0.00	1.43	0.00
time (sec)	N/A	0.163	0.099	2.351	0.000	0.307	0.000	0.276	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	29	14	13	44	31	25	27
N.S.	1	1.00	1.93	0.93	0.87	2.93	2.07	1.67	1.80
time (sec)	N/A	0.137	0.041	2.381	0.289	0.271	2.076	0.276	0.041

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	648	703	99	0	0	0	136	0	0
N.S.	1	1.08	0.15	0.00	0.00	0.00	0.21	0.00	0.00
time (sec)	N/A	0.577	11.106	0.000	0.000	0.000	1.863	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	617	669	176	0	0	0	99	0	0
N.S.	1	1.08	0.29	0.00	0.00	0.00	0.16	0.00	0.00
time (sec)	N/A	0.531	8.213	0.000	0.000	0.000	1.441	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	588	637	62	0	0	0	63	0	0
N.S.	1	1.08	0.11	0.00	0.00	0.00	0.11	0.00	0.00
time (sec)	N/A	0.485	5.662	0.000	0.000	0.000	1.089	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	740	803	162	0	0	0	0	0	0
N.S.	1	1.09	0.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.648	6.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	584	630	86	0	0	0	0	0	0
N.S.	1	1.08	0.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.496	10.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	818	878	252	0	0	0	0	0	0
N.S.	1	1.07	0.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.713	10.239	0.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	849	917	265	0	0	0	0	0	0
N.S.	1	1.08	0.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.724	10.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	668	726	110	0	0	0	139	0	0
N.S.	1	1.09	0.16	0.00	0.00	0.00	0.21	0.00	0.00
time (sec)	N/A	0.559	12.767	0.000	0.000	0.000	2.317	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	637	692	173	0	0	0	131	0	0
N.S.	1	1.09	0.27	0.00	0.00	0.00	0.21	0.00	0.00
time (sec)	N/A	0.532	9.542	0.000	0.000	0.000	2.097	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	608	660	68	0	0	0	100	0	0
N.S.	1	1.09	0.11	0.00	0.00	0.00	0.16	0.00	0.00
time (sec)	N/A	0.489	6.866	0.000	0.000	0.000	1.699	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	765	824	231	0	0	0	0	0	0
N.S.	1	1.08	0.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.649	7.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	775	836	235	0	0	0	0	0	0
N.S.	1	1.08	0.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.642	10.159	0.000	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	815	882	252	0	0	0	0	0	0
N.S.	1	1.08	0.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.677	10.168	0.000	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	659	714	98	0	0	0	165	0	0
N.S.	1	1.08	0.15	0.00	0.00	0.00	0.25	0.00	0.00
time (sec)	N/A	0.568	15.045	0.000	0.000	0.000	2.360	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	628	680	88	0	0	0	129	0	0
N.S.	1	1.08	0.14	0.00	0.00	0.00	0.21	0.00	0.00
time (sec)	N/A	0.527	15.038	0.000	0.000	0.000	1.856	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	597	646	158	0	0	0	94	0	0
N.S.	1	1.08	0.26	0.00	0.00	0.00	0.16	0.00	0.00
time (sec)	N/A	0.500	12.964	0.000	0.000	0.000	1.446	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	568	614	62	0	0	0	60	0	0
N.S.	1	1.08	0.11	0.00	0.00	0.00	0.11	0.00	0.00
time (sec)	N/A	0.458	10.021	0.000	0.000	0.000	0.952	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	162	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	5.261	0.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	787	842	234	0	0	0	0	0	0
N.S.	1	1.07	0.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.659	10.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	818	881	255	0	0	0	0	0	0
N.S.	1	1.08	0.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.697	10.135	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	623	675	76	0	0	0	0	0	0
N.S.	1	1.08	0.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.518	15.055	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	592	641	168	0	0	0	0	0	0
N.S.	1	1.08	0.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.506	14.230	0.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	561	611	53	0	0	0	60	0	0
N.S.	1	1.09	0.09	0.00	0.00	0.00	0.11	0.00	0.00
time (sec)	N/A	0.472	10.018	0.000	0.000	0.000	2.503	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	776	831	226	0	0	0	0	0	0
N.S.	1	1.07	0.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.663	6.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	807	863	236	0	0	0	0	0	0
N.S.	1	1.07	0.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.689	10.143	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	849	920	256	0	0	0	0	0	0
N.S.	1	1.08	0.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.728	10.182	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	653	706	96	0	0	0	0	0	0
N.S.	1	1.08	0.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.548	15.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	596	647	83	0	0	0	0	0	0
N.S.	1	1.09	0.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.531	15.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	24	24	33	42	0	0	27
N.S.	1	1.00	0.55	0.55	0.75	0.95	0.00	0.00	0.61
time (sec)	N/A	0.159	15.021	2.400	0.245	0.263	0.000	0.000	4.878

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	590	641	74	0	0	0	60	0	0
N.S.	1	1.09	0.13	0.00	0.00	0.00	0.10	0.00	0.00
time (sec)	N/A	0.496	10.027	0.000	0.000	0.000	4.637	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	796	859	248	0	0	0	0	0	0
N.S.	1	1.08	0.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.690	7.317	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	827	898	259	0	0	0	0	0	0
N.S.	1	1.09	0.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.713	10.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	252	252	163	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	5.409	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	166	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	5.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	153	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	4.944	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	162	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.027	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	156	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	5.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	118	704	0	1232	0	0	0
N.S.	1	1.00	1.04	6.23	0.00	10.90	0.00	0.00	0.00
time (sec)	N/A	0.182	0.031	64.534	0.000	0.689	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	1103	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	10.12	0.00	0.00	0.00
time (sec)	N/A	0.177	0.022	0.000	0.000	0.701	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	96	96	155	1033	0	285	0	0	0
N.S.	1	1.00	1.61	10.76	0.00	2.97	0.00	0.00	0.00
time (sec)	N/A	0.195	0.242	8.234	0.000	3.208	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	151	1552	0	315	0	0	0
N.S.	1	1.00	1.59	16.34	0.00	3.32	0.00	0.00	0.00
time (sec)	N/A	0.179	0.182	8.109	0.000	3.181	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	169	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	5.641	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	167	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	5.771	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	168	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	5.857	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	172	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	5.803	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	148	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	5.404	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	148	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	4.655	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	123	123	136	1066	0	1843	0	0	0
N.S.	1	1.00	1.11	8.67	0.00	14.98	0.00	0.00	0.00
time (sec)	N/A	0.183	4.505	76.625	0.000	192.780	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	123	123	136	547	0	1867	0	0	0
N.S.	1	1.00	1.11	4.45	0.00	15.18	0.00	0.00	0.00
time (sec)	N/A	0.183	4.148	81.705	0.000	193.663	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	119	119	136	1063	0	1771	0	0	0
N.S.	1	1.00	1.14	8.93	0.00	14.88	0.00	0.00	0.00
time (sec)	N/A	0.179	4.501	74.917	0.000	190.797	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	119	119	136	1061	0	1939	0	0	0
N.S.	1	1.00	1.14	8.92	0.00	16.29	0.00	0.00	0.00
time (sec)	N/A	0.179	4.572	73.391	0.000	191.298	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	124	623	0	1059	0	0	0
N.S.	1	1.00	1.77	8.90	0.00	15.13	0.00	0.00	0.00
time (sec)	N/A	0.159	4.122	6.474	0.000	1.169	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	137	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	4.786	0.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	125	539	0	269	0	0	0
N.S.	1	1.00	1.69	7.28	0.00	3.64	0.00	0.00	0.00
time (sec)	N/A	0.164	8.550	2.627	0.000	0.542	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	58	57	94	70	315	0	0	0
N.S.	1	0.73	0.72	1.19	0.89	3.99	0.00	0.00	0.00
time (sec)	N/A	0.176	0.102	2.436	0.232	0.287	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	58	54	94	0	303	0	0	0
N.S.	1	0.78	0.73	1.27	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	0.169	0.072	2.362	0.000	0.281	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	58	57	94	0	314	0	0	0
N.S.	1	0.76	0.75	1.24	0.00	4.13	0.00	0.00	0.00
time (sec)	N/A	0.167	0.091	2.355	0.000	0.297	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	312	243	412	0	232	0	0	0
N.S.	1	0.95	0.74	1.26	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.445	2.246	4.065	0.000	0.097	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	242	198	279	0	153	0	0	0
N.S.	1	0.97	0.80	1.12	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.335	1.058	3.732	0.000	0.088	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	206	86	101	0	114	0	0	0
N.S.	1	1.01	0.42	0.50	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.285	0.795	2.365	0.000	0.082	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	133	181	0	122	0	0	0
N.S.	1	1.00	1.58	2.15	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.179	1.897	2.367	0.000	0.092	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	240	243	418	0	333	0	0	0
N.S.	1	1.01	1.03	1.76	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.300	2.675	2.373	0.000	0.091	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	330	285	570	0	689	0	0	0
N.S.	1	1.07	0.92	1.84	0.00	2.23	0.00	0.00	0.00
time (sec)	N/A	0.403	2.871	2.394	0.000	0.106	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	392	302	566	0	316	0	0	0
N.S.	1	0.96	0.74	1.38	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.571	4.175	4.935	0.000	0.096	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	314	246	411	0	233	0	0	0
N.S.	1	0.93	0.73	1.22	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.441	2.236	4.045	0.000	0.097	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	264	199	310	0	166	0	0	0
N.S.	1	0.97	0.73	1.14	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.354	1.935	4.418	0.000	0.101	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	258	191	332	0	226	0	0	0
N.S.	1	0.97	0.72	1.24	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.357	3.634	3.344	0.000	0.095	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	233	232	421	0	294	0	0	0
N.S.	1	1.02	1.01	1.84	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.317	4.380	3.422	0.000	0.089	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	323	285	552	0	635	0	0	0
N.S.	1	1.03	0.90	1.75	0.00	2.02	0.00	0.00	0.00
time (sec)	N/A	0.429	4.594	3.411	0.000	0.100	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	230	127	252	0	149	0	0	0
N.S.	1	0.98	0.54	1.07	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.322	0.876	3.396	0.000	0.082	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	22	75	0	41	0	0	0
N.S.	1	1.00	0.58	1.97	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.158	0.118	2.753	0.000	0.075	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	19	15	38	0	38	17	17	0
N.S.	1	0.95	0.75	1.90	0.00	1.90	0.85	0.85	0.00
time (sec)	N/A	0.156	0.002	2.357	0.000	0.252	1.252	0.290	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	191	37	37	0	124	0	0	0
N.S.	1	1.05	0.20	0.20	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.275	0.631	2.366	0.000	0.082	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	60	78	0	67	0	0	0
N.S.	1	1.00	0.66	0.86	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.226	0.579	2.401	0.000	0.076	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	60	53	0	111	0	0	0
N.S.	1	1.00	0.40	0.35	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.256	0.588	2.383	0.000	0.082	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	23	0	36	34	0	0
N.S.	1	1.00	1.00	1.15	0.00	1.80	1.70	0.00	0.00
time (sec)	N/A	0.136	0.261	2.408	0.000	0.079	1.489	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	53	36	0	0
N.S.	1	1.00	1.00	0.86	0.00	2.52	1.71	0.00	0.00
time (sec)	N/A	0.138	0.264	2.404	0.000	0.080	1.539	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	29	0	66	34	0	0
N.S.	1	1.00	1.00	1.45	0.00	3.30	1.70	0.00	0.00
time (sec)	N/A	0.138	0.274	2.414	0.000	0.088	1.543	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	0	41	10	0	0
N.S.	1	1.00	1.00	1.25	0.00	10.25	2.50	0.00	0.00
time (sec)	N/A	0.136	0.254	2.382	0.000	0.083	1.370	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	76	36	0	0
N.S.	1	1.00	1.00	0.95	0.00	3.80	1.80	0.00	0.00
time (sec)	N/A	0.137	0.253	2.410	0.000	0.083	1.657	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	19	0	76	37	0	0
N.S.	1	1.00	1.00	0.90	0.00	3.62	1.76	0.00	0.00
time (sec)	N/A	0.138	0.265	2.414	0.000	0.088	1.755	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	78	36	0	0
N.S.	1	1.00	1.00	0.95	0.00	3.90	1.80	0.00	0.00
time (sec)	N/A	0.138	0.267	2.416	0.000	0.077	1.744	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	0	31	0	0	0
N.S.	1	1.00	0.92	1.08	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.159	0.250	2.384	0.000	0.079	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	27	0	36	0	0	0
N.S.	1	1.00	0.87	0.87	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.159	0.248	2.408	0.000	0.081	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	27	31	0	53	0	0	0
N.S.	1	1.00	0.77	0.89	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.161	0.264	2.396	0.000	0.075	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	27	31	0	53	0	0	0
N.S.	1	1.00	0.77	0.89	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.159	0.270	2.421	0.000	0.078	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	27	30	0	64	0	0	0
N.S.	1	1.00	0.21	0.23	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.227	0.243	2.378	0.000	0.079	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	27	26	0	64	0	0	0
N.S.	1	1.00	0.20	0.19	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	0.226	0.255	2.388	0.000	0.081	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	151	27	20	0	54	0	0	0
N.S.	1	1.02	0.18	0.14	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.247	0.250	2.394	0.000	0.088	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	35	32	0	36	0	0	0
N.S.	1	1.00	0.88	0.80	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.164	0.260	2.485	0.000	0.083	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	415	321	626	0	364	0	0	0
N.S.	1	0.98	0.76	1.48	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.583	3.650	7.294	0.000	0.091	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	335	260	435	0	271	0	0	0
N.S.	1	0.97	0.76	1.26	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.469	2.692	6.428	0.000	0.087	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	259	216	310	0	189	0	0	0
N.S.	1	1.00	0.83	1.19	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.358	2.044	4.509	0.000	0.094	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	203	86	158	0	130	0	0	0
N.S.	1	1.05	0.44	0.81	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.280	0.743	2.390	0.000	0.086	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	100	0	42	0	0	0
N.S.	1	1.00	0.99	1.15	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.180	0.562	3.013	0.000	0.077	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	258	112	248	0	187	0	0	0
N.S.	1	0.95	0.41	0.91	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.348	1.821	4.389	0.000	0.093	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	269	261	445	0	441	0	0	0
N.S.	1	1.05	1.02	1.75	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.332	2.957	5.629	0.000	0.097	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	365	301	573	0	845	0	0	0
N.S.	1	1.09	0.90	1.72	0.00	2.53	0.00	0.00	0.00
time (sec)	N/A	0.460	3.284	6.832	0.000	0.107	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	445	418	318	701	0	472	0	0	0
N.S.	1	0.94	0.71	1.58	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.576	5.175	9.533	0.000	0.103	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	330	256	509	0	338	0	0	0
N.S.	1	0.95	0.74	1.47	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.446	4.375	9.223	0.000	0.089	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	259	196	345	0	224	0	0	0
N.S.	1	1.00	0.76	1.34	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.358	3.733	3.345	0.000	0.102	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	136	188	0	133	0	0	0
N.S.	1	1.00	1.62	2.24	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.178	1.801	2.379	0.000	0.093	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	261	112	144	0	159	0	0	0
N.S.	1	1.35	0.58	0.74	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.339	1.762	4.378	0.000	0.089	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	254	224	354	0	407	0	0	0
N.S.	1	1.05	0.93	1.46	0.00	1.68	0.00	0.00	0.00
time (sec)	N/A	0.308	4.107	4.518	0.000	0.094	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	344	337	561	0	856	0	0	0
N.S.	1	1.07	1.04	1.74	0.00	2.65	0.00	0.00	0.00
time (sec)	N/A	0.433	5.399	6.708	0.000	0.126	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	100	0	42	0	0	0
N.S.	1	1.00	0.99	1.15	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.183	0.028	2.943	0.000	0.078	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	103	0	40	0	0	0
N.S.	1	1.00	1.00	1.18	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.215	0.612	3.574	0.000	0.082	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	89	103	0	40	0	0	0
N.S.	1	1.00	1.02	1.18	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.207	0.634	3.596	0.000	0.087	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	104	0	39	0	0	0
N.S.	1	1.00	1.00	1.18	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.207	0.636	4.310	0.000	0.088	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	0	9	19	0	0
N.S.	1	1.00	1.00	1.17	0.00	0.75	1.58	0.00	0.00
time (sec)	N/A	0.135	0.212	3.595	0.000	0.078	1.670	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	58	14	0	9	0	0	0
N.S.	1	1.00	5.80	1.40	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.135	10.024	2.912	0.000	0.089	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	0	9	19	0	0
N.S.	1	1.00	1.00	1.17	0.00	0.75	1.58	0.00	0.00
time (sec)	N/A	0.136	0.197	3.356	0.000	0.080	1.649	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	0	9	73	0	0
N.S.	1	1.00	1.00	1.00	0.00	0.90	7.30	0.00	0.00
time (sec)	N/A	0.148	10.019	2.467	0.000	0.079	11.024	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	0	9	19	0	0
N.S.	1	1.00	1.50	1.17	0.00	0.75	1.58	0.00	0.00
time (sec)	N/A	0.137	10.024	2.811	0.000	0.084	1.287	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	9	0	0	0
N.S.	1	1.00	1.00	1.08	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.136	0.188	2.494	0.000	0.075	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	26	8	0	68	19	0	0
N.S.	1	1.00	3.25	1.00	0.00	8.50	2.38	0.00	0.00
time (sec)	N/A	0.139	0.006	2.467	0.000	0.264	0.946	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	9	34	0	0
N.S.	1	1.00	1.00	1.08	0.00	0.75	2.83	0.00	0.00
time (sec)	N/A	0.137	0.191	2.528	0.000	0.080	1.430	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	0	9	39	0	0
N.S.	1	1.00	1.00	1.10	0.00	0.90	3.90	0.00	0.00
time (sec)	N/A	0.137	0.060	2.623	0.000	0.078	1.882	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	9	34	0	0
N.S.	1	1.00	1.00	1.08	0.00	0.75	2.83	0.00	0.00
time (sec)	N/A	0.136	0.195	2.619	0.000	0.073	1.450	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	19	17	0	11	0	0	0
N.S.	1	1.00	0.37	0.33	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.156	0.200	2.474	0.000	0.075	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	17	15	0	11	0	0	0
N.S.	1	1.00	0.35	0.31	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.155	0.070	2.506	0.000	0.089	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	19	17	0	11	0	0	0
N.S.	1	1.00	0.37	0.33	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.157	0.187	2.589	0.000	0.079	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	0	34	8	0	0
N.S.	1	1.00	1.00	1.00	0.00	4.25	1.00	0.00	0.00
time (sec)	N/A	0.138	0.005	2.482	0.000	0.264	0.971	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	19	15	0	11	0	0	0
N.S.	1	1.00	0.40	0.32	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.156	0.183	2.514	0.000	0.094	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	19	14	0	9	0	0	0
N.S.	1	1.00	1.90	1.40	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.136	10.023	2.682	0.000	0.075	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	0	9	76	0	0
N.S.	1	1.00	1.00	1.00	0.00	0.90	7.60	0.00	0.00
time (sec)	N/A	0.139	0.005	2.504	0.000	0.077	11.057	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	17	36	0	0
N.S.	1	1.00	1.00	0.95	0.00	0.85	1.80	0.00	0.00
time (sec)	N/A	0.138	0.188	3.147	0.000	0.087	1.594	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	0	10	41	0	0
N.S.	1	1.00	1.00	0.94	0.00	0.62	2.56	0.00	0.00
time (sec)	N/A	0.136	0.054	2.707	0.000	0.073	2.055	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	17	36	0	0
N.S.	1	1.00	1.00	0.95	0.00	0.85	1.80	0.00	0.00
time (sec)	N/A	0.142	0.185	3.285	0.000	0.077	1.605	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	0	9	0	0	0
N.S.	1	1.00	1.00	1.16	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.150	0.190	2.711	0.000	0.083	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	34	0	9	0	0	0
N.S.	1	1.00	1.00	1.13	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.150	0.068	2.514	0.000	0.081	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	0	9	0	0	0
N.S.	1	1.00	1.00	1.16	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.151	0.201	2.605	0.000	0.072	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	46	30	0	9	75	0	0
N.S.	1	1.00	1.84	1.20	0.00	0.36	3.00	0.00	0.00
time (sec)	N/A	0.140	0.060	2.528	0.000	0.080	10.873	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	34	0	9	0	0	0
N.S.	1	1.00	1.00	1.06	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.151	0.180	2.595	0.000	0.074	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	47	28	0	9	0	0	0
N.S.	1	1.00	3.92	2.33	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.132	10.025	2.748	0.000	0.080	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	24	0	34	0	0	0
N.S.	1	1.00	0.93	0.83	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.142	0.033	2.582	0.000	0.274	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	29	0	9	37	0	0
N.S.	1	1.00	1.25	0.91	0.00	0.28	1.16	0.00	0.00
time (sec)	N/A	0.147	0.190	3.289	0.000	0.074	1.684	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	36	27	0	9	42	0	0
N.S.	1	1.00	1.20	0.90	0.00	0.30	1.40	0.00	0.00
time (sec)	N/A	0.154	0.071	2.891	0.000	0.081	2.173	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	29	0	9	37	0	0
N.S.	1	1.00	1.25	0.91	0.00	0.28	1.16	0.00	0.00
time (sec)	N/A	0.150	0.184	3.231	0.000	0.075	1.639	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	39	36	0	11	0	0	0
N.S.	1	1.00	0.74	0.68	0.00	0.21	0.00	0.00	0.00
time (sec)	N/A	0.153	0.202	2.877	0.000	0.079	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	37	33	0	11	0	0	0
N.S.	1	1.00	0.73	0.65	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.159	0.070	2.565	0.000	0.082	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	39	36	0	11	0	0	0
N.S.	1	1.00	0.74	0.68	0.00	0.21	0.00	0.00	0.00
time (sec)	N/A	0.156	0.189	2.932	0.000	0.077	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	29	26	9	0	104	0	0	0
N.S.	1	1.04	0.93	0.32	0.00	3.71	0.00	0.00	0.00
time (sec)	N/A	0.151	0.036	2.494	0.000	0.264	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	53	33	0	17	0	0	0
N.S.	1	1.00	1.08	0.67	0.00	0.35	0.00	0.00	0.00
time (sec)	N/A	0.156	1.024	2.462	0.000	0.087	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	34	0	17	0	0	0
N.S.	1	1.00	1.26	1.10	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.160	0.199	2.451	0.000	0.077	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	30	48	30	0	9	73	0	0
N.S.	1	0.71	1.14	0.71	0.00	0.21	1.74	0.00	0.00
time (sec)	N/A	0.154	0.059	2.427	0.000	0.071	11.064	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	0	23	0	0	0
N.S.	1	1.00	1.00	0.85	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.158	0.196	2.489	0.000	0.083	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	0	16	44	0	0
N.S.	1	1.00	1.00	0.94	0.00	0.44	1.22	0.00	0.00
time (sec)	N/A	0.157	0.079	2.471	0.000	0.073	1.981	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	0	23	0	0	0
N.S.	1	1.00	1.00	0.85	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.153	0.211	2.467	0.000	0.071	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	104	0	130	0	0	0
N.S.	1	1.00	1.00	1.20	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.203	0.738	2.441	0.000	0.087	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	168	0	131	0	0	0
N.S.	1	1.00	1.00	1.87	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.201	0.758	2.476	0.000	0.084	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	167	0	128	0	0	0
N.S.	1	1.00	1.00	1.90	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.200	0.549	2.546	0.000	0.085	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	108	0	133	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.210	0.540	2.558	0.000	0.094	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	105	0	113	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.204	0.789	2.473	0.000	0.084	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	161	0	116	0	0	0
N.S.	1	1.00	1.00	1.81	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.200	0.770	2.492	0.000	0.085	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	162	0	115	0	0	0
N.S.	1	1.00	1.00	1.82	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.192	0.536	2.558	0.000	0.087	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	107	0	114	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.196	0.511	2.555	0.000	0.084	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	203	86	158	0	130	0	0	0
N.S.	1	1.05	0.44	0.81	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.273	0.028	2.444	0.000	0.087	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	213	89	104	0	135	0	0	0
N.S.	1	1.05	0.44	0.51	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.293	0.789	2.469	0.000	0.085	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	212	89	108	0	134	0	0	0
N.S.	1	1.04	0.44	0.53	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.292	0.770	2.472	0.000	0.084	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	222	92	165	0	135	0	0	0
N.S.	1	1.05	0.43	0.78	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.308	0.586	2.571	0.000	0.085	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	89	161	0	115	0	0	0
N.S.	1	1.00	0.47	0.85	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.318	0.785	2.546	0.000	0.080	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	90	107	0	114	0	0	0
N.S.	1	1.00	0.47	0.56	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.316	0.735	2.483	0.000	0.091	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	92	109	0	117	0	0	0
N.S.	1	1.00	0.47	0.56	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.323	0.581	2.573	0.000	0.085	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	93	166	0	120	0	0	0
N.S.	1	1.00	0.47	0.84	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.322	0.585	2.563	0.000	0.086	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	104	0	130	0	0	0
N.S.	1	1.00	1.00	1.20	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.201	0.772	2.474	0.000	0.083	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	168	0	131	0	0	0
N.S.	1	1.00	1.00	1.87	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.199	0.798	2.425	0.000	0.091	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	167	0	128	0	0	0
N.S.	1	1.00	1.00	1.90	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.196	0.629	2.480	0.000	0.083	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	108	0	133	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.200	0.599	2.413	0.000	0.084	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	160	0	127	0	0	0
N.S.	1	1.00	1.00	1.82	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.203	0.859	2.436	0.000	0.084	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	106	0	130	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.200	0.817	2.457	0.000	0.089	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	106	0	129	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.190	0.709	2.424	0.000	0.088	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	163	0	128	0	0	0
N.S.	1	1.00	1.00	1.81	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.194	0.752	2.433	0.000	0.085	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	206	86	101	0	114	0	0	0
N.S.	1	1.01	0.42	0.50	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.275	0.025	2.390	0.000	0.090	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	217	89	161	0	119	0	0	0
N.S.	1	1.01	0.42	0.75	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.296	0.833	2.424	0.000	0.080	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	216	89	162	0	118	0	0	0
N.S.	1	1.01	0.42	0.76	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.297	0.831	2.417	0.000	0.097	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	225	92	111	0	119	0	0	0
N.S.	1	1.01	0.41	0.50	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.301	0.762	2.492	0.000	0.093	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	89	161	0	115	0	0	0
N.S.	1	1.00	0.47	0.85	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.311	0.851	2.428	0.000	0.086	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	90	107	0	114	0	0	0
N.S.	1	1.00	0.47	0.56	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.308	0.795	2.447	0.000	0.080	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	92	109	0	117	0	0	0
N.S.	1	1.00	0.47	0.56	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.316	0.730	2.448	0.000	0.088	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	93	166	0	120	0	0	0
N.S.	1	1.00	0.47	0.84	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.319	0.588	2.454	0.000	0.085	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	37	38	0	35	0	0	0
N.S.	1	1.00	0.47	0.49	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.163	0.500	2.442	0.000	0.085	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	38	0	15	20	0	0
N.S.	1	1.00	1.03	0.97	0.00	0.38	0.51	0.00	0.00
time (sec)	N/A	0.162	0.503	3.719	0.000	0.085	1.322	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	53	0	16	0	0	0
N.S.	1	1.00	0.77	0.87	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.161	0.461	2.422	0.000	0.085	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	27	25	0	6	0	0	0
N.S.	1	1.00	4.50	4.17	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.130	0.226	2.483	0.000	0.075	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	24	28	0	73	0	0	0
N.S.	1	1.00	1.04	1.22	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.182	0.467	2.468	0.000	0.084	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	191	37	37	0	124	0	0	0
N.S.	1	1.05	0.20	0.20	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.272	0.008	2.430	0.000	0.081	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	35	37	0	72	0	0	0
N.S.	1	1.00	1.84	1.95	0.00	3.79	0.00	0.00	0.00
time (sec)	N/A	0.134	0.331	2.501	0.000	0.089	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	809	0	318	0	0	0
N.S.	1	1.00	1.00	8.52	0.00	3.35	0.00	0.00	0.00
time (sec)	N/A	0.240	2.238	2.888	0.000	0.137	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	95	1388	0	410	0	0	0
N.S.	1	1.00	1.01	14.77	0.00	4.36	0.00	0.00	0.00
time (sec)	N/A	0.215	2.275	2.725	0.000	0.102	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	478	540	102	1388	0	415	0	0	0
N.S.	1	1.13	0.21	2.90	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.682	2.276	2.689	0.000	0.107	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	102	872	0	287	0	0	0
N.S.	1	1.00	0.47	4.06	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.388	2.105	2.632	0.000	0.128	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	23	122	0	0	0	0	0	0
N.S.	1	0.37	1.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	1.330	0.000	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	48	117	0	118	0	0	0
N.S.	1	1.00	1.04	2.54	0.00	2.57	0.00	0.00	0.00
time (sec)	N/A	0.184	0.833	2.514	0.000	0.096	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	91	197	0	40	0	0	0
N.S.	1	1.00	1.94	4.19	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.195	1.980	2.610	0.000	0.079	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	119	187	0	289	0	0	0
N.S.	1	1.00	0.92	1.45	0.00	2.24	0.00	0.00	0.00
time (sec)	N/A	0.203	0.251	2.450	0.000	1.886	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	120	120	119	188	0	289	0	0	0
N.S.	1	1.00	0.99	1.57	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.185	0.239	2.472	0.000	1.934	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	119	0	0	378	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	2.93	0.00	0.00	0.00
time (sec)	N/A	0.201	0.273	0.000	0.000	5.275	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	123	0	0	388	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	3.13	0.00	0.00	0.00
time (sec)	N/A	0.191	0.355	0.000	0.000	5.254	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	121	0	0	383	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	3.19	0.00	0.00	0.00
time (sec)	N/A	0.199	0.306	0.000	0.000	4.027	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	121	0	0	381	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	3.18	0.00	0.00	0.00
time (sec)	N/A	0.187	0.311	0.000	0.000	4.051	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	119	0	0	451	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.192	0.322	0.000	0.000	24.396	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	123	0	0	459	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	3.70	0.00	0.00	0.00
time (sec)	N/A	0.190	0.328	0.000	0.000	25.300	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	138	0	104	0	0	0
N.S.	1	1.00	0.92	2.26	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.159	0.158	2.517	0.000	2.156	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	138	0	243	0	0	0
N.S.	1	1.00	0.92	2.26	0.00	3.98	0.00	0.00	0.00
time (sec)	N/A	0.157	0.155	2.500	0.000	2.069	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	67	0	0	274	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	3.56	0.00	0.00	0.00
time (sec)	N/A	0.166	0.172	0.000	0.000	5.884	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	69	0	0	273	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	3.46	0.00	0.00	0.00
time (sec)	N/A	0.170	0.186	0.000	0.000	5.867	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	375	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	4.41	0.00	0.00	0.00
time (sec)	N/A	0.177	0.202	0.000	0.000	4.091	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	373	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	4.39	0.00	0.00	0.00
time (sec)	N/A	0.166	0.205	0.000	0.000	4.885	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	88	0	0	457	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	4.52	0.00	0.00	0.00
time (sec)	N/A	0.180	0.210	0.000	0.000	21.579	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	90	0	0	469	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.178	0.194	0.000	0.000	20.416	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	117	0	91	0	0	0
N.S.	1	1.00	0.91	2.21	0.00	1.72	0.00	0.00	0.00
time (sec)	N/A	0.151	0.125	2.511	0.000	1.810	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	362	364	346	0	0	0	0	0	0
N.S.	1	1.01	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	9.123	0.000	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	302	309	348	0	0	0	0	0	0
N.S.	1	1.02	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	10.186	0.000	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	244	249	161	0	0	0	0	0	0
N.S.	1	1.02	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.368	9.242	0.000	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	199	215	160	0	0	0	0	0	0
N.S.	1	1.08	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.362	8.106	0.000	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	167	158	160	0	0	0	0	0	0
N.S.	1	0.95	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	7.771	0.000	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	144	120	0	0	0	0	0	0
N.S.	1	0.95	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.294	8.438	0.000	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	233	236	327	0	0	0	0	0	0
N.S.	1	1.01	1.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.365	10.493	0.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	254	246	331	0	0	0	0	0	0
N.S.	1	0.97	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	10.279	0.000	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	274	351	419	0	0	0	0	0	0
N.S.	1	1.28	1.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.549	10.696	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	304	321	431	0	0	0	0	0	0
N.S.	1	1.06	1.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.523	10.828	0.000	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	340	317	340	0	0	0	0	0	0
N.S.	1	0.93	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.472	10.347	0.000	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	279	282	341	0	0	0	0	0	0
N.S.	1	1.01	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	10.356	0.000	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	309	286	232	0	0	0	0	0	0
N.S.	1	0.93	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.424	10.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	278	251	232	0	0	0	0	0	0
N.S.	1	0.90	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.408	10.144	0.000	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	336	301	392	0	0	0	0	0	0
N.S.	1	0.90	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	10.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	292	267	336	0	0	0	0	0	0
N.S.	1	0.91	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.436	10.224	0.000	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	314	363	380	0	0	0	0	0	0
N.S.	1	1.16	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.541	10.331	0.000	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	345	335	387	0	0	0	0	0	0
N.S.	1	0.97	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.540	10.336	0.000	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	371	454	536	0	0	0	0	0	0
N.S.	1	1.22	1.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.696	10.700	0.000	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	419	424	550	0	0	0	0	0	0
N.S.	1	1.01	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.680	10.646	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	0
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	0.227	0.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	286	136	0	0	0	121	0	0
N.S.	1	0.97	0.46	0.00	0.00	0.00	0.41	0.00	0.00
time (sec)	N/A	0.466	7.865	0.000	0.000	0.000	20.723	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	175	106	0	0	0	88	0	0
N.S.	1	0.99	0.60	0.00	0.00	0.00	0.50	0.00	0.00
time (sec)	N/A	0.301	5.097	0.000	0.000	0.000	10.201	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	53	0	0
N.S.	1	0.91	0.97	0.00	0.00	0.00	0.57	0.00	0.00
time (sec)	N/A	0.197	0.070	0.000	0.000	0.000	4.763	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	22	0	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.50	0.00	0.93
time (sec)	N/A	0.158	0.002	0.000	0.000	0.000	1.218	0.000	5.188

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.175	0.195	0.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	0.222	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.174	0.256	0.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	71	0	91	0	0	131
N.S.	1	1.00	0.98	1.34	0.00	1.72	0.00	0.00	2.47
time (sec)	N/A	0.165	0.375	3.913	0.000	0.289	0.000	0.000	5.932

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [340] had the largest ratio of [.6666670000000000010]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	17	0.118
2	A	2	2	1.00	17	0.118
3	A	2	2	1.00	17	0.118
4	A	2	2	1.00	15	0.133
5	A	2	2	1.00	17	0.118
6	A	2	2	1.00	17	0.118
7	A	3	3	1.00	17	0.176
8	A	2	2	1.00	19	0.105
9	A	2	2	1.00	19	0.105
10	A	2	2	1.00	17	0.118
11	A	2	2	1.00	19	0.105
12	A	2	2	1.00	19	0.105
13	A	3	3	1.11	19	0.158
14	A	2	2	1.00	19	0.105
15	A	2	2	1.00	19	0.105
16	A	2	2	1.00	17	0.118
17	A	2	2	1.00	19	0.105
18	A	2	2	1.00	19	0.105
19	A	2	2	1.00	19	0.105
20	A	2	2	1.00	19	0.105
21	A	2	2	1.00	19	0.105
22	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	2	2	1.00	17	0.118
24	A	2	2	1.00	19	0.105
25	A	3	3	1.17	19	0.158
26	A	4	4	1.24	19	0.211
27	A	2	2	1.00	19	0.105
28	A	2	2	1.00	19	0.105
29	A	2	2	1.00	19	0.105
30	A	2	2	1.00	19	0.105
31	A	2	2	1.00	17	0.118
32	A	4	4	1.16	19	0.211
33	A	6	6	1.14	19	0.316
34	A	7	7	1.20	19	0.368
35	A	2	2	1.00	19	0.105
36	A	2	2	1.00	19	0.105
37	A	2	2	1.00	19	0.105
38	A	3	3	1.11	19	0.158
39	A	3	3	1.00	17	0.176
40	A	6	6	1.22	19	0.316
41	A	8	8	1.19	19	0.421
42	A	10	10	1.18	19	0.526
43	A	3	3	1.00	15	0.200
44	A	5	5	1.11	15	0.333
45	A	7	6	0.93	21	0.286
46	A	6	5	0.94	21	0.238
47	A	5	4	0.95	19	0.211
48	A	4	3	1.00	11	0.273
49	A	6	5	1.00	21	0.238
50	A	4	3	1.00	21	0.143
51	A	5	4	0.99	21	0.190
52	A	8	7	1.12	21	0.333
53	A	8	7	0.85	21	0.333
54	A	7	6	0.83	21	0.286
55	A	6	5	0.89	19	0.263
56	A	5	4	1.05	11	0.364

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	8	7	1.05	21	0.333
58	A	7	6	1.08	21	0.286
59	A	5	4	1.05	21	0.190
60	A	6	5	0.93	21	0.238
61	A	9	8	1.10	21	0.381
62	A	9	8	0.74	21	0.381
63	A	8	7	0.77	21	0.333
64	A	7	6	0.85	19	0.316
65	A	6	5	1.07	11	0.455
66	A	10	9	1.08	21	0.429
67	A	9	8	1.04	21	0.381
68	A	9	8	1.12	21	0.381
69	A	6	5	1.08	21	0.238
70	A	7	6	0.89	21	0.286
71	A	5	4	1.00	19	0.211
72	A	6	5	1.00	17	0.294
73	A	6	5	1.00	21	0.238
74	A	6	5	1.09	21	0.238
75	A	5	4	1.07	21	0.190
76	A	4	3	1.00	19	0.158
77	A	3	2	1.00	11	0.182
78	A	3	2	1.00	21	0.095
79	A	4	3	1.00	21	0.143
80	A	6	5	1.11	21	0.238
81	A	8	7	1.05	21	0.333
82	A	7	6	1.05	21	0.286
83	A	6	5	1.22	21	0.238
84	A	4	3	1.00	19	0.158
85	A	1	1	1.00	11	0.091
86	A	4	3	1.00	21	0.143
87	A	6	5	1.07	21	0.238
88	A	8	7	1.11	21	0.333
89	A	8	7	1.07	21	0.333
90	A	7	6	1.09	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	5	4	1.10	21	0.190
92	A	2	2	1.00	19	0.105
93	A	2	2	1.00	11	0.182
94	A	7	6	1.14	21	0.286
95	A	8	7	1.13	21	0.333
96	A	11	10	1.13	21	0.476
97	A	5	5	0.83	21	0.238
98	A	4	4	0.86	21	0.190
99	A	3	3	0.97	19	0.158
100	A	2	2	1.00	11	0.182
101	A	4	3	1.00	21	0.143
102	A	4	3	1.00	21	0.143
103	A	5	4	0.99	21	0.190
104	A	6	5	0.93	21	0.238
105	A	2	2	1.00	26	0.077
106	A	3	2	1.00	19	0.105
107	A	3	2	1.00	21	0.095
108	A	2	2	1.00	15	0.133
109	A	11	10	1.08	24	0.417
110	A	9	8	1.08	24	0.333
111	A	7	6	1.08	22	0.273
112	A	7	6	1.09	24	0.250
113	A	7	6	1.08	24	0.250
114	A	11	10	1.07	24	0.417
115	A	13	12	1.08	24	0.500
116	A	12	11	1.09	24	0.458
117	A	10	9	1.09	24	0.375
118	A	8	7	1.09	22	0.318
119	A	9	8	1.08	24	0.333
120	A	9	8	1.08	24	0.333
121	A	11	10	1.08	24	0.417
122	A	12	11	1.08	24	0.458
123	A	10	9	1.08	24	0.375
124	A	8	7	1.08	24	0.292

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	6	5	1.08	22	0.227
126	A	1	1	1.00	24	0.042
127	A	9	8	1.07	24	0.333
128	A	11	10	1.08	24	0.417
129	A	10	9	1.08	24	0.375
130	A	8	7	1.08	24	0.292
131	A	6	5	1.09	22	0.227
132	A	9	8	1.07	24	0.333
133	A	11	10	1.07	24	0.417
134	A	13	12	1.08	24	0.500
135	A	12	11	1.08	24	0.458
136	A	8	7	1.09	24	0.292
137	A	3	3	1.00	24	0.125
138	A	7	6	1.09	22	0.273
139	A	11	10	1.08	24	0.417
140	A	13	12	1.09	24	0.500
141	A	1	1	1.00	26	0.038
142	A	1	1	1.00	24	0.042
143	A	1	1	1.00	23	0.043
144	A	1	1	1.00	24	0.042
145	A	1	1	1.00	22	0.045
146	A	1	1	1.00	19	0.053
147	A	1	1	1.00	19	0.053
148	A	1	1	1.00	24	0.042
149	A	1	1	1.00	22	0.045
150	A	1	1	1.00	27	0.037
151	A	1	1	1.00	28	0.036
152	A	1	1	1.00	29	0.034
153	A	1	1	1.00	30	0.033
154	A	1	1	1.00	26	0.038
155	A	1	1	1.00	26	0.038
156	A	1	1	1.00	23	0.043
157	A	1	1	1.00	23	0.043
158	A	1	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	1	1	1.00	23	0.043
160	A	1	1	1.00	17	0.059
161	A	1	1	1.00	21	0.048
162	A	1	1	1.00	21	0.048
163	A	3	3	0.73	29	0.103
164	A	3	3	0.78	29	0.103
165	A	3	3	0.76	29	0.103
166	A	8	8	0.95	23	0.348
167	A	6	6	0.97	23	0.261
168	A	4	4	1.01	23	0.174
169	A	1	1	1.00	23	0.043
170	A	5	5	1.01	23	0.217
171	A	7	7	1.07	23	0.304
172	A	9	9	0.96	23	0.391
173	A	7	7	0.93	23	0.304
174	A	5	5	0.97	23	0.217
175	A	6	6	0.97	23	0.261
176	A	4	4	1.02	23	0.174
177	A	7	7	1.03	23	0.304
178	A	6	6	0.98	23	0.261
179	A	3	3	1.00	23	0.130
180	A	2	2	0.95	23	0.087
181	A	4	4	1.05	23	0.174
182	A	5	5	1.00	23	0.217
183	A	4	4	1.00	21	0.190
184	A	1	1	1.00	23	0.043
185	A	1	1	1.00	23	0.043
186	A	1	1	1.00	23	0.043
187	A	1	1	1.00	21	0.048
188	A	1	1	1.00	21	0.048
189	A	1	1	1.00	21	0.048
190	A	1	1	1.00	23	0.043
191	A	4	4	1.00	21	0.190
192	A	3	3	1.00	23	0.130

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	3	3	1.00	23	0.130
194	A	3	3	1.00	23	0.130
195	A	4	4	1.00	21	0.190
196	A	4	4	1.00	21	0.190
197	A	4	4	1.02	23	0.174
198	A	2	2	1.00	23	0.087
199	A	10	10	0.98	23	0.435
200	A	8	8	0.97	23	0.348
201	A	6	6	1.00	23	0.261
202	A	4	4	1.05	23	0.174
203	A	1	1	1.00	23	0.043
204	A	6	6	0.95	23	0.261
205	A	5	5	1.05	23	0.217
206	A	7	7	1.09	23	0.304
207	A	10	10	0.94	23	0.435
208	A	8	8	0.95	23	0.348
209	A	6	6	1.00	23	0.261
210	A	1	1	1.00	23	0.043
211	A	6	6	1.35	23	0.261
212	A	5	5	1.05	23	0.217
213	A	8	8	1.07	23	0.348
214	A	1	1	1.00	23	0.043
215	A	3	3	1.00	24	0.125
216	A	3	3	1.00	24	0.125
217	A	3	3	1.00	25	0.120
218	A	1	1	1.00	23	0.043
219	A	1	1	1.00	23	0.043
220	A	1	1	1.00	23	0.043
221	A	2	2	1.00	23	0.087
222	A	1	1	1.00	21	0.048
223	A	1	1	1.00	23	0.043
224	A	2	2	1.00	23	0.087
225	A	1	1	1.00	23	0.043
226	A	1	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
227	A	1	1	1.00	23	0.043
228	A	1	1	1.00	21	0.048
229	A	1	1	1.00	21	0.048
230	A	1	1	1.00	21	0.048
231	A	2	2	1.00	21	0.095
232	A	1	1	1.00	19	0.053
233	A	1	1	1.00	21	0.048
234	A	2	2	1.00	21	0.095
235	A	1	1	1.00	21	0.048
236	A	1	1	1.00	21	0.048
237	A	1	1	1.00	21	0.048
238	A	2	2	1.00	21	0.095
239	A	3	3	1.00	21	0.143
240	A	2	2	1.00	21	0.095
241	A	1	1	1.00	21	0.048
242	A	2	2	1.00	19	0.105
243	A	1	1	1.00	21	0.048
244	A	2	2	1.00	21	0.095
245	A	2	2	1.00	21	0.095
246	A	3	3	1.00	21	0.143
247	A	2	2	1.00	21	0.095
248	A	1	1	1.00	23	0.043
249	A	1	1	1.00	23	0.043
250	A	1	1	1.00	23	0.043
251	A	2	2	1.04	23	0.087
252	A	1	1	1.00	21	0.048
253	A	2	2	1.00	23	0.087
254	A	2	2	0.71	23	0.087
255	A	2	2	1.00	23	0.087
256	A	3	3	1.00	23	0.130
257	A	2	2	1.00	23	0.087
258	A	3	3	1.00	24	0.125
259	A	3	3	1.00	27	0.111
260	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
261	A	3	3	1.00	28	0.107
262	A	3	3	1.00	25	0.120
263	A	3	3	1.00	26	0.115
264	A	3	3	1.00	26	0.115
265	A	3	3	1.00	27	0.111
266	A	4	4	1.05	23	0.174
267	A	4	4	1.05	26	0.154
268	A	4	4	1.04	26	0.154
269	A	4	4	1.05	29	0.138
270	A	7	7	1.00	24	0.292
271	A	7	7	1.00	25	0.280
272	A	7	7	1.00	27	0.259
273	A	7	7	1.00	28	0.250
274	A	3	3	1.00	24	0.125
275	A	3	3	1.00	27	0.111
276	A	3	3	1.00	25	0.120
277	A	3	3	1.00	28	0.107
278	A	3	3	1.00	25	0.120
279	A	3	3	1.00	26	0.115
280	A	3	3	1.00	26	0.115
281	A	3	3	1.00	27	0.111
282	A	4	4	1.01	23	0.174
283	A	4	4	1.01	26	0.154
284	A	4	4	1.01	26	0.154
285	A	4	4	1.01	29	0.138
286	A	7	7	1.00	24	0.292
287	A	7	7	1.00	25	0.280
288	A	7	7	1.00	27	0.259
289	A	7	7	1.00	28	0.250
290	A	1	1	1.00	23	0.043
291	A	2	2	1.00	23	0.087
292	A	1	1	1.00	21	0.048
293	A	1	1	1.00	23	0.043
294	A	4	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	4	4	1.05	23	0.174
296	A	1	1	1.00	23	0.043
297	A	1	1	1.00	59	0.017
298	A	1	1	1.00	59	0.017
299	A	4	4	1.13	59	0.068
300	A	3	3	1.00	59	0.051
301	C	1	1	0.37	21	0.048
302	A	2	2	1.00	26	0.077
303	A	1	1	1.00	41	0.024
304	A	1	1	1.00	21	0.048
305	A	1	1	1.00	21	0.048
306	A	1	1	1.00	21	0.048
307	A	1	1	1.00	23	0.043
308	A	1	1	1.00	23	0.043
309	A	1	1	1.00	23	0.043
310	A	1	1	1.00	23	0.043
311	A	1	1	1.00	25	0.040
312	A	1	1	1.00	21	0.048
313	A	1	1	1.00	21	0.048
314	A	1	1	1.00	21	0.048
315	A	1	1	1.00	23	0.043
316	A	1	1	1.00	25	0.040
317	A	1	1	1.00	25	0.040
318	A	1	1	1.00	25	0.040
319	A	1	1	1.00	27	0.037
320	A	1	1	1.00	19	0.053
321	A	13	12	1.01	21	0.571
322	A	13	12	1.02	21	0.571
323	A	8	7	1.02	21	0.333
324	A	9	8	1.08	21	0.381
325	A	4	3	0.95	21	0.143
326	A	6	5	0.95	21	0.238
327	A	7	6	1.01	21	0.286
328	A	10	9	0.97	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	12	11	1.28	21	0.524
330	A	13	12	1.06	21	0.571
331	A	10	9	0.93	21	0.429
332	A	11	10	1.01	21	0.476
333	A	10	9	0.93	21	0.429
334	A	11	10	0.90	21	0.476
335	A	10	9	0.90	21	0.429
336	A	11	10	0.91	21	0.476
337	A	12	11	1.16	21	0.524
338	A	13	12	0.97	21	0.571
339	A	14	13	1.22	21	0.619
340	A	15	14	1.01	21	0.667
341	A	3	3	1.00	19	0.158
342	A	7	7	0.97	19	0.368
343	A	5	5	0.99	19	0.263
344	A	3	3	0.91	17	0.176
345	A	2	2	1.00	9	0.222
346	A	2	2	1.00	19	0.105
347	A	2	2	1.00	19	0.105
348	A	2	2	1.00	19	0.105
349	A	1	1	1.00	50	0.020

CHAPTER 3

Listing of integrals

3.1 $\int (a+bx^2)(c+dx^2)^4\,dx$

3.1.1 Optimal result

Integrand size = 17, antiderivative size = 94

$$\int (a+bx^2)(c+dx^2)^4\,dx = ac^4x+\frac{1}{3}c^3(bc+4ad)x^3+\frac{2}{5}c^2d(2bc+3ad)x^5 +\frac{2}{7}cd^2(3bc+2ad)x^7+\frac{1}{9}d^3(4bc+ad)x^9+\frac{1}{11}bd^4x^{11}$$

output
```
a*c^4*x+1/3*c^3*(4*a*d+b*c)*x^3+2/5*c^2*d*(3*a*d+2*b*c)*x^5+2/7*c*d^2*(2*a
*d+3*b*c)*x^7+1/9*d^3*(a*d+4*b*c)*x^9+1/11*b*d^4*x^11
```

3.1.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)^4\,dx = ac^4x+\frac{1}{3}c^3(bc+4ad)x^3+\frac{2}{5}c^2d(2bc+3ad)x^5 +\frac{2}{7}cd^2(3bc+2ad)x^7+\frac{1}{9}d^3(4bc+ad)x^9+\frac{1}{11}bd^4x^{11}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^4,x]
```

output
```
a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^5)/5 + (2
*c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/9 + (b*d^4*x^11)/1
1
```

3.1.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^4 dx$$

$$\Big\downarrow \ 290$$

$$\int \left(c^3x^2(4ad+bc)+2c^2dx^4(3ad+2bc)+d^3x^8(ad+4bc)+2cd^2x^6(2ad+3bc)+ac^4+bd^4x^{10}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}c^3x^3(4ad+bc)+\frac{2}{5}c^2dx^5(3ad+2bc)+\frac{1}{9}d^3x^9(ad+4bc)+\frac{2}{7}cd^2x^7(2ad+3bc)+ac^4x+\frac{1}{11}bd^4x^{11}$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^4,x]
```

output
```
a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^5)/5 + (2
*c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/9 + (b*d^4*x^11)/1
1
```

3.1.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result
norman	$\frac{b\,d^4x^{11}}{11}+\left(\frac{1}{9}a\,d^4+\frac{4}{9}bc\,d^3\right)x^9+\left(\frac{4}{7}ac\,d^3+\frac{6}{7}b\,c^2d^2\right)x^7+\left(\frac{6}{5}a\,c^2d^2+\frac{4}{5}b\,c^3d\right)x^5+\left(\frac{4}{3}a\,c^3d+\frac{1}{3}b\right.$
default	$\frac{b\,d^4x^{11}}{11}+\frac{(a\,d^4+4bc\,d^3)x^9}{9}+\frac{(4ac\,d^3+6b\,c^2d^2)x^7}{7}+\frac{(6a\,c^2d^2+4b\,c^3d)x^5}{5}+\frac{(4a\,c^3d+b\,c^4)x^3}{3}+a\,c^4x$
gosper	$\frac{1}{11}b\,d^4x^{11}+\frac{1}{9}x^9a\,d^4+\frac{4}{9}x^9bc\,d^3+\frac{4}{7}x^7ac\,d^3+\frac{6}{7}x^7b\,c^2d^2+\frac{6}{5}x^5a\,c^2d^2+\frac{4}{5}x^5b\,c^3d+\frac{4}{3}x^3a\,c^3d+$
risch	$\frac{1}{11}b\,d^4x^{11}+\frac{1}{9}x^9a\,d^4+\frac{4}{9}x^9bc\,d^3+\frac{4}{7}x^7ac\,d^3+\frac{6}{7}x^7b\,c^2d^2+\frac{6}{5}x^5a\,c^2d^2+\frac{4}{5}x^5b\,c^3d+\frac{4}{3}x^3a\,c^3d+$
parallelrisch	$\frac{1}{11}b\,d^4x^{11}+\frac{1}{9}x^9a\,d^4+\frac{4}{9}x^9bc\,d^3+\frac{4}{7}x^7ac\,d^3+\frac{6}{7}x^7b\,c^2d^2+\frac{6}{5}x^5a\,c^2d^2+\frac{4}{5}x^5b\,c^3d+\frac{4}{3}x^3a\,c^3d+$

input
```
int((b*x^2+a)*(d*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/11*b*d^4*x^11+(1/9*a*d^4+4/9*b*c*d^3)*x^9+(4/7*a*c*d^3+6/7*b*c^2*d^2)*x^
7+(6/5*a*c^2*d^2+4/5*b*c^3*d)*x^5+(4/3*a*c^3*d+1/3*b*c^4)*x^3+a*c^4*x
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^4\,dx=\frac{1}{11}\,bd^4x^{11}+\frac{1}{9}\left(4\,bcd^3+ad^4\right)x^9+\frac{2}{7}\left(3\,bc^2d^2+2\,acd^3\right)x^7+ac^4x+\frac{2}{5}\left(2\,bc^3d+3\,ac^2d^2\right)x^5+\frac{1}{3}\left(bc^4+4\,ac^3d\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="fricas")
```

output
```
1/11*b*d^4*x^11 + 1/9*(4*b*c*d^3 + a*d^4)*x^9 + 2/7*(3*b*c^2*d^2 + 2*a*c*d
^3)*x^7 + a*c^4*x + 2/5*(2*b*c^3*d + 3*a*c^2*d^2)*x^5 + 1/3*(b*c^4 + 4*a*c
^3*d)*x^3
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int (a+bx^2)(c+dx^2)^4\,dx = ac^4x + \frac{bd^4x^{11}}{11} + x^9\left(\frac{ad^4}{9} + \frac{4bcd^3}{9}\right) + x^7\cdot\left(\frac{4acd^3}{7} + \frac{6bc^2d^2}{7}\right) + x^5\cdot\left(\frac{6ac^2d^2}{5} + \frac{4bc^3d}{5}\right) + x^3\cdot\left(\frac{4ac^3d}{3} + \frac{bc^4}{3}\right)$$

input

```
integrate((b*x**2+a)*(d*x**2+c)**4,x)
```

output

```
a*c**4*x + b*d**4*x**11/11 + x**9*(a*d**4/9 + 4*b*c*d**3/9) + x**7*(4*a*c*
d**3/7 + 6*b*c**2*d**2/7) + x**5*(6*a*c**2*d**2/5 + 4*b*c**3*d/5) + x**3*(
4*a*c**3*d/3 + b*c**4/3)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^2)(c+dx^2)^4\,dx = \frac{1}{11}bd^4x^{11} + \frac{1}{9}(4\,bcd^3 + ad^4)x^9 + \frac{2}{7}(3\,bc^2d^2 + 2\,acd^3)x^7 + ac^4x + \frac{2}{5}(2\,bc^3d + 3\,ac^2d^2)x^5 + \frac{1}{3}(bc^4 + 4\,ac^3d)x^3$$

input

```
integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="maxima")
```

output

```
1/11*b*d^4*x^11 + 1/9*(4*b*c*d^3 + a*d^4)*x^9 + 2/7*(3*b*c^2*d^2 + 2*a*c*d
^3)*x^7 + a*c^4*x + 2/5*(2*b*c^3*d + 3*a*c^2*d^2)*x^5 + 1/3*(b*c^4 + 4*a*c
^3*d)*x^3
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int (a+bx^2)(c+dx^2)^4\,dx = \frac{1}{11}bd^4x^{11} + \frac{4}{9}bcd^3x^9 + \frac{1}{9}ad^4x^9 + \frac{6}{7}bc^2d^2x^7 + \frac{4}{7}acd^3x^7 + \frac{4}{5}bc^3dx^5 + \frac{6}{5}ac^2d^2x^5 + \frac{1}{3}bc^4x^3 + \frac{4}{3}ac^3dx^3 + ac^4x$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="giac")
```

output
```
1/11*b*d^4*x^11 + 4/9*b*c*d^3*x^9 + 1/9*a*d^4*x^9 + 6/7*b*c^2*d^2*x^7 + 4/
7*a*c*d^3*x^7 + 4/5*b*c^3*d*x^5 + 6/5*a*c^2*d^2*x^5 + 1/3*b*c^4*x^3 + 4/3*
a*c^3*d*x^3 + a*c^4*x
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int (a+bx^2)\,(c+dx^2)^4\,dx = x^3\left(\frac{b\,c^4}{3}+\frac{4\,a\,d\,c^3}{3}\right)+x^9\left(\frac{a\,d^4}{9}+\frac{4\,b\,c\,d^3}{9}\right)+\frac{b\,d^4\,x^{11}}{11} + a\,c^4\,x+\frac{2\,c^2\,d\,x^5\,(3\,a\,d+2\,b\,c)}{5}+\frac{2\,c\,d^2\,x^7\,(2\,a\,d+3\,b\,c)}{7}$$

input
```
int((a + b*x^2)*(c + d*x^2)^4,x)
```

output
```
x^3*((b*c^4)/3 + (4*a*c^3*d)/3) + x^9*((a*d^4)/9 + (4*b*c*d^3)/9) + (b*d^4
*x^11)/11 + a*c^4*x + (2*c^2*d*x^5*(3*a*d + 2*b*c))/5 + (2*c*d^2*x^7*(2*a*
d + 3*b*c))/7
```

3.2 $\int (a+bx^2)(c+dx^2)^3\,dx$

3.2.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^2)(c+dx^2)^3\,dx = ac^3x+\frac{1}{3}c^2(bc+3ad)x^3+\frac{3}{5}cd(bc+ad)x^5+\frac{1}{7}d^2(3bc+ad)x^7+\frac{1}{9}bd^3x^9$$

output
```
a*c^3*x+1/3*c^2*(3*a*d+b*c)*x^3+3/5*c*d*(a*d+b*c)*x^5+1/7*d^2*(a*d+3*b*c)*
x^7+1/9*b*d^3*x^9
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)^3\,dx = ac^3x+\frac{1}{3}c^2(bc+3ad)x^3+\frac{3}{5}cd(bc+ad)x^5+\frac{1}{7}d^2(3bc+ad)x^7+\frac{1}{9}bd^3x^9$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^3,x]
```

output
```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2*(3*
b*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

3.2.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(c+dx^2)^3\,dx$$

$$\Big\downarrow \text{290}$$

$$\int \left(c^2x^2(3ad+bc)+d^2x^6(ad+3bc)+3cdx^4(ad+bc)+ac^3+bd^3x^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^3,x]
```

output
```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2*(3*
b*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

3.2.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result	size
norman	$\frac{b\,d^3x^9}{9}+\left(\frac{1}{7}a\,d^3+\frac{3}{7}bc\,d^2\right)x^7+\left(\frac{3}{5}ac\,d^2+\frac{3}{5}b\,c^2d\right)x^5+\left(a\,c^2d+\frac{1}{3}c^3b\right)x^3+a\,c^3x$	71
default	$\frac{b\,d^3x^9}{9}+\frac{(a\,d^3+3bc\,d^2)x^7}{7}+\frac{(3ac\,d^2+3b\,c^2d)x^5}{5}+\frac{(3a\,c^2d+c^3b)x^3}{3}+a\,c^3x$	73
gosper	$\frac{1}{9}b\,d^3x^9+\frac{1}{7}x^7a\,d^3+\frac{3}{7}x^7bc\,d^2+\frac{3}{5}x^5ac\,d^2+\frac{3}{5}x^5b\,c^2d+x^3a\,c^2d+\frac{1}{3}x^3c^3b+a\,c^3x$	74
risch	$\frac{1}{9}b\,d^3x^9+\frac{1}{7}x^7a\,d^3+\frac{3}{7}x^7bc\,d^2+\frac{3}{5}x^5ac\,d^2+\frac{3}{5}x^5b\,c^2d+x^3a\,c^2d+\frac{1}{3}x^3c^3b+a\,c^3x$	74
parallelrisch	$\frac{1}{9}b\,d^3x^9+\frac{1}{7}x^7a\,d^3+\frac{3}{7}x^7bc\,d^2+\frac{3}{5}x^5ac\,d^2+\frac{3}{5}x^5b\,c^2d+x^3a\,c^2d+\frac{1}{3}x^3c^3b+a\,c^3x$	74

input
```
int((b*x^2+a)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*b*d^3*x^9+(1/7*a*d^3+3/7*b*c*d^2)*x^7+(3/5*a*c*d^2+3/5*b*c^2*d)*x^5+(a
*c^2*d+1/3*c^3*b)*x^3+a*c^3*x
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^2)\left(c+dx^2\right)^3\,dx=\frac{1}{9}\,bd^3x^9+\frac{1}{7}\left(3\,bcd^2+ad^3\right)x^7+\frac{3}{5}\left(bc^2d+acd^2\right)x^5+ac^3x+\frac{1}{3}\left(bc^3+3\,ac^2d\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/9*b*d^3*x^9 + 1/7*(3*b*c*d^2 + a*d^3)*x^7 + 3/5*(b*c^2*d + a*c*d^2)*x^5
+ a*c^3*x + 1/3*(b*c^3 + 3*a*c^2*d)*x^3
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int (a+bx^2)(c+dx^2)^3\,dx = ac^3x + \frac{bd^3x^9}{9} + x^7\left(\frac{ad^3}{7} + \frac{3bcd^2}{7}\right) + x^5 \cdot \left(\frac{3acd^2}{5} + \frac{3bc^2d}{5}\right) + x^3\left(ac^2d + \frac{bc^3}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3,x)
```

output
```
a*c**3*x + b*d**3*x**9/9 + x**7*(a*d**3/7 + 3*b*c*d**2/7) + x**5*(3*a*c*d*
*2/5 + 3*b*c**2*d/5) + x**3*(a*c**2*d + b*c**3/3)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)^3\,dx = \frac{1}{9}bd^3x^9 + \frac{1}{7}(3bcd^2 + ad^3)x^7 + \frac{3}{5}(bc^2d + acd^2)x^5 + ac^3x + \frac{1}{3}(bc^3 + 3ac^2d)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/9*b*d^3*x^9 + 1/7*(3*b*c*d^2 + a*d^3)*x^7 + 3/5*(b*c^2*d + a*c*d^2)*x^5
+ a*c^3*x + 1/3*(b*c^3 + 3*a*c^2*d)*x^3
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int (a+bx^2)(c+dx^2)^3\,dx = \frac{1}{9}bd^3x^9 + \frac{3}{7}bcd^2x^7 + \frac{1}{7}ad^3x^7 + \frac{3}{5}bc^2dx^5 + \frac{3}{5}acd^2x^5 + \frac{1}{3}bc^3x^3 + ac^2dx^3 + ac^3x$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/9*b*d^3*x^9 + 3/7*b*c*d^2*x^7 + 1/7*a*d^3*x^7 + 3/5*b*c^2*d*x^5 + 3/5*a*
c*d^2*x^5 + 1/3*b*c^3*x^3 + a*c^2*d*x^3 + a*c^3*x
```

3.2.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int (a+bx^2)\left(c+dx^2\right)^3\,dx = x^3\left(\frac{b\,c^3}{3}+a\,d\,c^2\right)+x^7\left(\frac{a\,d^3}{7}+\frac{3\,b\,c\,d^2}{7}\right)+\frac{b\,d^3\,x^9}{9}+a\,c^3\,x+\frac{3\,c\,d\,x^5\,(a\,d+b\,c)}{5}$$

input
```
int((a + b*x^2)*(c + d*x^2)^3,x)
```

output
```
x^3*((b*c^3)/3 + a*c^2*d) + x^7*((a*d^3)/7 + (3*b*c*d^2)/7) + (b*d^3*x^9)/
9 + a*c^3*x + (3*c*d*x^5*(a*d + b*c))/5
```

3.3 $\int (a+bx^2)(c+dx^2)^2 \, dx$

3.3.1 Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^2)(c+dx^2)^2 \, dx = ac^2x + \frac{1}{3}c(bc+2ad)x^3 + \frac{1}{5}d(2bc+ad)x^5 + \frac{1}{7}bd^2x^7$$

output
```
a*c^2*x+1/3*c*(2*a*d+b*c)*x^3+1/5*d*(a*d+2*b*c)*x^5+1/7*b*d^2*x^7
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)^2 \, dx = ac^2x + \frac{1}{3}c(bc+2ad)x^3 + \frac{1}{5}d(2bc+ad)x^5 + \frac{1}{7}bd^2x^7$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^2,x]
```

output
```
a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*d^2*x^7)/
7
```

3.3.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right) \left(c+dx^2\right)^2 \, dx$$

$$\Big\downarrow \text{290}$$

$$\int \left(dx^4(ad+2bc)+cx^2(2ad+bc)+ac^2+bd^2x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}dx^5(ad+2bc)+\frac{1}{3}cx^3(2ad+bc)+ac^2x+\frac{1}{7}bd^2x^7$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^2,x]
```

output
```
a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*d^2*x^7)/
7
```

3.3.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b\,d^{2} x^{7}}{7}+\frac{\left(a\,d^{2}+2 b c d \right) x^{5}}{5}+\frac{\left(2 a c d +b\,c^{2}\right) x^{3}}{3}+a\,c^{2} x$	49
norman	$\frac{b\,d^{2} x^{7}}{7}+\left(\frac{1}{5} a\,d^{2}+\frac{2}{5} b c d \right) x^{5}+\left(\frac{2}{3} a c d +\frac{1}{3} b\,c^{2}\right) x^{3}+a\,c^{2} x$	49
gosper	$\frac{1}{7} b\,d^{2} x^{7}+\frac{1}{5} x^{5} a\,d^{2}+\frac{2}{5} x^{5} b c d +\frac{2}{3} x^{3} a c d +\frac{1}{3} x^{3} b\,c^{2}+a\,c^{2} x$	51
risch	$\frac{1}{7} b\,d^{2} x^{7}+\frac{1}{5} x^{5} a\,d^{2}+\frac{2}{5} x^{5} b c d +\frac{2}{3} x^{3} a c d +\frac{1}{3} x^{3} b\,c^{2}+a\,c^{2} x$	51
parallelrisch	$\frac{1}{7} b\,d^{2} x^{7}+\frac{1}{5} x^{5} a\,d^{2}+\frac{2}{5} x^{5} b c d +\frac{2}{3} x^{3} a c d +\frac{1}{3} x^{3} b\,c^{2}+a\,c^{2} x$	51

input
```
int((b*x^2+a)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*b*d^2*x^7+1/5*(a*d^2+2*b*c*d)*x^5+1/3*(2*a*c*d+b*c^2)*x^3+a*c^2*x
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx=\frac{1}{7}\,bd^2x^7+\frac{1}{5}\,(2\,bcd+ad^2)x^5+ac^2x+\frac{1}{3}\,(bc^2+2\,acd)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/7*b*d^2*x^7 + 1/5*(2*b*c*d + a*d^2)*x^5 + a*c^2*x + 1/3*(b*c^2 + 2*a*c*d
)*x^3
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx=ac^2x+\frac{bd^2x^7}{7}+x^5\left(\frac{ad^2}{5}+\frac{2bcd}{5}\right)+x^3\cdot\left(\frac{2acd}{3}+\frac{bc^2}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2,x)
```

output
```
a*c**2*x + b*d**2*x**7/7 + x**5*(a*d**2/5 + 2*b*c*d/5) + x**3*(2*a*c*d/3 +
 b*c**2/3)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^2)(c+dx^2)^2\,dx = \frac{1}{7}\,bd^2x^7 + \frac{1}{5}\,(2\,bcd+ad^2)x^5 + ac^2x + \frac{1}{3}\,(bc^2+2\,acd)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/7*b*d^2*x^7 + 1/5*(2*b*c*d + a*d^2)*x^5 + a*c^2*x + 1/3*(b*c^2 + 2*a*c*d
)*x^3
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)^2\,dx = \frac{1}{7}\,bd^2x^7 + \frac{2}{5}\,bcdx^5 + \frac{1}{5}\,ad^2x^5 + \frac{1}{3}\,bc^2x^3 + \frac{2}{3}\,acdx^3 + ac^2x$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/7*b*d^2*x^7 + 2/5*b*c*d*x^5 + 1/5*a*d^2*x^5 + 1/3*b*c^2*x^3 + 2/3*a*c*d*
x^3 + a*c^2*x
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^2)(c+dx^2)^2\,dx = x^3\left(\frac{b\,c^2}{3}+\frac{2\,a\,d\,c}{3}\right) + x^5\left(\frac{a\,d^2}{5}+\frac{2\,b\,c\,d}{5}\right) + \frac{b\,d^2\,x^7}{7} + a\,c^2\,x$$

input
```
int((a + b*x^2)*(c + d*x^2)^2,x)
```

output
```
x^3*((b*c^2)/3 + (2*a*c*d)/3) + x^5*((a*d^2)/5 + (2*b*c*d)/5) + (b*d^2*x^7
)/7 + a*c^2*x
```

3.4 $\int (a+bx^2)(c+dx^2)\,dx$

3.4.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (a+bx^2)(c+dx^2)\,dx = acx + \frac{1}{3}(bc+ad)x^3 + \frac{1}{5}bdx^5$$

output
```
a*c*x+1/3*(a*d+b*c)*x^3+1/5*b*d*x^5
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)\,dx = acx + \frac{1}{3}(bc+ad)x^3 + \frac{1}{5}bdx^5$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2),x]
```

output
```
a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

3.4.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(c+dx^2)\,dx$$

$$\Big\downarrow \text{290}$$

$$\int (x^2(ad+bc)+ac+bdx^4)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(ad+bc)+acx+\frac{1}{5}bdx^5$$

input
```
Int[(a + b*x^2)*(c + d*x^2),x]
```

output
```
a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

3.4.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.4.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^3}{3}+\frac{bd\,x^5}{5}$	25
norman	$\frac{bd\,x^5}{5}+\left(\frac{ad}{3}+\frac{bc}{3}\right)x^3+acx$	26
gosper	$\frac{1}{5}bd\,x^5+\frac{1}{3}x^3ad+\frac{1}{3}bc\,x^3+acx$	27
risch	$\frac{1}{5}bd\,x^5+\frac{1}{3}x^3ad+\frac{1}{3}bc\,x^3+acx$	27
parallelrisch	$\frac{1}{5}bd\,x^5+\frac{1}{3}x^3ad+\frac{1}{3}bc\,x^3+acx$	27

input
```
int((b*x^2+a)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/3*(a*d+b*c)*x^3+1/5*b*d*x^5
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^2)(c+dx^2)\,dx=\frac{1}{5}\,bdx^5+\frac{1}{3}\,(bc+ad)x^3+acx$$

input
```
integrate((b*x^2+a)*(d*x^2+c),x, algorithm="fricas")
```

output
```
1/5*b*d*x^5 + 1/3*(b*c + a*d)*x^3 + a*c*x
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^2)(c+dx^2)\,dx=acx+\frac{bdx^5}{5}+x^3\left(\frac{ad}{3}+\frac{bc}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c),x)
```

output
```
a*c*x + b*d*x**5/5 + x**3*(a*d/3 + b*c/3)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^2)(c+dx^2)\,dx = \frac{1}{5}bdx^5 + \frac{1}{3}(bc+ad)x^3 + acx$$

input
```
integrate((b*x^2+a)*(d*x^2+c),x, algorithm="maxima")
```

output
```
1/5*b*d*x^5 + 1/3*(b*c + a*d)*x^3 + a*c*x
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^2)(c+dx^2)\,dx = \frac{1}{5}bdx^5 + \frac{1}{3}bcx^3 + \frac{1}{3}adx^3 + acx$$

input
```
integrate((b*x^2+a)*(d*x^2+c),x, algorithm="giac")
```

output
```
1/5*b*d*x^5 + 1/3*b*c*x^3 + 1/3*a*d*x^3 + a*c*x
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx^2)(c+dx^2)\,dx = \frac{b\,d\,x^5}{5} + \left(\frac{a\,d}{3} + \frac{b\,c}{3}\right)x^3 + a\,c\,x$$

input
```
int((a + b*x^2)*(c + d*x^2),x)
```

output
```
x^3*((a*d)/3 + (b*c)/3) + a*c*x + (b*d*x^5)/5
```

3.5 $\int \frac{a+bx^2}{c+dx^2} \, dx$

3.5.1 Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{a+bx^2}{c+dx^2} \, dx = \frac{bx}{d} - \frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

output
```
b*x/d-(-a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/d^(3/2)/c^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{c+dx^2} \, dx = \frac{bx}{d} - \frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

input
```
Integrate[(a + b*x^2)/(c + d*x^2),x]
```

output
```
(b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2))
```

3.5.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{c+dx^2} \, dx$$
$$\Big\downarrow \; 299$$
$$\frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{dx^2+c}dx}{d}$$
$$\Big\downarrow \; 218$$
$$\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

input
```
Int[(a + b*x^2)/(c + d*x^2),x]
```

output
```
(b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2))
```

3.5.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.5.4 Maple [A] (verified)

Time = 8.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{bx}{d}+\frac{(ad-bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d\sqrt{cd}}$	34
risch	$\frac{bx}{d}-\frac{\ln\left(dx+\sqrt{-cd}\right)a}{2\sqrt{-cd}}+\frac{\ln\left(dx+\sqrt{-cd}\right)bc}{2d\sqrt{-cd}}+\frac{\ln\left(-dx+\sqrt{-cd}\right)a}{2\sqrt{-cd}}-\frac{\ln\left(-dx+\sqrt{-cd}\right)bc}{2d\sqrt{-cd}}$	98

input
```
int((b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
b*x/d+1/d*(a*d-b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.48

$$\int \frac{a+bx^2}{c+dx^2}\,dx = \left[\frac{2\,bcdx+(bc-ad)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)}{2\,cd^2},\frac{bcdx-(bc-ad)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}x}{c}\right)}{cd^2}\right]$$

input
```
integrate((b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/2*(2*b*c*d*x + (b*c - a*d)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/
(d*x^2 + c)))/(c*d^2), (b*c*d*x - (b*c - a*d)*sqrt(c*d)*arctan(sqrt(c*d)*x
/c))/(c*d^2)]
```

3.5.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{a+bx^2}{c+dx^2}\,dx = \frac{bx}{d}-\frac{\sqrt{-\frac{1}{cd^3}}(ad-bc)\log\left(-cd\sqrt{-\frac{1}{cd^3}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{cd^3}}(ad-bc)\log\left(cd\sqrt{-\frac{1}{cd^3}}+x\right)}{2}$$

input
```
integrate((b*x**2+a)/(d*x**2+c),x)
```

output
```
b*x/d - sqrt(-1/(c*d**3))*(a*d - b*c)*log(-c*d*sqrt(-1/(c*d**3)) + x)/2 +
sqrt(-1/(c*d**3))*(a*d - b*c)*log(c*d*sqrt(-1/(c*d**3)) + x)/2
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{c+dx^2}\,dx = \frac{bx}{d} - \frac{(bc-ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d}$$

input
```
integrate((b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
b*x/d - (b*c - a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d)
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{c+dx^2}\,dx = \frac{bx}{d} - \frac{(bc-ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d}$$

input
```
integrate((b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
b*x/d - (b*c - a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{a+bx^2}{c+dx^2}\,dx = \frac{b\,x}{d} + \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,(a\,d-b\,c)}{\sqrt{c}\,d^{3/2}}$$

input
```
int((a + b*x^2)/(c + d*x^2),x)
```

output
```
(b*x)/d + (atan((d^(1/2)*x)/c^(1/2))*(a*d - b*c))/(c^(1/2)*d^(3/2))
```

3.6 $\int \frac{a+bx^2}{(c+dx^2)^2}\,dx$

3.6.1 Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{a+bx^2}{(c+dx^2)^2}\,dx = -\frac{(bc-ad)x}{2cd\,(c+dx^2)} + \frac{(bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}$$

output
```
-1/2*(-a*d+b*c)*x/c/d/(d*x^2+c)+1/2*(a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/c^
(3/2)/d^(3/2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{(c+dx^2)^2}\,dx = -\frac{(bc-ad)x}{2cd\,(c+dx^2)} + \frac{(bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}$$

input
```
Integrate[(a + b*x^2)/(c + d*x^2)^2,x]
```

output
```
-1/2*((b*c - a*d)*x)/(c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(2*c^(3/2)*d^(3/2))
```

3.6.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)^2} \, dx$$

$$\Big\downarrow \ 298$$

$$\frac{(ad+bc)\int \frac{1}{dx^2+c}dx}{2cd}-\frac{x(bc-ad)}{2cd\,(c+dx^2)}$$

$$\Big\downarrow \ 218$$

$$\frac{(ad+bc)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}-\frac{x(bc-ad)}{2cd\,(c+dx^2)}$$

input
```
Int[(a + b*x^2)/(c + d*x^2)^2,x]
```

output
```
-1/2*((b*c - a*d)*x)/(c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(2*c^(3/2)*d^(3/2))
```

3.6.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.6.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{(ad-bc)x}{2cd(d\,x^2+c)}+\frac{(ad+bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2cd\sqrt{cd}}$	57
risch	$\frac{(ad-bc)x}{2cd(d\,x^2+c)}-\frac{\ln\left(dx+\sqrt{-cd}\right)a}{4\sqrt{-cd}\,c}-\frac{\ln\left(dx+\sqrt{-cd}\right)b}{4\sqrt{-cd}\,d}+\frac{\ln\left(-dx+\sqrt{-cd}\right)a}{4\sqrt{-cd}\,c}+\frac{\ln\left(-dx+\sqrt{-cd}\right)b}{4\sqrt{-cd}\,d}$	122

input
```
int((b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*d-b*c)/c/d*x/(d*x^2+c)+1/2*(a*d+b*c)/c/d/(c*d)^(1/2)*arctan(d*x/(c*
d)^(1/2))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.89

$$\int \frac{a+bx^2}{(c+dx^2)^2}\,dx$$
$$=\left[-\frac{(bc^2+acd+(bcd+ad^2)x^2)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)+2\left(bc^2d-acd^2\right)x}{4\left(c^2d^3x^2+c^3d^2\right)},\ \frac{(bc^2+acd+(bcd+ad^2)x^2}{2\,(c}\right.$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[-1/4*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqr
t(-c*d)*x - c)/(d*x^2 + c)) + 2*(b*c^2*d - a*c*d^2)*x)/(c^2*d^3*x^2 + c^3*
d^2), 1/2*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(sqrt(c*d
)*x/c) - (b*c^2*d - a*c*d^2)*x)/(c^2*d^3*x^2 + c^3*d^2)]
```

3.6.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^2}\,dx = \frac{x(ad-bc)}{2c^2d+2cd^2x^2} - \frac{\sqrt{-\frac{1}{c^3d^3}}(ad+bc)\log\left(-c^2d\sqrt{-\frac{1}{c^3d^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^3}}(ad+bc)\log\left(c^2d\sqrt{-\frac{1}{c^3d^3}}+x\right)}{4}$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
x*(a*d - b*c)/(2*c**2*d + 2*c*d**2*x**2) - sqrt(-1/(c**3*d**3))*(a*d + b*c
)*log(-c**2*d*sqrt(-1/(c**3*d**3)) + x)/4 + sqrt(-1/(c**3*d**3))*(a*d + b*
c)*log(c**2*d*sqrt(-1/(c**3*d**3)) + x)/4
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^2}\,dx = -\frac{(bc-ad)x}{2\left(cd^2x^2+c^2d\right)} + \frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}cd}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*c - a*d)*x/(c*d^2*x^2 + c^2*d) + 1/2*(b*c + a*d)*arctan(d*x/sqrt(c
*d))/(sqrt(c*d)*c*d)
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^2}\,dx = \frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}cd} - \frac{bcx-adx}{2\left(dx^2+c\right)cd}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d) - 1/2*(b*c*x - a*d*x
)/((d*x^2 + c)*c*d)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 4.47 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^2}\,dx=\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(a\,d+b\,c\right)}{2\,c^{3/2}\,d^{3/2}}+\frac{x\,\left(a\,d-b\,c\right)}{2\,c\,d\,\left(d\,x^2+c\right)}$$

input
```
int((a + b*x^2)/(c + d*x^2)^2,x)
```

output
```
(atan((d^(1/2)*x)/c^(1/2))*(a*d + b*c))/(2*c^(3/2)*d^(3/2)) + (x*(a*d - b*
c))/(2*c*d*(c + d*x^2))
```

3.7 $\int \frac{a+bx^2}{(c+dx^2)^3} \, dx$

3.7.1 Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{a+bx^2}{(c+dx^2)^3} \, dx = -\frac{(bc-ad)x}{4cd(c+dx^2)^2} + \frac{(bc+3ad)x}{8c^2d(c+dx^2)} + \frac{(bc+3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}$$

output
```
-1/4*(-a*d+b*c)*x/c/d/(d*x^2+c)^2+1/8*(3*a*d+b*c)*x/c^2/d/(d*x^2+c)+1/8*(3
*a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/d^(3/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{(c+dx^2)^3} \, dx = \frac{x(bc(-c+dx^2)+ad(5c+3dx^2))}{8c^2d(c+dx^2)^2} + \frac{(bc+3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}$$

input
```
Integrate[(a + b*x^2)/(c + d*x^2)^3,x]
```

output
```
(x*(b*c*(-c + d*x^2) + a*d*(5*c + 3*d*x^2)))/(8*c^2*d*(c + d*x^2)^2) + ((b
*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(3/2))
```

3.7.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{298}$$

$$\frac{(3ad+bc)\int \frac{1}{(dx^2+c)^2}dx}{4cd}-\frac{x(bc-ad)}{4cd\,(c+dx^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{(3ad+bc)\left(\frac{\int \frac{1}{dx^2+c}dx}{2c}+\frac{x}{2c(c+dx^2)}\right)}{4cd}-\frac{x(bc-ad)}{4cd\,(c+dx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(3ad+bc)\left(\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x}{2c(c+dx^2)}\right)}{4cd}-\frac{x(bc-ad)}{4cd\,(c+dx^2)^2}$$

input
```
Int[(a + b*x^2)/(c + d*x^2)^3,x]
```

output
```
-1/4*((b*c - a*d)*x)/(c*d*(c + d*x^2)^2) + ((b*c + 3*a*d)*(x/(2*c*(c + d*x
^2)) + ArcTan[(Sqrt[d]*x)/Sqrt[c]]/(2*c^(3/2)*Sqrt[d])))/(4*c*d)
```

3.7.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.7.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{(3ad+bc)x^3}{8c^2}+\frac{(5ad-bc)x}{8cd}}{(d\,x^2+c)^2}+\frac{(3ad+bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2d\sqrt{cd}}$	77
risch	$\frac{\frac{(3ad+bc)x^3}{8c^2}+\frac{(5ad-bc)x}{8cd}}{(d\,x^2+c)^2}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a}{16\sqrt{-cd}\,c^2}-\frac{\ln\left(dx+\sqrt{-cd}\right)b}{16\sqrt{-cd}\,dc}+\frac{3\ln\left(-dx+\sqrt{-cd}\right)a}{16\sqrt{-cd}\,c^2}+\frac{\ln\left(-dx+\sqrt{-cd}\right)b}{16\sqrt{-cd}\,dc}$	147

input
```
int((b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*a*d+b*c)/c^2*x^3+1/8*(5*a*d-b*c)/c/d*x)/(d*x^2+c)^2+1/8*(3*a*d+b*c
)/c^2/d/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.26

$$\int \frac{a+bx^2}{(c+dx^2)^3}\,dx$$
$$= \left[\frac{2\,(bc^2d^2+3\,acd^3)x^3-((bcd^2+3\,ad^3)x^4+bc^3+3\,ac^2d+2\,(bc^2d+3\,acd^2)x^2)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)}{16\,(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2)}\right.$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(b*c^2*d^2 + 3*a*c*d^3)*x^3 - ((b*c*d^2 + 3*a*d^3)*x^4 + b*c^3 +
3*a*c^2*d + 2*(b*c^2*d + 3*a*c*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c
*d)*x - c)/(d*x^2 + c)) - 2*(b*c^3*d - 5*a*c^2*d^2)*x)/(c^3*d^4*x^4 + 2*c^
4*d^3*x^2 + c^5*d^2), 1/8*((b*c^2*d^2 + 3*a*c*d^3)*x^3 + ((b*c*d^2 + 3*a*d
^3)*x^4 + b*c^3 + 3*a*c^2*d + 2*(b*c^2*d + 3*a*c*d^2)*x^2)*sqrt(c*d)*arcta
n(sqrt(c*d)*x/c) - (b*c^3*d - 5*a*c^2*d^2)*x)/(c^3*d^4*x^4 + 2*c^4*d^3*x^2
 + c^5*d^2)]
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.63

$$\int \frac{a+bx^2}{(c+dx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{c^5d^3}}\cdot(3ad+bc)\log\left(-c^3d\sqrt{-\frac{1}{c^5d^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{c^5d^3}}\cdot(3ad+bc)\log\left(c^3d\sqrt{-\frac{1}{c^5d^3}}+x\right)}{16} + \frac{x^3\cdot(3ad^2+bcd)+x(5acd-bc^2)}{8c^4d+16c^3d^2x^2+8c^2d^3x^4}$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
-sqrt(-1/(c**5*d**3))*(3*a*d + b*c)*log(-c**3*d*sqrt(-1/(c**5*d**3)) + x)/
16 + sqrt(-1/(c**5*d**3))*(3*a*d + b*c)*log(c**3*d*sqrt(-1/(c**5*d**3)) +
x)/16 + (x**3*(3*a*d**2 + b*c*d) + x*(5*a*c*d - b*c**2))/(8*c**4*d + 16*c*
*3*d**2*x**2 + 8*c**2*d**3*x**4)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{(c+dx^2)^3}\,dx = \frac{(bcd+3\,ad^2)x^3-(bc^2-5\,acd)x}{8\,(c^2d^3x^4+2\,c^3d^2x^2+c^4d)} + \frac{(bc+3\,ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((b*c*d + 3*a*d^2)*x^3 - (b*c^2 - 5*a*c*d)*x)/(c^2*d^3*x^4 + 2*c^3*d^2
*x^2 + c^4*d) + 1/8*(b*c + 3*a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d)
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{(c+dx^2)^3}\,dx = \frac{(bc+3\,ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d} + \frac{bcdx^3+3\,ad^2x^3-bc^2x+5\,acdx}{8\,(dx^2+c)^2c^2d}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/8*(b*c + 3*a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d) + 1/8*(b*c*d*x^3
 + 3*a*d^2*x^3 - b*c^2*x + 5*a*c*d*x)/((d*x^2 + c)^2*c^2*d)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{(c+dx^2)^3}\,dx = \frac{\frac{x^3\,(3\,a\,d+b\,c)}{8\,c^2}+\frac{x\,(5\,a\,d-b\,c)}{8\,c\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,(3\,a\,d+b\,c)}{8\,c^{5/2}\,d^{3/2}}$$

input
```
int((a + b*x^2)/(c + d*x^2)^3,x)
```

output
```
((x^3*(3*a*d + b*c))/(8*c^2) + (x*(5*a*d - b*c))/(8*c*d))/(c^2 + d^2*x^4 +
 2*c*d*x^2) + (atan((d^(1/2)*x)/c^(1/2))*(3*a*d + b*c))/(8*c^(5/2)*d^(3/2)
)
```

3.8 $\int (a+bx^2)^2 (c+dx^2)^3 \, dx$

3.8.1 Optimal result

Integrand size = 19, antiderivative size = 122

$$\int (a+bx^2)^2 (c+dx^2)^3 \, dx = a^2c^3x + \frac{1}{3}ac^2(2bc+3ad)x^3 + \frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5 + \frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7 + \frac{1}{9}bd^2(3bc+2ad)x^9 + \frac{1}{11}b^2d^3x^{11}$$

output
```
a^2*c^3*x+1/3*a*c^2*(3*a*d+2*b*c)*x^3+1/5*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*
x^5+1/7*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^7+1/9*b*d^2*(2*a*d+3*b*c)*x^9+1/
11*b^2*d^3*x^11
```

3.8.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (c+dx^2)^3 \, dx = a^2c^3x + \frac{1}{3}ac^2(2bc+3ad)x^3 + \frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5 + \frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7 + \frac{1}{9}bd^2(3bc+2ad)x^9 + \frac{1}{11}b^2d^3x^{11}$$

input
```
Integrate[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*
c + 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

3.8.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$$

$$\Big\downarrow \; 290$$

$$\int \left(dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3+ac^2x^2(3ad+2bc)+bd^2x^8(2ad+3bc)+b^2d$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

input
```
Int[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*
c + 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

3.8.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.8.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

method	result
norman	$\frac{b^2d^3x^{11}}{11} + \left(\frac{2}{9}ab\,d^3 + \frac{1}{3}b^2c\,d^2\right)x^9 + \left(\frac{1}{7}a^2d^3 + \frac{6}{7}abc\,d^2 + \frac{3}{7}b^2c^2d\right)x^7 + \left(\frac{3}{5}c\,a^2d^2 + \frac{6}{5}ab\,c^2d + \frac{1}{5}b^2c$
default	$\frac{b^2d^3x^{11}}{11} + \frac{(2ab\,d^3+3b^2c\,d^2)x^9}{9} + \frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^7}{7} + \frac{(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^5}{5} + \frac{(3a^2c^2d+2ab\,c^3)x^3}{3} + c$
gosper	$\frac{1}{11}b^2d^3x^{11} + \frac{2}{9}x^9ab\,d^3 + \frac{1}{3}x^9b^2c\,d^2 + \frac{1}{7}x^7a^2d^3 + \frac{6}{7}x^7abc\,d^2 + \frac{3}{7}x^7b^2c^2d + \frac{3}{5}x^5c\,a^2d^2 + \frac{6}{5}x^5ab\,c^2$
risch	$\frac{1}{11}b^2d^3x^{11} + \frac{2}{9}x^9ab\,d^3 + \frac{1}{3}x^9b^2c\,d^2 + \frac{1}{7}x^7a^2d^3 + \frac{6}{7}x^7abc\,d^2 + \frac{3}{7}x^7b^2c^2d + \frac{3}{5}x^5c\,a^2d^2 + \frac{6}{5}x^5ab\,c^2$
parallelrisch	$\frac{1}{11}b^2d^3x^{11} + \frac{2}{9}x^9ab\,d^3 + \frac{1}{3}x^9b^2c\,d^2 + \frac{1}{7}x^7a^2d^3 + \frac{6}{7}x^7abc\,d^2 + \frac{3}{7}x^7b^2c^2d + \frac{3}{5}x^5c\,a^2d^2 + \frac{6}{5}x^5ab\,c^2$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/11*b^2*d^3*x^11+(2/9*a*b*d^3+1/3*b^2*c*d^2)*x^9+(1/7*a^2*d^3+6/7*a*b*c*d
^2+3/7*b^2*c^2*d)*x^7+(3/5*c*a^2*d^2+6/5*a*b*c^2*d+1/5*b^2*c^3)*x^5+(a^2*c
^2*d+2/3*a*b*c^3)*x^3+a^2*c^3*x
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx = {} & \frac{1}{11}b^2d^3x^{11} + \frac{1}{9}\left(3\,b^2cd^2 + 2\,abd^3\right)x^9 \\ & + \frac{1}{7}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^7 + a^2c^3x \\ & + \frac{1}{5}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^5 + \frac{1}{3}\left(2\,abc^3 + 3\,a^2c^2d\right)x^3\end{aligned}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
 6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a
^2*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx = a^2c^3x + \frac{b^2d^3x^{11}}{11} + x^9 \cdot \left(\frac{2abd^3}{9} + \frac{b^2cd^2}{3}\right) + x^7\left(\frac{a^2d^3}{7} + \frac{6abcd^2}{7} + \frac{3b^2c^2d}{7}\right) + x^5 \cdot \left(\frac{3a^2cd^2}{5} + \frac{6abc^2d}{5} + \frac{b^2c^3}{5}\right) + x^3\left(a^2c^2d + \frac{2abc^3}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

output
```
a**2*c**3*x + b**2*d**3*x**11/11 + x**9*(2*a*b*d**3/9 + b**2*c*d**2/3) + x
**7*(a**2*d**3/7 + 6*a*b*c*d**2/7 + 3*b**2*c**2*d/7) + x**5*(3*a**2*c*d**2
/5 + 6*a*b*c**2*d/5 + b**2*c**3/5) + x**3*(a**2*c**2*d + 2*a*b*c**3/3)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx = \frac{1}{11}\,b^2d^3x^{11} + \frac{1}{9}\left(3\,b^2cd^2 + 2\,abd^3\right)x^9 + \frac{1}{7}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^7 + a^2c^3x + \frac{1}{5}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^5 + \frac{1}{3}\left(2\,abc^3 + 3\,a^2c^2d\right)x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
 6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a
^2*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx = \frac{1}{11}\,b^2d^3x^{11} + \frac{1}{3}\,b^2cd^2x^9 + \frac{2}{9}\,abd^3x^9 + \frac{3}{7}\,b^2c^2dx^7 + \frac{6}{7}\,abcd^2x^7 + \frac{1}{7}\,a^2d^3x^7 + \frac{1}{5}\,b^2c^3x^5 + \frac{6}{5}\,abc^2dx^5 + \frac{3}{5}\,a^2cd^2x^5 + \frac{2}{3}\,abc^3x^3 + a^2c^2dx^3 + a^2c^3x$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/11*b^2*d^3*x^11 + 1/3*b^2*c*d^2*x^9 + 2/9*a*b*d^3*x^9 + 3/7*b^2*c^2*d*x^
7 + 6/7*a*b*c*d^2*x^7 + 1/7*a^2*d^3*x^7 + 1/5*b^2*c^3*x^5 + 6/5*a*b*c^2*d*
x^5 + 3/5*a^2*c*d^2*x^5 + 2/3*a*b*c^3*x^3 + a^2*c^2*d*x^3 + a^2*c^3*x
```

3.8.9 Mupad [B] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx = x^5 \left(\frac{3\,a^2\,c\,d^2}{5} + \frac{6\,a\,b\,c^2\,d}{5} + \frac{b^2\,c^3}{5}\right) + x^7 \left(\frac{a^2\,d^3}{7} + \frac{6\,a\,b\,c\,d^2}{7} + \frac{3\,b^2\,c^2\,d}{7}\right) + a^2\,c^3\,x + \frac{b^2\,d^3\,x^{11}}{11} + \frac{a\,c^2\,x^3\,(3\,a\,d+2\,b\,c)}{3} + \frac{b\,d^2\,x^9\,(2\,a\,d+3\,b\,c)}{9}$$

input
```
int((a + b*x^2)^2*(c + d*x^2)^3,x)
```

output
```
x^5*((b^2*c^3)/5 + (3*a^2*c*d^2)/5 + (6*a*b*c^2*d)/5) + x^7*((a^2*d^3)/7 +
 (3*b^2*c^2*d)/7 + (6*a*b*c*d^2)/7) + a^2*c^3*x + (b^2*d^3*x^11)/11 + (a*c
^2*x^3*(3*a*d + 2*b*c))/3 + (b*d^2*x^9*(2*a*d + 3*b*c))/9
```

3.9 $\int (a+bx^2)^2 (c+dx^2)^2 \, dx$

3.9.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int (a+bx^2)^2 (c+dx^2)^2 \, dx = a^2c^2x + \frac{2}{3}ac(bc+ad)x^3 + \frac{1}{5}(b^2c^2+4abcd+a^2d^2)x^5 + \frac{2}{7}bd(bc+ad)x^7 + \frac{1}{9}b^2d^2x^9$$

output
```
a^2*c^2*x+2/3*a*c*(a*d+b*c)*x^3+1/5*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^5+2/7*b*
d*(a*d+b*c)*x^7+1/9*b^2*d^2*x^9
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (c+dx^2)^2 \, dx = a^2c^2x + \frac{2}{3}ac(bc+ad)x^3 + \frac{1}{5}(b^2c^2+4abcd+a^2d^2)x^5 + \frac{2}{7}bd(bc+ad)x^7 + \frac{1}{9}b^2d^2x^9$$

input
```
Integrate[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x
^5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

3.9.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$$

$$\Big\downarrow \; 290$$

$$\int \left(x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2+2bdx^6(ad+bc)+2acx^2(ad+bc)+b^2d^2x^8\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

input
```
Int[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x
^5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

3.9.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.9.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

method	result
norman	$\frac{b^{2} d^{2} x^{9}}{9}+\left(\frac{2}{7} a b \,d^{2}+\frac{2}{7} b^{2} c d \right) x^{7}+\left(\frac{1}{5} a^{2} d^{2}+\frac{4}{5} a b c d +\frac{1}{5} b^{2} c^{2}\right) x^{5}+\left(\frac{2}{3} a^{2} c d +\frac{2}{3} b \,c^{2} a \right) x^{3}+a^{2} c^{2} x$
default	$\frac{b^{2} d^{2} x^{9}}{9}+\frac{\left(2 a b \,d^{2}+2 b^{2} c d \right) x^{7}}{7}+\frac{\left(a^{2} d^{2}+4 a b c d +b^{2} c^{2}\right) x^{5}}{5}+\frac{\left(2 a^{2} c d +2 b \,c^{2} a \right) x^{3}}{3}+a^{2} c^{2} x$
gosper	$\frac{1}{9} b^{2} d^{2} x^{9}+\frac{2}{7} x^{7} a b \,d^{2}+\frac{2}{7} x^{7} b^{2} c d +\frac{1}{5} x^{5} a^{2} d^{2}+\frac{4}{5} x^{5} a b c d +\frac{1}{5} x^{5} b^{2} c^{2}+\frac{2}{3} x^{3} a^{2} c d +\frac{2}{3} x^{3} b \,c^{2} a +a^{2} c^{2}$
risch	$\frac{1}{9} b^{2} d^{2} x^{9}+\frac{2}{7} x^{7} a b \,d^{2}+\frac{2}{7} x^{7} b^{2} c d +\frac{1}{5} x^{5} a^{2} d^{2}+\frac{4}{5} x^{5} a b c d +\frac{1}{5} x^{5} b^{2} c^{2}+\frac{2}{3} x^{3} a^{2} c d +\frac{2}{3} x^{3} b \,c^{2} a +a^{2} c^{2}$
parallelrisch	$\frac{1}{9} b^{2} d^{2} x^{9}+\frac{2}{7} x^{7} a b \,d^{2}+\frac{2}{7} x^{7} b^{2} c d +\frac{1}{5} x^{5} a^{2} d^{2}+\frac{4}{5} x^{5} a b c d +\frac{1}{5} x^{5} b^{2} c^{2}+\frac{2}{3} x^{3} a^{2} c d +\frac{2}{3} x^{3} b \,c^{2} a +a^{2} c^{2}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^2*d^2*x^9+(2/7*a*b*d^2+2/7*b^2*c*d)*x^7+(1/5*a^2*d^2+4/5*a*b*c*d+1/5
*b^2*c^2)*x^5+(2/3*a^2*c*d+2/3*b*c^2*a)*x^3+a^2*c^2*x
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = \frac{1}{9} \, b^{2} d^{2} x^{9} + \frac{2}{7} \, {\left(b^{2} c d + a b d^{2}\right)} x^{7} + \frac{1}{5} \, {\left(b^{2} c^{2} + 4 \, a b c d + a^{2} d^{2}\right)} x^{5} + a^{2} c^{2} x + \frac{2}{3} \, {\left(a b c^{2} + a^{2} c d\right)} x^{3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = a^2c^2x + \frac{b^2d^2x^9}{9} + x^7 \cdot \left(\frac{2abd^2}{7} + \frac{2b^2cd}{7}\right) + x^5\left(\frac{a^2d^2}{5} + \frac{4abcd}{5} + \frac{b^2c^2}{5}\right) + x^3 \cdot \left(\frac{2a^2cd}{3} + \frac{2abc^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

output
```
a**2*c**2*x + b**2*d**2*x**9/9 + x**7*(2*a*b*d**2/7 + 2*b**2*c*d/7) + x**5
*(a**2*d**2/5 + 4*a*b*c*d/5 + b**2*c**2/5) + x**3*(2*a**2*c*d/3 + 2*a*b*c*
*2/3)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = \frac{1}{9}\, b^2d^2x^9 + \frac{2}{7}\left(b^2cd + abd^2\right)x^7 + \frac{1}{5}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^5 + a^2c^2x + \frac{2}{3}\left(abc^2 + a^2cd\right)x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = \frac{1}{9}\, b^2d^2x^9 + \frac{2}{7}\, b^2cdx^7 + \frac{2}{7}\, abd^2x^7 + \frac{1}{5}\, b^2c^2x^5 + \frac{4}{5}\, abcdx^5 + \frac{1}{5}\, a^2d^2x^5 + \frac{2}{3}\, abc^2x^3 + \frac{2}{3}\, a^2cdx^3 + a^2c^2x$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/9*b^2*d^2*x^9 + 2/7*b^2*c*d*x^7 + 2/7*a*b*d^2*x^7 + 1/5*b^2*c^2*x^5 + 4/
5*a*b*c*d*x^5 + 1/5*a^2*d^2*x^5 + 2/3*a*b*c^2*x^3 + 2/3*a^2*c*d*x^3 + a^2*
c^2*x
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = x^5 \left(\frac{a^2\,d^2}{5}+\frac{4\,a\,b\,c\,d}{5}+\frac{b^2\,c^2}{5}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^9}{9} + \frac{2\,a\,c\,x^3\,\left(a\,d+b\,c\right)}{3}+\frac{2\,b\,d\,x^7\,\left(a\,d+b\,c\right)}{7}$$

input
```
int((a + b*x^2)^2*(c + d*x^2)^2,x)
```

output
```
x^5*((a^2*d^2)/5 + (b^2*c^2)/5 + (4*a*b*c*d)/5) + a^2*c^2*x + (b^2*d^2*x^9
)/9 + (2*a*c*x^3*(a*d + b*c))/3 + (2*b*d*x^7*(a*d + b*c))/7
```

3.10 $\int (a+bx^2)^2 (c+dx^2)\, dx$

3.10.1 Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^2)^2 (c+dx^2)\, dx = a^2cx + \frac{1}{3}a(2bc+ad)x^3 + \frac{1}{5}b(bc+2ad)x^5 + \frac{1}{7}b^2dx^7$$

output
```
a^2*c*x+1/3*a*(a*d+2*b*c)*x^3+1/5*b*(2*a*d+b*c)*x^5+1/7*b^2*d*x^7
```

3.10.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (c+dx^2)\, dx = a^2cx + \frac{1}{3}a(2bc+ad)x^3 + \frac{1}{5}b(bc+2ad)x^5 + \frac{1}{7}b^2dx^7$$

input
```
Integrate[(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/
7
```

3.10.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^2 (c+dx^2) \, dx$$

$$\Big\downarrow \text{ 290}$$

$$\int \left(a^2c+bx^4(2ad+bc)+ax^2(ad+2bc)+b^2dx^6\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

input
```
Int[(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/
7
```

3.10.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.10.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 d\,x^7}{7}+\frac{(2abd+b^2c)x^5}{5}+\frac{(a^2d+2abc)x^3}{3}+a^2cx$	49
norman	$\frac{b^2 d\,x^7}{7}+\left(\frac{2}{5}abd+\frac{1}{5}b^2c\right)x^5+\left(\frac{1}{3}a^2d+\frac{2}{3}abc\right)x^3+a^2cx$	49
gosper	$\frac{1}{7}b^2d\,x^7+\frac{2}{5}x^5abd+\frac{1}{5}x^5b^2c+\frac{1}{3}x^3a^2d+\frac{2}{3}abc\,x^3+a^2cx$	51
risch	$\frac{1}{7}b^2d\,x^7+\frac{2}{5}x^5abd+\frac{1}{5}x^5b^2c+\frac{1}{3}x^3a^2d+\frac{2}{3}abc\,x^3+a^2cx$	51
parallelrisch	$\frac{1}{7}b^2d\,x^7+\frac{2}{5}x^5abd+\frac{1}{5}x^5b^2c+\frac{1}{3}x^3a^2d+\frac{2}{3}abc\,x^3+a^2cx$	51

input
```
int((b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/7*b^2*d*x^7+1/5*(2*a*b*d+b^2*c)*x^5+1/3*(a^2*d+2*a*b*c)*x^3+a^2*c*x
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^2\left(c+dx^2\right)\,dx=\frac{1}{7}\,b^2dx^7+\frac{1}{5}\,\left(b^2c+2\,abd\right)x^5+a^2cx+\frac{1}{3}\,\left(2\,abc+a^2d\right)x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

output
```
1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d
)*x^3
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)^2\left(c+dx^2\right)\,dx=a^2cx+\frac{b^2dx^7}{7}+x^5\cdot\left(\frac{2abd}{5}+\frac{b^2c}{5}\right)+x^3\left(\frac{a^2d}{3}+\frac{2abc}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c),x)
```

output
```
a**2*c*x + b**2*d*x**7/7 + x**5*(2*a*b*d/5 + b**2*c/5) + x**3*(a**2*d/3 +
2*a*b*c/3)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\, dx = \frac{1}{7}\, b^2 d x^7 + \frac{1}{5}\left(b^2 c + 2\, abd\right) x^5 + a^2 c x + \frac{1}{3}\left(2\, abc + a^2 d\right) x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

output
```
1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d
)*x^3
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\, dx = \frac{1}{7}\, b^2 d x^7 + \frac{1}{5}\, b^2 c x^5 + \frac{2}{5}\, abd x^5 + \frac{2}{3}\, abc x^3 + \frac{1}{3}\, a^2 d x^3 + a^2 c x$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

output
```
1/7*b^2*d*x^7 + 1/5*b^2*c*x^5 + 2/5*a*b*d*x^5 + 2/3*a*b*c*x^3 + 1/3*a^2*d*
x^3 + a^2*c*x
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\, dx = x^3 \left(\frac{d\, a^2}{3} + \frac{2\, b\, c\, a}{3}\right) + x^5 \left(\frac{c\, b^2}{5} + \frac{2\, a\, d\, b}{5}\right) + \frac{b^2\, d\, x^7}{7} + a^2\, c\, x$$

input
```
int((a + b*x^2)^2*(c + d*x^2),x)
```

output
```
x^3*((a^2*d)/3 + (2*a*b*c)/3) + x^5*((b^2*c)/5 + (2*a*b*d)/5) + (b^2*d*x^7
)/7 + a^2*c*x
```

3.11 $\int \frac{(a+bx^2)^2}{c+dx^2}\,dx$

3.11.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^3}{3d}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}$$

output
```
-b*(-2*a*d+b*c)*x/d^2+1/3*b^2*x^3/d+(-a*d+b*c)^2*arctan(x*d^(1/2)/c^(1/2))
/d^(5/2)/c^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{bx(-3bc+6ad+bdx^2)}{3d^2}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/(c + d*x^2),x]
```

output
```
(b*x*(-3*b*c + 6*a*d + b*d*x^2))/(3*d^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

3.11.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx$$

$$\Big\downarrow \ 300$$

$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{d^2(c+dx^2)}-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^2}{d}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^3}{3d}$$

input
```
Int[(a + b*x^2)^2/(c + d*x^2),x]
```

output
```
-((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt
[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

3.11.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.11.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result
default	$\frac{b\left(\frac{1}{3}bd\,x^{3}+2adx-bcx\right)}{d^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d^{2}\sqrt{cd}}$
risch	$\frac{b^{2}x^{3}}{3d}+\frac{2bax}{d}-\frac{b^{2}cx}{d^{2}}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^{2}}{2\sqrt{-cd}}+\frac{\ln\left(dx+\sqrt{-cd}\right)abc}{d\sqrt{-cd}}-\frac{\ln\left(dx+\sqrt{-cd}\right)b^{2}c^{2}}{2d^{2}\sqrt{-cd}}+\frac{\ln\left(-dx+\sqrt{-cd}\right)a^{2}}{2\sqrt{-cd}}-\frac{\ln\left(-dx+\sqrt{-\right.}}{d\sqrt{-c}}$

input
```
int((b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
b/d^2*(1/3*b*d*x^3+2*a*d*x-b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2/(c*d)^(1
/2)*arctan(d*x/(c*d)^(1/2))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx$$
$$= \left[\frac{2\,b^2cd^2x^3 - 3\,(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{-cd}\log\left(\frac{dx^2-2\,\sqrt{-cd}x-c}{dx^2+c}\right) - 6\,(b^2c^2d - 2\,abcd^2)x}{6\,cd^3}, \frac{b^2cd^2x^3 + 3\,(b^2c^2}{}\right.$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/6*(2*b^2*c*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*log((
d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(b^2*c^2*d - 2*a*b*c*d^2)*x)/
(c*d^3), 1/3*(b^2*c*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*
arctan(sqrt(c*d)*x/c) - 3*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c*d^3)]
```

3.11.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(56) = 112$.

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.73

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2x^3}{3d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right) - \frac{\sqrt{-\frac{1}{cd^5}}(ad-bc)^2\log\left(-\frac{cd^2\sqrt{-\frac{1}{cd^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^5}}(ad-bc)^2\log\left(\frac{cd^2\sqrt{-\frac{1}{cd^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}$$

input

```
integrate((b*x**2+a)**2/(d*x**2+c),x)
```

output

```
b**2*x**3/(3*d) + x*(2*a*b/d - b**2*c/d**2) - sqrt(-1/(c*d**5))*(a*d - b*c
)**2*log(-c*d**2*sqrt(-1/(c*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d +
 b**2*c**2) + x)/2 + sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(c*d**2*sqrt(-1/(
c*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{(b^2c^2 - 2\,abcd + a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^2} + \frac{b^2dx^3 - 3\,(b^2c - 2\,abd)x}{3\,d^2}$$

input

```
integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output

```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/
3*(b^2*d*x^3 - 3*(b^2*c - 2*a*b*d)*x)/d^2
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^2}+\frac{b^2d^2x^3-3\,b^2cdx+6\,abd^2x}{3\,d^3}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/
3*(b^2*d^2*x^3 - 3*b^2*c*d*x + 6*a*b*d^2*x)/d^3
```

3.11.9 Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2\,x^3}{3\,d}-x\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)^2}{\sqrt{c}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)^2}{\sqrt{c}\,d^{5/2}}$$

input
```
int((a + b*x^2)^2/(c + d*x^2),x)
```

output
```
(b^2*x^3)/(3*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((d^(1/2)*x*(a*d - b*
c)^2)/(c^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(c^(1/2)*d
^(5/2))
```

3.12 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx$

3.12.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{2cd^2(c+dx^2)} - \frac{(bc-ad)(3bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}$$

output
```
b^2*x/d^2+1/2*(-a*d+b*c)^2*x/c/d^2/(d*x^2+c)-1/2*(-a*d+b*c)*(a*d+3*b*c)*ar
ctan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(5/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{2cd^2(c+dx^2)} - \frac{(3b^2c^2-2abcd-a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

output
```
(b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((3*b^2*c^2 - 2*a*
b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

3.12.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}dx$$

$$\Big\downarrow \ 300$$

$$\int \left(\frac{b^2}{d^2}-\frac{-a^2d^2+2bdx^2(bc-ad)+b^2c^2}{d^2\left(c+dx^2\right)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{(bc-ad)(ad+3bc)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x}{d^2}$$

input
```
Int[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

output
```
(b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(3*b*
c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

3.12.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.12.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

method	result
default	$\frac{b^2x}{d^2}+\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)x}{2c\left(d\,x^2+c\right)}+\frac{\left(a^2d^2+2abcd-3b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}}{d^2}$
risch	$\frac{b^2x}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{2c\,d^2(d\,x^2+c)}-\frac{\ln(dx+\sqrt{-cd})a^2}{4\sqrt{-cd}\,c}-\frac{\ln(dx+\sqrt{-cd})ab}{2d\sqrt{-cd}}+\frac{3c\ln(dx+\sqrt{-cd})b^2}{4d^2\sqrt{-cd}}+\frac{\ln(-dx+\sqrt{-cd})a^2}{4\sqrt{-cd}\,c}+\frac{\ln(-dx}{2}$

input
```
int((b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*x/d^2+1/d^2*(1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/(d*x^2+c)+1/2*(a^2*d^
2+2*a*b*c*d-3*b^2*c^2)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.12.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(70) = 140.

Time = 0.26 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.68

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx$$
$$= \left[\frac{4\,b^2c^2d^2x^3+(3\,b^2c^3-2\,abc^2d-a^2cd^2+(3\,b^2c^2d-2\,abcd^2-a^2d^3)x^2)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)+2\,(3\,b}{4\,(c^2d^4x^2+c^3d^3)}\right.$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*b^2*c^2*d^2*x^3 + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^
2*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x -
 c)/(d*x^2 + c)) + 2*(3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4
*x^2 + c^3*d^3), 1/2*(2*b^2*c^2*d^2*x^3 - (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c
*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*
d)*x/c) + (3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^2 + c^3*
d^3)]
```

3.12.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(73) = 146$.

Time = 0.40 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.88

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x}{d^2} + \frac{x(a^2d^2-2abcd+b^2c^2)}{2c^2d^2+2cd^3x^2} - \frac{\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)\log\left(-\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)\log\left(\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*d**
3*x**2) - sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(-c**2*d**2*sq
rt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b*
*2*c**2) + x)/4 + sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(c**2*
d**2*sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d
 - 3*b**2*c**2) + x)/4
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{2\,(cd^3x^2+c^2d^2)} + \frac{b^2x}{d^2} - \frac{(3\,b^2c^2-2\,abcd-a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^2 + c^2*d^2) + b^2*x/d^2 -
1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d
^2)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x}{d^2} - \frac{(3\,b^2c^2 - 2\,abcd - a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\,(dx^2+c)cd^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
b^2*x/d^2 - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(s
qrt(c*d)*c*d^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c
*d^2)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2\,x}{d^2} + \frac{x\,(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2)}{2\,c\,(d^3\,x^2 + c\,d^2)} + \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{\sqrt{c}\,(a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2)}\right)\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{2\,c^{3/2}\,d^{5/2}}$$

input
```
int((a + b*x^2)^2/(c + d*x^2)^2,x)
```

output
```
(b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(c*d^2 + d^3*x^2))
+ (atan((d^(1/2)*x*(a*d - b*c)*(a*d + 3*b*c))/(c^(1/2)*(a^2*d^2 - 3*b^2*c^
2 + 2*a*b*c*d)))*(a*d - b*c)*(a*d + 3*b*c))/(2*c^(3/2)*d^(5/2))
```

3.13 $\int \frac{(a+bx^2)^2}{(c+dx^2)^3} dx$

3.13.1 Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3} dx = -\frac{(bc-ad)x(a+bx^2)}{4cd(c+dx^2)^2} + \frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8(c+dx^2)} + \frac{(3b^2c^2+2abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

output
```
-1/4*(-a*d+b*c)*x*(b*x^2+a)/c/d/(d*x^2+c)^2+3/8*(a^2/c^2-b^2/d^2)*x/(d*x^2
+c)+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/
d^(5/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3} dx = -\frac{(bc-ad)x(ad(5c+3dx^2)+bc(3c+5dx^2))}{8c^2d^2(c+dx^2)^2} + \frac{(3b^2c^2+2abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

output
```
-1/8*((b*c - a*d)*x*(a*d*(5*c + 3*d*x^2) + b*c*(3*c + 5*d*x^2)))/(c^2*d^2*
(c + d*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(8*c^(5/2)*d^(5/2))
```

3.13.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {315, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}dx$$

$$\Big\downarrow \text{315}$$

$$\frac{\int \frac{b(3bc+ad)x^2+a(bc+3ad)}{\left(dx^2+c\right)^2}dx}{4cd}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{2}\left(\frac{3a^2d}{c}+2ab+\frac{3b^2c}{d}\right)\int \frac{1}{dx^2+c}dx-\frac{3x\left(\frac{b^2c}{d}-\frac{a^2d}{c}\right)}{2(c+dx^2)}}{4cd}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\left(\frac{3a^2d}{c}+2ab+\frac{3b^2c}{d}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}}-\frac{3x\left(\frac{b^2c}{d}-\frac{a^2d}{c}\right)}{2(c+dx^2)}}{4cd}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

input
```
Int[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

output
```
-1/4*((b*c - a*d)*x*(a + b*x^2))/(c*d*(c + d*x^2)^2) + ((-3*((b^2*c)/d - (
a^2*d)/c)*x)/(2*(c + d*x^2)) + ((2*a*b + (3*b^2*c)/d + (3*a^2*d)/c)*ArcTan
[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]*Sqrt[d]))/(4*c*d)
```

3.13.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.13.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

method	result
default	$\frac{\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x^3}{8c^2d}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x}{8c\,d^2}}{\left(d\,x^2+c\right)^2}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2d^2\sqrt{cd}}$
risch	$\frac{\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x^3}{8c^2d}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x}{8c\,d^2}}{\left(d\,x^2+c\right)^2}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a^2}{16\sqrt{-cd}\,c^2}-\frac{\ln\left(dx+\sqrt{-cd}\right)ab}{8\sqrt{-cd}\,dc}-\frac{3\ln\left(dx+\sqrt{-cd}\right)b^2}{16\sqrt{-cd}\,d^2}+\frac{3\ln(-dx+}{16\sqrt{-}}$

input
```
int((b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c^2/d*x^3+1/8*(5*a^2*d^2-2*a*b*c*d-3*
b^2*c^2)/c/d^2*x)/(d*x^2+c)^2+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)/c^2/d^2/
(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(102) = 204.

Time = 0.28 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.87

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx$$
$$= \left[-\frac{2\,(5\,b^2c^3d^2 - 2\,abc^2d^3 - 3\,a^2cd^4)x^3 + (3\,b^2c^4 + 2\,abc^3d + 3\,a^2c^2d^2 + (3\,b^2c^2d^2 + 2\,abcd^3 + 3\,a^2d^4)x^4 + 2}{16\,(c^3d^5x^4 + 2\,c^4d}\right.$$
$$-\frac{(5\,b^2c^3d^2 - 2\,abc^2d^3 - 3\,a^2cd^4)x^3 - (3\,b^2c^4 + 2\,abc^3d + 3\,a^2c^2d^2 + (3\,b^2c^2d^2 + 2\,abcd^3 + 3\,a^2d^4)x^4 + 2\,(3}{8\,(c^3d^5x^4 + 2\,c^4d^4x^2 +}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(5*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 + (3*b^2*c^4 +
 2*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x
^4 + 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*
x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(3*b^2*c^4*d + 2*a*b*c^3*d^2 -
5*a^2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^3), -1/8*((5*b^2*c^
3*d^2 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 - (3*b^2*c^4 + 2*a*b*c^3*d + 3*a^
2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d
 + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (3*
b^2*c^4*d + 2*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2
 + c^5*d^3)]
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(110) = 220.

Time = 0.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{c^5d^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(-c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{c^5d^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16} + \frac{x^3\cdot(3a^2d^3+2abcd^2-5b^2c^2d)+x(5a^2cd^2-2abc^2d-3b^2c^3)}{8c^4d^2+16c^3d^3x^2+8c^2d^4x^4}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
-sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-c**3*d*
*2*sqrt(-1/(c**5*d**5)) + x)/16 + sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*
b*c*d + 3*b**2*c**2)*log(c**3*d**2*sqrt(-1/(c**5*d**5)) + x)/16 + (x**3*(3
*a**2*d**3 + 2*a*b*c*d**2 - 5*b**2*c**2*d) + x*(5*a**2*c*d**2 - 2*a*b*c**2
*d - 3*b**2*c**3))/(8*c**4*d**2 + 16*c**3*d**3*x**2 + 8*c**2*d**4*x**4)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{(5\,b^2c^2d - 2\,abcd^2 - 3\,a^2d^3)x^3 + (3\,b^2c^3 + 2\,abc^2d - 5\,a^2cd^2)x}{8\,(c^2d^4x^4 + 2\,c^3d^3x^2 + c^4d^2)} + \frac{(3\,b^2c^2 + 2\,abcd + 3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/8*((5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (3*b^2*c^3 + 2*a*b*c^2
*d - 5*a^2*c*d^2)*x)/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) + 1/8*(3*b^2*
c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^2)
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{(3\,b^2c^2 + 2\,abcd + 3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^2} - \frac{5\,b^2c^2dx^3 - 2\,abcd^2x^3 - 3\,a^2d^3x^3 + 3\,b^2c^3x + 2\,abc^2dx - 5\,a^2cd^2x}{8\,(dx^2+c)^2c^2d^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c
^2*d^2) - 1/8*(5*b^2*c^2*d*x^3 - 2*a*b*c*d^2*x^3 - 3*a^2*d^3*x^3 + 3*b^2*c
^3*x + 2*a*b*c^2*d*x - 5*a^2*c*d^2*x)/((d*x^2 + c)^2*c^2*d^2)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2)}{8\,c^{5/2}\,d^{5/2}} - \frac{\frac{x\,(-5\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2)}{8\,c\,d^2}-\frac{x^3\,(3\,a^2\,d^2+2\,a\,b\,c\,d-5\,b^2\,c^2)}{8\,c^2\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}$$

input
```
int((a + b*x^2)^2/(c + d*x^2)^3,x)
```

output
```
(atan((d^(1/2)*x)/c^(1/2))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(8*c^(5/2)
*d^(5/2)) - ((x*(3*b^2*c^2 - 5*a^2*d^2 + 2*a*b*c*d))/(8*c*d^2) - (x^3*(3*a
^2*d^2 - 5*b^2*c^2 + 2*a*b*c*d))/(8*c^2*d))/(c^2 + d^2*x^4 + 2*c*d*x^2)
```

3.14 $\int (a+bx^2)^3 (c+dx^2)^3 \, dx$

3.14.1 Optimal result

Integrand size = 19, antiderivative size = 154

$$\int (a+bx^2)^3 (c+dx^2)^3 \, dx = a^3c^3x + a^2c^2(bc+ad)x^3 + \frac{3}{5}ac(b^2c^2+3abcd+a^2d^2)\,x^5 + \frac{1}{7}(bc+ad)\,(b^2c^2+8abcd+a^2d^2)\,x^7 + \frac{1}{3}bd(b^2c^2+3abcd+a^2d^2)\,x^9 + \frac{3}{11}b^2d^2(bc+ad)x^{11} + \frac{1}{13}b^3d^3x^{13}$$

output
```
a^3*c^3*x+a^2*c^2*(a*d+b*c)*x^3+3/5*a*c*(a^2*d^2+3*a*b*c*d+b^2*c^2)*x^5+1/
7*(a*d+b*c)*(a^2*d^2+8*a*b*c*d+b^2*c^2)*x^7+1/3*b*d*(a^2*d^2+3*a*b*c*d+b^2
*c^2)*x^9+3/11*b^2*d^2*(a*d+b*c)*x^11+1/13*b^3*d^3*x^13
```

3.14.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\int (a+bx^2)^3 (c+dx^2)^3 \, dx = a^3c^3x + a^2c^2(bc+ad)x^3 + \frac{3}{5}ac(b^2c^2+3abcd+a^2d^2)\,x^5 + \frac{1}{7}(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3)\,x^7 + \frac{1}{3}bd(b^2c^2+3abcd+a^2d^2)\,x^9 + \frac{3}{11}b^2d^2(bc+ad)x^{11} + \frac{1}{13}b^3d^3x^{13}$$

input
```
Integrate[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

output
```
a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^
2)*x^5)/5 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^7)/7 +
(b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^2*(b*c + a*d)*x^11)
/11 + (b^3*d^3*x^13)/13
```

3.14.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \, dx$$

$$\Big\downarrow \text{ 290}$$

$$\int \left(a^3c^3+3bdx^8\left(a^2d^2+3abcd+b^2c^2\right)+x^6(ad+bc)\left(a^2d^2+8abcd+b^2c^2\right)+3acx^4\left(a^2d^2+3abcd+b^2c^2\right)+3a^2c$$

$$\Big\downarrow \text{ 2009}$$

$$a^3c^3x+\frac{1}{3}bdx^9\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{7}x^7(ad+bc)\left(a^2d^2+8abcd+b^2c^2\right)+\frac{3}{5}acx^5\left(a^2d^2+3abcd+b^2c^2\right)+a^2c^2x^3(ad+bc)+\frac{3}{11}b^2d^2x^{11}(ad+bc)+\frac{1}{13}b^3d^3x^{13}$$

input
```
Int[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

output
```
a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^
2)*x^5)/5 + ((b*c + a*d)*(b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d*(b^
2*c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^2*(b*c + a*d)*x^11)/11 + (b
^3*d^3*x^13)/13
```

3.14.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.14.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11

method	result
norman	$\frac{b^3d^3x^{13}}{13}+\left(\frac{3}{11}a\,b^2d^3+\frac{3}{11}b^3c\,d^2\right)x^{11}+\left(\frac{1}{3}a^2b\,d^3+a\,b^2c\,d^2+\frac{1}{3}b^3c^2d\right)x^9+\left(\frac{1}{7}a^3d^3+\frac{9}{7}a^2bc\,d^2+\right.$
default	$\frac{b^3d^3x^{13}}{13}+\frac{(3a\,b^2d^3+3b^3c\,d^2)x^{11}}{11}+\frac{(3a^2b\,d^3+9a\,b^2c\,d^2+3b^3c^2d)x^9}{9}+\frac{(a^3d^3+9a^2bc\,d^2+9a\,b^2c^2d+b^3c^3)x^7}{7}+\frac{(3a^3c\,d^2+}{}$
gosper	$\frac{1}{13}b^3d^3x^{13}+\frac{3}{11}x^{11}a\,b^2d^3+\frac{3}{11}x^{11}b^3c\,d^2+\frac{1}{3}x^9a^2b\,d^3+x^9a\,b^2c\,d^2+\frac{1}{3}x^9b^3c^2d+\frac{1}{7}x^7a^3d^3+\frac{9}{7}x^7$
risch	$\frac{1}{13}b^3d^3x^{13}+\frac{3}{11}x^{11}a\,b^2d^3+\frac{3}{11}x^{11}b^3c\,d^2+\frac{1}{3}x^9a^2b\,d^3+x^9a\,b^2c\,d^2+\frac{1}{3}x^9b^3c^2d+\frac{1}{7}x^7a^3d^3+\frac{9}{7}x^7$
parallelrisch	$\frac{1}{13}b^3d^3x^{13}+\frac{3}{11}x^{11}a\,b^2d^3+\frac{3}{11}x^{11}b^3c\,d^2+\frac{1}{3}x^9a^2b\,d^3+x^9a\,b^2c\,d^2+\frac{1}{3}x^9b^3c^2d+\frac{1}{7}x^7a^3d^3+\frac{9}{7}x^7$

input
```
int((b*x^2+a)^3*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^3*d^3*x^13+(3/11*a*b^2*d^3+3/11*b^3*c*d^2)*x^11+(1/3*a^2*b*d^3+a*b^
2*c*d^2+1/3*b^3*c^2*d)*x^9+(1/7*a^3*d^3+9/7*a^2*b*c*d^2+9/7*a*b^2*c^2*d+1/
7*b^3*c^3)*x^7+(3/5*a^3*c*d^2+9/5*a^2*b*c^2*d+3/5*b^2*c^3*a)*x^5+(a^3*c^2*
d+a^2*b*c^3)*x^3+a^3*c^3*x
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08

$$\begin{aligned}\int (a+bx^2)^3 (c+dx^2)^3\, dx &= \frac{1}{13}b^3d^3x^{13}+\frac{3}{11}\left(b^3cd^2+ab^2d^3\right)x^{11}\\ &\quad+\frac{1}{3}\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)x^9\\ &\quad+\frac{1}{7}\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)x^7+a^3c^3x\\ &\quad+\frac{3}{5}\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)x^5+\left(a^2bc^3+a^3c^2d\right)x^3\end{aligned}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/13*b^3*d^3*x^13 + 3/11*(b^3*c*d^2 + a*b^2*d^3)*x^11 + 1/3*(b^3*c^2*d + 3
*a*b^2*c*d^2 + a^2*b*d^3)*x^9 + 1/7*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d
^2 + a^3*d^3)*x^7 + a^3*c^3*x + 3/5*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2
)*x^5 + (a^2*b*c^3 + a^3*c^2*d)*x^3
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.23

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \,dx = a^3c^3x + \frac{b^3d^3x^{13}}{13} + x^{11}\cdot\left(\frac{3ab^2d^3}{11} + \frac{3b^3cd^2}{11}\right) + x^9\left(\frac{a^2bd^3}{3} + ab^2cd^2 + \frac{b^3c^2d}{3}\right) + x^7\left(\frac{a^3d^3}{7} + \frac{9a^2bcd^2}{7} + \frac{9ab^2c^2d}{7} + \frac{b^3c^3}{7}\right) + x^5\cdot\left(\frac{3a^3cd^2}{5} + \frac{9a^2bc^2d}{5} + \frac{3ab^2c^3}{5}\right) + x^3\left(a^3c^2d + a^2bc^3\right)$$

input
```
integrate((b*x**2+a)**3*(d*x**2+c)**3,x)
```

output
```
a**3*c**3*x + b**3*d**3*x**13/13 + x**11*(3*a*b**2*d**3/11 + 3*b**3*c*d**2
/11) + x**9*(a**2*b*d**3/3 + a*b**2*c*d**2 + b**3*c**2*d/3) + x**7*(a**3*d
**3/7 + 9*a**2*b*c*d**2/7 + 9*a*b**2*c**2*d/7 + b**3*c**3/7) + x**5*(3*a**
3*c*d**2/5 + 9*a**2*b*c**2*d/5 + 3*a*b**2*c**3/5) + x**3*(a**3*c**2*d + a*
*2*b*c**3)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \,dx = \frac{1}{13}\,b^3d^3x^{13} + \frac{3}{11}\left(b^3cd^2 + ab^2d^3\right)x^{11} + \frac{1}{3}\left(b^3c^2d + 3\,ab^2cd^2 + a^2bd^3\right)x^9 + \frac{1}{7}\left(b^3c^3 + 9\,ab^2c^2d + 9\,a^2bcd^2 + a^3d^3\right)x^7 + a^3c^3x + \frac{3}{5}\left(ab^2c^3 + 3\,a^2bc^2d + a^3cd^2\right)x^5 + \left(a^2bc^3 + a^3c^2d\right)x^3$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/13*b^3*d^3*x^13 + 3/11*(b^3*c*d^2 + a*b^2*d^3)*x^11 + 1/3*(b^3*c^2*d + 3
*a*b^2*c*d^2 + a^2*b*d^3)*x^9 + 1/7*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d
^2 + a^3*d^3)*x^7 + a^3*c^3*x + 3/5*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2
)*x^5 + (a^2*b*c^3 + a^3*c^2*d)*x^3
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21

$$\int (a+bx^2)^3 (c+dx^2)^3 \, dx = \frac{1}{13}b^3d^3x^{13} + \frac{3}{11}b^3cd^2x^{11} + \frac{3}{11}ab^2d^3x^{11} + \frac{1}{3}b^3c^2dx^9 + ab^2cd^2x^9 + \frac{1}{3}a^2bd^3x^9 + \frac{1}{7}b^3c^3x^7 + \frac{9}{7}ab^2c^2dx^7 + \frac{9}{7}a^2bcd^2x^7 + \frac{1}{7}a^3d^3x^7 + \frac{3}{5}ab^2c^3x^5 + \frac{9}{5}a^2bc^2dx^5 + \frac{3}{5}a^3cd^2x^5 + a^2bc^3x^3 + a^3c^2dx^3 + a^3c^3x$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/13*b^3*d^3*x^13 + 3/11*b^3*c*d^2*x^11 + 3/11*a*b^2*d^3*x^11 + 1/3*b^3*c^
2*d*x^9 + a*b^2*c*d^2*x^9 + 1/3*a^2*b*d^3*x^9 + 1/7*b^3*c^3*x^7 + 9/7*a*b^
2*c^2*d*x^7 + 9/7*a^2*b*c*d^2*x^7 + 1/7*a^3*d^3*x^7 + 3/5*a*b^2*c^3*x^5 +
9/5*a^2*b*c^2*d*x^5 + 3/5*a^3*c*d^2*x^5 + a^2*b*c^3*x^3 + a^3*c^2*d*x^3 +
a^3*c^3*x
```

3.14.9 Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int (a+bx^2)^3 (c+dx^2)^3 \, dx = x^7 \left(\frac{a^3 d^3}{7} + \frac{9 a^2 b c d^2}{7} + \frac{9 a b^2 c^2 d}{7} + \frac{b^3 c^3}{7}\right) + a^3 c^3 x + \frac{b^3 d^3 x^{13}}{13} + \frac{3 a c x^5 (a^2 d^2 + 3 a b c d + b^2 c^2)}{5} + \frac{b d x^9 (a^2 d^2 + 3 a b c d + b^2 c^2)}{3} + a^2 c^2 x^3 (a d + b c) + \frac{3 b^2 d^2 x^{11} (a d + b c)}{11}$$

input
```
int((a + b*x^2)^3*(c + d*x^2)^3,x)
```

output
```
x^7*((a^3*d^3)/7 + (b^3*c^3)/7 + (9*a*b^2*c^2*d)/7 + (9*a^2*b*c*d^2)/7) +
a^3*c^3*x + (b^3*d^3*x^13)/13 + (3*a*c*x^5*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d)
)/5 + (b*d*x^9*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/3 + a^2*c^2*x^3*(a*d + b*c
) + (3*b^2*d^2*x^11*(a*d + b*c))/11
```

3.14. $\int (a+bx^2)^3 (c+dx^2)^3 \, dx$

3.15 $\int (a+bx^2)^3 (c+dx^2)^2 \, dx$

3.15.1 Optimal result

Integrand size = 19, antiderivative size = 122

$$\int (a+bx^2)^3 (c+dx^2)^2 \, dx = a^3c^2x+\frac{1}{3}a^2c(3bc+2ad)x^3+\frac{1}{5}a(3b^2c^2+6abcd+a^2d^2)\,x^5 +\frac{1}{7}b(b^2c^2+6abcd+3a^2d^2)\,x^7+\frac{1}{9}b^2d(2bc+3ad)x^9+\frac{1}{11}b^3d^2x^{11}$$

output
```
a^3*c^2*x+1/3*a^2*c*(2*a*d+3*b*c)*x^3+1/5*a*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*
x^5+1/7*b*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^7+1/9*b^2*d*(3*a*d+2*b*c)*x^9+1/
11*b^3*d^2*x^11
```

3.15.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^3 (c+dx^2)^2 \, dx = a^3c^2x+\frac{1}{3}a^2c(3bc+2ad)x^3+\frac{1}{5}a(3b^2c^2+6abcd+a^2d^2)\,x^5 +\frac{1}{7}b(b^2c^2+6abcd+3a^2d^2)\,x^7+\frac{1}{9}b^2d(2bc+3ad)x^9+\frac{1}{11}b^3d^2x^{11}$$

input
```
Integrate[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

output
```
a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (b^2*d*(2*b*
c + 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

3.15.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^2 \, dx$$

$$\Big\downarrow \text{ 290}$$

$$\int \left(a^3c^2+bx^6\left(3a^2d^2+6abcd+b^2c^2\right)+ax^4\left(a^2d^2+6abcd+3b^2c^2\right)+a^2cx^2(2ad+3bc)+b^2dx^8(3ad+2bc)+b^3d$$

$$\Big\downarrow \text{ 2009}$$

$$a^3c^2x+\frac{1}{7}bx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}ax^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}a^2cx^3(2ad+3bc)+\frac{1}{9}b^2dx^9(3ad+2bc)+\frac{1}{11}b^3d^2x^{11}$$

input
```
Int[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

output
```
a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (b^2*d*(2*b*
c + 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

3.15.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.15.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

method	result
norman	$\frac{b^3 d^2 x^{11}}{11}+\left(\frac{1}{3} a\, b^2 d^2+\frac{2}{9} b^3 dc\right) x^9+\left(\frac{3}{7} a^2 b\, d^2+\frac{6}{7} a\, b^2 cd+\frac{1}{7} c^2 b^3\right) x^7+\left(\frac{1}{5} a^3 d^2+\frac{6}{5} a^2 bcd+\frac{3}{5} a\, b^2 c$
default	$\frac{b^3 d^2 x^{11}}{11}+\frac{(3 a\, b^2 d^2+2 b^3 dc) x^9}{9}+\frac{(3 a^2 b\, d^2+6 a\, b^2 cd+c^2 b^3) x^7}{7}+\frac{(a^3 d^2+6 a^2 bcd+3 a\, b^2 c^2) x^5}{5}+\frac{(2 a^3 cd+3 a^2 b\, c^2) x^3}{3}+a$
gosper	$\frac{1}{11} b^3 d^2 x^{11}+\frac{1}{3} x^9 a\, b^2 d^2+\frac{2}{9} x^9 b^3 dc+\frac{3}{7} x^7 a^2 b\, d^2+\frac{6}{7} x^7 a\, b^2 cd+\frac{1}{7} b^3 c^2 x^7+\frac{1}{5} x^5 a^3 d^2+\frac{6}{5} x^5 a^2 bcd$
risch	$\frac{1}{11} b^3 d^2 x^{11}+\frac{1}{3} x^9 a\, b^2 d^2+\frac{2}{9} x^9 b^3 dc+\frac{3}{7} x^7 a^2 b\, d^2+\frac{6}{7} x^7 a\, b^2 cd+\frac{1}{7} b^3 c^2 x^7+\frac{1}{5} x^5 a^3 d^2+\frac{6}{5} x^5 a^2 bcd$
parallelrisch	$\frac{1}{11} b^3 d^2 x^{11}+\frac{1}{3} x^9 a\, b^2 d^2+\frac{2}{9} x^9 b^3 dc+\frac{3}{7} x^7 a^2 b\, d^2+\frac{6}{7} x^7 a\, b^2 cd+\frac{1}{7} b^3 c^2 x^7+\frac{1}{5} x^5 a^3 d^2+\frac{6}{5} x^5 a^2 bcd$

input
```
int((b*x^2+a)^3*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*b^3*d^2*x^11+(1/3*a*b^2*d^2+2/9*b^3*d*c)*x^9+(3/7*a^2*b*d^2+6/7*a*b^2
*c*d+1/7*c^2*b^3)*x^7+(1/5*a^3*d^2+6/5*a^2*b*c*d+3/5*a*b^2*c^2)*x^5+(2/3*a
^3*c*d+a^2*b*c^2)*x^3+a^3*c^2*x
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^2\right)^3\left(c+dx^2\right)^2\,dx=\frac{1}{11}b^3d^2x^{11}+\frac{1}{9}\left(2\,b^3cd+3\,ab^2d^2\right)x^9+\frac{1}{7}\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^7+a^3c^2x+\frac{1}{5}\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^5+\frac{1}{3}\left(3\,a^2bc^2+2\,a^3cd\right)x^3$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/11*b^3*d^2*x^11 + 1/9*(2*b^3*c*d + 3*a*b^2*d^2)*x^9 + 1/7*(b^3*c^2 + 6*a
*b^2*c*d + 3*a^2*b*d^2)*x^7 + a^3*c^2*x + 1/5*(3*a*b^2*c^2 + 6*a^2*b*c*d +
 a^3*d^2)*x^5 + 1/3*(3*a^2*b*c^2 + 2*a^3*c*d)*x^3
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^2 \, dx = a^3c^2x + \frac{b^3d^2x^{11}}{11} + x^9\left(\frac{ab^2d^2}{3} + \frac{2b^3cd}{9}\right) + x^7\cdot\left(\frac{3a^2bd^2}{7} + \frac{6ab^2cd}{7} + \frac{b^3c^2}{7}\right) + x^5\left(\frac{a^3d^2}{5} + \frac{6a^2bcd}{5} + \frac{3ab^2c^2}{5}\right) + x^3\cdot\left(\frac{2a^3cd}{3} + a^2bc^2\right)$$

input
```
integrate((b*x**2+a)**3*(d*x**2+c)**2,x)
```

output
```
a**3*c**2*x + b**3*d**2*x**11/11 + x**9*(a*b**2*d**2/3 + 2*b**3*c*d/9) + x
**7*(3*a**2*b*d**2/7 + 6*a*b**2*c*d/7 + b**3*c**2/7) + x**5*(a**3*d**2/5 +
 6*a**2*b*c*d/5 + 3*a*b**2*c**2/5) + x**3*(2*a**3*c*d/3 + a**2*b*c**2)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^2 \, dx = \frac{1}{11}\,b^3d^2x^{11} + \frac{1}{9}\left(2\,b^3cd + 3\,ab^2d^2\right)x^9 + \frac{1}{7}\left(b^3c^2 + 6\,ab^2cd + 3\,a^2bd^2\right)x^7 + a^3c^2x + \frac{1}{5}\left(3\,ab^2c^2 + 6\,a^2bcd + a^3d^2\right)x^5 + \frac{1}{3}\left(3\,a^2bc^2 + 2\,a^3cd\right)x^3$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/11*b^3*d^2*x^11 + 1/9*(2*b^3*c*d + 3*a*b^2*d^2)*x^9 + 1/7*(b^3*c^2 + 6*a
*b^2*c*d + 3*a^2*b*d^2)*x^7 + a^3*c^2*x + 1/5*(3*a*b^2*c^2 + 6*a^2*b*c*d +
 a^3*d^2)*x^5 + 1/3*(3*a^2*b*c^2 + 2*a^3*c*d)*x^3
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int (a+bx^2)^3 (c+dx^2)^2 \, dx = \frac{1}{11} b^3 d^2 x^{11} + \frac{2}{9} b^3 c d x^9 + \frac{1}{3} a b^2 d^2 x^9 + \frac{1}{7} b^3 c^2 x^7 + \frac{6}{7} a b^2 c d x^7 + \frac{3}{7} a^2 b d^2 x^7 + \frac{3}{5} a b^2 c^2 x^5 + \frac{6}{5} a^2 b c d x^5 + \frac{1}{5} a^3 d^2 x^5 + a^2 b c^2 x^3 + \frac{2}{3} a^3 c d x^3 + a^3 c^2 x$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/11*b^3*d^2*x^11 + 2/9*b^3*c*d*x^9 + 1/3*a*b^2*d^2*x^9 + 1/7*b^3*c^2*x^7
+ 6/7*a*b^2*c*d*x^7 + 3/7*a^2*b*d^2*x^7 + 3/5*a*b^2*c^2*x^5 + 6/5*a^2*b*c*
d*x^5 + 1/5*a^3*d^2*x^5 + a^2*b*c^2*x^3 + 2/3*a^3*c*d*x^3 + a^3*c^2*x
```

3.15.9 Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int (a+bx^2)^3 (c+dx^2)^2 \, dx = x^5 \left(\frac{a^3 d^2}{5} + \frac{6 a^2 b c d}{5} + \frac{3 a b^2 c^2}{5}\right) + x^7 \left(\frac{3 a^2 b d^2}{7} + \frac{6 a b^2 c d}{7} + \frac{b^3 c^2}{7}\right) + a^3 c^2 x + \frac{b^3 d^2 x^{11}}{11} + \frac{a^2 c x^3 (2 a d + 3 b c)}{3} + \frac{b^2 d x^9 (3 a d + 2 b c)}{9}$$

input
```
int((a + b*x^2)^3*(c + d*x^2)^2,x)
```

output
```
x^5*((a^3*d^2)/5 + (3*a*b^2*c^2)/5 + (6*a^2*b*c*d)/5) + x^7*((b^3*c^2)/7 +
 (3*a^2*b*d^2)/7 + (6*a*b^2*c*d)/7) + a^3*c^2*x + (b^3*d^2*x^11)/11 + (a^2
*c*x^3*(2*a*d + 3*b*c))/3 + (b^2*d*x^9*(3*a*d + 2*b*c))/9
```

3.16 $\int (a+bx^2)^3 (c+dx^2)\, dx$

3.16.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^2)^3 (c+dx^2)\, dx = a^3cx+\frac{1}{3}a^2(3bc+ad)x^3+\frac{3}{5}ab(bc+ad)x^5+\frac{1}{7}b^2(bc+3ad)x^7+\frac{1}{9}b^3dx^9$$

output
```
a^3*c*x+1/3*a^2*(a*d+3*b*c)*x^3+3/5*a*b*(a*d+b*c)*x^5+1/7*b^2*(3*a*d+b*c)*
x^7+1/9*b^3*d*x^9
```

3.16.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^3 (c+dx^2)\, dx = a^3cx+\frac{1}{3}a^2(3bc+ad)x^3+\frac{3}{5}ab(bc+ad)x^5+\frac{1}{7}b^2(bc+3ad)x^7+\frac{1}{9}b^3dx^9$$

input
```
Integrate[(a + b*x^2)^3*(c + d*x^2),x]
```

output
```
a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 + (b^2*(b*
c + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

3.16.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right) \, dx$$

$$\Big\downarrow \text{290}$$

$$\int \left(a^3c+a^2x^2(ad+3bc)+b^2x^6(3ad+bc)+3abx^4(ad+bc)+b^3dx^8\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^3cx+\frac{1}{3}a^2x^3(ad+3bc)+\frac{1}{7}b^2x^7(3ad+bc)+\frac{3}{5}abx^5(ad+bc)+\frac{1}{9}b^3dx^9$$

input
```
Int[(a + b*x^2)^3*(c + d*x^2),x]
```

output
```
a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 + (b^2*(b*
c + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

3.16.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.16.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result	size
norman	$\frac{b^3 d x^9}{9}+\left(\frac{3}{7} a b^2 d+\frac{1}{7} b^3 c\right) x^7+\left(\frac{3}{5} a^2 b d+\frac{3}{5} a b^2 c\right) x^5+\left(\frac{1}{3} a^3 d+a^2 b c\right) x^3+a^3 c x$	71
default	$\frac{b^3 d x^9}{9}+\frac{(3 a b^2 d+b^3 c) x^7}{7}+\frac{(3 a^2 b d+3 a b^2 c) x^5}{5}+\frac{(a^3 d+3 a^2 b c) x^3}{3}+a^3 c x$	73
gosper	$\frac{1}{9} b^3 d x^9+\frac{3}{7} x^7 a b^2 d+\frac{1}{7} x^7 b^3 c+\frac{3}{5} x^5 a^2 b d+\frac{3}{5} a b^2 c x^5+\frac{1}{3} x^3 a^3 d+x^3 a^2 b c+a^3 c x$	74
risch	$\frac{1}{9} b^3 d x^9+\frac{3}{7} x^7 a b^2 d+\frac{1}{7} x^7 b^3 c+\frac{3}{5} x^5 a^2 b d+\frac{3}{5} a b^2 c x^5+\frac{1}{3} x^3 a^3 d+x^3 a^2 b c+a^3 c x$	74
parallelrisch	$\frac{1}{9} b^3 d x^9+\frac{3}{7} x^7 a b^2 d+\frac{1}{7} x^7 b^3 c+\frac{3}{5} x^5 a^2 b d+\frac{3}{5} a b^2 c x^5+\frac{1}{3} x^3 a^3 d+x^3 a^2 b c+a^3 c x$	74

input
```
int((b*x^2+a)^3*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/9*b^3*d*x^9+(3/7*a*b^2*d+1/7*b^3*c)*x^7+(3/5*a^2*b*d+3/5*a*b^2*c)*x^5+(1
/3*a^3*d+a^2*b*c)*x^3+a^3*c*x
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)\,dx=\frac{1}{9}\,b^3dx^9+\frac{1}{7}\,(b^3c+3\,ab^2d)x^7+\frac{3}{5}\,(ab^2c+a^2bd)x^5+a^3cx+\frac{1}{3}\,(3\,a^2bc+a^3d)x^3$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="fricas")
```

output
```
1/9*b^3*d*x^9 + 1/7*(b^3*c + 3*a*b^2*d)*x^7 + 3/5*(a*b^2*c + a^2*b*d)*x^5
+ a^3*c*x + 1/3*(3*a^2*b*c + a^3*d)*x^3
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)\,dx = a^3cx + \frac{b^3dx^9}{9} + x^7\cdot\left(\frac{3ab^2d}{7}+\frac{b^3c}{7}\right) + x^5\cdot\left(\frac{3a^2bd}{5}+\frac{3ab^2c}{5}\right) + x^3\left(\frac{a^3d}{3}+a^2bc\right)$$

input
```
integrate((b*x**2+a)**3*(d*x**2+c),x)
```

output
```
a**3*c*x + b**3*d*x**9/9 + x**7*(3*a*b**2*d/7 + b**3*c/7) + x**5*(3*a**2*b
*d/5 + 3*a*b**2*c/5) + x**3*(a**3*d/3 + a**2*b*c)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)\,dx = \frac{1}{9}\,b^3dx^9 + \frac{1}{7}\,\left(b^3c+3\,ab^2d\right)x^7 + \frac{3}{5}\,\left(ab^2c+a^2bd\right)x^5 + a^3cx + \frac{1}{3}\,\left(3\,a^2bc+a^3d\right)x^3$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="maxima")
```

output
```
1/9*b^3*d*x^9 + 1/7*(b^3*c + 3*a*b^2*d)*x^7 + 3/5*(a*b^2*c + a^2*b*d)*x^5
+ a^3*c*x + 1/3*(3*a^2*b*c + a^3*d)*x^3
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)\,dx = \frac{1}{9}\,b^3dx^9 + \frac{1}{7}\,b^3cx^7 + \frac{3}{7}\,ab^2dx^7 + \frac{3}{5}\,ab^2cx^5 + \frac{3}{5}\,a^2bdx^5 + a^2bcx^3 + \frac{1}{3}\,a^3dx^3 + a^3cx$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="giac")
```

output
```
1/9*b^3*d*x^9 + 1/7*b^3*c*x^7 + 3/7*a*b^2*d*x^7 + 3/5*a*b^2*c*x^5 + 3/5*a^
2*b*d*x^5 + a^2*b*c*x^3 + 1/3*a^3*d*x^3 + a^3*c*x
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int (a+bx^2)^3 (c+dx^2) \, dx = x^7 \left(\frac{c\,b^3}{7}+\frac{3\,a\,d\,b^2}{7}\right)+x^3 \left(\frac{d\,a^3}{3}+b\,c\,a^2\right) + \frac{b^3\,d\,x^9}{9}+a^3\,c\,x+\frac{3\,a\,b\,x^5\,(a\,d+b\,c)}{5}$$

input
```
int((a + b*x^2)^3*(c + d*x^2),x)
```

output
```
x^7*((b^3*c)/7 + (3*a*b^2*d)/7) + x^3*((a^3*d)/3 + a^2*b*c) + (b^3*d*x^9)/
9 + a^3*c*x + (3*a*b*x^5*(a*d + b*c))/5
```

3.17 $\int \frac{(a+bx^2)^3}{c+dx^2}\,dx$

3.17.1 Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = \frac{b(b^2c^2-3abcd+3a^2d^2)\,x}{d^3} - \frac{b^2(bc-3ad)x^3}{3d^2} + \frac{b^3x^5}{5d} - \frac{(bc-ad)^3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}$$

output
```
b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)*x/d^3-1/3*b^2*(-3*a*d+b*c)*x^3/d^2+1/5*b^3
*x^5/d-(-a*d+b*c)^3*arctan(x*d^(1/2)/c^(1/2))/d^(7/2)/c^(1/2)
```

3.17.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = \frac{bx(45a^2d^2+15abd(-3c+dx^2)+b^2(15c^2-5cdx^2+3d^2x^4))}{15d^3} - \frac{(bc-ad)^3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}$$

input
```
Integrate[(a + b*x^2)^3/(c + d*x^2),x]
```

output
```
(b*x*(45*a^2*d^2 + 15*a*b*d*(-3*c + d*x^2) + b^2*(15*c^2 - 5*c*d*x^2 + 3*d
^2*x^4)))/(15*d^3) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*
d^(7/2))
```

3.17.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^3}{c+dx^2}dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{b\left(3a^2d^2-3abcd+b^2c^2\right)}{d^3}+\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{d^3\left(c+dx^2\right)}-\frac{b^2x^2(bc-3ad)}{d^2}+\frac{b^3x^4}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx\left(3a^2d^2-3abcd+b^2c^2\right)}{d^3}-\frac{(bc-ad)^3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}-\frac{b^2x^3(bc-3ad)}{3d^2}+\frac{b^3x^5}{5d}$$

input
```
Int[(a + b*x^2)^3/(c + d*x^2),x]
```

output
```
(b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x)/d^3 - (b^2*(b*c - 3*a*d)*x^3)/(3*d
^2) + (b^3*x^5)/(5*d) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[
c]*d^(7/2))
```

3.17.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.17.4 Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.18

method	result
default	$\frac{b\left(\frac{1}{5}b^2d^2x^5+x^3ab\,d^2-\frac{1}{3}x^3b^2cd+3a^2d^2x-3abcdx+b^2c^2x\right)}{d^3}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d^3\sqrt{cd}}$
risch	$\frac{b^3x^5}{5d}+\frac{b^2x^3a}{d}-\frac{b^3x^3c}{3d^2}+\frac{3b\,a^2x}{d}-\frac{3b^2acx}{d^2}+\frac{b^3c^2x}{d^3}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^3}{2\sqrt{-cd}}+\frac{3\ln\left(dx+\sqrt{-cd}\right)a^2bc}{2d\sqrt{-cd}}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a\,b^2c^2}{2d^2\sqrt{-cd}}$

input
```
int((b*x^2+a)^3/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
b/d^3*(1/5*b^2*d^2*x^5+x^3*a*b*d^2-1/3*x^3*b^2*c*d+3*a^2*d^2*x-3*a*b*c*d*x
+b^2*c^2*x)+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^3/(c*d)^(1/2)*
arctan(d*x/(c*d)^(1/2))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.96

$$\int \frac{\left(a+bx^2\right)^3}{c+dx^2}\,dx = \left[\frac{6\,b^3cd^3x^5-10\,(b^3c^2d^2-3\,ab^2cd^3)x^3+15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)}{30\,cd^4}\right.$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/30*(6*b^3*c*d^3*x^5 - 10*(b^3*c^2*d^2 - 3*a*b^2*c*d^3)*x^3 + 15*(b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d)*log((d*x^2 - 2*sqr
t(-c*d)*x - c)/(d*x^2 + c)) + 30*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*
d^3)*x)/(c*d^4), 1/15*(3*b^3*c*d^3*x^5 - 5*(b^3*c^2*d^2 - 3*a*b^2*c*d^3)*x
^3 - 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)*arct
an(sqrt(c*d)*x/c) + 15*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3)*x)/(c
*d^4)]
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(92) = 184$.

Time = 0.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.43

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = \frac{b^3x^5}{5d} + x^3\left(\frac{ab^2}{d} - \frac{b^3c}{3d^2}\right) + x\left(\frac{3a^2b}{d} - \frac{3ab^2c}{d^2} + \frac{b^3c^2}{d^3}\right) - \frac{\sqrt{-\frac{1}{cd^7}}(ad-bc)^3\log\left(-\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3} + x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^7}}(ad-bc)^3\log\left(\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3} + x\right)}{2}$$

input
```
integrate((b*x**2+a)**3/(d*x**2+c),x)
```

output
```
b**3*x**5/(5*d) + x**3*(a*b**2/d - b**3*c/(3*d**2)) + x*(3*a**2*b/d - 3*a*
b**2*c/d**2 + b**3*c**2/d**3) - sqrt(-1/(c*d**7))*(a*d - b*c)**3*log(-c*d*
*3*sqrt(-1/(c*d**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**
2*c**2*d - b**3*c**3) + x)/2 + sqrt(-1/(c*d**7))*(a*d - b*c)**3*log(c*d**3
*sqrt(-1/(c*d**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*
c**2*d - b**3*c**3) + x)/2
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = -\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^3} + \frac{3\,b^3d^2x^5-5\,(b^3cd-3\,ab^2d^2)x^3+15\,(b^3c^2-3\,ab^2cd+3\,a^2bd^2)x}{15\,d^3}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

output
```
-(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))
/(sqrt(c*d)*d^3) + 1/15*(3*b^3*d^2*x^5 - 5*(b^3*c*d - 3*a*b^2*d^2)*x^3 + 1
5*(b^3*c^2 - 3*a*b^2*c*d + 3*a^2*b*d^2)*x)/d^3
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = -\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^3} + \frac{3\,b^3d^4x^5-5\,b^3cd^3x^3+15\,ab^2d^4x^3+15\,b^3c^2d^2x-45\,ab^2cd^3x+45\,a^2bd^4x}{15\,d^5}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

output
```
-(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))
/(sqrt(c*d)*d^3) + 1/15*(3*b^3*d^4*x^5 - 5*b^3*c*d^3*x^3 + 15*a*b^2*d^4*x^
3 + 15*b^3*c^2*d^2*x - 45*a*b^2*c*d^3*x + 45*a^2*b*d^4*x)/d^5
```

3.17.9 Mupad [B] (verification not implemented)

Time = 4.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = x^3\left(\frac{a\,b^2}{d}-\frac{b^3\,c}{3\,d^2}\right)+x\left(\frac{3\,a^2\,b}{d}-\frac{c\left(\frac{3\,a\,b^2}{d}-\frac{b^3\,c}{d^2}\right)}{d}\right)+\frac{b^3\,x^5}{5\,d}+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)^3}{\sqrt{c}\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)\,(a\,d-b\,c)^3}{\sqrt{c}\,d^{7/2}}$$

input
```
int((a + b*x^2)^3/(c + d*x^2),x)
```

output
```
x^3*((a*b^2)/d - (b^3*c)/(3*d^2)) + x*((3*a^2*b)/d - (c*((3*a*b^2)/d - (b^
3*c)/d^2))/d) + (b^3*x^5)/(5*d) + (atan((d^(1/2)*x*(a*d - b*c)^3)/(c^(1/2)
*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)^3)/(c^(
1/2)*d^(7/2))
```

3.18 $\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx$

3.18.1 Optimal result

Integrand size = 19, antiderivative size = 107

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx = -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3(c+dx^2)} + \frac{(bc-ad)^2(5bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}$$

output
```
-b^2*(-3*a*d+2*b*c)*x/d^3+1/3*b^3*x^3/d^2-1/2*(-a*d+b*c)^3*x/c/d^3/(d*x^2+
c)+1/2*(-a*d+b*c)^2*(a*d+5*b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(7/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx = -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3(c+dx^2)} + \frac{(bc-ad)^2(5bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}$$

input
```
Integrate[(a + b*x^2)^3/(c + d*x^2)^2,x]
```

output
```
-((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)^3*x)/(2*
c*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(3/2)*d^(7/2))
```

3.18.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{3bdx^2(bc-ad)^2+(ad+2bc)(bc-ad)^2}{d^3(c+dx^2)^2}+\frac{b^3x^2}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ad+5bc)(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}-\frac{b^2x(2bc-3ad)}{d^3}-\frac{x(bc-ad)^3}{2cd^3(c+dx^2)}+\frac{b^3x^3}{3d^2}$$

input
```
Int[(a + b*x^2)^3/(c + d*x^2)^2,x]
```

output

```
-((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)^3*x)/(2*
c*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(3/2)*d^(7/2))
```

3.18.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.18.4 Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

method	result
default	$\frac{b^2\left(\frac{1}{3}bd\,x^3+3adx-2bcx\right)}{d^3}+\frac{\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{2c\left(d\,x^2+c\right)}+\frac{\left(a^3d^3+3a^2bc\,d^2-9a\,b^2c^2d+5b^3c^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}}{d^3}$
risch	$\frac{b^3x^3}{3d^2}+\frac{3b^2ax}{d^2}-\frac{2b^3cx}{d^3}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{2c\,d^3(d\,x^2+c)}-\frac{\ln(dx+\sqrt{-cd})a^3}{4\sqrt{-cd}\,c}-\frac{3\ln(dx+\sqrt{-cd})a^2b}{4d\sqrt{-cd}}+\frac{9c\ln(dx+\sqrt{-cd}}{4d^2\sqrt{-cd}}$

input
```
int((b*x^2+a)^3/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/d^3*(1/3*b*d*x^3+3*a*d*x-2*b*c*x)+1/d^3*(1/2*(a^3*d^3-3*a^2*b*c*d^2+3*
a*b^2*c^2*d-b^3*c^3)/c*x/(d*x^2+c)+1/2*(a^3*d^3+3*a^2*b*c*d^2-9*a*b^2*c^2*
d+5*b^3*c^3)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(93) = 186.

Time = 0.25 (sec) , antiderivative size = 444, normalized size of antiderivative = 4.15

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx$$
$$=\left[\frac{4\,b^3c^2d^3x^5-4\,(5\,b^3c^3d^2-9\,ab^2c^2d^3)x^3-3\,(5\,b^3c^4-9\,ab^2c^3d+3\,a^2bc^2d^2+a^3cd^3+(5\,b^3c^3d-9\,ab^2c^2d^2}{12\,(c^2d^5x^2+c}\right.$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*b^3*c^2*d^3*x^5 - 4*(5*b^3*c^3*d^2 - 9*a*b^2*c^2*d^3)*x^3 - 3*(5*
b^3*c^4 - 9*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 + a^3*c*d^3 + (5*b^3*c^3*d - 9*a
*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + a^3*d^4)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqr
t(-c*d)*x - c)/(d*x^2 + c)) - 6*(5*b^3*c^4*d - 9*a*b^2*c^3*d^2 + 3*a^2*b*c
^2*d^3 - a^3*c*d^4)*x)/(c^2*d^5*x^2 + c^3*d^4), 1/6*(2*b^3*c^2*d^3*x^5 - 2
*(5*b^3*c^3*d^2 - 9*a*b^2*c^2*d^3)*x^3 + 3*(5*b^3*c^4 - 9*a*b^2*c^3*d + 3*
a^2*b*c^2*d^2 + a^3*c*d^3 + (5*b^3*c^3*d - 9*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3
 + a^3*d^4)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - 3*(5*b^3*c^4*d - 9*a*b^
2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x)/(c^2*d^5*x^2 + c^3*d^4)]
```

3.18.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(95) = 190$.

Time = 0.64 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.93

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx = \frac{b^3x^3}{3d^2} + x\left(\frac{3ab^2}{d^2} - \frac{2b^3c}{d^3}\right) + \frac{x(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3)}{2c^2d^3 + 2cd^4x^2} - \frac{\sqrt{-\frac{1}{c^3d^7}}(ad-bc)^2(ad+5bc)\log\left(-\frac{c^2d^3\sqrt{-\frac{1}{c^3d^7}}(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3} + x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^7}}(ad-bc)^2(ad+5bc)\log\left(\frac{c^2d^3\sqrt{-\frac{1}{c^3d^7}}(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3} + x\right)}{4}$$

input
```
integrate((b*x**2+a)**3/(d*x**2+c)**2,x)
```

output
```
b**3*x**3/(3*d**2) + x*(3*a*b**2/d**2 - 2*b**3*c/d**3) + x*(a**3*d**3 - 3*
a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2*c**2*d**3 + 2*c*d**4*x**2)
 - sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)*log(-c**2*d**3*sqrt(-
1/(c**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 -
 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/4 + sqrt(-1/(c**3*d**7))*(a*d - b*c)*
*2*(a*d + 5*b*c)*log(c**2*d**3*sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a*d +
5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/
4
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx = -\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{2\,(cd^4x^2+c^2d^3)} + \frac{b^3dx^3-3\,(2\,b^3c-3\,ab^2d)x}{3\,d^3} + \frac{(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^3}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(c*d^4*x^2 + c^
2*d^3) + 1/3*(b^3*d*x^3 - 3*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 1/2*(5*b^3*c^3
- 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d
)*c*d^3)
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx = \frac{(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^3} - \frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\,(dx^2+c)cd^3} + \frac{b^3d^4x^3-6\,b^3cd^3x+9\,ab^2d^4x}{3\,d^6}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(d*x/sqrt(
c*d))/(sqrt(c*d)*c*d^3) - 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2
*x - a^3*d^3*x)/((d*x^2 + c)*c*d^3) + 1/3*(b^3*d^4*x^3 - 6*b^3*c*d^3*x + 9
*a*b^2*d^4*x)/d^6
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx = x\left(\frac{3\,a\,b^2}{d^2}-\frac{2\,b^3\,c}{d^3}\right)+\frac{b^3\,x^3}{3\,d^2}+\frac{x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{2\,c\,(d^4\,x^2+c\,d^3)} + \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)^2\,(a\,d+5\,b\,c)}{\sqrt{c}\,(a^3\,d^3+3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+5\,b^3\,c^3)}\right)\,(a\,d-b\,c)^2\,(a\,d+5\,b\,c)}{2\,c^{3/2}\,d^{7/2}}$$

input
```
int((a + b*x^2)^3/(c + d*x^2)^2,x)
```

output
```
x*((3*a*b^2)/d^2 - (2*b^3*c)/d^3) + (b^3*x^3)/(3*d^2) + (x*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*c*(c*d^3 + d^4*x^2)) + (atan((d^(
1/2)*x*(a*d - b*c)^2*(a*d + 5*b*c))/(c^(1/2)*(a^3*d^3 + 5*b^3*c^3 - 9*a*b^
2*c^2*d + 3*a^2*b*c*d^2)))*(a*d - b*c)^2*(a*d + 5*b*c))/(2*c^(3/2)*d^(7/2)
)
```

3.19 $\int \frac{(a+bx^2)^3}{(c+dx^2)^3} dx$

3.19.1 Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3} dx = \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3(c+dx^2)^2} + \frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3(c+dx^2)} - \frac{3(bc-ad)(4b^2c^2+(bc+ad)^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}$$

output
```
b^3*x/d^3-1/4*(-a*d+b*c)^3*x/c/d^3/(d*x^2+c)^2+3/8*(-a*d+b*c)^2*(a*d+3*b*c
)*x/c^2/d^3/(d*x^2+c)-3/8*(-a*d+b*c)*(4*b^2*c^2+(a*d+b*c)^2)*arctan(x*d^(1
/2)/c^(1/2))/c^(5/2)/d^(7/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3} dx = \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3(c+dx^2)^2} + \frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3(c+dx^2)} - \frac{3(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}$$

input
```
Integrate[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

output
```
(b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c - a*d)^2
*(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(5*b^3*c^3 - 3*a*b^2*c^2*d
- a^2*b*c*d^2 - a^3*d^3)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(7/2))
```

3.19.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3} \, dx$$

$$\Big\downarrow \ 300$$

$$\int \left(\frac{b^3}{d^3} - \frac{-a^3d^3 + 3b^2d^2x^4(bc-ad) + 3bdx^2(bc-ad)(ad+bc) + b^3c^3}{d^3\,(c+dx^2)^3}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3(bc-ad)\left((ad+bc)^2+4b^2c^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}} + \frac{3x(bc-ad)^2(ad+3bc)}{8c^2d^3\,(c+dx^2)} - \frac{x(bc-ad)^3}{4cd^3\,(c+dx^2)^2} + \frac{b^3x}{d^3}$$

input
```
Int[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

output
```
(b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c - a*d)^2
*(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(b*c - a*d)*(4*b^2*c^2 + (b
*c + a*d)^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(7/2))
```

3.19.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.19.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.28

method	result
default	$\frac{b^3 x}{d^3}+\frac{\frac{\frac{3d\left(a^3d^3+a^2bc\,d^2-5a\,b^2c^2d+3b^3c^3\right)x^3}{8c^2}+\frac{\left(5a^3d^3-3a^2bc\,d^2-9a\,b^2c^2d+7b^3c^3\right)x}{8c}}{\left(d\,x^2+c\right)^2}+\frac{3\left(a^3d^3+a^2bc\,d^2+3a\,b^2c^2d-5b^3c^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2\sqrt{cd}}}{d^3}$
risch	$\frac{b^3 x}{d^3}+\frac{\frac{3d\left(a^3d^3+a^2bc\,d^2-5a\,b^2c^2d+3b^3c^3\right)x^3}{8c^2}+\frac{\left(5a^3d^3-3a^2bc\,d^2-9a\,b^2c^2d+7b^3c^3\right)x}{8c}}{d^3\left(d\,x^2+c\right)^2}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a^3}{16\sqrt{-cd}\,c^2}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a^2b}{16d\sqrt{-cd}\,c}-$

input
```
int((b*x^2+a)^3/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
b^3*x/d^3+1/d^3*((3/8*d*(a^3*d^3+a^2*b*c*d^2-5*a*b^2*c^2*d+3*b^3*c^3)/c^2*
x^3+1/8*(5*a^3*d^3-3*a^2*b*c*d^2-9*a*b^2*c^2*d+7*b^3*c^3)/c*x)/(d*x^2+c)^2
+3/8*(a^3*d^3+a^2*b*c*d^2+3*a*b^2*c^2*d-5*b^3*c^3)/c^2/(c*d)^(1/2)*arctan(
d*x/(c*d)^(1/2)))
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(116) = 232.

Time = 0.26 (sec) , antiderivative size = 618, normalized size of antiderivative = 4.75

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3}\,dx$$
$$= \left[\frac{16\,b^3c^3d^3x^5+2\,(25\,b^3c^4d^2-15\,ab^2c^3d^3+3\,a^2bc^2d^4+3\,a^3cd^5)x^3+3\,(5\,b^3c^5-3\,ab^2c^4d-a^2bc^3d^2-a^3c^2d\right.$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*b^3*c^3*d^3*x^5 + 2*(25*b^3*c^4*d^2 - 15*a*b^2*c^3*d^3 + 3*a^2*b
*c^2*d^4 + 3*a^3*c*d^5)*x^3 + 3*(5*b^3*c^5 - 3*a*b^2*c^4*d - a^2*b*c^3*d^2
 - a^3*c^2*d^3 + (5*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 - a^2*b*c*d^4 - a^3*d^5)
*x^4 + 2*(5*b^3*c^4*d - 3*a*b^2*c^3*d^2 - a^2*b*c^2*d^3 - a^3*c*d^4)*x^2)*
sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(15*b^3*c^5*d
 - 9*a*b^2*c^4*d^2 - 3*a^2*b*c^3*d^3 + 5*a^3*c^2*d^4)*x)/(c^3*d^6*x^4 + 2*
c^4*d^5*x^2 + c^5*d^4), 1/8*(8*b^3*c^3*d^3*x^5 + (25*b^3*c^4*d^2 - 15*a*b^
2*c^3*d^3 + 3*a^2*b*c^2*d^4 + 3*a^3*c*d^5)*x^3 - 3*(5*b^3*c^5 - 3*a*b^2*c^
4*d - a^2*b*c^3*d^2 - a^3*c^2*d^3 + (5*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 - a^2
*b*c*d^4 - a^3*d^5)*x^4 + 2*(5*b^3*c^4*d - 3*a*b^2*c^3*d^2 - a^2*b*c^2*d^3
 - a^3*c*d^4)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (15*b^3*c^5*d - 9*a*b
^2*c^4*d^2 - 3*a^2*b*c^3*d^3 + 5*a^3*c^2*d^4)*x)/(c^3*d^6*x^4 + 2*c^4*d^5*
x^2 + c^5*d^4)]
```

3.19.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(122) = 244$.

Time = 1.03 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.25

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3}\,dx = \frac{b^3x}{d^3} - \frac{3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)\left(a^2d^2+2abcd+5b^2c^2\right)\log\left(-\frac{3c^3d^3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)(a^2d^2+2abcd+5b^2c^2)}{3a^3d^3+3a^2bcd^2+9ab^2c^2d-15b^3c^3}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)\left(a^2d^2+2abcd+5b^2c^2\right)\log\left(\frac{3c^3d^3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)(a^2d^2+2abcd+5b^2c^2)}{3a^3d^3+3a^2bcd^2+9ab^2c^2d-15b^3c^3}+x\right)}{16} + \frac{x^3\cdot\left(3a^3d^4+3a^2bcd^3-15ab^2c^2d^2+9b^3c^3d\right)+x\left(5a^3cd^3-3a^2bc^2d^2-9ab^2c^3d+7b^3c^4\right)}{8c^4d^3+16c^3d^4x^2+8c^2d^5x^4}$$

input
```
integrate((b*x**2+a)**3/(d*x**2+c)**3,x)
```

output
```
b**3*x/d**3 - 3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 + 2*a*b*c*d +
5*b**2*c**2)*log(-3*c**3*d**3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2
+ 2*a*b*c*d + 5*b**2*c**2)/(3*a**3*d**3 + 3*a**2*b*c*d**2 + 9*a*b**2*c**2*
d - 15*b**3*c**3) + x)/16 + 3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2
+ 2*a*b*c*d + 5*b**2*c**2)*log(3*c**3*d**3*sqrt(-1/(c**5*d**7))*(a*d - b*c
)*(a**2*d**2 + 2*a*b*c*d + 5*b**2*c**2)/(3*a**3*d**3 + 3*a**2*b*c*d**2 + 9
*a*b**2*c**2*d - 15*b**3*c**3) + x)/16 + (x**3*(3*a**3*d**4 + 3*a**2*b*c*d
**3 - 15*a*b**2*c**2*d**2 + 9*b**3*c**3*d) + x*(5*a**3*c*d**3 - 3*a**2*b*c
**2*d**2 - 9*a*b**2*c**3*d + 7*b**3*c**4))/(8*c**4*d**3 + 16*c**3*d**4*x**
2 + 8*c**2*d**5*x**4)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3}\,dx$$
$$= \frac{b^3x}{d^3} + \frac{3\,(3\,b^3c^3d-5\,ab^2c^2d^2+a^2bcd^3+a^3d^4)x^3+(7\,b^3c^4-9\,ab^2c^3d-3\,a^2bc^2d^2+5\,a^3cd^3)x}{8\,(c^2d^5x^4+2\,c^3d^4x^2+c^4d^3)} - \frac{3\,(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^3}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
b^3*x/d^3 + 1/8*(3*(3*b^3*c^3*d - 5*a*b^2*c^2*d^2 + a^2*b*c*d^3 + a^3*d^4)
*x^3 + (7*b^3*c^4 - 9*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x)/(c^2
*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3) - 3/8*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2
*b*c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^3)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3}\,dx = \frac{b^3x}{d^3} - \frac{3\left(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^3} + \frac{9\,b^3c^3dx^3-15\,ab^2c^2d^2x^3+3\,a^2bcd^3x^3+3\,a^3d^4x^3+7\,b^3c^4x-9\,ab^2c^3dx-3\,a^2bc^2d^2x+5\,a^3cd^3x}{8\,(dx^2+c)^2c^2d^3}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
b^3*x/d^3 - 3/8*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*arctan
(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^3) + 1/8*(9*b^3*c^3*d*x^3 - 15*a*b^2*c^2*
d^2*x^3 + 3*a^2*b*c*d^3*x^3 + 3*a^3*d^4*x^3 + 7*b^3*c^4*x - 9*a*b^2*c^3*d*
x - 3*a^2*b*c^2*d^2*x + 5*a^3*c*d^3*x)/((d*x^2 + c)^2*c^2*d^3)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3}\,dx = \frac{\frac{x\,\left(5\,a^3\,d^3-3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+7\,b^3\,c^3\right)}{8\,c}+\frac{3\,x^3\,\left(a^3\,d^4+a^2\,b\,c\,d^3-5\,a\,b^2\,c^2\,d^2+3\,b^3\,c^3\,d\right)}{8\,c^2}}{c^2\,d^3+2\,c\,d^4\,x^2+d^5\,x^4} + \frac{b^3\,x}{d^3} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{d}\,x\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2+2\,a\,b\,c\,d+5\,b^2\,c^2\right)}{\sqrt{c}\,\left(a^3\,d^3+a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-5\,b^3\,c^3\right)}\right)\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2+2\,a\,b\,c\,d+5\,b^2\,c^2\right)}{8\,c^{5/2}\,d^{7/2}}$$

input
```
int((a + b*x^2)^3/(c + d*x^2)^3,x)
```

output
```
((x*(5*a^3*d^3 + 7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(8*c) + (3*x^
3*(a^3*d^4 + 3*b^3*c^3*d - 5*a*b^2*c^2*d^2 + a^2*b*c*d^3))/(8*c^2))/(c^2*d
^3 + d^5*x^4 + 2*c*d^4*x^2) + (b^3*x)/d^3 + (3*atan((d^(1/2)*x*(a*d - b*c)
*(a^2*d^2 + 5*b^2*c^2 + 2*a*b*c*d))/(c^(1/2)*(a^3*d^3 - 5*b^3*c^3 + 3*a*b^
2*c^2*d + a^2*b*c*d^2)))*(a*d - b*c)*(a^2*d^2 + 5*b^2*c^2 + 2*a*b*c*d))/(8
*c^(5/2)*d^(7/2))
```

3.20 $\int \frac{(c+dx^2)^4}{a+bx^2}\,dx$

3.20.1 Optimal result

Integrand size = 19, antiderivative size = 142

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx = \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{3b^3} + \frac{d^3(4bc-ad)x^5}{5b^2}+\frac{d^4x^7}{7b}+\frac{(bc-ad)^4\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{9/2}}$$

output
```
d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/3*d^2*(a^2*d^2-4*a*b*
c*d+6*b^2*c^2)*x^3/b^3+1/5*d^3*(-a*d+4*b*c)*x^5/b^2+1/7*d^4*x^7/b+(-a*d+b*
c)^4*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)/a^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx = \frac{dx\left(-105a^3d^3+35a^2bd^2(12c+dx^2)-7ab^2d(90c^2+20cdx^2+3d^2x^4)+3b^3(140c^3+70c^2dx^2+28cd^2x^4+\right.}{105b^4} + \frac{(bc-ad)^4\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{9/2}}$$

input
```
Integrate[(c + d*x^2)^4/(a + b*x^2),x]
```

output
```
(d*x*(-105*a^3*d^3 + 35*a^2*b*d^2*(12*c + d*x^2) - 7*a*b^2*d*(90*c^2 + 20*
c*d*x^2 + 3*d^2*x^4) + 3*b^3*(140*c^3 + 70*c^2*d*x^2 + 28*c*d^2*x^4 + 5*d^
3*x^6)))/(105*b^4) + ((b*c - a*d)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*
b^(9/2))
```

3.20.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx$$

$$\Big\downarrow \ 300$$

$$\int \left(\frac{d(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^2\left(a^2d^2-4abcd+6b^2c^2\right)}{b^3}+\frac{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d}{b^4\left(a+bx^2\right)}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^3\left(a^2d^2-4abcd+6b^2c^2\right)}{3b^3}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^4}{\sqrt{a}b^{9/2}}+\frac{d^3x^5(4bc-ad)}{5b^2}+\frac{d^4x^7}{7b}$$

input
```
Int[(c + d*x^2)^4/(a + b*x^2),x]
```

output
```
(d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^
2 - 4*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) + (d^3*(4*b*c - a*d)*x^5)/(5*b^2) +
(d^4*x^7)/(7*b) + ((b*c - a*d)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(
9/2))
```

3.20.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.20.4 Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.38

method	result
default	$-\frac{d\left(-\frac{b^3d^3x^7}{7}+\frac{\left((ad-2bc)b^2d^2-2b^3c\,d^2\right)x^5}{5}+\frac{\left(2(ad-2bc)b^2cd-bd\left(a^2d^2-2abcd+2b^2c^2\right)\right)x^3}{3}+(ad-2bc)\left(a^2d^2-2abcd+2b^2c^2\right)x\right)}{b^4}+\frac{(a^4a$
risch	$\frac{d^4x^7}{7b}-\frac{d^4x^5a}{5b^2}+\frac{4d^3x^5c}{5b}-\frac{4d^3ac\,x^3}{3b^2}+\frac{2d^2c^2x^3}{b}+\frac{d^4a^2x^3}{3b^3}-\frac{d^4a^3x}{b^4}+\frac{4d^3a^2cx}{b^3}-\frac{6d^2a\,c^2x}{b^2}+\frac{4d\,c^3x}{b}-\frac{\ln\left(bx+\sqrt{-ab}\right.}{2b^4\sqrt{-a}}$

input
```
int((d*x^2+c)^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-d/b^4*(-1/7*b^3*d^3*x^7+1/5*((a*d-2*b*c)*b^2*d^2-2*b^3*c*d^2)*x^5+1/3*(2*
(a*d-2*b*c)*b^2*c*d-b*d*(a^2*d^2-2*a*b*c*d+2*b^2*c^2))*x^3+(a*d-2*b*c)*(a^
2*d^2-2*a*b*c*d+2*b^2*c^2)*x)+(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 428, normalized size of antiderivative = 3.01

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx$$
$$=\left[\frac{30\,ab^4d^4x^7+42\,(4\,ab^4cd^3-a^2b^3d^4)x^5+70\,(6\,ab^4c^2d^2-4\,a^2b^3cd^3+a^3b^2d^4)x^3-105\,(b^4c^4-4\,ab^3c^3d+6}{21}\right.$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/210*(30*a*b^4*d^4*x^7 + 42*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^5 + 70*(6*a*
b^4*c^2*d^2 - 4*a^2*b^3*c*d^3 + a^3*b^2*d^4)*x^3 - 105*(b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a*b)*log((b*x^2
 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 210*(4*a*b^4*c^3*d - 6*a^2*b^3*c^2*d
^2 + 4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)/(a*b^5), 1/105*(15*a*b^4*d^4*x^7 + 21
*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^5 + 35*(6*a*b^4*c^2*d^2 - 4*a^2*b^3*c*d^3
 + a^3*b^2*d^4)*x^3 + 105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 105*(4*a*b^4*c^3
*d - 6*a^2*b^3*c^2*d^2 + 4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)/(a*b^5)]
```

3.20.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(136) = 272$.

Time = 0.48 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.30

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx = x^5\left(-\frac{ad^4}{5b^2}+\frac{4cd^3}{5b}\right)+x^3\left(\frac{a^2d^4}{3b^3}-\frac{4acd^3}{3b^2}+\frac{2c^2d^2}{b}\right) + x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right) - \frac{\sqrt{-\frac{1}{ab^9}}(ad-bc)^4\log\left(-\frac{ab^4\sqrt{-\frac{1}{ab^9}}(ad-bc)^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab^9}}(ad-bc)^4\log\left(\frac{ab^4\sqrt{-\frac{1}{ab^9}}(ad-bc)^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+x\right)}{2} + \frac{d^4x^7}{7b}$$

input
```
integrate((d*x**2+c)**4/(b*x**2+a),x)
```

output
```
x**5*(-a*d**4/(5*b**2) + 4*c*d**3/(5*b)) + x**3*(a**2*d**4/(3*b**3) - 4*a*
c*d**3/(3*b**2) + 2*c**2*d**2/b) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b**3
 - 6*a*c**2*d**2/b**2 + 4*c**3*d/b) - sqrt(-1/(a*b**9))*(a*d - b*c)**4*log
(-a*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6
*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) + x)/2 + sqrt(-1/(a*b*
*9))*(a*d - b*c)**4*log(a*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**4/(a**4*d**4
 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)
+ x)/2 + d**4*x**7/(7*b)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx = \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^3d^4x^7 + 21\,(4\,b^3cd^3 - ab^2d^4)x^5 + 35\,(6\,b^3c^2d^2 - 4\,ab^2cd^3 + a^2bd^4)x^3 + 105\,(4\,b^3c^3d - 6\,ab^2c^2d^2 + 4\,a}{105\,b^4}$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*ar
ctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^3*d^4*x^7 + 21*(4*b^3*c*
d^3 - a*b^2*d^4)*x^5 + 35*(6*b^3*c^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^3
+ 105*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx = \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^6d^4x^7 + 84\,b^6cd^3x^5 - 21\,ab^5d^4x^5 + 210\,b^6c^2d^2x^3 - 140\,ab^5cd^3x^3 + 35\,a^2b^4d^4x^3 + 420\,b^6c^3dx - 630\,a}{105\,b^7}$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*ar
ctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*d^4*x^7 + 84*b^6*c*d^3
*x^5 - 21*a*b^5*d^4*x^5 + 210*b^6*c^2*d^2*x^3 - 140*a*b^5*c*d^3*x^3 + 35*a
^2*b^4*d^4*x^3 + 420*b^6*c^3*d*x - 630*a*b^5*c^2*d^2*x + 420*a^2*b^4*c*d^3
*x - 105*a^3*b^3*d^4*x)/b^7
```

3.20.9 Mupad [B] (verification not implemented)

Time = 4.75 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.52

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx = x\left(\frac{4\,c^3\,d}{b}-\frac{a\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{b}+\frac{6\,c^2\,d^2}{b}\right)}{b}\right)-x^5\left(\frac{a\,d^4}{5\,b^2}-\frac{4\,c\,d^3}{5\,b}\right)+x^3\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{3\,b}+\frac{2\,c^2\,d^2}{b}\right)+\frac{d^4\,x^7}{7\,b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^4}{\sqrt{a}\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}\right)\,(a\,d-b\,c)^4}{\sqrt{a}\,b^{9/2}}$$

input
```
int((c + d*x^2)^4/(a + b*x^2),x)
```

output
```
x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b
) - x^5*((a*d^4)/(5*b^2) - (4*c*d^3)/(5*b)) + x^3*((a*((a*d^4)/b^2 - (4*c*
d^3)/b))/(3*b) + (2*c^2*d^2)/b) + (d^4*x^7)/(7*b) + (atan((b^(1/2)*x*(a*d
- b*c)^4)/(a^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3)))*(a*d - b*c)^4)/(a^(1/2)*b^(9/2))
```

3.21 $\int \frac{(c+dx^2)^3}{a+bx^2} \, dx$

3.21.1 Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{(c+dx^2)^3}{a+bx^2} \, dx = \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

output
```
d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/3*d^2*(-a*d+3*b*c)*x^3/b^2+1/5*d^3
*x^5/b+(-a*d+b*c)^3*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^2)^3}{a+bx^2} \, dx = \frac{dx(15a^2d^2-5abd(9c+dx^2)+3b^2(15c^2+5cdx^2+d^2x^4))}{15b^3}+\frac{(bc-ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

input
```
Integrate[(c + d*x^2)^3/(a + b*x^2),x]
```

output
```
(d*x*(15*a^2*d^2 - 5*a*b*d*(9*c + d*x^2) + 3*b^2*(15*c^2 + 5*c*d*x^2 + d^2
*x^4)))/(15*b^3) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^
(7/2))
```

3.21.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^3}{a+bx^2}dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{d\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{b^3\left(a+bx^2\right)}+\frac{d^2x^2(3bc-ad)}{b^2}+\frac{d^3x^4}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^3}{\sqrt{a}b^{7/2}}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{d^3x^5}{5b}$$

input
```
Int[(c + d*x^2)^3/(a + b*x^2),x]
```

output
```
(d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^3)/(3*b
^2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
a]*b^(7/2))
```

3.21.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.21.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.18

method	result
default	$\frac{d\left(\frac{1}{5}b^2d^2x^5-\frac{1}{3}x^3ab\,d^2+x^3b^2cd+a^2d^2x-3abcdx+3b^2c^2x\right)}{b^3}+\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{d^3x^5}{5b}-\frac{d^3x^3a}{3b^2}+\frac{d^2x^3c}{b}+\frac{d^3a^2x}{b^3}-\frac{3d^2acx}{b^2}+\frac{3d\,c^2x}{b}-\frac{\ln\left(bx-\sqrt{-ab}\right)a^3d^3}{2b^3\sqrt{-ab}}+\frac{3\ln\left(bx-\sqrt{-ab}\right)a^2c\,d^2}{2b^2\sqrt{-ab}}-\frac{3\ln\left(bx-\sqrt{-ab}\right)a}{2b\sqrt{-ab}}$

input
```
int((d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
d/b^3*(1/5*b^2*d^2*x^5-1/3*x^3*a*b*d^2+x^3*b^2*c*d+a^2*d^2*x-3*a*b*c*d*x+3
*b^2*c^2*x)+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^3/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.98

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx$$
$$=\left[\frac{6\,ab^3d^3x^5+10\,(3\,ab^3cd^2-a^2b^2d^3)x^3+15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{30\,ab^4}\right.$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/30*(6*a*b^3*d^3*x^5 + 10*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 + 15*(b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a*b)*log((b*x^2 + 2*sqr
t(-a*b)*x - a)/(b*x^2 + a)) + 30*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*
d^3)*x)/(a*b^4), 1/15*(3*a*b^3*d^3*x^5 + 5*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x
^3 + 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a) + 15*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x)/(a
*b^4)]
```

3.21.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(92) = 184$.

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.43

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = x^3\left(-\frac{ad^3}{3b^2}+\frac{cd^2}{b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right) + \frac{\sqrt{-\frac{1}{ab^7}}(ad-bc)^3\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^7}}(ad-bc)^3\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x^5}{5b}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a),x)
```

output
```
x**3*(-a*d**3/(3*b**2) + c*d**2/b) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b**2 +
 3*c**2*d/b) + sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(-a*b**3*sqrt(-1/(a*b**
7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c
**3) + x)/2 - sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(a*b**3*sqrt(-1/(a*b**7)
)*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**
3) + x)/2 + d**3*x**5/(5*b)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^2d^3x^5+5\,(3\,b^2cd^2-abd^3)x^3+15\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)x}{15\,b^3}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^3) + 1/15*(3*b^2*d^3*x^5 + 5*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 15
*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4d^3x^5+15\,b^4cd^2x^3-5\,ab^3d^3x^3+45\,b^4c^2dx-45\,ab^3cd^2x+15\,a^2b^2d^3x}{15\,b^5}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^3) + 1/15*(3*b^4*d^3*x^5 + 15*b^4*c*d^2*x^3 - 5*a*b^3*d^3*x^3
 + 45*b^4*c^2*d*x - 45*a*b^3*c*d^2*x + 15*a^2*b^2*d^3*x)/b^5
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = x\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^3\left(\frac{a\,d^3}{3\,b^2}-\frac{c\,d^2}{b}\right)+\frac{d^3\,x^5}{5\,b}-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^3}{\sqrt{a}\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)\,(a\,d-b\,c)^3}{\sqrt{a}\,b^{7/2}}$$

input
```
int((c + d*x^2)^3/(a + b*x^2),x)
```

output
```
x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^3*((a*d^3)/(3*b^2)
 - (c*d^2)/b) + (d^3*x^5)/(5*b) - (atan((b^(1/2)*x*(a*d - b*c)^3)/(a^(1/2)
*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)^3)/(a^(
1/2)*b^(7/2))
```

3.22 $\int \frac{(c+dx^2)^2}{a+bx^2} dx$

3.22.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{(c+dx^2)^2}{a+bx^2} dx = \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^3}{3b} + \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

output
```
d*(-a*d+2*b*c)*x/b^2+1/3*d^2*x^3/b+(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))/
b^(5/2)/a^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^2)^2}{a+bx^2} dx = \frac{dx(6bc-3ad+bdx^2)}{3b^2} + \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

input
```
Integrate[(c + d*x^2)^2/(a + b*x^2),x]
```

output
```
(d*x*(6*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

3.22.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{a+bx^2} dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{b^2(a+bx^2)}+\frac{d(2bc-ad)}{b^2}+\frac{d^2x^2}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^2}{\sqrt{a}b^{5/2}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^3}{3b}$$

input
```
Int[(c + d*x^2)^2/(a + b*x^2),x]
```

output
```
(d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

3.22.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.22.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result
default	$-\frac{d\left(-\frac{1}{3}bd\,x^3+adx-2bcx\right)}{b^2}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
risch	$\frac{d^2x^3}{3b}-\frac{d^2ax}{b^2}+\frac{2dcx}{b}-\frac{\ln\left(bx+\sqrt{-ab}\right)a^2d^2}{2b^2\sqrt{-ab}}+\frac{\ln\left(bx+\sqrt{-ab}\right)acd}{b\sqrt{-ab}}-\frac{\ln\left(bx+\sqrt{-ab}\right)c^2}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)a^2d^2}{2b^2\sqrt{-ab}}-\frac{\ln(-bx+\sqrt{}}{b\sqrt{-}}$

input
```
int((d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-d/b^2*(-1/3*b*d*x^3+a*d*x-2*b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.87

$$\int \frac{\left(c+dx^2\right)^2}{a+bx^2}\,dx$$
$$=\left[\frac{2\,ab^2d^2x^3-3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+6\left(2\,ab^2cd-a^2bd^2\right)x}{6\,ab^3},\ \frac{ab^2d^2x^3+3\left(b^2c^2\right.}{}\right.$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*a*b^2*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*log((
b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(2*a*b^2*c*d - a^2*b*d^2)*x)/
(a*b^3), 1/3*(a*b^2*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a) + 3*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a*b^3)]
```

3.22.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(56) = 112$.

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.73

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right) - \frac{\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2\log\left(-\frac{ab^2\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2\log\left(\frac{ab^2\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{d^2x^3}{3b}$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a),x)
```

output
```
x*(-a*d**2/b**2 + 2*c*d/b) - sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(-a*b**2*
sqrt(-1/(a*b**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/
2 + sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(a*b**2*sqrt(-1/(a*b**5))*(a*d - b
*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x**3/(3*b)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{bd^2x^3+3\,(2\,bcd-ad^2)x}{3\,b^2}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/
3*(b*d^2*x^3 + 3*(2*b*c*d - a*d^2)*x)/b^2
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2d^2x^3+6\,b^2cdx-3\,abd^2x}{3\,b^3}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/
3*(b^2*d^2*x^3 + 6*b^2*c*d*x - 3*a*b*d^2*x)/b^3
```

3.22.9 Mupad [B] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \frac{d^2\,x^3}{3\,b} - x\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^2}{\sqrt{a}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)^2}{\sqrt{a}\,b^{5/2}}$$

input
```
int((c + d*x^2)^2/(a + b*x^2),x)
```

output
```
(d^2*x^3)/(3*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((b^(1/2)*x*(a*d - b*
c)^2)/(a^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(a^(1/2)*b
^(5/2))
```

3.23 $\int \frac{c+dx^2}{a+bx^2} dx$

3.23.1 Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{c+dx^2}{a+bx^2}\,dx=\frac{dx}{b}+\frac{(bc-ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

output
```
d*x/b+(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2}{a+bx^2}\,dx=\frac{dx}{b}-\frac{(-bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

input
```
Integrate[(c + d*x^2)/(a + b*x^2),x]
```

output
```
(d*x)/b - ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

3.23.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{a+bx^2}\,dx$$

$$\Big\downarrow \; 299$$

$$\frac{(bc-ad)\int \frac{1}{bx^2+a}dx}{b}+\frac{dx}{b}$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)}{\sqrt{a}b^{3/2}}+\frac{dx}{b}$$

input
```
Int[(c + d*x^2)/(a + b*x^2),x]
```

output
```
(d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

3.23.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.23.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{dx}{b}+\frac{(-ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	34
risch	$\frac{dx}{b}-\frac{\ln(bx-\sqrt{-ab})ad}{2b\sqrt{-ab}}+\frac{\ln(bx-\sqrt{-ab})c}{2\sqrt{-ab}}+\frac{\ln(-bx-\sqrt{-ab})ad}{2b\sqrt{-ab}}-\frac{\ln(-bx-\sqrt{-ab})c}{2\sqrt{-ab}}$	106

input
```
int((d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
d*x/b+(-a*d+b*c)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.51

$$\int\frac{c+dx^2}{a+bx^2}\,dx = \left[\frac{2\,abdx+\sqrt{-ab}(bc-ad)\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab^2},\frac{abdx+\sqrt{ab}(bc-ad)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab^2}\right]$$

input
```
integrate((d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*a*b*d*x + sqrt(-a*b)*(b*c - a*d)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)))/(a*b^2), (a*b*d*x + sqrt(a*b)*(b*c - a*d)*arctan(sqrt(a*b)*x
/a))/(a*b^2)]
```

3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int\frac{c+dx^2}{a+bx^2}\,dx = \frac{\sqrt{-\frac{1}{ab^3}}(ad-bc)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^3}}(ad-bc)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}+\frac{dx}{b}$$

input
```
integrate((d*x**2+c)/(b*x**2+a),x)
```

output
```
sqrt(-1/(a*b**3))*(a*d - b*c)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/
(a*b**3))*(a*d - b*c)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + d*x/b
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{c + dx^2}{a + bx^2} \, dx = \frac{dx}{b} + \frac{(bc - ad) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{c + dx^2}{a + bx^2} \, dx = \frac{dx}{b} + \frac{(bc - ad) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \frac{d\,x}{b} - \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d-b\,c)}{\sqrt{a}\,b^{3/2}}$$

input
```
int((c + d*x^2)/(a + b*x^2),x)
```

output
```
(d*x)/b - (atan((b^(1/2)*x)/a^(1/2))*(a*d - b*c))/(a^(1/2)*b^(3/2))
```

3.24 $\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx$

3.24.1 Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

output
```
arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)/a^(1/2)-arctan(x*d^(1/2)/c^(1
/2))*d^(1/2)/(-a*d+b*c)/c^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx=\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}}}{bc-ad}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/Sqrt[c])/(b*c - a*d)
```

3.24.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {303, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)}dx$$

$$\Big\downarrow \text{303}$$

$$\frac{b\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{d\int \frac{1}{dx^2+c}dx}{bc-ad}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

3.24.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

3.24.4 Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)\sqrt{ab}}+\frac{d \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)\sqrt{cd}}$	55
risch	$\frac{\sqrt{-cd}\,\ln\left(dx+\sqrt{-cd}\right)}{2c(ad-bc)}-\frac{\sqrt{-cd}\,\ln\left(dx-\sqrt{-cd}\right)}{2c(ad-bc)}+\frac{\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{2a(ad-bc)}-\frac{\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{2a(ad-bc)}$	136

input
```
int(1/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-b/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)/(c*d)^(1/2)*a
rctan(d*x/(c*d)^(1/2))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.17

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\right.$$
$$-\frac{2\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2\,(bc-ad)}, \frac{2\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + sqrt(-
d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2
*sqrt(d/c)*arctan(x*sqrt(d/c)) + sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
- a)/(b*x^2 + a)))/(b*c - a*d), 1/2*(2*sqrt(b/a)*arctan(x*sqrt(b/a)) - sqr
t(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), (sqr
t(b/a)*arctan(x*sqrt(b/a)) - sqrt(d/c)*arctan(x*sqrt(d/c)))/(b*c - a*d)]
```

3.24.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(60) = 120$.

Time = 2.93 (sec) , antiderivative size = 712, normalized size of antiderivative = 10.17

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{\sqrt{-\frac{b}{a}}\log\left(x + \frac{-\frac{a^4cd^3\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{a^3bc^2d^2\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{a^2b^2c^3d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{a^2d^2\sqrt{-\frac{b}{a}}}{ad-bc} - \frac{ab^3c^4\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{b^2c^2\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2(ad-bc)} - \frac{\sqrt{-\frac{b}{a}}\log\left(x + \frac{\frac{a^4cd^3\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{a^3bc^2d^2\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{a^2b^2c^3d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{a^2d^2\sqrt{-\frac{b}{a}}}{ad-bc} + \frac{ab^3c^4\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{b^2c^2\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2(ad-bc)} + \frac{\sqrt{-\frac{d}{c}}\log\left(x + \frac{-\frac{a^4cd^3\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{a^3bc^2d^2\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{a^2b^2c^3d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{a^2d^2\sqrt{-\frac{d}{c}}}{ad-bc} - \frac{ab^3c^4\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{b^2c^2\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2(ad-bc)} - \frac{\sqrt{-\frac{d}{c}}\log\left(x + \frac{\frac{a^4cd^3\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{a^3bc^2d^2\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{a^2b^2c^3d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{a^2d^2\sqrt{-\frac{d}{c}}}{ad-bc} + \frac{ab^3c^4\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{b^2c^2\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2(ad-bc)}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c),x)
```

output
```
sqrt(-b/a)*log(x + (-a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 + a**3*b*c**
2*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d
- b*c)**3 - a**2*d**2*sqrt(-b/a)/(a*d - b*c) - a*b**3*c**4*(-b/a)**(3/2)/(
a*d - b*c)**3 - b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) -
 sqrt(-b/a)*log(x + (a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 - a**3*b*c**
2*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d
- b*c)**3 + a**2*d**2*sqrt(-b/a)/(a*d - b*c) + a*b**3*c**4*(-b/a)**(3/2)/(
a*d - b*c)**3 + b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) +
 sqrt(-d/c)*log(x + (-a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 + a**3*b*c*
*2*d**2*(-d/c)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d
 - b*c)**3 - a**2*d**2*sqrt(-d/c)/(a*d - b*c) - a*b**3*c**4*(-d/c)**(3/2)/
(a*d - b*c)**3 - b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c))
- sqrt(-d/c)*log(x + (a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 - a**3*b*c*
*2*d**2*(-d/c)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d
 - b*c)**3 + a**2*d**2*sqrt(-d/c)/(a*d - b*c) + a*b**3*c**4*(-d/c)**(3/2)/
(a*d - b*c)**3 + b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}(bc-ad)} - \frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/
((b*c - a*d)*sqrt(c*d))
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}(bc-ad)} - \frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/
((b*c - a*d)*sqrt(c*d))
```

3.24.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.93

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{\ln\left(b\,x-\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,a^2\,d-2\,a\,b\,c} - \frac{\ln\left(d\,x+\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,\left(b\,c^2-a\,c\,d\right)} - \frac{\ln\left(b\,x+\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,\left(a^2\,d-a\,b\,c\right)} + \frac{\ln\left(d\,x-\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,b\,c^2-2\,a\,c\,d}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
(log(b*x - (-a*b)^(1/2))*(-a*b)^(1/2))/(2*a^2*d - 2*a*b*c) - (log(d*x + (-
c*d)^(1/2))*(-c*d)^(1/2))/(2*(b*c^2 - a*c*d)) - (log(b*x + (-a*b)^(1/2))*(
-a*b)^(1/2))/(2*(a^2*d - a*b*c)) + (log(d*x - (-c*d)^(1/2))*(-c*d)^(1/2))/
(2*b*c^2 - 2*a*c*d)
```

3.25 $\int \frac{1}{(a+bx^2)(c+dx^2)^2} dx$

3.25.1 Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2} dx = -\frac{dx}{2c(bc-ad)(c+dx^2)} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2} - \frac{\sqrt{d}(3bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}$$

output
```
-1/2*d*x/c/(-a*d+b*c)/(d*x^2+c)+b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b*
c)^2/a^(1/2)-1/2*(-a*d+3*b*c)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(3/2)/(-
a*d+b*c)^2
```

3.25.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2} dx = \frac{\frac{d(-bc+ad)x}{c(c+dx^2)} + \frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{d}(-3bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{2(bc-ad)^2}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((d*(-(b*c) + a*d)*x)/(c*(c + d*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/Sqrt[a] + (Sqrt[d]*(-3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/
2))/(2*(b*c - a*d)^2)
```

3.25.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {316, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx$$

$$\Big\downarrow \ 316$$

$$\frac{\int \frac{-bdx^2+2bc-ad}{(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx}{2c(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{2b^2c\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{d(3bc-ad)\int \frac{1}{dx^2+c}dx}{bc-ad}}{2c(bc-ad)}-\frac{dx}{2c(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{2b^{3/2}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}(3bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{2c(bc-ad)}-\frac{dx}{2c(c+dx^2)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
-1/2*(d*x)/(c*(b*c - a*d)*(c + d*x^2)) + ((2*b^(3/2)*c*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*(3*b*c - a*d)*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(2*c*(b*c - a*d))
```

3.25.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.25.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{b^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\left(a d-b c\right)^{2} \sqrt{a b}}+\frac{d \left(\frac{\left(a d-b c\right) x}{2 c \left(d \,x^{2}+c\right)}+\frac{\left(a d-3 b c\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 c \sqrt{c d}}\right)}{\left(a d-b c\right)^{2}}$	93
risch	Expression too large to display	1039

input
```
int(1/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)^2*(1/2*(a*
d-b*c)/c*x/(d*x^2+c)+1/2*(a*d-3*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 711, normalized size of antiderivative = 6.52

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx$$
$$= \left[\frac{2\,(bcdx^2+bc^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-(3\,bc^2-acd+(3\,bcd-ad^2)x^2)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{4\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^3d-2\,abc^2d^2+a^2cd^3)x^2)}\right.$$
$$-\frac{(3\,bc^2-acd+(3\,bcd-ad^2)x^2)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-(bcdx^2+bc^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+(bcd-}{2\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^3d-2\,abc^2d^2+a^2cd^3)x^2)}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*log((d
*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2*c^4
- 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)
, -1/2*((3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(
d/c)) - (b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)) + (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (
b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/4*(4*(b*c*d*x^2 + b*c^2)*sq
rt(b/a)*arctan(x*sqrt(b/a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sq
rt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^
2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 +
a^2*c*d^3)*x^2), 1/2*(2*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*arctan(x*sqrt(b/a))
- (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
- (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2
*a*b*c^2*d^2 + a^2*c*d^3)*x^2)]
```

3.25.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}} - \frac{dx}{2\,(bc^3-ac^2d+(bc^2d-acd^2)x^2)} - \frac{(3\,bcd-ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{cd}}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/
2*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) - 1/2*(3*b*c*d - a*d^2)*
arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}} - \frac{(3\,bcd-ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{cd}} - \frac{dx}{2\,(bc^2-acd)(dx^2+c)}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/
2*(3*b*c*d - a*d^2)*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*
d^2)*sqrt(c*d)) - 1/2*d*x/((b*c^2 - a*c*d)*(d*x^2 + c))
```

3.25.9 Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 3637, normalized size of antiderivative = 33.37

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
(d*x)/(2*c*(c + d*x^2)*(a*d - b*c)) - (atan((((-c^3*d)^(1/2)*(a*d - 3*b*c)
*((x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2
*d^2 - 2*a*b*c^3*d)) - (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d
^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^
5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) - (x*(-c^3*d)^(1/2)*(a*
d - 3*b*c)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^
3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^
2*c^2*d^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^
(1/2)*(a*d - 3*b*c))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^
2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)) + ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a
^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 -
2*a*b*c^3*d)) + (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32
*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3
*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + (x*(-c^3*d)^(1/2)*(a*d - 3*b
*c)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c
^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(
a*d - 3*b*c))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^2*c^5 +
 a^2*c^3*d^2 - 2*a*b*c^4*d)))/(((a*b^4*d^4)/2 - (3*b^5*c*d^3)/2)/(b^3*c^5
- a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + ((-c^3*d)^(1/2)*(a*d...
```

3.26 $\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx$

3.26.1 Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{dx}{4c(bc-ad)(c+dx^2)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3} - \frac{\sqrt{d}(15b^2c^2-10abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}$$

output
```
-1/4*d*x/c/(-a*d+b*c)/(d*x^2+c)^2-1/8*d*(-3*a*d+7*b*c)*x/c^2/(-a*d+b*c)^2/
(d*x^2+c)+b^(5/2)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b*c)^3/a^(1/2)-1/8*(3*a^
2*d^2-10*a*b*c*d+15*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(5/2)/(-a
*d+b*c)^3
```

3.26.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{1}{8}\left(\frac{dx(ad(5c+3dx^2)-bc(9c+7dx^2))}{c^2(bc-ad)^2(c+dx^2)^2} - \frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(-bc+ad)^3} - \frac{\sqrt{d}(15b^2c^2-10abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3}\right)$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
((d*x*(a*d*(5*c + 3*d*x^2) - b*c*(9*c + 7*d*x^2)))/(c^2*(b*c - a*d)^2*(c +
 d*x^2)^2) - (8*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-(b*c) + a*
d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(c^(5/2)*(b*c - a*d)^3))/8
```

3.26.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {316, 402, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{-3bdx^2+4bc-3ad}{(bx^2+a)(dx^2+c)^2}dx}{4c(bc-ad)}-\frac{dx}{4c(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{8b^2c^2-7abdc+3a^2d^2-bd(7bc-3ad)x^2}{(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx(7bc-3ad)}{2c(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{8b^3c^2\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{d\left(3a^2d^2-10abcd+15b^2c^2\right)\int \frac{1}{dx^2+c}dx}{bc-ad}}{2c(bc-ad)}-\frac{dx(7bc-3ad)}{2c(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{8b^{5/2}c^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{2c(bc-ad)}-\frac{dx(7bc-3ad)}{2c(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(c+dx^2)^2(bc-ad)}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
-1/4*(d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) + (-1/2*(d*(7*b*c - 3*a*d)*x)/(c*
(b*c - a*d)*(c + d*x^2)) + ((8*b^(5/2)*c^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(S
qrt[a]*(b*c - a*d)) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTa
n[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(2*c*(b*c - a*d)))/(4*c*(b*
c - a*d))
```

3.26.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.26.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^3\sqrt{ab}}+\frac{d\left(\frac{\frac{d(3a^2d^2-10abcd+7b^2c^2)x^3}{8c^2}+\frac{(5a^2d^2-14abcd+9b^2c^2)x}{8c}}{(dx^2+c)^2}+\frac{(3a^2d^2-10abcd+15b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2\sqrt{cd}}\right)}{(ad-bc)^3}$	158
risch	Expression too large to display	2285

input
```
int(1/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-b^3/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)^3*((1/8*d
*(3*a^2*d^2-10*a*b*c*d+7*b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-14*a*b*c*d+9*b^2*
c^2)/c*x)/(d*x^2+c)^2+1/8*(3*a^2*d^2-10*a*b*c*d+15*b^2*c^2)/c^2/(c*d)^(1/2
)*arctan(d*x/(c*d)^(1/2)))
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(138) = 276.

Time = 0.75 (sec) , antiderivative size = 1585, normalized size of antiderivative = 9.91

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 + 8*(b^2*c^2*d^2*
x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
- a)/(b*x^2 + a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c
^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2
+ 3*a^2*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 +
 c)) + 2*(9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^
2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 +
 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a
^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*
a^2*d^4)*x^3 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^
2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a
^2*c*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 4*(b^2*c^2*d^2*x^4 + 2*b^2*
c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
+ a)) + (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2
*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 +
3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^
2*b*c^4*d^3 - a^3*c^3*d^4)*x^2), -1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 +
3*a^2*d^4)*x^3 - 16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a
)*arctan(x*sqrt(b/a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b
^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c...
```

3.26.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.26.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(138) = 276.

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.73

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx$$
$$= \frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}} - \frac{(15\,b^2c^2d-10\,abcd^2+3\,a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\sqrt{cd}}$$
$$- \frac{(7\,bcd^2-3\,ad^3)x^3+(9\,bc^2d-5\,acd^2)x}{8\,(b^2c^6-2\,abc^5d+a^2c^4d^2+(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4)x^4+2\,(b^2c^5d-2\,abc^4d^2+a^2c^3d^3)x^2)}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqr
t(c*d)) - 1/8*((7*b*c*d^2 - 3*a*d^3)*x^3 + (9*b*c^2*d - 5*a*c*d^2)*x)/(b^2
*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*
d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}}$$
$$- \frac{(15\,b^2c^2d-10\,abcd^2+3\,a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\sqrt{cd}}$$
$$- \frac{7\,bcd^2x^3-3\,ad^3x^3+9\,bc^2dx-5\,acd^2x}{8\,(b^2c^4-2\,abc^3d+a^2c^2d^2)(dx^2+c)^2}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqr
t(c*d)) - 1/8*(7*b*c*d^2*x^3 - 3*a*d^3*x^3 + 9*b*c^2*d*x - 5*a*c*d^2*x)/((
b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 6033, normalized size of antiderivative = 37.71

$$\int \frac{1}{(a+bx^2)\left(c+dx^2\right)^3} \, dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
((x^3*(3*a*d^3 - 7*b*c*d^2))/(8*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*
(5*a*d^2 - 9*b*c*d))/(8*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*x^4
 + 2*c*d*x^2) - (atan((((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^
3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*
c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))
- ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*
c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d
^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64
*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b
^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-a*b^5)^(1/2)*(256
*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*
c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d
^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*
a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2
- 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*
d^2)))*1i)/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) + (
(-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 -
60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^
3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + ((-a*b^5)^(1/2)*((256*
b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^...
```

3.27 $\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx$

3.27.1 Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx = \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^3}{3b^4} + \frac{d^4(5bc-2ad)x^5}{5b^3} + \frac{d^5x^7}{7b^2} + \frac{(bc-ad)^5x}{2ab^5\,(a+bx^2)} + \frac{(bc-ad)^4(bc+9ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}$$

output
```
d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/3*d^3*(3
*a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^3/b^4+1/5*d^4*(-2*a*d+5*b*c)*x^5/b^3+1/7
*d^5*x^7/b^2+1/2*(-a*d+b*c)^5*x/a/b^5/(b*x^2+a)+1/2*(-a*d+b*c)^4*(9*a*d+b*
c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(11/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx = \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^3}{3b^4} + \frac{d^4(5bc-2ad)x^5}{5b^3} + \frac{d^5x^7}{7b^2} + \frac{(bc-ad)^5x}{2ab^5\,(a+bx^2)} + \frac{(bc-ad)^4(bc+9ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}$$

input
```
Integrate[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

output
```
(d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (
d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) + (d^4*(5*b*c - 2*a
*d)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c - a*d)^5*x)/(2*a*b^5*(a + b*x
^2)) + ((b*c - a*d)^4*(b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2
)*b^(11/2))
```

3.27.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^5}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 300}$$

$$\int \left(\frac{d^3x^2\left(3a^2d^2-10abcd+10b^2c^2\right)}{b^4}+\frac{d^2\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{5bdx^2(bc-ad)^4+(4ad+}{b^5\left(a+bx^2\right)^2}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^4(9ad+bc)}{2a^{3/2}b^{11/2}}+\frac{d^3x^3\left(3a^2d^2-10abcd+10b^2c^2\right)}{3b^4}+\frac{d^2x\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{x(bc-ad)^5}{2ab^5\left(a+bx^2\right)}+\frac{d^4x^5(5bc-2ad)}{5b^3}+\frac{d^5x^7}{7b^2}$$

input
```
Int[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

output
```
(d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (
d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) + (d^4*(5*b*c - 2*a
*d)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c - a*d)^5*x)/(2*a*b^5*(a + b*x
^2)) + ((b*c - a*d)^4*(b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2
)*b^(11/2))
```

3.27.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.27.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.51

method	result
default	$-\frac{d^2\left(-\frac{1}{7}b^3d^3x^7+\frac{2}{5}a\,b^2d^3x^5-b^3c\,d^2x^5-a^2b\,d^3x^3+\frac{10}{3}a\,b^2c\,d^2x^3-\frac{10}{3}b^3c^2d\,x^3+4a^3d^3x-15a^2bc\,d^2x+20a\,b^2c^2dx-10b^3c^3x\right)}{b^5}+\frac{-\frac{(a^5}{}}{}$
risch	$\frac{d^5x^7}{7b^2}-\frac{2d^5a\,x^5}{5b^3}+\frac{d^4c\,x^5}{b^2}+\frac{d^5a^2x^3}{b^4}-\frac{10d^4ac\,x^3}{3b^3}+\frac{10d^3c^2x^3}{3b^2}-\frac{4d^5a^3x}{b^5}+\frac{15d^4a^2cx}{b^4}-\frac{20d^3a\,c^2x}{b^3}+\frac{10d^2c^3x}{b^2}-\frac{(a^5d}{}$

input
```
int((d*x^2+c)^5/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-d^2/b^5*(-1/7*b^3*d^3*x^7+2/5*a*b^2*d^3*x^5-b^3*c*d^2*x^5-a^2*b*d^3*x^3+1
0/3*a*b^2*c*d^2*x^3-10/3*b^3*c^2*d*x^3+4*a^3*d^3*x-15*a^2*b*c*d^2*x+20*a*b
^2*c^2*d*x-10*b^3*c^3*x)+1/b^5*(-1/2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2
*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/a*x/(b*x^2+a)+1/2*(9*a^5*d^
5-35*a^4*b*c*d^4+50*a^3*b^2*c^2*d^3-30*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d+b^5*c
^5)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(174) = 348.

Time = 0.29 (sec) , antiderivative size = 810, normalized size of antiderivative = 4.22

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{60\,a^2b^5d^5x^9+12\,(35\,a^2b^5cd^4-9\,a^3b^4d^5)x^7+28\,(50\,a^2b^5c^2d^3-35\,a^3b^4cd^4+9\,a^4b^3d^5)x^5+140\,(30\,a^2b^5c^3}{}\right.$$

input
```
integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/420*(60*a^2*b^5*d^5*x^9 + 12*(35*a^2*b^5*c*d^4 - 9*a^3*b^4*d^5)*x^7 + 2
8*(50*a^2*b^5*c^2*d^3 - 35*a^3*b^4*c*d^4 + 9*a^4*b^3*d^5)*x^5 + 140*(30*a^
2*b^5*c^3*d^2 - 50*a^3*b^4*c^2*d^3 + 35*a^4*b^3*c*d^4 - 9*a^5*b^2*d^5)*x^3
 - 105*(a*b^5*c^5 + 5*a^2*b^4*c^4*d - 30*a^3*b^3*c^3*d^2 + 50*a^4*b^2*c^2*
d^3 - 35*a^5*b*c*d^4 + 9*a^6*d^5 + (b^6*c^5 + 5*a*b^5*c^4*d - 30*a^2*b^4*c
^3*d^2 + 50*a^3*b^3*c^2*d^3 - 35*a^4*b^2*c*d^4 + 9*a^5*b*d^5)*x^2)*sqrt(-a
*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 210*(a*b^6*c^5 - 5*a^2
*b^5*c^4*d + 30*a^3*b^4*c^3*d^2 - 50*a^4*b^3*c^2*d^3 + 35*a^5*b^2*c*d^4 -
9*a^6*b*d^5)*x)/(a^2*b^7*x^2 + a^3*b^6), 1/210*(30*a^2*b^5*d^5*x^9 + 6*(35
*a^2*b^5*c*d^4 - 9*a^3*b^4*d^5)*x^7 + 14*(50*a^2*b^5*c^2*d^3 - 35*a^3*b^4*
c*d^4 + 9*a^4*b^3*d^5)*x^5 + 70*(30*a^2*b^5*c^3*d^2 - 50*a^3*b^4*c^2*d^3 +
 35*a^4*b^3*c*d^4 - 9*a^5*b^2*d^5)*x^3 + 105*(a*b^5*c^5 + 5*a^2*b^4*c^4*d
- 30*a^3*b^3*c^3*d^2 + 50*a^4*b^2*c^2*d^3 - 35*a^5*b*c*d^4 + 9*a^6*d^5 + (
b^6*c^5 + 5*a*b^5*c^4*d - 30*a^2*b^4*c^3*d^2 + 50*a^3*b^3*c^2*d^3 - 35*a^4
*b^2*c*d^4 + 9*a^5*b*d^5)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 105*(a*b^
6*c^5 - 5*a^2*b^5*c^4*d + 30*a^3*b^4*c^3*d^2 - 50*a^4*b^3*c^2*d^3 + 35*a^5
*b^2*c*d^4 - 9*a^6*b*d^5)*x)/(a^2*b^7*x^2 + a^3*b^6)]
```

3.27.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(185) = 370$.

Time = 1.10 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.61

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx = x^5\left(-\frac{2ad^5}{5b^3}+\frac{cd^4}{b^2}\right)+x^3\left(\frac{a^2d^5}{b^4}-\frac{10acd^4}{3b^3}+\frac{10c^2d^3}{3b^2}\right)$$
$$+x\left(-\frac{4a^3d^5}{b^5}+\frac{15a^2cd^4}{b^4}-\frac{20ac^2d^3}{b^3}+\frac{10c^3d^2}{b^2}\right)$$
$$+\frac{x(-a^5d^5+5a^4bcd^4-10a^3b^2c^2d^3+10a^2b^3c^3d^2-5ab^4c^4d+b^5c^5)}{2a^2b^5+2ab^6x^2}$$
$$-\frac{\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4\cdot(9ad+bc)\log\left(-\frac{a^2b^5\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4\cdot(9ad+bc)}{9a^5d^5-35a^4bcd^4+50a^3b^2c^2d^3-30a^2b^3c^3d^2+5ab^4c^4d+b^5c^5}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4\cdot(9ad+bc)\log\left(\frac{a^2b^5\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4\cdot(9ad+bc)}{9a^5d^5-35a^4bcd^4+50a^3b^2c^2d^3-30a^2b^3c^3d^2+5ab^4c^4d+b^5c^5}+x\right)}{4}$$
$$+\frac{d^5x^7}{7b^2}$$

input
```
integrate((d*x**2+c)**5/(b*x**2+a)**2,x)
```

output
```
x**5*(-2*a*d**5/(5*b**3) + c*d**4/b**2) + x**3*(a**2*d**5/b**4 - 10*a*c*d*
*4/(3*b**3) + 10*c**2*d**3/(3*b**2)) + x*(-4*a**3*d**5/b**5 + 15*a**2*c*d*
*4/b**4 - 20*a*c**2*d**3/b**3 + 10*c**3*d**2/b**2) + x*(-a**5*d**5 + 5*a**
4*b*c*d**4 - 10*a**3*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b**4*c*
*4*d + b**5*c**5)/(2*a**2*b**5 + 2*a*b**6*x**2) - sqrt(-1/(a**3*b**11))*(a
*d - b*c)**4*(9*a*d + b*c)*log(-a**2*b**5*sqrt(-1/(a**3*b**11))*(a*d - b*c
)**4*(9*a*d + b*c)/(9*a**5*d**5 - 35*a**4*b*c*d**4 + 50*a**3*b**2*c**2*d**
3 - 30*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + b**5*c**5) + x)/4 + sqrt(-1
/(a**3*b**11))*(a*d - b*c)**4*(9*a*d + b*c)*log(a**2*b**5*sqrt(-1/(a**3*b*
*11))*(a*d - b*c)**4*(9*a*d + b*c)/(9*a**5*d**5 - 35*a**4*b*c*d**4 + 50*a*
*3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + b**5*c**5)
+ x)/4 + d**5*x**7/(7*b**2)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.53

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx = \frac{(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5)x}{2\,(ab^6x^2 + a^2b^5)}$$
$$+ \frac{15\,b^3d^5x^7 + 21\,(5\,b^3cd^4 - 2\,ab^2d^5)x^5 + 35\,(10\,b^3c^2d^3 - 10\,ab^2cd^4 + 3\,a^2bd^5)x^3 + 105\,(10\,b^3c^3d^2 - 20\,ab^2c}{105\,b^5}$$
$$+ \frac{(b^5c^5 + 5\,ab^4c^4d - 30\,a^2b^3c^3d^2 + 50\,a^3b^2c^2d^3 - 35\,a^4bcd^4 + 9\,a^5d^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^5}$$

input
```
integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5
*a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^2 + a^2*b^5) + 1/105*(15*b^3*d^5*x^7 +
21*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^5 + 35*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 +
 3*a^2*b*d^5)*x^3 + 105*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^
4 - 4*a^3*d^5)*x)/b^5 + 1/2*(b^5*c^5 + 5*a*b^4*c^4*d - 30*a^2*b^3*c^3*d^2
+ 50*a^3*b^2*c^2*d^3 - 35*a^4*b*c*d^4 + 9*a^5*d^5)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a*b^5)
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx$$
$$= \frac{(b^5c^5 + 5\,ab^4c^4d - 30\,a^2b^3c^3d^2 + 50\,a^3b^2c^2d^3 - 35\,a^4bcd^4 + 9\,a^5d^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab^5}$$
$$+ \frac{b^5c^5x - 5\,ab^4c^4dx + 10\,a^2b^3c^3d^2x - 10\,a^3b^2c^2d^3x + 5\,a^4bcd^4x - a^5d^5x}{2\,(bx^2+a)ab^5}$$
$$+ \frac{15\,b^{12}d^5x^7 + 105\,b^{12}cd^4x^5 - 42\,ab^{11}d^5x^5 + 350\,b^{12}c^2d^3x^3 - 350\,ab^{11}cd^4x^3 + 105\,a^2b^{10}d^5x^3 + 1050\,b^{12}c^3d^2}{105\,b^{14}}$$

input
```
integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^5*c^5 + 5*a*b^4*c^4*d - 30*a^2*b^3*c^3*d^2 + 50*a^3*b^2*c^2*d^3 - 3
5*a^4*b*c*d^4 + 9*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^5) + 1/2*(
b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x
+ 5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^2 + a)*a*b^5) + 1/105*(15*b^12*d^5*x^
7 + 105*b^12*c*d^4*x^5 - 42*a*b^11*d^5*x^5 + 350*b^12*c^2*d^3*x^3 - 350*a*
b^11*c*d^4*x^3 + 105*a^2*b^10*d^5*x^3 + 1050*b^12*c^3*d^2*x - 2100*a*b^11*
c^2*d^3*x + 1575*a^2*b^10*c*d^4*x - 420*a^3*b^9*d^5*x)/b^14
```

3.27.9 Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.01

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx$$
$$= x\left(\frac{10\,c^3\,d^2}{b^2} - \frac{2\,a\left(\frac{2\,a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b} - \frac{a^2\,d^5}{b^4} + \frac{10\,c^2\,d^3}{b^2}\right)}{b} + \frac{a^2\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b^2}\right)$$
$$- x^5\left(\frac{2\,a\,d^5}{5\,b^3} - \frac{c\,d^4}{b^2}\right) + x^3\left(\frac{2\,a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{3\,b} - \frac{a^2\,d^5}{3\,b^4} + \frac{10\,c^2\,d^3}{3\,b^2}\right) + \frac{d^5\,x^7}{7\,b^2}$$
$$- \frac{x\,(a^5\,d^5 - 5\,a^4\,b\,c\,d^4 + 10\,a^3\,b^2\,c^2\,d^3 - 10\,a^2\,b^3\,c^3\,d^2 + 5\,a\,b^4\,c^4\,d - b^5\,c^5)}{2\,a\,(b^6\,x^2 + a\,b^5)}$$
$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^4\,(9\,a\,d+b\,c)}{\sqrt{a}\,(9\,a^5\,d^5-35\,a^4\,b\,c\,d^4+50\,a^3\,b^2\,c^2\,d^3-30\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d+b^5\,c^5)}\right)\,(a\,d-b\,c)^4\,(9\,a\,d+b\,c)}{2\,a^{3/2}\,b^{11/2}}$$

input
```
int((c + d*x^2)^5/(a + b*x^2)^2,x)
```

output
```
x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2
*d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b
^2) - x^5*((2*a*d^5)/(5*b^3) - (c*d^4)/b^2) + x^3*((2*a*((2*a*d^5)/b^3 - (
5*c*d^4)/b^2))/(3*b) - (a^2*d^5)/(3*b^4) + (10*c^2*d^3)/(3*b^2)) + (d^5*x^
7)/(7*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d
^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(2*a*(a*b^5 + b^6*x^2)) + (atan((b^(1
/2)*x*(a*d - b*c)^4*(9*a*d + b*c))/(a^(1/2)*(9*a^5*d^5 + b^5*c^5 - 30*a^2*
b^3*c^3*d^2 + 50*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 35*a^4*b*c*d^4)))*(a*d
- b*c)^4*(9*a*d + b*c))/(2*a^(3/2)*b^(11/2))
```

3.27. $\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx$

3.28 $\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx$

3.28.1 Optimal result

Integrand size = 19, antiderivative size = 142

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx = \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{2d^3(2bc-ad)x^3}{3b^3}+\frac{d^4x^5}{5b^2}+\frac{(bc-ad)^4x}{2ab^4\,(a+bx^2)}+\frac{(bc-ad)^3(bc+7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}$$

output
```
d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+2/3*d^3*(-a*d+2*b*c)*x^3/b^3+1/5
*d^4*x^5/b^2+1/2*(-a*d+b*c)^4*x/a/b^4/(b*x^2+a)+1/2*(-a*d+b*c)^3*(7*a*d+b*
c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(9/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx = \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{2d^3(2bc-ad)x^3}{3b^3}+\frac{d^4x^5}{5b^2}+\frac{(bc-ad)^4x}{2ab^4\,(a+bx^2)}+\frac{(bc-ad)^3(bc+7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}$$

input
```
Integrate[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

output
```
(d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^3
)/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b^4*(a + b*x^2)) +
((b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(9/
2))
```

3.28.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{d^2(3a^2d^2-8abcd+6b^2c^2)}{b^4}+\frac{4bdx^2(bc-ad)^3+(3ad+bc)(bc-ad)^3}{b^4(a+bx^2)^2}+\frac{2d^3x^2(2bc-ad)}{b^3}+\frac{d^4x^4}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^3(7ad+bc)}{2a^{3/2}b^{9/2}}+\frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4}+\frac{x(bc-ad)^4}{2ab^4(a+bx^2)}+\frac{2d^3x^3(2bc-ad)}{3b^3}+\frac{d^4x^5}{5b^2}$$

input
```
Int[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

output

```
(d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^3
)/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b^4*(a + b*x^2)) +
((b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(9/
2))
```

3.28.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.28.4 Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.45

method	result
default	$\frac{d^2\left(\frac{1}{5}b^2d^2x^5-\frac{2}{3}x^3ab\,d^2+\frac{4}{3}x^3b^2cd+3a^2d^2x-8abcdx+6b^2c^2x\right)}{b^4}-\frac{-\frac{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(7a^4d^4-20a^3bc\,d^3+\right.}{}}{b^4}$
risch	$\frac{d^4x^5}{5b^2}-\frac{2d^4x^3a}{3b^3}+\frac{4d^3x^3c}{3b^2}+\frac{3d^4a^2x}{b^4}-\frac{8d^3acx}{b^3}+\frac{6d^2c^2x}{b^2}+\frac{(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)x}{2a\,b^4(b\,x^2+a)}-\frac{7a^3\ln(bx-}{4b^4\sqrt{}}$

input
```
int((d*x^2+c)^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2/b^4*(1/5*b^2*d^2*x^5-2/3*x^3*a*b*d^2+4/3*x^3*b^2*c*d+3*a^2*d^2*x-8*a*b
*c*d*x+6*b^2*c^2*x)-1/b^4*(-1/2*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4
*a*b^3*c^3*d+b^4*c^4)/a*x/(b*x^2+a)+1/2*(7*a^4*d^4-20*a^3*b*c*d^3+18*a^2*b
^2*c^2*d^2-4*a*b^3*c^3*d-b^4*c^4)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(126) = 252.

Time = 0.26 (sec) , antiderivative size = 612, normalized size of antiderivative = 4.31

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{12\,a^2b^4d^4x^7+4\,(20\,a^2b^4cd^3-7\,a^3b^3d^4)x^5+20\,(18\,a^2b^4c^2d^2-20\,a^3b^3cd^3+7\,a^4b^2d^4)x^3+15\,(ab^4c^4+4\,a}{}\right.$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*a^2*b^4*d^4*x^7 + 4*(20*a^2*b^4*c*d^3 - 7*a^3*b^3*d^4)*x^5 + 20*
(18*a^2*b^4*c^2*d^2 - 20*a^3*b^3*c*d^3 + 7*a^4*b^2*d^4)*x^3 + 15*(a*b^4*c^
4 + 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 20*a^4*b*c*d^3 - 7*a^5*d^4 + (b
^5*c^4 + 4*a*b^4*c^3*d - 18*a^2*b^3*c^2*d^2 + 20*a^3*b^2*c*d^3 - 7*a^4*b*d
^4)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 30*(a*
b^5*c^4 - 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 20*a^4*b^2*c*d^3 + 7*a^5*
b*d^4)*x)/(a^2*b^6*x^2 + a^3*b^5), 1/30*(6*a^2*b^4*d^4*x^7 + 2*(20*a^2*b^4
*c*d^3 - 7*a^3*b^3*d^4)*x^5 + 10*(18*a^2*b^4*c^2*d^2 - 20*a^3*b^3*c*d^3 +
7*a^4*b^2*d^4)*x^3 + 15*(a*b^4*c^4 + 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2
+ 20*a^4*b*c*d^3 - 7*a^5*d^4 + (b^5*c^4 + 4*a*b^4*c^3*d - 18*a^2*b^3*c^2*d
^2 + 20*a^3*b^2*c*d^3 - 7*a^4*b*d^4)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a)
+ 15*(a*b^5*c^4 - 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 20*a^4*b^2*c*d^3
+ 7*a^5*b*d^4)*x)/(a^2*b^6*x^2 + a^3*b^5)]
```

3.28.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(133) = 266$.

Time = 0.82 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.84

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx$$
$$= x^3\left(-\frac{2ad^4}{3b^3}+\frac{4cd^3}{3b^2}\right)+x\left(\frac{3a^2d^4}{b^4}-\frac{8acd^3}{b^3}+\frac{6c^2d^2}{b^2}\right)$$
$$+\frac{x(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{2a^2b^4+2ab^5x^2}$$
$$+\frac{\sqrt{-\frac{1}{a^3b^9}}(ad-bc)^3\cdot(7ad+bc)\log\left(-\frac{a^2b^4\sqrt{-\frac{1}{a^3b^9}}(ad-bc)^3\cdot(7ad+bc)}{7a^4d^4-20a^3bcd^3+18a^2b^2c^2d^2-4ab^3c^3d-b^4c^4}+x\right)}{4}$$
$$-\frac{\sqrt{-\frac{1}{a^3b^9}}(ad-bc)^3\cdot(7ad+bc)\log\left(\frac{a^2b^4\sqrt{-\frac{1}{a^3b^9}}(ad-bc)^3\cdot(7ad+bc)}{7a^4d^4-20a^3bcd^3+18a^2b^2c^2d^2-4ab^3c^3d-b^4c^4}+x\right)}{4}+\frac{d^4x^5}{5b^2}$$

input
```
integrate((d*x**2+c)**4/(b*x**2+a)**2,x)
```

output
```
x**3*(-2*a*d**4/(3*b**3) + 4*c*d**3/(3*b**2)) + x*(3*a**2*d**4/b**4 - 8*a*
c*d**3/b**3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*
b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(2*a**2*b**4 + 2*a*b**5*x**2
) + sqrt(-1/(a**3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)*log(-a**2*b**4*sqrt(
-1/(a**3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)/(7*a**4*d**4 - 20*a**3*b*c*d*
*3 + 18*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d - b**4*c**4) + x)/4 - sqrt(-
1/(a**3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)*log(a**2*b**4*sqrt(-1/(a**3*b*
*9))*(a*d - b*c)**3*(7*a*d + b*c)/(7*a**4*d**4 - 20*a**3*b*c*d**3 + 18*a**
2*b**2*c**2*d**2 - 4*a*b**3*c**3*d - b**4*c**4) + x)/4 + d**4*x**5/(5*b**2
)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx = \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)x}{2\,(ab^5x^2 + a^2b^4)} + \frac{3\,b^2d^4x^5 + 10\,(2\,b^2cd^3 - abd^4)x^3 + 15\,(6\,b^2c^2d^2 - 8\,abcd^3 + 3\,a^2d^4)x}{15\,b^4} + \frac{(b^4c^4 + 4\,ab^3c^3d - 18\,a^2b^2c^2d^2 + 20\,a^3bcd^3 - 7\,a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^4}$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)*x/(a*b^5*x^2 + a^2*b^4) + 1/15*(3*b^2*d^4*x^5 + 10*(2*b^2*c*d^3 - a*b*d^
4)*x^3 + 15*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 1/2*(b^4*c^
4 + 4*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 7*a^4*d^4)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.55

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx$$
$$= \frac{(b^4c^4 + 4\,ab^3c^3d - 18\,a^2b^2c^2d^2 + 20\,a^3bcd^3 - 7\,a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab^4} + \frac{b^4c^4x - 4\,ab^3c^3dx + 6\,a^2b^2c^2d^2x - 4\,a^3bcd^3x + a^4d^4x}{2\,(bx^2+a)ab^4} + \frac{3\,b^8d^4x^5 + 20\,b^8cd^3x^3 - 10\,ab^7d^4x^3 + 90\,b^8c^2d^2x - 120\,ab^7cd^3x + 45\,a^2b^6d^4x}{15\,b^{10}}$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^4*c^4 + 4*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 7*a^4
*d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/2*(b^4*c^4*x - 4*a*b^3*c
^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*d^4*x)/((b*x^2 + a)*a
*b^4) + 1/15*(3*b^8*d^4*x^5 + 20*b^8*c*d^3*x^3 - 10*a*b^7*d^4*x^3 + 90*b^8
*c^2*d^2*x - 120*a*b^7*c*d^3*x + 45*a^2*b^6*d^4*x)/b^10
```

3.28.9 Mupad [B] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.84

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx$$
$$= x\left(\frac{2\,a\left(\frac{2\,a\,d^4}{b^3} - \frac{4\,c\,d^3}{b^2}\right)}{b} - \frac{a^2\,d^4}{b^4} + \frac{6\,c^2\,d^2}{b^2}\right) - x^3\left(\frac{2\,a\,d^4}{3\,b^3} - \frac{4\,c\,d^3}{3\,b^2}\right) + \frac{d^4\,x^5}{5\,b^2} + \frac{x\,(a^4\,d^4 - 4\,a^3\,b\,c\,d^3 + 6\,a^2\,b^2\,c^2\,d^2 - 4\,a\,b^3\,c^3\,d + b^4\,c^4)}{2\,a\,(b^5\,x^2 + a\,b^4)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^3\,(7\,a\,d+b\,c)}{\sqrt{a}\,(-7\,a^4\,d^4+20\,a^3\,b\,c\,d^3-18\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d+b^4\,c^4)}\right)(a\,d-b\,c)^3\,(7\,a\,d+b\,c)}{2\,a^{3/2}\,b^{9/2}}$$

input
```
int((c + d*x^2)^4/(a + b*x^2)^2,x)
```

output
```
x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b
^2) - x^3*((2*a*d^4)/(3*b^3) - (4*c*d^3)/(3*b^2)) + (d^4*x^5)/(5*b^2) + (x
*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/
(2*a*(a*b^4 + b^5*x^2)) + (atan((b^(1/2)*x*(a*d - b*c)^3*(7*a*d + b*c))/(a
^(1/2)*(b^4*c^4 - 7*a^4*d^4 - 18*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 20*a^3*
b*c*d^3)))*(a*d - b*c)^3*(7*a*d + b*c))/(2*a^(3/2)*b^(9/2))
```

3.28. $\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx$

3.29 $\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx$

3.29.1 Optimal result

Integrand size = 19, antiderivative size = 106

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^3}{3b^2} + \frac{(bc-ad)^3x}{2ab^3(a+bx^2)} + \frac{(bc-ad)^2(bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

output
```
d^2*(-2*a*d+3*b*c)*x/b^3+1/3*d^3*x^3/b^2+1/2*(-a*d+b*c)^3*x/a/b^3/(b*x^2+a
)+1/2*(-a*d+b*c)^2*(5*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(7/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^3}{3b^2} + \frac{(bc-ad)^3x}{2ab^3(a+bx^2)} + \frac{(bc-ad)^2(bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

input
```
Integrate[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

output
```
(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b
^3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*a^(3/2)*b^(7/2))
```

3.29.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \ 300$$

$$\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{3bdx^2(bc-ad)^2+(2ad+bc)(bc-ad)^2}{b^3(a+bx^2)^2}+\frac{d^3x^2}{b^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5ad+bc)(bc-ad)^2}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3(a+bx^2)}+\frac{d^3x^3}{3b^2}$$

input
```
Int[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

output
```
(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b
^3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*a^(3/2)*b^(7/2))
```

3.29.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.29.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.31

method	result
default	$-\frac{d^2\left(-\frac{1}{3}bd\,x^3+2adx-3bcx\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(5a^3d^3-9a^2bc\,d^2+3a\,b^2c^2d+b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^3}$
risch	$\frac{d^3x^3}{3b^2}-\frac{2d^3ax}{b^3}+\frac{3d^2cx}{b^2}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{2a\,b^3(b\,x^2+a)}-\frac{5a^2\ln(bx+\sqrt{-ab})d^3}{4b^3\sqrt{-ab}}+\frac{9a\ln(bx+\sqrt{-ab})c\,d^2}{4b^2\sqrt{-ab}}-\frac{3\ln(bx+\sqrt{}}{4b\sqrt{}}$

input
```
int((d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-d^2/b^3*(-1/3*b*d*x^3+2*a*d*x-3*b*c*x)+1/b^3*(-1/2*(a^3*d^3-3*a^2*b*c*d^2
+3*a*b^2*c^2*d-b^3*c^3)/a*x/(b*x^2+a)+1/2*(5*a^3*d^3-9*a^2*b*c*d^2+3*a*b^2
*c^2*d+b^3*c^3)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(92) = 184.

Time = 0.28 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.17

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{4\,a^2b^3d^3x^5+4\,(9\,a^2b^3cd^2-5\,a^3b^2d^3)x^3-3\,(ab^3c^3+3\,a^2b^2c^2d-9\,a^3bcd^2+5\,a^4d^3+(b^4c^3+3\,ab^3c^2d-9}{12\,(a^2b^5x^2+a}\right.$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*a^2*b^3*d^3*x^5 + 4*(9*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 - 3*(a*
b^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3
*c^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqr
t(-a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 9*a^3*b^2*c
*d^2 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2*a^2*b^3*d^3*x^5 + 2
*(9*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 + 3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d -
9*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 5
*a^3*b*d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c^3 - 3*a^2*b^
3*c^2*d + 9*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

3.29.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(95) = 190$.

Time = 0.58 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.96

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{2a^2b^3+2ab^4x^2} - \frac{\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2\cdot(5ad+bc)\log\left(-\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2\cdot(5ad+bc)}{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2\cdot(5ad+bc)\log\left(\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2\cdot(5ad+bc)}{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3}+x\right)}{4} + \frac{d^3x^3}{3b^2}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

output
```
x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a
*b**2*c**2*d + b**3*c**3)/(2*a**2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b*
*7))*(a*d - b*c)**2*(5*a*d + b*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d
 - b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d
+ b**3*c**3) + x)/4 + sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)*lo
g(a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)/(5*a**3*d**3
 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3) + x)/4 + d**3*x**3/(3*b*
*2)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{2\,(ab^4x^2+a^2b^3)} + \frac{bd^3x^3+3\,(3\,bcd^2-2\,ad^3)x}{3\,b^3} + \frac{(b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^2 + a^2
*b^3) + 1/3*(b*d^3*x^3 + 3*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/2*(b^3*c^3 + 3
*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a*b^3)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{(b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3} + \frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\,(bx^2+a)ab^3} + \frac{b^4d^3x^3+9\,b^4cd^2x-6\,ab^3d^3x}{3\,b^6}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*a*b^3) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2
*x - a^3*d^3*x)/((b*x^2 + a)*a*b^3) + 1/3*(b^4*d^3*x^3 + 9*b^4*c*d^2*x - 6
*a*b^3*d^3*x)/b^6
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.72

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{d^3\,x^3}{3\,b^2} - x\left(\frac{2\,a\,d^3}{b^3} - \frac{3\,c\,d^2}{b^2}\right) - \frac{x\,(a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3)}{2\,a\,(b^4\,x^2 + a\,b^3)} + \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^2\,(5\,a\,d+b\,c)}{\sqrt{a}\,(5\,a^3\,d^3-9\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3)}\right)\,(a\,d-b\,c)^2\,(5\,a\,d+b\,c)}{2\,a^{3/2}\,b^{7/2}}$$

input
```
int((c + d*x^2)^3/(a + b*x^2)^2,x)
```

output
```
(d^3*x^3)/(3*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 + b^4*x^2)) + (atan((b^(
1/2)*x*(a*d - b*c)^2*(5*a*d + b*c))/(a^(1/2)*(5*a^3*d^3 + b^3*c^3 + 3*a*b^
2*c^2*d - 9*a^2*b*c*d^2)))*(a*d - b*c)^2*(5*a*d + b*c))/(2*a^(3/2)*b^(7/2)
)
```

3.30 $\int \frac{(c+dx^2)^2}{(a+bx^2)^2} dx$

3.30.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2} dx = \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{2ab^2(a+bx^2)} + \frac{(bc-ad)(bc+3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}$$

output
```
d^2*x/b^2+1/2*(-a*d+b*c)^2*x/a/b^2/(b*x^2+a)+1/2*(-a*d+b*c)*(3*a*d+b*c)*ar
ctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2} dx = \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{2ab^2(a+bx^2)} + \frac{(b^2c^2+2abcd-3a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}$$

input
```
Integrate[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

output
```
(d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b^2*c^2 + 2*a*b*
c*d - 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

3.30.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{ 300}$$

$$\int \left(\frac{-a^2d^2+2bdx^2(bc-ad)+b^2c^2}{b^2\,(a+bx^2)^2}+\frac{d^2}{b^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)(3ad+bc)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\,(a+bx^2)}+\frac{d^2x}{b^2}$$

input
```
Int[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

output
```
(d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a*d)*(b*c
+ 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

3.30.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.30.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

method	result
default	$\frac{d^2x}{b^2}-\frac{-\frac{\left(a^2d^2-2abcd+b^2c^2\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(3a^2d^2-2abcd-b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}$
risch	$\frac{d^2x}{b^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{2a\,b^2(b\,x^2+a)}-\frac{3a\ln(bx-\sqrt{-ab})\,d^2}{4b^2\sqrt{-ab}}+\frac{\ln(bx-\sqrt{-ab})\,cd}{2b\sqrt{-ab}}+\frac{\ln(bx-\sqrt{-ab})\,c^2}{4\sqrt{-ab}\,a}+\frac{3a\ln(-bx-\sqrt{-ab})\,d^2}{4b^2\sqrt{-ab}}-\frac{\ln(-$

input
```
int((d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*x/b^2-1/b^2*(-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/(b*x^2+a)+1/2*(3*a^2
*d^2-2*a*b*c*d-b^2*c^2)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.62

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{4\,a^2b^2d^2x^3+(ab^2c^2+2\,a^2bcd-3\,a^3d^2+(b^3c^2+2\,ab^2cd-3\,a^2bd^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(ab}{4\,(a^2b^4x^2+a^3b^3)}\right.$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2
+ 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x -
 a)/(b*x^2 + a)) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4
*x^2 + a^3*b^3), 1/2*(2*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3
*d^2 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*
b)*x/a) + (a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^2 + a^3*
b^3)]
```

3.30.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(73) = 146$.

Time = 0.44 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.88

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{x(a^2d^2-2abcd+b^2c^2)}{2a^2b^2+2ab^3x^2} + \frac{\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4} - \frac{\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)\log\left(\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4} + \frac{d^2x}{b^2}$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

output
```
x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + sqrt
(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(-a**2*b**2*sqrt(-1/(a**3*b*
*5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/
4 - sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(a**2*b**2*sqrt(-1/(
a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2
) + x)/4 + d**2*x/b**2
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{2\,(ab^3x^2+a^2b^2)} + \frac{d^2x}{b^2} + \frac{(b^2c^2+2\,abcd-3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^2 + a^2*b^2) + d^2*x/b^2 +
1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b
^2)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2x}{b^2} + \frac{(b^2c^2+2\,abcd-3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2} + \frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\,(bx^2+a)ab^2}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
d^2*x/b^2 + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*a*b^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a
*b^2)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2\,x}{b^2} + \frac{x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,a\,(b^3\,x^2+a\,b^2)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)\,(3\,a\,d+b\,c)}{\sqrt{a}\,(-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)\,(3\,a\,d+b\,c)}{2\,a^{3/2}\,b^{5/2}}$$

input
```
int((c + d*x^2)^2/(a + b*x^2)^2,x)
```

output
```
(d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*a*(a*b^2 + b^3*x^2))
+ (atan((b^(1/2)*x*(a*d - b*c)*(3*a*d + b*c))/(a^(1/2)*(b^2*c^2 - 3*a^2*d^
2 + 2*a*b*c*d)))*(a*d - b*c)*(3*a*d + b*c))/(2*a^(3/2)*b^(5/2))
```

3.31 $\int \frac{c+dx^2}{(a+bx^2)^2}\,dx$

3.31.1 Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{(bc-ad)x}{2ab\,(a+bx^2)} + \frac{(bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

output
```
1/2*(-a*d+b*c)*x/a/b/(b*x^2+a)+1/2*(a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(
3/2)/b^(3/2)
```

3.31.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = -\frac{(-bc+ad)x}{2ab\,(a+bx^2)} + \frac{(bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

input
```
Integrate[(c + d*x^2)/(a + b*x^2)^2,x]
```

output
```
-1/2*((-(b*c) + a*d)*x)/(a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

3.31.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \; 298$$

$$\frac{(ad+bc)\int \frac{1}{bx^2+a}dx}{2ab}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(ad+bc)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(c + d*x^2)/(a + b*x^2)^2,x]
```

output
```
((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(3/2)*b^(3/2))
```

3.31.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.31.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{(ad-bc)x}{2ab(b\,x^2+a)}+\frac{(ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$	57
risch	$-\frac{(ad-bc)x}{2ab(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)d}{4\sqrt{-ab}\,b}-\frac{\ln\left(bx+\sqrt{-ab}\right)c}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)d}{4\sqrt{-ab}\,b}+\frac{\ln\left(-bx+\sqrt{-ab}\right)c}{4\sqrt{-ab}\,a}$	122

input
```
int((d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*d-b*c)/a/b*x/(b*x^2+a)+1/2*(a*d+b*c)/a/b/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.87

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx$$
$$=\left[-\frac{(abc+a^2d+(b^2c+abd)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,(ab^2c-a^2bd)x}{4\,(a^2b^3x^2+a^3b^2)},\frac{(abc+a^2d+(b^2c+abd)x^2}{2\,(a}\right.$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqr
t(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*
b^2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b
)*x/a) + (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

3.31.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{x(-ad+bc)}{2a^2b+2ab^2x^2} - \frac{\sqrt{-\frac{1}{a^3b^3}}(ad+bc)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^3}}(ad+bc)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

output
```
x*(-a*d + b*c)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(a*d + b*
c)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(a*d + b
*c)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{(bc-ad)x}{2\,(ab^2x^2+a^2b)} + \frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c - a*d)*x/(a*b^2*x^2 + a^2*b) + 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a*b)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab} + \frac{bcx-adx}{2\,(bx^2+a)ab}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(b*c*x - a*d*x
)/((b*x^2 + a)*a*b)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d+b\,c)}{2\,a^{3/2}\,b^{3/2}} - \frac{x\,(a\,d-b\,c)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

input
```
int((c + d*x^2)/(a + b*x^2)^2,x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(a*d + b*c))/(2*a^(3/2)*b^(3/2)) - (x*(a*d - b*
c))/(2*a*b*(a + b*x^2))
```

3.32 $\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx$

3.32.1 Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{bx}{2a(bc-ad)(a+bx^2)} + \frac{\sqrt{b}(bc-3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}$$

output
```
1/2*b*x/a/(-a*d+b*c)/(b*x^2+a)+1/2*(-3*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))*
b^(1/2)/a^(3/2)/(-a*d+b*c)^2+d^(3/2)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^
2/c^(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = -\frac{bx}{2a(-bc+ad)(a+bx^2)} - \frac{\sqrt{b}(-bc+3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(-bc+ad)^2} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}$$

input
```
Integrate[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
-1/2*(b*x)/(a*(-(b*c) + a*d)*(a + b*x^2)) - (Sqrt[b]*(-(b*c) + 3*a*d)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*ArcTan[(S
qrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

3.32.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {316, 25, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)} dx$$

$$\Big\downarrow \text{316}$$

$$\frac{bx}{2a(a+bx^2)(bc-ad)} - \frac{\int -\frac{bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)} + \frac{bx}{2a(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{2ad^2\int \frac{1}{dx^2+c}dx}{bc-ad} + \frac{b(bc-3ad)\int \frac{1}{bx^2+a}dx}{bc-ad}}{2a(bc-ad)} + \frac{bx}{2a(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2ad^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-3ad)}{\sqrt{a}(bc-ad)}}{2a(bc-ad)} + \frac{bx}{2a(a+bx^2)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + ((Sqrt[b]*(b*c - 3*a*d)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) + (2*a*d^(3/2)*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(Sqrt[c]*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.32.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{b\left(\frac{(ad-bc)x}{2a\left(b\,x^{2}+a\right)}+\frac{(3ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^{2}}+\frac{d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^{2}\sqrt{cd}}$	95
risch	Expression too large to display	1035

input
```
int(1/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*d-b*c)^2*b*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(3*a*d-b*c)/a/(a*b)^(1/2
)*arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1
/2))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 699, normalized size of antiderivative = 6.47

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx$$
$$= \left[-\frac{(abc-3a^2d+(b^2c-3abd)x^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2(abdx^2+a^2d)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right.}{4(a^2b^2c^2-2a^3bcd+a^4d^2+(ab^3c^2-2a^2b^2cd+a^3bd^2)x^2)}\right.$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/4*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2
*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a*b*d*x^2 + a^2*d)*sqrt(-d/c)*log((
d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b^2*c - a*b*d)*x)/(a^2*b^2
*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2
), 1/4*(4*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (a*b*c - 3*a
^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)) + 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2
 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/2*((a*b*c - 3*a^2*d + (
b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (a*b*d*x^2 + a^2*d)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (b^2*c - a*b*
d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d +
a^3*b*d^2)*x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*
arctan(x*sqrt(b/a)) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c))
+ (b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2
*a^2*b^2*c*d + a^3*b*d^2)*x^2)]
```

3.32.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}} + \frac{bx}{2\,(a^2bc-a^3d+(ab^2c-a^2bd)x^2)} + \frac{(b^2c-3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{ab}}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/
2*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2) + 1/2*(b^2*c - 3*a*b*d)*
arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}} + \frac{(b^2c-3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{ab}} + \frac{bx}{2\,(abc-a^2d)(bx^2+a)}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/
2*(b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*
d^2)*sqrt(a*b)) + 1/2*b*x/((a*b*c - a^2*d)*(b*x^2 + a))
```

3.32.9 Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 3649, normalized size of antiderivative = 33.79

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
(atan((((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6
*a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^
7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c
^3*d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*
a^4*b*c*d^2) - (x*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^
3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 3
2*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^
3*b^2*c^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3
*b^2*c^2 - 2*a^4*b*c*d)))*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) +
((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*
c*d^4))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^7 - 2*a
*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4
+ 32*a^4*b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c
*d^2) + (x*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6
 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b
^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c
^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^
2 - 2*a^4*b*c*d)))*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))/(((3*a*b
^3*d^5)/2 - (b^4*c*d^4)/2)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^
4*b*c*d^2) - ((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^...
```

3.33 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.33.1 Optimal result

Integrand size = 19, antiderivative size = 167

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{d(bc+ad)x}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} + \frac{b^{3/2}(bc-5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3} + \frac{d^{3/2}(5bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}$$

output
```
1/2*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)+1/2*b*x/a/(-a*d+b*c)/(b*x^2+a
)/(d*x^2+c)+1/2*b^(3/2)*(-5*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a
*d+b*c)^3+1/2*d^(3/2)*(-a*d+5*b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d
+b*c)^3
```

3.33.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{1}{2}\left(\frac{b^{3/2}(-bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(-bc+ad)^3} + \frac{(bc-ad)x\left(\frac{b^2}{a^2+abx^2}+\frac{d^2}{c^2+cdx^2}\right)+\frac{d^{3/2}(5bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{(bc-ad)^3}\right)$$

input
```
Integrate[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((b^(3/2)*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) +
 a*d)^3) + ((b*c - a*d)*x*(b^2/(a^2 + a*b*x^2) + d^2/(c^2 + c*d*x^2)) + (d
^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c - a*d)^3)/
2
```

3.33.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {316, 25, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}dx$$

$$\Big\downarrow \text{ 316}$$

$$\frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\int -\frac{3bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)^2}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)^2}dx}{2a(bc-ad)}+\frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\int \frac{2\left(b^2c^2-4abdc+a^2d^2+bd(bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{b^2c^2-4abdc+a^2d^2+bd(bc+ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\frac{b^2c(bc-5ad)\int \frac{1}{bx^2+a}dx}{bc-ad}+\frac{ad^2(5bc-ad)\int \frac{1}{dx^2+c}dx}{bc-ad}}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{b^{3/2}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-5ad)}{\sqrt{a}(bc-ad)}+\frac{ad^{3/2}(5bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(b*x)/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + ((d*(b*c + a*d)*x)/(c*(b
*c - a*d)*(c + d*x^2)) + ((b^(3/2)*c*(b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(Sqrt[a]*(b*c - a*d)) + (a*d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(c*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.33.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{b^2\left(\frac{(ad-bc)x}{2a\left(b\,x^2+a\right)}+\frac{(5ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^3}+\frac{d^2\left(\frac{(ad-bc)x}{2c\left(d\,x^2+c\right)}+\frac{(ad-5bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}\right)}{(ad-bc)^3}$	133
risch	Expression too large to display	2124

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/(a*d-b*c)^3*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(5*a*d-b*c)/a/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^3*(1/2*(a*d-b*c)/c*x/(d*x^2+c)+1/2
*(a*d-5*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(143) = 286.

Time = 0.72 (sec) , antiderivative size = 1681, normalized size of antiderivative = 10.07

$$\int \frac{1}{(a+bx^2)^2\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^
2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*
sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (5*a^2*b*c^2*
d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b
*c*d^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^
2 + c)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^
5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2
*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b
^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d^
3)*x^3 + 2*(5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 +
(5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c
)) + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c
^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqr
t(-b/a) - a)/(b*x^2 + a)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d
^3)*x)/(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a
*b^4*c^4*d - 3*a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a
*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^
3*c^2*d - a^2*b*d^3)*x^3 + 2*(a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a
*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)
*arctan(x*sqrt(b/a)) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^...
```

3.33.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(143) = 286$.

Time = 0.29 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^2}\,dx = \frac{(b^3c - 5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3)\sqrt{ab}} + \frac{(5\,bcd^2 - ad^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^4 - 3\,ab^2c^3d + 3\,a^2bc^2d^2 - a^3cd^3)\sqrt{cd}} + \frac{(b^2cd + abd^2)x^3 + (b^2c^2 + a^2d^2)x}{2\,(a^2b^2c^4 - 2\,a^3bc^3d + a^4c^2d^2 + (ab^3c^3d - 2\,a^2b^2c^2d^2 + a^3bcd^3)x^4 + (ab^3c^4 - a^2b^2c^3d - a^3bc^2d^2 + a^4cd^3)}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*
d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d
*x/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqr
t(c*d)) + 1/2*((b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c
^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*
b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2
)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.39

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^2}\,dx = \frac{(b^3c - 5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3)\sqrt{ab}} + \frac{(5\,bcd^2 - ad^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^4 - 3\,ab^2c^3d + 3\,a^2bc^2d^2 - a^3cd^3)\sqrt{cd}} + \frac{b^2cdx^3 + abd^2x^3 + b^2c^2x + a^2d^2x}{2\,(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2)(bdx^4 + bcx^2 + adx^2 + ac)}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*
d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d
*x/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqr
t(c*d)) + 1/2*(b^2*c*d*x^3 + a*b*d^2*x^3 + b^2*c^2*x + a^2*d^2*x)/((a*b^2*
c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))
```

3.33.9 Mupad [B] (verification not implemented)

Time = 6.50 (sec) , antiderivative size = 6183, normalized size of antiderivative = 37.02

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
((x*(a^2*d^2 + b^2*c^2))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^
3*(a*d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^2*(a*d +
b*c) + b*d*x^4) + (atan(((((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^
4 - 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4
- 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^
9*d^2 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a
^4*b^7*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^
3*d^8 - 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d -
 6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^
4*d^4) - (x*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^
8*c^8*d^3 + 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6
+ 144*a^7*b^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*
d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^
2*d^4 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d -
b*c)*(-a^3*b^3)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5
*b*c*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 +
 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*
a*b^6*c^3*d^4 - 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 +
a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + ((
(2*a*b^10*c^9*d^2 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*...
```

3.34 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

3.34.1 Optimal result

Integrand size = 19, antiderivative size = 230

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2}+\frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2}+\frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3\left(c+dx^2\right)}+\frac{b^{5/2}(bc-7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(35b^2c^2-14abcd+3a^2d^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}$$

output
```
1/4*d*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*b*x/a/(-a*d+b*c)/(b*x
^2+a)/(d*x^2+c)^2+1/8*d*(-a*d+4*b*c)*(3*a*d+b*c)*x/a/c^2/(-a*d+b*c)^3/(d*x
^2+c)+1/2*b^(5/2)*(-7*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c
)^4+1/8*d^(3/2)*(3*a^2*d^2-14*a*b*c*d+35*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2)
)/c^(5/2)/(-a*d+b*c)^4
```

3.34.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^3} \, dx = \frac{1}{8} \left(-\frac{4b^3 x}{a(-bc+ad)^3 (a+bx^2)} + \frac{2d^2 x}{c(bc-ad)^2 (c+dx^2)^2} + \frac{d^2(11bc-3ad)x}{c^2(bc-ad)^3 (c+dx^2)} + \frac{4b^{5/2}(bc-7ad) \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^4} + \frac{d^{3/2}(35b^2c^2-14abcd+3a^2d^2) \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^4} \right)$$

input
```
Integrate[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((-4*b^3*x)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2*x)/(c*(b*c - a*d)^2*
(c + d*x^2)^2) + (d^2*(11*b*c - 3*a*d)*x)/(c^2*(b*c - a*d)^3*(c + d*x^2))
+ (4*b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*
d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(c^(5/2)*(b*c - a*d)^4))/8
```

3.34.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {316, 25, 402, 27, 402, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^3} \, dx$$

$$\Big\downarrow \; 316$$

$$\frac{bx}{2a(a+bx^2)(c+dx^2)^2(bc-ad)} - \frac{\int -\frac{5bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)^3}dx}{2a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{5bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)^3}dx}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 402

$$\frac{\frac{\int \frac{2\left(2b^2c^2-8abdc+3a^2d^2+3bd(2bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{4c(bc-ad)}+\frac{dx(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{2b^2c^2-8abdc+3a^2d^2+3bd(2bc+ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{2c(bc-ad)}+\frac{dx(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{4b^3c^3-24ab^2dc^2+11a^2bd^2c-3a^3d^3+bd(4bc-ad)(bc+3ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{dx(4bc-ad)(3ad+bc)}{2c\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{dx(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{ad^2\left(3a^2d^2-14abcd+35b^2c^2\right)\int \frac{1}{dx^2+c}dx}{bc-ad}+\frac{4b^3c^2(bc-7ad)\int \frac{1}{bx^2+a}dx}{bc-ad}}{2c(bc-ad)}+\frac{dx(4bc-ad)(3ad+bc)}{2c\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{dx(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{ad^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}+\frac{4b^{5/2}c^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-7ad)}{\sqrt{a}(bc-ad)}}{2c(bc-ad)}+\frac{dx(4bc-ad)(3ad+bc)}{2c\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{dx(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

3.34. $\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx$

output
```
(b*x)/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((d*(2*b*c + a*d)*x)/(
2*c*(b*c - a*d)*(c + d*x^2)^2) + ((d*(4*b*c - a*d)*(b*c + 3*a*d)*x)/(2*c*(
b*c - a*d)*(c + d*x^2)) + ((4*b^(5/2)*c^2*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) + (a*d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3
*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(2*c*(b*c -
a*d)))/(2*c*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.34.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

method	result
default	$-\frac{b^3\left(\frac{(ad-bc)x}{2a\left(b\,x^2+a\right)}+\frac{(7ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^4}+\frac{d^2\left(\frac{\frac{d\left(3a^2d^2-14abcd+11b^2c^2\right)x^3}{8c^2}+\frac{\left(5a^2d^2-18abcd+13b^2c^2\right)x}{8c}}{\left(d\,x^2+c\right)^2}+\frac{\left(3a^2d^2-14abcd+35b^2c^2\right.}{8c^2\sqrt{cd}}\right.}{(ad-bc)^4}$
risch	Expression too large to display

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-b^3/(a*d-b*c)^4*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(7*a*d-b*c)/a/(a*b)^(1/2
)*arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^4*((1/8*d*(3*a^2*d^2-14*a*b*c*d+1
1*b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-18*a*b*c*d+13*b^2*c^2)/c*x)/(d*x^2+c)^2+
1/8*(3*a^2*d^2-14*a*b*c*d+35*b^2*c^2)/c^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/
2)))
```

3.34.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. 2(204) = 408.

Time = 2.34 (sec) , antiderivative size = 3239, normalized size of antiderivative = 14.08

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5
)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d
^4 + 3*a^4*d^5)*x^3 - 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*
b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^
4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a)*log((b*
x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^
3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5
)*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)
*x^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d
^4)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4
*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c
*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*
d^3 + a^6*c^4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4
 - 4*a^4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6
*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d
^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5
*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^
3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c
^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + (35*a^2*b^2*
c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b...
```

3.34.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^3}\, dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(204) = 408$.

Time = 0.30 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.30

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{(b^4c-7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4)\sqrt{ab}} + \frac{(35\,b^2c^2d^2-14\,abcd^3+3\,a^2d^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^6-4\,ab^3c^5d+6\,a^2b^2c^4d^2-4\,a^3bc^3d^3+a^4c^2d^4)\sqrt{cd}} + \frac{(4\,b^3c^2d^2+11\,ab^2cd^3-3\,a^2bd^4)x^5+(8\,b^3c^3d+13\,ab^2c^2}{8\,(a^2b^3c^7-3\,a^3b^2c^6d+3\,a^4bc^5d^2-a^5c^4d^3+(ab^4c^5d^2-3\,a^2b^3c^4d^3+3\,a^3b^2c^3d^4-a^4bc^2d^5)x^6+(2\,ab^4c^6}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^3*c^3*
d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35*b^2*
c^2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 - 4*a*
b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d))
+ 1/8*((4*b^3*c^2*d^2 + 11*a*b^2*c*d^3 - 3*a^2*b*d^4)*x^5 + (8*b^3*c^3*d +
 13*a*b^2*c^2*d^2 + 6*a^2*b*c*d^3 - 3*a^3*d^4)*x^3 + (4*b^3*c^4 + 13*a^2*b
*c^2*d^2 - 5*a^3*c*d^3)*x)/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5*d^
2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4 -
 a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*d
^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3
*b^2*c^5*d^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2)
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{b^3x}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)(bx^2+a)} + \frac{(b^4c-7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4)\sqrt{ab}} + \frac{(35\,b^2c^2d^2-14\,abcd^3+3\,a^2d^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^6-4\,ab^3c^5d+6\,a^2b^2c^4d^2-4\,a^3bc^3d^3+a^4c^2d^4)\sqrt{cd}} + \frac{11\,bcd^3x^3-3\,ad^4x^3+13\,bc^2d^2x-5\,acd^3x}{8\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)(dx^2+c)^2}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b*x^2
+ a)) + 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*
b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*
(35*b^2*c^2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^
6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqr
t(c*d)) + 1/8*(11*b*c*d^3*x^3 - 3*a*d^4*x^3 + 13*b*c^2*d^2*x - 5*a*c*d^3*x
)/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2
)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 8649, normalized size of antiderivative = 37.60

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
(atan(((((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b
^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^
7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 +
15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) - (((2*a*b^
13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*
b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b
^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*
d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 -
 a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2
- 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8
*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b
^2*c^2 - 14*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 512
0*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a
^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b
^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*
b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6
 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7
*d^3 + 15*a^6*b^2*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14
*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^
2 - 4*a*b^3*c^8*d))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*...
```

3.35 $\int \frac{(c+dx^2)^5}{(a+bx^2)^3} dx$

3.35.1 Optimal result

Integrand size = 19, antiderivative size = 196

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^3} dx = \frac{d^3(10b^2c^2-15abcd+6a^2d^2)x}{b^5} + \frac{d^4(5bc-3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5(a+bx^2)^2} + \frac{(bc-ad)^4(3bc+17ad)x}{8a^2b^5(a+bx^2)} + \frac{(bc-ad)^3(3b^2c^2+14abcd+63a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}$$

output
```
d^3*(6*a^2*d^2-15*a*b*c*d+10*b^2*c^2)*x/b^5+1/3*d^4*(-3*a*d+5*b*c)*x^3/b^4
+1/5*d^5*x^5/b^3+1/4*(-a*d+b*c)^5*x/a/b^5/(b*x^2+a)^2+1/8*(-a*d+b*c)^4*(17
*a*d+3*b*c)*x/a^2/b^5/(b*x^2+a)+1/8*(-a*d+b*c)^3*(63*a^2*d^2+14*a*b*c*d+3*
b^2*c^2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(11/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^3} dx = \frac{d^3(10b^2c^2-15abcd+6a^2d^2)x}{b^5} + \frac{d^4(5bc-3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5(a+bx^2)^2} + \frac{(bc-ad)^4(3bc+17ad)x}{8a^2b^5(a+bx^2)} + \frac{(bc-ad)^3(3b^2c^2+14abcd+63a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}$$

input
```
Integrate[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

output
```
(d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c - 3*a*d)*x
^3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^2)^2
) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b^5*(a + b*x^2)) + ((b*c - a
*d)^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(5/2)*b^(11/2))
```

3.35.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^5}{\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \ 300$$

$$\int \left(\frac{(bc-ad)^3\left(6a^2d^2+3abcd+b^2c^2\right)+10b^2d^2x^4(bc-ad)^3+5bdx^2(bc-ad)^3(3ad+bc)}{b^5\left(a+bx^2\right)^3}+\frac{d^3\left(6a^2d^2-15abcd\right.}{b^5}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{x(17ad+3bc)(bc-ad)^4}{8a^2b^5\left(a+bx^2\right)}+\frac{d^3x\left(6a^2d^2-15abcd+10b^2c^2\right)}{b^5}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(63a^2d^2+14abcd+3b^2c^2\right)(bc-ad)^3}{8a^{5/2}b^{11/2}}+\frac{x(bc-ad)^5}{4ab^5\left(a+bx^2\right)^2}+\frac{d^4x^3(5bc-3ad)}{3b^4}+\frac{d^5x^5}{5b^3}$$

input
```
Int[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

output
```
(d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c - 3*a*d)*x
^3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^2)^2
) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b^5*(a + b*x^2)) + ((b*c - a
*d)^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(5/2)*b^(11/2))
```

3.35.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.35.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.59

method	result
default	$\frac{d^3\left(\frac{1}{5}b^2d^2x^5-x^3ab\,d^2+\frac{5}{3}x^3b^2cd+6a^2d^2x-15abcdx+10b^2c^2x\right)}{b^5}-\frac{-\frac{b\left(17a^5d^5-65a^4bc\,d^4+90a^3b^2c^2d^3-50a^2b^3c^3d^2+5a\,b^4c^4d+3b^5c^5\right)x^3}{8a^2}\cdots}{\left(b\,x^2+a\cdots\right.}$
risch	$\frac{d^5x^5}{5b^3}-\frac{d^5x^3a}{b^4}+\frac{5d^4x^3c}{3b^3}+\frac{6d^5a^2x}{b^5}-\frac{15d^4acx}{b^4}+\frac{10d^3c^2x}{b^3}+\frac{\frac{b\left(17a^5d^5-65a^4bc\,d^4+90a^3b^2c^2d^3-50a^2b^3c^3d^2+5a\,b^4c^4d+3b^5c^5\right)x\cdots}{8a^2}}{b^5(b\,x\cdots}$

input
```
int((d*x^2+c)^5/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
d^3/b^5*(1/5*b^2*d^2*x^5-x^3*a*b*d^2+5/3*x^3*b^2*c*d+6*a^2*d^2*x-15*a*b*c*
d*x+10*b^2*c^2*x)-1/b^5*((-1/8*b*(17*a^5*d^5-65*a^4*b*c*d^4+90*a^3*b^2*c^2
*d^3-50*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d+3*b^5*c^5)/a^2*x^3-5/8*(3*a^5*d^5-11
*a^4*b*c*d^4+14*a^3*b^2*c^2*d^3-6*a^2*b^3*c^3*d^2-a*b^4*c^4*d+b^5*c^5)/a*x
)/(b*x^2+a)^2+1/8*(63*a^5*d^5-175*a^4*b*c*d^4+150*a^3*b^2*c^2*d^3-30*a^2*b
^3*c^3*d^2-5*a*b^4*c^4*d-3*b^5*c^5)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
))
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(178) = 356.

Time = 0.26 (sec) , antiderivative size = 1044, normalized size of antiderivative = 5.33

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx$$

$$= \left[\frac{48\,a^3b^5d^5x^9+16\,(25\,a^3b^5cd^4-9\,a^4b^4d^5)x^7+16\,(150\,a^3b^5c^2d^3-175\,a^4b^4cd^4+63\,a^5b^3d^5)x^5+10\,(9\,ab^7c^5\cdots}{\cdots}\right.$$

input
```
integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/240*(48*a^3*b^5*d^5*x^9 + 16*(25*a^3*b^5*c*d^4 - 9*a^4*b^4*d^5)*x^7 + 1
6*(150*a^3*b^5*c^2*d^3 - 175*a^4*b^4*c*d^4 + 63*a^5*b^3*d^5)*x^5 + 10*(9*a
*b^7*c^5 + 15*a^2*b^6*c^4*d - 150*a^3*b^5*c^3*d^2 + 750*a^4*b^4*c^2*d^3 -
875*a^5*b^3*c*d^4 + 315*a^6*b^2*d^5)*x^3 + 15*(3*a^2*b^5*c^5 + 5*a^3*b^4*c
^4*d + 30*a^4*b^3*c^3*d^2 - 150*a^5*b^2*c^2*d^3 + 175*a^6*b*c*d^4 - 63*a^7
*d^5 + (3*b^7*c^5 + 5*a*b^6*c^4*d + 30*a^2*b^5*c^3*d^2 - 150*a^3*b^4*c^2*d
^3 + 175*a^4*b^3*c*d^4 - 63*a^5*b^2*d^5)*x^4 + 2*(3*a*b^6*c^5 + 5*a^2*b^5*
c^4*d + 30*a^3*b^4*c^3*d^2 - 150*a^4*b^3*c^2*d^3 + 175*a^5*b^2*c*d^4 - 63*
a^6*b*d^5)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) +
 30*(5*a^2*b^6*c^5 - 5*a^3*b^5*c^4*d - 30*a^4*b^4*c^3*d^2 + 150*a^5*b^3*c^
2*d^3 - 175*a^6*b^2*c*d^4 + 63*a^7*b*d^5)*x)/(a^3*b^8*x^4 + 2*a^4*b^7*x^2
+ a^5*b^6), 1/120*(24*a^3*b^5*d^5*x^9 + 8*(25*a^3*b^5*c*d^4 - 9*a^4*b^4*d^
5)*x^7 + 8*(150*a^3*b^5*c^2*d^3 - 175*a^4*b^4*c*d^4 + 63*a^5*b^3*d^5)*x^5
+ 5*(9*a*b^7*c^5 + 15*a^2*b^6*c^4*d - 150*a^3*b^5*c^3*d^2 + 750*a^4*b^4*c^
2*d^3 - 875*a^5*b^3*c*d^4 + 315*a^6*b^2*d^5)*x^3 + 15*(3*a^2*b^5*c^5 + 5*a
^3*b^4*c^4*d + 30*a^4*b^3*c^3*d^2 - 150*a^5*b^2*c^2*d^3 + 175*a^6*b*c*d^4
- 63*a^7*d^5 + (3*b^7*c^5 + 5*a*b^6*c^4*d + 30*a^2*b^5*c^3*d^2 - 150*a^3*b
^4*c^2*d^3 + 175*a^4*b^3*c*d^4 - 63*a^5*b^2*d^5)*x^4 + 2*(3*a*b^6*c^5 + 5*
a^2*b^5*c^4*d + 30*a^3*b^4*c^3*d^2 - 150*a^4*b^3*c^2*d^3 + 175*a^5*b^2*c*d
^4 - 63*a^6*b*d^5)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 15*(5*a^2*b^6...
```

3.35.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(187) = 374$.

Time = 7.52 (sec) , antiderivative size = 615, normalized size of antiderivative = 3.14

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx = x^3\left(-\frac{ad^5}{b^4}+\frac{5cd^4}{3b^3}\right)+x\left(\frac{6a^2d^5}{b^5}-\frac{15acd^4}{b^4}+\frac{10c^2d^3}{b^3}\right)$$
$$+\frac{\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3\cdot(63a^2d^2+14abcd+3b^2c^2)\log\left(-\frac{a^3b^5\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3\cdot(63a^2d^2+14abcd+3b^2c^2)}{63a^5d^5-175a^4bcd^4+150a^3b^2c^2d^3-30a^2b^3c^3d^2-5ab^4c^4d-3b^5c^5}\right.}{16}$$
$$-\frac{\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3\cdot(63a^2d^2+14abcd+3b^2c^2)\log\left(\frac{a^3b^5\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3\cdot(63a^2d^2+14abcd+3b^2c^2)}{63a^5d^5-175a^4bcd^4+150a^3b^2c^2d^3-30a^2b^3c^3d^2-5ab^4c^4d-3b^5c^5}+\right.}{16}$$
$$+\frac{x^3\cdot(17a^5bd^5-65a^4b^2cd^4+90a^3b^3c^2d^3-50a^2b^4c^3d^2+5ab^5c^4d+3b^6c^5)+x(15a^6d^5-55a^5bcd^4+70a^4}{8a^4b^5+16a^3b^6x^2+8a^2b^7x^4}$$
$$+\frac{d^5x^5}{5b^3}$$

input
```
integrate((d*x**2+c)**5/(b*x**2+a)**3,x)
```

output
```
x**3*(-a*d**5/b**4 + 5*c*d**4/(3*b**3)) + x*(6*a**2*d**5/b**5 - 15*a*c*d**
4/b**4 + 10*c**2*d**3/b**3) + sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**
2*d**2 + 14*a*b*c*d + 3*b**2*c**2)*log(-a**3*b**5*sqrt(-1/(a**5*b**11))*(a
*d - b*c)**3*(63*a**2*d**2 + 14*a*b*c*d + 3*b**2*c**2)/(63*a**5*d**5 - 175
*a**4*b*c*d**4 + 150*a**3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 - 5*a*b*
*4*c**4*d - 3*b**5*c**5) + x)/16 - sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(6
3*a**2*d**2 + 14*a*b*c*d + 3*b**2*c**2)*log(a**3*b**5*sqrt(-1/(a**5*b**11)
)*(a*d - b*c)**3*(63*a**2*d**2 + 14*a*b*c*d + 3*b**2*c**2)/(63*a**5*d**5 -
 175*a**4*b*c*d**4 + 150*a**3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 - 5*
a*b**4*c**4*d - 3*b**5*c**5) + x)/16 + (x**3*(17*a**5*b*d**5 - 65*a**4*b**
2*c*d**4 + 90*a**3*b**3*c**2*d**3 - 50*a**2*b**4*c**3*d**2 + 5*a*b**5*c**4
*d + 3*b**6*c**5) + x*(15*a**6*d**5 - 55*a**5*b*c*d**4 + 70*a**4*b**2*c**2
*d**3 - 30*a**3*b**3*c**3*d**2 - 5*a**2*b**4*c**4*d + 5*a*b**5*c**5))/(8*a
**4*b**5 + 16*a**3*b**6*x**2 + 8*a**2*b**7*x**4) + d**5*x**5/(5*b**3)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.70

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx$$
$$= \frac{(3\,b^6c^5 + 5\,ab^5c^4d - 50\,a^2b^4c^3d^2 + 90\,a^3b^3c^2d^3 - 65\,a^4b^2cd^4 + 17\,a^5bd^5)x^3 + 5\,(ab^5c^5 - a^2b^4c^4d - 6\,a^3b^3c^3a}{8\,(a^2b^7x^4 + 2\,a^3b^6x^2 + a^4b^5)}$$
$$+ \frac{3\,b^2d^5x^5 + 5\,(5\,b^2cd^4 - 3\,abd^5)x^3 + 15\,(10\,b^2c^2d^3 - 15\,abcd^4 + 6\,a^2d^5)x}{15\,b^5}$$
$$+ \frac{(3\,b^5c^5 + 5\,ab^4c^4d + 30\,a^2b^3c^3d^2 - 150\,a^3b^2c^2d^3 + 175\,a^4bcd^4 - 63\,a^5d^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^5}$$

input
```
integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((3*b^6*c^5 + 5*a*b^5*c^4*d - 50*a^2*b^4*c^3*d^2 + 90*a^3*b^3*c^2*d^3
- 65*a^4*b^2*c*d^4 + 17*a^5*b*d^5)*x^3 + 5*(a*b^5*c^5 - a^2*b^4*c^4*d - 6*
a^3*b^3*c^3*d^2 + 14*a^4*b^2*c^2*d^3 - 11*a^5*b*c*d^4 + 3*a^6*d^5)*x)/(a^2
*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5) + 1/15*(3*b^2*d^5*x^5 + 5*(5*b^2*c*d^4
 - 3*a*b*d^5)*x^3 + 15*(10*b^2*c^2*d^3 - 15*a*b*c*d^4 + 6*a^2*d^5)*x)/b^5
+ 1/8*(3*b^5*c^5 + 5*a*b^4*c^4*d + 30*a^2*b^3*c^3*d^2 - 150*a^3*b^2*c^2*d^
3 + 175*a^4*b*c*d^4 - 63*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^5
)
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.73

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx$$
$$= \frac{(3\,b^5c^5+5\,ab^4c^4d+30\,a^2b^3c^3d^2-150\,a^3b^2c^2d^3+175\,a^4bcd^4-63\,a^5d^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^5}$$
$$+\frac{3\,b^6c^5x^3+5\,ab^5c^4dx^3-50\,a^2b^4c^3d^2x^3+90\,a^3b^3c^2d^3x^3-65\,a^4b^2cd^4x^3+17\,a^5bd^5x^3+5\,ab^5c^5x-5\,a^2b^4c}{8\,(bx^2+a)^2a^2b^5}$$
$$+\frac{3\,b^{12}d^5x^5+25\,b^{12}cd^4x^3-15\,ab^{11}d^5x^3+150\,b^{12}c^2d^3x-225\,ab^{11}cd^4x+90\,a^2b^{10}d^5x}{15\,b^{15}}$$

input
```
integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*b^5*c^5 + 5*a*b^4*c^4*d + 30*a^2*b^3*c^3*d^2 - 150*a^3*b^2*c^2*d^3
+ 175*a^4*b*c*d^4 - 63*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^5)
+ 1/8*(3*b^6*c^5*x^3 + 5*a*b^5*c^4*d*x^3 - 50*a^2*b^4*c^3*d^2*x^3 + 90*a^3
*b^3*c^2*d^3*x^3 - 65*a^4*b^2*c*d^4*x^3 + 17*a^5*b*d^5*x^3 + 5*a*b^5*c^5*x
 - 5*a^2*b^4*c^4*d*x - 30*a^3*b^3*c^3*d^2*x + 70*a^4*b^2*c^2*d^3*x - 55*a^
5*b*c*d^4*x + 15*a^6*d^5*x)/((b*x^2 + a)^2*a^2*b^5) + 1/15*(3*b^12*d^5*x^5
 + 25*b^12*c*d^4*x^3 - 15*a*b^11*d^5*x^3 + 150*b^12*c^2*d^3*x - 225*a*b^11
*c*d^4*x + 90*a^2*b^10*d^5*x)/b^15
```

3.35.9 Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.09

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx$$
$$= \frac{\frac{5\,x\,\left(3\,a^5\,d^5-11\,a^4\,b\,c\,d^4+14\,a^3\,b^2\,c^2\,d^3-6\,a^2\,b^3\,c^3\,d^2-a\,b^4\,c^4\,d+b^5\,c^5\right)}{8\,a}+\frac{x^3\,\left(17\,a^5\,b\,d^5-65\,a^4\,b^2\,c\,d^4+90\,a^3\,b^3\,c^2\,d^3-50\,a^2\,b^4\,c^3\,d^2+5\,a\,b^5\,c^4\,d+\right.}{8\,a^2}}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}$$
$$-x^3\,\left(\frac{a\,d^5}{b^4}-\frac{5\,c\,d^4}{3\,b^3}\right)+x\,\left(\frac{3\,a\,\left(\frac{3\,a\,d^5}{b^4}-\frac{5\,c\,d^4}{b^3}\right)}{b}-\frac{3\,a^2\,d^5}{b^5}+\frac{10\,c^2\,d^3}{b^3}\right)+\frac{d^5\,x^5}{5\,b^3}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^3\,\left(63\,a^2\,d^2+14\,a\,b\,c\,d+3\,b^2\,c^2\right)}{\sqrt{a}\,\left(-63\,a^5\,d^5+175\,a^4\,b\,c\,d^4-150\,a^3\,b^2\,c^2\,d^3+30\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d+3\,b^5\,c^5\right)}\right)\,{\left(a\,d-b\,c\right)}^3\,\left(63\,a^2\,d^2+14\,a\,b\,c\,d+3\,b\right.}{8\,a^{5/2}\,b^{11/2}}$$

input
```
int((c + d*x^2)^5/(a + b*x^2)^3,x)
```

output
```
((5*x*(3*a^5*d^5 + b^5*c^5 - 6*a^2*b^3*c^3*d^2 + 14*a^3*b^2*c^2*d^3 - a*b^
4*c^4*d - 11*a^4*b*c*d^4))/(8*a) + (x^3*(3*b^6*c^5 + 17*a^5*b*d^5 - 65*a^4
*b^2*c*d^4 - 50*a^2*b^4*c^3*d^2 + 90*a^3*b^3*c^2*d^3 + 5*a*b^5*c^4*d))/(8*
a^2))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) - x^3*((a*d^5)/b^4 - (5*c*d^4)/(3*
b^3)) + x*((3*a*((3*a*d^5)/b^4 - (5*c*d^4)/b^3))/b - (3*a^2*d^5)/b^5 + (10
*c^2*d^3)/b^3) + (d^5*x^5)/(5*b^3) + (atan((b^(1/2)*x*(a*d - b*c)^3*(63*a^
2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(a^(1/2)*(3*b^5*c^5 - 63*a^5*d^5 + 30*a^2
*b^3*c^3*d^2 - 150*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d + 175*a^4*b*c*d^4))*(a
*d - b*c)^3*(63*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(8*a^(5/2)*b^(11/2))
```

3.36 $\int \frac{(c+dx^2)^4}{(a+bx^2)^3} dx$

3.36.1 Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3} dx = \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4(a+bx^2)^2} + \frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4(a+bx^2)} + \frac{(bc-ad)^2(3b^2c^2+10abcd+35a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}$$

output
```
d^3*(-3*a*d+4*b*c)*x/b^4+1/3*d^4*x^3/b^3+1/4*(-a*d+b*c)^4*x/a/b^4/(b*x^2+a
)^2+1/8*(-a*d+b*c)^3*(13*a*d+3*b*c)*x/a^2/b^4/(b*x^2+a)+1/8*(-a*d+b*c)^2*(
35*a^2*d^2+10*a*b*c*d+3*b^2*c^2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(9/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3} dx = \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4(a+bx^2)^2} + \frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4(a+bx^2)} + \frac{(bc-ad)^2(3b^2c^2+10abcd+35a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}$$

input
```
Integrate[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

output
```
(d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*x)/(4*a*b
^4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(8*a^2*b^4*(a + b*x
^2)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

3.36.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{3a^4d^4-4a^3bcd^3+6b^2d^2x^4(bc-ad)^2+4bdx^2(bc-ad)^2(2ad+bc)+b^4c^4}{b^4(a+bx^2)^3}+\frac{d^3(4bc-3ad)}{b^4}+\frac{d^4x^2}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(bc-ad)^3(13ad+3bc)}{8a^2b^4(a+bx^2)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^2\left(35a^2d^2+10abcd+3b^2c^2\right)}{8a^{5/2}b^{9/2}}+\frac{d^3x(4bc-3ad)}{b^4}+\frac{x(bc-ad)^4}{4ab^4(a+bx^2)^2}+\frac{d^4x^3}{3b^3}$$

input
```
Int[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

output
```
(d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*x)/(4*a*b
^4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(8*a^2*b^4*(a + b*x
^2)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

3.36.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.36.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.44

method	result
default	$-\frac{d^3\left(-\frac{1}{3}bd\,x^3+3adx-4bcx\right)}{b^4}+\frac{\frac{-\frac{b\left(13a^4d^4-36a^3bc\,d^3+30a^2b^2c^2d^2-4a\,b^3c^3d-3b^4c^4\right)x^3}{8a^2}-\frac{\left(11a^4d^4-28a^3bc\,d^3+18a^2b^2c^2d^2+4a\,b^3c^3d-5b}{8a}}{\left(b\,x^2+a\right)^2}}{b^4}$
risch	$\frac{d^4x^3}{3b^3}-\frac{3d^4ax}{b^4}+\frac{4d^3cx}{b^3}+\frac{-\frac{b\left(13a^4d^4-36a^3bc\,d^3+30a^2b^2c^2d^2-4a\,b^3c^3d-3b^4c^4\right)x^3}{8a^2}-\frac{\left(11a^4d^4-28a^3bc\,d^3+18a^2b^2c^2d^2+4a\,b^3c^3d-5b^4c^4}{8a}}{b^4\left(b\,x^2+a\right)^2}$

input
```
int((d*x^2+c)^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-d^3/b^4*(-1/3*b*d*x^3+3*a*d*x-4*b*c*x)+1/b^4*((-1/8*b*(13*a^4*d^4-36*a^3*
b*c*d^3+30*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d-3*b^4*c^4)/a^2*x^3-1/8*(11*a^4*d^
4-28*a^3*b*c*d^3+18*a^2*b^2*c^2*d^2+4*a*b^3*c^3*d-5*b^4*c^4)/a*x)/(b*x^2+a
)^2+1/8*(35*a^4*d^4-60*a^3*b*c*d^3+18*a^2*b^2*c^2*d^2+4*a*b^3*c^3*d+3*b^4*
c^4)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(144) = 288.

Time = 0.26 (sec) , antiderivative size = 817, normalized size of antiderivative = 5.11

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^3}\,dx$$
$$=\left[\frac{16\,a^3b^4d^4x^7+16\,(12\,a^3b^4cd^3-7\,a^4b^3d^4)x^5+2\,(9\,ab^6c^4+12\,a^2b^5c^3d-90\,a^3b^4c^2d^2+300\,a^4b^3cd^3-175\,a}{}\right.$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*a^3*b^4*d^4*x^7 + 16*(12*a^3*b^4*c*d^3 - 7*a^4*b^3*d^4)*x^5 + 2*
(9*a*b^6*c^4 + 12*a^2*b^5*c^3*d - 90*a^3*b^4*c^2*d^2 + 300*a^4*b^3*c*d^3 -
 175*a^5*b^2*d^4)*x^3 - 3*(3*a^2*b^4*c^4 + 4*a^3*b^3*c^3*d + 18*a^4*b^2*c^
2*d^2 - 60*a^5*b*c*d^3 + 35*a^6*d^4 + (3*b^6*c^4 + 4*a*b^5*c^3*d + 18*a^2*
b^4*c^2*d^2 - 60*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(3*a*b^5*c^4 + 4*
a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 60*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)
*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*a^2*b^5*c
^4 - 4*a^3*b^4*c^3*d - 18*a^4*b^3*c^2*d^2 + 60*a^5*b^2*c*d^3 - 35*a^6*b*d^
4)*x)/(a^3*b^7*x^4 + 2*a^4*b^6*x^2 + a^5*b^5), 1/24*(8*a^3*b^4*d^4*x^7 + 8
*(12*a^3*b^4*c*d^3 - 7*a^4*b^3*d^4)*x^5 + (9*a*b^6*c^4 + 12*a^2*b^5*c^3*d
- 90*a^3*b^4*c^2*d^2 + 300*a^4*b^3*c*d^3 - 175*a^5*b^2*d^4)*x^3 + 3*(3*a^2
*b^4*c^4 + 4*a^3*b^3*c^3*d + 18*a^4*b^2*c^2*d^2 - 60*a^5*b*c*d^3 + 35*a^6*
d^4 + (3*b^6*c^4 + 4*a*b^5*c^3*d + 18*a^2*b^4*c^2*d^2 - 60*a^3*b^3*c*d^3 +
 35*a^4*b^2*d^4)*x^4 + 2*(3*a*b^5*c^4 + 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d
^2 - 60*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a)
 + 3*(5*a^2*b^5*c^4 - 4*a^3*b^4*c^3*d - 18*a^4*b^3*c^2*d^2 + 60*a^5*b^2*c*
d^3 - 35*a^6*b*d^4)*x)/(a^3*b^7*x^4 + 2*a^4*b^6*x^2 + a^5*b^5)]
```

3.36.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(150) = 300$.

Time = 1.83 (sec) , antiderivative size = 515, normalized size of antiderivative = 3.22

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx = x\left(-\frac{3ad^4}{b^4}+\frac{4cd^3}{b^3}\right) - \frac{\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2\cdot(35a^2d^2+10abcd+3b^2c^2)\log\left(-\frac{a^3b^4\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2\cdot(35a^2d^2+10abcd+3b^2c^2)}{35a^4d^4-60a^3bcd^3+18a^2b^2c^2d^2+4ab^3c^3d+3b^4c^4}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2\cdot(35a^2d^2+10abcd+3b^2c^2)\log\left(\frac{a^3b^4\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2\cdot(35a^2d^2+10abcd+3b^2c^2)}{35a^4d^4-60a^3bcd^3+18a^2b^2c^2d^2+4ab^3c^3d+3b^4c^4}+x\right)}{16} + \frac{x^3(-13a^4bd^4+36a^3b^2cd^3-30a^2b^3c^2d^2+4ab^4c^3d+3b^5c^4)+x(-11a^5d^4+28a^4bcd^3-18a^3b^2c^2d^2-4a}{8a^4b^4+16a^3b^5x^2+8a^2b^6x^4} + \frac{d^4x^3}{3b^3}$$

input
```
integrate((d*x**2+c)**4/(b*x**2+a)**3,x)
```

output
```
x*(-3*a*d**4/b**4 + 4*c*d**3/b**3) - sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(
35*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)*log(-a**3*b**4*sqrt(-1/(a**5*b**9
))*(a*d - b*c)**2*(35*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)/(35*a**4*d**4
- 60*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d**2 + 4*a*b**3*c**3*d + 3*b**4*c**
4) + x)/16 + sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(35*a**2*d**2 + 10*a*b*c*
d + 3*b**2*c**2)*log(a**3*b**4*sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(35*a**
2*d**2 + 10*a*b*c*d + 3*b**2*c**2)/(35*a**4*d**4 - 60*a**3*b*c*d**3 + 18*a
**2*b**2*c**2*d**2 + 4*a*b**3*c**3*d + 3*b**4*c**4) + x)/16 + (x**3*(-13*a
**4*b*d**4 + 36*a**3*b**2*c*d**3 - 30*a**2*b**3*c**2*d**2 + 4*a*b**4*c**3*
d + 3*b**5*c**4) + x*(-11*a**5*d**4 + 28*a**4*b*c*d**3 - 18*a**3*b**2*c**2
*d**2 - 4*a**2*b**3*c**3*d + 5*a*b**4*c**4))/(8*a**4*b**4 + 16*a**3*b**5*x
**2 + 8*a**2*b**6*x**4) + d**4*x**3/(3*b**3)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx$$
$$= \frac{(3\,b^5c^4 + 4\,ab^4c^3d - 30\,a^2b^3c^2d^2 + 36\,a^3b^2cd^3 - 13\,a^4bd^4)x^3 + (5\,ab^4c^4 - 4\,a^2b^3c^3d - 18\,a^3b^2c^2d^2 + 28\,a^4bc}{8\,(a^2b^6x^4 + 2\,a^3b^5x^2 + a^4b^4)}$$
$$+ \frac{bd^4x^3 + 3\,(4\,bcd^3 - 3\,ad^4)x}{3\,b^4}$$
$$+ \frac{(3\,b^4c^4 + 4\,ab^3c^3d + 18\,a^2b^2c^2d^2 - 60\,a^3bcd^3 + 35\,a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^4}$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((3*b^5*c^4 + 4*a*b^4*c^3*d - 30*a^2*b^3*c^2*d^2 + 36*a^3*b^2*c*d^3 -
13*a^4*b*d^4)*x^3 + (5*a*b^4*c^4 - 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 +
28*a^4*b*c*d^3 - 11*a^5*d^4)*x)/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4) +
1/3*(b*d^4*x^3 + 3*(4*b*c*d^3 - 3*a*d^4)*x)/b^4 + 1/8*(3*b^4*c^4 + 4*a*b^3
*c^3*d + 18*a^2*b^2*c^2*d^2 - 60*a^3*b*c*d^3 + 35*a^4*d^4)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^2*b^4)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx = \frac{(3\,b^4c^4+4\,ab^3c^3d+18\,a^2b^2c^2d^2-60\,a^3bcd^3+35\,a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^4}$$
$$+\frac{3\,b^5c^4x^3+4\,ab^4c^3dx^3-30\,a^2b^3c^2d^2x^3+36\,a^3b^2cd^3x^3-13\,a^4bd^4x^3+5\,ab^4c^4x-4\,a^2b^3c^3dx-18\,a^3b^2c^2d}{8\,(bx^2+a)^2a^2b^4}$$
$$+\frac{b^6d^4x^3+12\,b^6cd^3x-9\,ab^5d^4x}{3\,b^9}$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*b^4*c^4 + 4*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 60*a^3*b*c*d^3 + 35*
a^4*d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^4) + 1/8*(3*b^5*c^4*x^3 +
4*a*b^4*c^3*d*x^3 - 30*a^2*b^3*c^2*d^2*x^3 + 36*a^3*b^2*c*d^3*x^3 - 13*a^4
*b*d^4*x^3 + 5*a*b^4*c^4*x - 4*a^2*b^3*c^3*d*x - 18*a^3*b^2*c^2*d^2*x + 28
*a^4*b*c*d^3*x - 11*a^5*d^4*x)/((b*x^2 + a)^2*a^2*b^4) + 1/3*(b^6*d^4*x^3
+ 12*b^6*c*d^3*x - 9*a*b^5*d^4*x)/b^9
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx = \frac{d^4\,x^3}{3\,b^3}-x\left(\frac{3\,a\,d^4}{b^4}-\frac{4\,c\,d^3}{b^3}\right)$$
$$-\frac{\frac{x\,(11\,a^4\,d^4-28\,a^3\,b\,c\,d^3+18\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d-5\,b^4\,c^4)}{8\,a}-\frac{x^3\,(-13\,a^4\,b\,d^4+36\,a^3\,b^2\,c\,d^3-30\,a^2\,b^3\,c^2\,d^2+4\,a\,b^4\,c^3\,d+3\,b^5\,c^4)}{8\,a^2}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^2\,(35\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2)}{\sqrt{a}\,(35\,a^4\,d^4-60\,a^3\,b\,c\,d^3+18\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d+3\,b^4\,c^4)}\right)\,(a\,d-b\,c)^2\,(35\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2)}{8\,a^{5/2}\,b^{9/2}}$$

input
```
int((c + d*x^2)^4/(a + b*x^2)^3,x)
```

output
```
(d^4*x^3)/(3*b^3) - x*((3*a*d^4)/b^4 - (4*c*d^3)/b^3) - ((x*(11*a^4*d^4 -
5*b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 28*a^3*b*c*d^3))/(8*a) -
(x^3*(3*b^5*c^4 - 13*a^4*b*d^4 + 36*a^3*b^2*c*d^3 - 30*a^2*b^3*c^2*d^2 + 4
*a*b^4*c^3*d))/(8*a^2))/(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + (atan((b^(1/2)
*x*(a*d - b*c)^2*(35*a^2*d^2 + 3*b^2*c^2 + 10*a*b*c*d))/(a^(1/2)*(35*a^4*d
^4 + 3*b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 60*a^3*b*c*d^3)))*(a
*d - b*c)^2*(35*a^2*d^2 + 3*b^2*c^2 + 10*a*b*c*d))/(8*a^(5/2)*b^(9/2))
```

3.36. $\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx$

3.37 $\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx$

3.37.1 Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx = \frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3(a+bx^2)^2}+\frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3(a+bx^2)} + \frac{3(bc-ad)\left(4a^2d^2+(bc+ad)^2\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

output
```
d^3*x/b^3+1/4*(-a*d+b*c)^3*x/a/b^3/(b*x^2+a)^2+3/8*(-a*d+b*c)^2*(3*a*d+b*c
)*x/a^2/b^3/(b*x^2+a)+3/8*(-a*d+b*c)*(4*a^2*d^2+(a*d+b*c)^2)*arctan(x*b^(1
/2)/a^(1/2))/a^(5/2)/b^(7/2)
```

3.37.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx = \frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3(a+bx^2)^2}+\frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3(a+bx^2)} + \frac{3(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

input
```
Integrate[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

output
```
(d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c - a*d)^2
*(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b^3*c^3 + a*b^2*c^2*d + 3*
a^2*b*c*d^2 - 5*a^3*d^3)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

3.37.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^3}\,dx$$

$$\Big\downarrow \ 300$$

$$\int \left(\frac{-a^3d^3+3b^2d^2x^4(bc-ad)+3bdx^2(bc-ad)(ad+bc)+b^3c^3}{b^3\left(a+bx^2\right)^3}+\frac{d^3}{b^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{3x(bc-ad)^2(3ad+bc)}{8a^2b^3\left(a+bx^2\right)}+\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)\left(4a^2d^2+(ad+bc)^2\right)}{8a^{5/2}b^{7/2}}+\frac{x(bc-ad)^3}{4ab^3\left(a+bx^2\right)^2}+\frac{d^3x}{b^3}$$

input
```
Int[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

output
```
(d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c - a*d)^2
*(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b*c - a*d)*(4*a^2*d^2 + (b
*c + a*d)^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

3.37.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.37.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.31

method	result
default	$\frac{d^3x}{b^3}-\frac{\frac{-\frac{3b\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)x^3}{8a^2}-\frac{\left(7a^3d^3-9a^2bc\,d^2-3a\,b^2c^2d+5b^3c^3\right)x}{8a}}{\left(b\,x^2+a\right)^2}+\frac{3\left(5a^3d^3-3a^2bc\,d^2-a\,b^2c^2d-b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}}{b^3}$
risch	$\frac{d^3x}{b^3}+\frac{\frac{3b\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)x^3}{8a^2}+\frac{\left(7a^3d^3-9a^2bc\,d^2-3a\,b^2c^2d+5b^3c^3\right)x}{8a}}{b^3\left(b\,x^2+a\right)^2}-\frac{15a\ln\left(bx-\sqrt{-ab}\right)d^3}{16b^3\sqrt{-ab}}+\frac{9\ln\left(bx-\sqrt{-ab}\right)c\,d^2}{16b^2\sqrt{-ab}}$

input
```
int((d*x^2+c)^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
d^3*x/b^3-1/b^3*((-3/8*b*(3*a^3*d^3-5*a^2*b*c*d^2+a*b^2*c^2*d+b^3*c^3)/a^2
*x^3-1/8*(7*a^3*d^3-9*a^2*b*c*d^2-3*a*b^2*c^2*d+5*b^3*c^3)/a*x)/(b*x^2+a)^
2+3/8*(5*a^3*d^3-3*a^2*b*c*d^2-a*b^2*c^2*d-b^3*c^3)/a^2/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2)))
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(116) = 232.

Time = 0.28 (sec) , antiderivative size = 606, normalized size of antiderivative = 4.66

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^3}\,dx$$

$$=\left[\frac{16\,a^3b^3d^3x^5+2\left(3\,ab^5c^3+3\,a^2b^4c^2d-15\,a^3b^3cd^2+25\,a^4b^2d^3\right)x^3+3\left(a^2b^3c^3+a^3b^2c^2d+3\,a^4bcd^2-5\,a^5\right.}{}\right.$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*a^3*b^3*d^3*x^5 + 2*(3*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 15*a^3*b^3*
c*d^2 + 25*a^4*b^2*d^3)*x^3 + 3*(a^2*b^3*c^3 + a^3*b^2*c^2*d + 3*a^4*b*c*d
^2 - 5*a^5*d^3 + (b^5*c^3 + a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)
*x^4 + 2*(a*b^4*c^3 + a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*
sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^4*c^
3 - 3*a^3*b^3*c^2*d - 9*a^4*b^2*c*d^2 + 15*a^5*b*d^3)*x)/(a^3*b^6*x^4 + 2*
a^4*b^5*x^2 + a^5*b^4), 1/8*(8*a^3*b^3*d^3*x^5 + (3*a*b^5*c^3 + 3*a^2*b^4*
c^2*d - 15*a^3*b^3*c*d^2 + 25*a^4*b^2*d^3)*x^3 + 3*(a^2*b^3*c^3 + a^3*b^2*
c^2*d + 3*a^4*b*c*d^2 - 5*a^5*d^3 + (b^5*c^3 + a*b^4*c^2*d + 3*a^2*b^3*c*d
^2 - 5*a^3*b^2*d^3)*x^4 + 2*(a*b^4*c^3 + a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 -
 5*a^4*b*d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (5*a^2*b^4*c^3 - 3*a^
3*b^3*c^2*d - 9*a^4*b^2*c*d^2 + 15*a^5*b*d^3)*x)/(a^3*b^6*x^4 + 2*a^4*b^5*
x^2 + a^5*b^4)]
```

3.37.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(122) = 244$.

Time = 1.01 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.25

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx$$
$$= \frac{3\sqrt{-\frac{1}{a^5b^7}}(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)\log\left(-\frac{3a^3b^3\sqrt{-\frac{1}{a^5b^7}}(ad-bc)(5a^2d^2+2abcd+b^2c^2)}{15a^3d^3-9a^2bcd^2-3ab^2c^2d-3b^3c^3}+x\right)}{16}$$
$$-\frac{3\sqrt{-\frac{1}{a^5b^7}}(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)\log\left(\frac{3a^3b^3\sqrt{-\frac{1}{a^5b^7}}(ad-bc)(5a^2d^2+2abcd+b^2c^2)}{15a^3d^3-9a^2bcd^2-3ab^2c^2d-3b^3c^3}+x\right)}{16}$$
$$+\frac{x^3\cdot\left(9a^3bd^3-15a^2b^2cd^2+3ab^3c^2d+3b^4c^3\right)+x\left(7a^4d^3-9a^3bcd^2-3a^2b^2c^2d+5ab^3c^3\right)}{8a^4b^3+16a^3b^4x^2+8a^2b^5x^4}$$
$$+\frac{d^3x}{b^3}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**3,x)
```

output
```
3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d + b**2*c**2)*l
og(-3*a**3*b**3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d
+ b**2*c**2)/(15*a**3*d**3 - 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 3*b**3*c*
*3) + x)/16 - 3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d
+ b**2*c**2)*log(3*a**3*b**3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2
 + 2*a*b*c*d + b**2*c**2)/(15*a**3*d**3 - 9*a**2*b*c*d**2 - 3*a*b**2*c**2*
d - 3*b**3*c**3) + x)/16 + (x**3*(9*a**3*b*d**3 - 15*a**2*b**2*c*d**2 + 3*
a*b**3*c**2*d + 3*b**4*c**3) + x*(7*a**4*d**3 - 9*a**3*b*c*d**2 - 3*a**2*b
**2*c**2*d + 5*a*b**3*c**3))/(8*a**4*b**3 + 16*a**3*b**4*x**2 + 8*a**2*b**
5*x**4) + d**3*x/b**3
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx$$
$$= \frac{d^3x}{b^3} + \frac{3\,(b^4c^3+ab^3c^2d-5\,a^2b^2cd^2+3\,a^3bd^3)x^3+(5\,ab^3c^3-3\,a^2b^2c^2d-9\,a^3bcd^2+7\,a^4d^3)x}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)} + \frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
d^3*x/b^3 + 1/8*(3*(b^4*c^3 + a*b^3*c^2*d - 5*a^2*b^2*c*d^2 + 3*a^3*b*d^3)
*x^3 + (5*a*b^3*c^3 - 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 7*a^4*d^3)*x)/(a^2
*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3) + 3/8*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b
*c*d^2 - 5*a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx = \frac{d^3x}{b^3} + \frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3} + \frac{3\,b^4c^3x^3+3\,ab^3c^2dx^3-15\,a^2b^2cd^2x^3+9\,a^3bd^3x^3+5\,ab^3c^3x-3\,a^2b^2c^2dx-9\,a^3bcd^2x+7\,a^4d^3x}{8\,(bx^2+a)^2a^2b^3}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
d^3*x/b^3 + 3/8*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*arctan
(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c^3*x^3 + 3*a*b^3*c^2*d*x
^3 - 15*a^2*b^2*c*d^2*x^3 + 9*a^3*b*d^3*x^3 + 5*a*b^3*c^3*x - 3*a^2*b^2*c^
2*d*x - 9*a^3*b*c*d^2*x + 7*a^4*d^3*x)/((b*x^2 + a)^2*a^2*b^3)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 4.75 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.85

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx = \frac{\frac{x\,(7\,a^3\,d^3-9\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+5\,b^3\,c^3)}{8\,a}+\frac{3\,x^3\,(3\,a^3\,b\,d^3-5\,a^2\,b^2\,c\,d^2+a\,b^3\,c^2\,d+b^4\,c^3)}{8\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4} + \frac{d^3\,x}{b^3} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)\,(5\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2)}{\sqrt{a}\,(-5\,a^3\,d^3+3\,a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3)}\right)\,(a\,d-b\,c)\,(5\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2)}{8\,a^{5/2}\,b^{7/2}}$$

input
```
int((c + d*x^2)^3/(a + b*x^2)^3,x)
```

output
```
((x*(7*a^3*d^3 + 5*b^3*c^3 - 3*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(8*a) + (3*x^
3*(b^4*c^3 + 3*a^3*b*d^3 - 5*a^2*b^2*c*d^2 + a*b^3*c^2*d))/(8*a^2))/(a^2*b
^3 + b^5*x^4 + 2*a*b^4*x^2) + (d^3*x)/b^3 + (3*atan((b^(1/2)*x*(a*d - b*c)
*(5*a^2*d^2 + b^2*c^2 + 2*a*b*c*d))/(a^(1/2)*(b^3*c^3 - 5*a^3*d^3 + a*b^2*
c^2*d + 3*a^2*b*c*d^2)))*(a*d - b*c)*(5*a^2*d^2 + b^2*c^2 + 2*a*b*c*d))/(8
*a^(5/2)*b^(7/2))
```

3.38 $\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx$

3.38.1 Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = \frac{3\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{8\,(a+bx^2)}+\frac{(bc-ad)x(c+dx^2)}{4ab\,(a+bx^2)^2}+\frac{(3b^2c^2+2abcd+3a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

output
```
3/8*(c^2/a^2-d^2/b^2)*x/(b*x^2+a)+1/4*(-a*d+b*c)*x*(d*x^2+c)/a/b/(b*x^2+a)
^2+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b
^(5/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = \frac{(bc-ad)x(3a^2d+3b^2cx^2+5ab(c+dx^2))}{8a^2b^2\,(a+bx^2)^2}+\frac{(3b^2c^2+2abcd+3a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

input
```
Integrate[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

output
```
((b*c - a*d)*x*(3*a^2*d + 3*b^2*c*x^2 + 5*a*b*(c + d*x^2)))/(8*a^2*b^2*(a
+ b*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(8*a^(5/2)*b^(5/2))
```

3.38.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {315, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\int \frac{d(bc+3ad)x^2+c(3bc+ad)}{\left(bx^2+a\right)^2}dx}{4ab}+\frac{x\left(c+dx^2\right)(bc-ad)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{1}{2}\left(\frac{3bc^2}{a}+\frac{3ad^2}{b}+2cd\right)\int \frac{1}{bx^2+a}dx+\frac{3x\left(\frac{bc^2}{a}-\frac{ad^2}{b}\right)}{2(a+bx^2)}}{4ab}+\frac{x\left(c+dx^2\right)(bc-ad)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(\frac{3bc^2}{a}+\frac{3ad^2}{b}+2cd\right)}{2\sqrt{a}\sqrt{b}}+\frac{3x\left(\frac{bc^2}{a}-\frac{ad^2}{b}\right)}{2(a+bx^2)}}{4ab}+\frac{x\left(c+dx^2\right)(bc-ad)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

output
```
((b*c - a*d)*x*(c + d*x^2))/(4*a*b*(a + b*x^2)^2) + ((3*((b*c^2)/a - (a*d^
2)/b)*x)/(2*(a + b*x^2)) + (((3*b*c^2)/a + 2*c*d + (3*a*d^2)/b)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]))/(4*a*b)
```

3.38.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.38.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

method	result
default	$\frac{-\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x^3}{8a^2b}-\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x}{8a\,b^2}}{\left(b\,x^2+a\right)^2}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b^2\sqrt{ab}}$
risch	$\frac{-\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x^3}{8a^2b}-\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x}{8a\,b^2}}{\left(b\,x^2+a\right)^2}-\frac{3\ln\left(bx+\sqrt{-ab}\right)d^2}{16\sqrt{-ab}\,b^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)cd}{8\sqrt{-ab}\,ba}-\frac{3\ln\left(bx+\sqrt{-ab}\right)c^2}{16\sqrt{-ab}\,a^2}+\frac{3\ln(-bx\cdots}{16\sqrt{\cdots}}$

input
```
int((d*x^2+c)^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*(5*a^2*d^2-2*a*b*c*d-3*b^2*c^2)/a^2/b*x^3-1/8*(3*a^2*d^2+2*a*b*c*d-5
*b^2*c^2)/a/b^2*x)/(b*x^2+a)^2+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)/a^2/b^2
/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(102) = 204$.

Time = 0.26 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.87

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{2\,(3\,ab^4c^2+2\,a^2b^3cd-5\,a^3b^2d^2)x^3-(3\,a^2b^2c^2+2\,a^3bcd+3\,a^4d^2+(3\,b^4c^2+2\,ab^3cd+3\,a^2b^2d^2)x^4+2\,(}{16\,(a^3b^5x^4+2\,a^4b^4x}\right.$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(3*a*b^4*c^2 + 2*a^2*b^3*c*d - 5*a^3*b^2*d^2)*x^3 - (3*a^2*b^2*c^
2 + 2*a^3*b*c*d + 3*a^4*d^2 + (3*b^4*c^2 + 2*a*b^3*c*d + 3*a^2*b^2*d^2)*x^
4 + 2*(3*a*b^3*c^2 + 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x^2)*sqrt(-a*b)*log((b*x
^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^3*c^2 - 2*a^3*b^2*c*d -
 3*a^4*b*d^2)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 + a^5*b^3), 1/8*((3*a*b^4*c^
2 + 2*a^2*b^3*c*d - 5*a^3*b^2*d^2)*x^3 + (3*a^2*b^2*c^2 + 2*a^3*b*c*d + 3*
a^4*d^2 + (3*b^4*c^2 + 2*a*b^3*c*d + 3*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 +
 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (5*a^
2*b^3*c^2 - 2*a^3*b^2*c*d - 3*a^4*b*d^2)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 +
 a^5*b^3)]
```

3.38.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(110) = 220$.

Time = 0.55 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.92

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16} + \frac{x^3(-5a^2bd^2+2ab^2cd+3b^3c^2)+x(-3a^3d^2-2a^2bcd+5ab^2c^2)}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*b**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-a**3*b*
*2*sqrt(-1/(a**5*b**5)) + x)/16 + sqrt(-1/(a**5*b**5))*(3*a**2*d**2 + 2*a*
b*c*d + 3*b**2*c**2)*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/16 + (x**3*(-
5*a**2*b*d**2 + 2*a*b**2*c*d + 3*b**3*c**2) + x*(-3*a**3*d**2 - 2*a**2*b*c
*d + 5*a*b**2*c**2))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**2*b**4*x**4)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = \frac{(3\,b^3c^2+2\,ab^2cd-5\,a^2bd^2)x^3+(5\,ab^2c^2-2\,a^2bcd-3\,a^3d^2)x}{8\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)} + \frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^2}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((3*b^3*c^2 + 2*a*b^2*c*d - 5*a^2*b*d^2)*x^3 + (5*a*b^2*c^2 - 2*a^2*b*
c*d - 3*a^3*d^2)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(3*b^2*c
^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = \frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^2} + \frac{3\,b^3c^2x^3+2\,ab^2cdx^3-5\,a^2bd^2x^3+5\,ab^2c^2x-2\,a^2bcdx-3\,a^3d^2x}{8\,(bx^2+a)^2a^2b^2}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a
^2*b^2) + 1/8*(3*b^3*c^2*x^3 + 2*a*b^2*c*d*x^3 - 5*a^2*b*d^2*x^3 + 5*a*b^2
*c^2*x - 2*a^2*b*c*d*x - 3*a^3*d^2*x)/((b*x^2 + a)^2*a^2*b^2)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2)}{8\,a^{5/2}\,b^{5/2}} - \frac{\frac{x\,(3\,a^2\,d^2+2\,a\,b\,c\,d-5\,b^2\,c^2)}{8\,a\,b^2}-\frac{x^3\,(-5\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2)}{8\,a^2\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

input
```
int((c + d*x^2)^2/(a + b*x^2)^3,x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(8*a^(5/2)
*b^(5/2)) - ((x*(3*a^2*d^2 - 5*b^2*c^2 + 2*a*b*c*d))/(8*a*b^2) - (x^3*(3*b
^2*c^2 - 5*a^2*d^2 + 2*a*b*c*d))/(8*a^2*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

3.39 $\int \frac{c+dx^2}{(a+bx^2)^3}\,dx$

3.39.1 Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx=\frac{(bc-ad)x}{4ab\,(a+bx^2)^2}+\frac{(3bc+ad)x}{8a^2b\,(a+bx^2)}+\frac{(3bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

output
```
1/4*(-a*d+b*c)*x/a/b/(b*x^2+a)^2+1/8*(a*d+3*b*c)*x/a^2/b/(b*x^2+a)+1/8*(a*
d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx=\frac{x(-a^2d+3b^2cx^2+ab(5c+dx^2))}{8a^2b\,(a+bx^2)^2}+\frac{(3bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

input
```
Integrate[(c + d*x^2)/(a + b*x^2)^3,x]
```

output
```
(x*(-(a^2*d) + 3*b^2*c*x^2 + a*b*(5*c + d*x^2)))/(8*a^2*b*(a + b*x^2)^2) +
 ((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

3.39.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \; 298$$

$$\frac{(ad+3bc)\int \frac{1}{(bx^2+a)^2}dx}{4ab}+\frac{x(bc-ad)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \; 215$$

$$\frac{(ad+3bc)\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}\right)}{4ab}+\frac{x(bc-ad)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}\right)(ad+3bc)}{4ab}+\frac{x(bc-ad)}{4ab\,(a+bx^2)^2}$$

input
```
Int[(c + d*x^2)/(a + b*x^2)^3,x]
```

output
```
((b*c - a*d)*x)/(4*a*b*(a + b*x^2)^2) + ((3*b*c + a*d)*(x/(2*a*(a + b*x^2)
) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a*b)
```

3.39.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.39.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{(ad+3bc)x^3}{8a^2}-\frac{(ad-5bc)x}{8ab}}{(b\,x^2+a)^2}+\frac{(ad+3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$	76
risch	$\frac{\frac{(ad+3bc)x^3}{8a^2}-\frac{(ad-5bc)x}{8ab}}{(b\,x^2+a)^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)d}{16\sqrt{-ab}\,ba}-\frac{3\ln\left(bx+\sqrt{-ab}\right)c}{16\sqrt{-ab}\,a^2}+\frac{\ln\left(-bx+\sqrt{-ab}\right)d}{16\sqrt{-ab}\,ba}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)c}{16\sqrt{-ab}\,a^2}$	146

input
```
int((d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(a*d+3*b*c)/a^2*x^3-1/8*(a*d-5*b*c)/a/b*x)/(b*x^2+a)^2+1/8*(a*d+3*b*c
)/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.27

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx$$

$$= \left[\frac{2\,(3\,ab^3c+a^2b^2d)x^3-((3\,b^3c+ab^2d)x^4+3\,a^2bc+a^3d+2\,(3\,ab^2c+a^2bd)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}\right.$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(3*a*b^3*c + a^2*b^2*d)*x^3 - ((3*b^3*c + a*b^2*d)*x^4 + 3*a^2*b*
c + a^3*d + 2*(3*a*b^2*c + a^2*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^2*c - a^3*b*d)*x)/(a^3*b^4*x^4 + 2*a^
4*b^3*x^2 + a^5*b^2), 1/8*((3*a*b^3*c + a^2*b^2*d)*x^3 + ((3*b^3*c + a*b^2
*d)*x^4 + 3*a^2*b*c + a^3*d + 2*(3*a*b^2*c + a^2*b*d)*x^2)*sqrt(a*b)*arcta
n(sqrt(a*b)*x/a) + (5*a^2*b^2*c - a^3*b*d)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2
 + a^5*b^2)]
```

3.39.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.63

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^3}}(ad+3bc)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^3}}(ad+3bc)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{x^3(abd+3b^2c)+x(-a^2d+5abc)}{8a^4b+16a^3b^2x^2+8a^2b^3x^4}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*b**3))*(a*d + 3*b*c)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/
16 + sqrt(-1/(a**5*b**3))*(a*d + 3*b*c)*log(a**3*b*sqrt(-1/(a**5*b**3)) +
x)/16 + (x**3*(a*b*d + 3*b**2*c) + x*(-a**2*d + 5*a*b*c))/(8*a**4*b + 16*a
**3*b**2*x**2 + 8*a**2*b**3*x**4)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx = \frac{(3\,b^2c+abd)x^3+(5\,abc-a^2d)x}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)} + \frac{(3\,bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((3*b^2*c + a*b*d)*x^3 + (5*a*b*c - a^2*d)*x)/(a^2*b^3*x^4 + 2*a^3*b^2
*x^2 + a^4*b) + 1/8*(3*b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx = \frac{(3\,bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{3\,b^2cx^3+abdx^3+5\,abcx-a^2dx}{8\,(bx^2+a)^2a^2b}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(3*b^2*c*x
^3 + a*b*d*x^3 + 5*a*b*c*x - a^2*d*x)/((b*x^2 + a)^2*a^2*b)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx = \frac{\frac{x^3\,(a\,d+3\,b\,c)}{8\,a^2}-\frac{x\,(a\,d-5\,b\,c)}{8\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d+3\,b\,c)}{8\,a^{5/2}\,b^{3/2}}$$

input
```
int((c + d*x^2)/(a + b*x^2)^3,x)
```

output
```
((x^3*(a*d + 3*b*c))/(8*a^2) - (x*(a*d - 5*b*c))/(8*a*b))/(a^2 + b^2*x^4 +
 2*a*b*x^2) + (atan((b^(1/2)*x)/a^(1/2))*(a*d + 3*b*c))/(8*a^(5/2)*b^(3/2)
)
```

3.40 $\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx$

3.40.1 Optimal result

Integrand size = 19, antiderivative size = 161

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx = \frac{bx}{4a(bc-ad)(a+bx^2)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2(a+bx^2)} + \frac{\sqrt{b}(3b^2c^2-10abcd+15a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^3} - \frac{d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3}$$

output
```
1/4*b*x/a/(-a*d+b*c)/(b*x^2+a)^2+1/8*b*(-7*a*d+3*b*c)*x/a^2/(-a*d+b*c)^2/(
b*x^2+a)+1/8*(15*a^2*d^2-10*a*b*c*d+3*b^2*c^2)*arctan(x*b^(1/2)/a^(1/2))*b
^(1/2)/a^(5/2)/(-a*d+b*c)^3-d^(5/2)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^3
/c^(1/2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx = -\frac{\frac{b(-bc+ad)x(-5abc+9a^2d-3b^2cx^2+7abdx^2)}{a^2(a+bx^2)^2} + \frac{\sqrt{b}(3b^2c^2-10abcd+15a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}} - \frac{8d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}}}{8(-bc+ad)^3}$$

input
```
Integrate[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

output
```
-1/8*((b*(-(b*c) + a*d)*x*(-5*a*b*c + 9*a^2*d - 3*b^2*c*x^2 + 7*a*b*d*x^2)
)/(a^2*(a + b*x^2)^2) + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2) - (8*d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/Sqrt[c])/(-(b*c) + a*d)^3
```

3.40.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {316, 25, 402, 25, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^3 (c+dx^2)} dx$$

$$\Big\downarrow \text{ 316}$$

$$\frac{bx}{4a(a+bx^2)^2(bc-ad)} - \frac{\int -\frac{3bdx^2+3bc-4ad}{(bx^2+a)^2(dx^2+c)}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3bdx^2+3bc-4ad}{(bx^2+a)^2(dx^2+c)}dx}{4a(bc-ad)} + \frac{bx}{4a(a+bx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{bx(3bc-7ad)}{2a(a+bx^2)(bc-ad)} - \frac{\int -\frac{3b^2c^2-7abdc+8a^2d^2+bd(3bc-7ad)x^2}{(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)}}{4a(bc-ad)} + \frac{bx}{4a(a+bx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{3b^2c^2-7abdc+8a^2d^2+bd(3bc-7ad)x^2}{(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)} + \frac{bx(3bc-7ad)}{2a(a+bx^2)(bc-ad)}}{4a(bc-ad)} + \frac{bx}{4a(a+bx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\frac{b\left(15a^2d^2-10abcd+3b^2c^2\right)\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{8a^2d^3\int \frac{1}{dx^2+c}dx}{bc-ad}}{2a(bc-ad)}+\frac{bx(3bc-7ad)}{2a(a+bx^2)(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(15a^2d^2-10abcd+3b^2c^2\right)}{\sqrt{a}(bc-ad)}-\frac{8a^2d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{2a(bc-ad)}+\frac{bx(3bc-7ad)}{2a(a+bx^2)(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

output
```
(b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2) + ((b*(3*b*c - 7*a*d)*x)/(2*a*(b*c -
 a*d)*(a + b*x^2)) + ((Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (8*a^2*d^(5/2)*ArcTan[(Sqr
t[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(2*a*(b*c - a*d)))/(4*a*(b*c - a*
d))
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.40.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{b\left(\frac{\frac{b\left(7a^2d^2-10abcd+3b^2c^2\right)x^3}{8a^2}+\frac{\left(9a^2d^2-14abcd+5b^2c^2\right)x}{8a}}{\left(b\,x^2+a\right)^2}+\frac{\left(15a^2d^2-10abcd+3b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(ad-bc)^3}+\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^3\sqrt{cd}}$	158
risch	Expression too large to display	2285

input
```
int(1/(b*x^2+a)^3/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-b/(a*d-b*c)^3*((1/8*b*(7*a^2*d^2-10*a*b*c*d+3*b^2*c^2)/a^2*x^3+1/8*(9*a^2
*d^2-14*a*b*c*d+5*b^2*c^2)/a*x)/(b*x^2+a)^2+1/8*(15*a^2*d^2-10*a*b*c*d+3*b
^2*c^2)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))+d^3/(a*d-b*c)^3/(c*d)^(1/
2)*arctan(d*x/(c*d)^(1/2))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(139) = 278.

Time = 0.80 (sec) , antiderivative size = 1587, normalized size of antiderivative = 9.86

$$\int \frac{1}{(a+bx^2)^3\,(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/16*(2*(3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 - (3*a^2*b^2*c^2 -
 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15*a^2*b^2*d^2)*x
^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sqrt(-b/a)*log((
b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 8*(a^2*b^2*d^2*x^4 + 2*a^3*b*
d^2*x^2 + a^4*d^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 +
c)) + 2*(5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a
^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d +
3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*
a^5*b^2*c*d^2 - a^6*b*d^3)*x^2), 1/16*(2*(3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2
*b^2*d^2)*x^3 - 16*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(d/c)
*arctan(x*sqrt(d/c)) - (3*a^2*b^2*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4
*c^2 - 10*a*b^3*c*d + 15*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*
d + 15*a^3*b*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^
2 + a)) + 2*(5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 -
 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*
d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d
+ 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x^2), 1/8*((3*b^4*c^2 - 10*a*b^3*c*d + 7*a^
2*b^2*d^2)*x^3 + (3*a^2*b^2*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 -
 10*a*b^3*c*d + 15*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15
*a^3*b*d^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 4*(a^2*b^2*d^2*x^4 + 2...
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**3/(d*x**2+c),x)
```

output
```
Timed out
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.73

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx = -\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{cd}} + \frac{(3\,b^3c^2-10\,ab^2cd+15\,a^2bd^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{ab}} + \frac{(3\,b^3c-7\,ab^2d)x^3+(5\,ab^2c-9\,a^2bd)x}{8\,(a^4b^2c^2-2\,a^5bcd+a^6d^2+(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2)x^4+2\,(a^3b^3c^2-2\,a^4b^2cd+a^5bd^2)x^2)}$$

input
```
integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

output
```
-d^3*arctan(d*x/sqrt(c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*sqrt(c*d)) + 1/8*(3*b^3*c^2 - 10*a*b^2*c*d + 15*a^2*b*d^2)*arctan(b*
x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sq
rt(a*b)) + 1/8*((3*b^3*c - 7*a*b^2*d)*x^3 + (5*a*b^2*c - 9*a^2*b*d)*x)/(a^
4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2 + (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2
*d^2)*x^4 + 2*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x^2)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx = -\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{cd}} + \frac{(3\,b^3c^2-10\,ab^2cd+15\,a^2bd^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{ab}} + \frac{3\,b^3cx^3-7\,ab^2dx^3+5\,ab^2cx-9\,a^2bdx}{8\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)(bx^2+a)^2}$$

input
```
integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

output
```
-d^3*arctan(d*x/sqrt(c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*sqrt(c*d)) + 1/8*(3*b^3*c^2 - 10*a*b^2*c*d + 15*a^2*b*d^2)*arctan(b*
x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sq
rt(a*b)) + 1/8*(3*b^3*c*x^3 - 7*a*b^2*d*x^3 + 5*a*b^2*c*x - 9*a^2*b*d*x)/(
(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(b*x^2 + a)^2)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 6.62 (sec) , antiderivative size = 6033, normalized size of antiderivative = 37.47

$$\int \frac{1}{(a+bx^2)^3 (c+dx^2)} \, dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)^3*(c + d*x^2)),x)
```

output
```
((x^3*(3*b^3*c - 7*a*b^2*d))/(8*a^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*
(5*b^2*c - 9*a*b*d))/(8*a*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a^2 + b^2*x^4
 + 2*a*b*x^2) + (atan(((((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^
3*d^4 - 300*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c
^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) - ((-c*d^5)^(1/
2)*((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^
3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*b^6
*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 + a^
4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15
*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) - (x*(-c*d^5)^(1/2)*(256*a^11*b^2*d^9 -
 1280*a^10*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a
^6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^
4*c^2*d^7))/(64*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a
^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3
))))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))*(-c*d^5)
^(1/2)*1i)/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)) + (
((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^
6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6
*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) + ((-c*d^5)^(1/2)*((256*a^10*b^2*d^10 -
 1760*a^9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*...
```

3.41 $\int \frac{1}{(a+bx^2)^3 (c+dx^2)^2} \, dx$

3.41.1 Optimal result

Integrand size = 19, antiderivative size = 236

$$\begin{aligned}\int \frac{1}{(a+bx^2)^3 (c+dx^2)^2} \, dx &= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3 (c+dx^2)} + \frac{bx}{4a(bc-ad)(a+bx^2)^2 (c+dx^2)} \\ &\quad + \frac{3b(bc-3ad)x}{8a^2(bc-ad)^2 (a+bx^2)(c+dx^2)} \\ &\quad + \frac{b^{3/2}(3b^2c^2-14abcd+35a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^4} \\ &\quad - \frac{d^{5/2}(7bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^4}\end{aligned}$$

output
```
1/8*d*(-4*a*d+b*c)*(a*d+3*b*c)*x/a^2/c/(-a*d+b*c)^3/(d*x^2+c)+1/4*b*x/a/(-
a*d+b*c)/(b*x^2+a)^2/(d*x^2+c)+3/8*b*(-3*a*d+b*c)*x/a^2/(-a*d+b*c)^2/(b*x^
2+a)/(d*x^2+c)+1/8*b^(3/2)*(35*a^2*d^2-14*a*b*c*d+3*b^2*c^2)*arctan(x*b^(1
/2)/a^(1/2))/a^(5/2)/(-a*d+b*c)^4-1/2*d^(5/2)*(-a*d+7*b*c)*arctan(x*d^(1/2
)/c^(1/2))/c^(3/2)/(-a*d+b*c)^4
```

3.41.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx = \frac{1}{8}\left(\frac{2b^2x}{a(bc-ad)^2(a+bx^2)^2} + \frac{b^2(-3bc+11ad)x}{a^2(-bc+ad)^3(a+bx^2)} - \frac{4d^3x}{c(bc-ad)^3(c+dx^2)} + \frac{b^{3/2}(3b^2c^2-14abcd+35a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^4} + \frac{4d^{5/2}(-7bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^4}\right)$$

input
```
Integrate[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

output
```
((2*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^2)^2) + (b^2*(-3*b*c + 11*a*d)*x)/(a^
2*(-(b*c) + a*d)^3*(a + b*x^2)) - (4*d^3*x)/(c*(b*c - a*d)^3*(c + d*x^2))
+ (b^(3/2)*(3*b^2*c^2 - 14*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(a^(5/2)*(b*c - a*d)^4) + (4*d^(5/2)*(-7*b*c + a*d)*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/(c^(3/2)*(b*c - a*d)^4))/8
```

3.41.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {316, 25, 402, 25, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{316}$$

$$\frac{bx}{4a(a+bx^2)^2(c+dx^2)(bc-ad)} - \frac{\int -\frac{5bdx^2+3bc-4ad}{(bx^2+a)^2(dx^2+c)^2}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5bdx^2+3bc-4ad}{(bx^2+a)^2(dx^2+c)^2}dx}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 402

$$\frac{\frac{3bx(bc-3ad)}{2a(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\int -\frac{3b^2c^2-5abdc+8a^2d^2+9bd(bc-3ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{2a(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{3b^2c^2-5abdc+8a^2d^2+9bd(bc-3ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{2a(bc-ad)}+\frac{3bx(bc-3ad)}{2a(a+bx^2)(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{2\left(3b^3c^3-11ab^2dc^2+24a^2bd^2c-4a^3d^3+bd(bc-4ad)(3bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{dx(bc-4ad)(ad+3bc)}{c\left(c+dx^2\right)(bc-ad)}}{2a(bc-ad)}+\frac{3bx(bc-3ad)}{2a(a+bx^2)(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{3b^3c^3-11ab^2dc^2+24a^2bd^2c-4a^3d^3+bd(bc-4ad)(3bc+ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{c(bc-ad)}+\frac{dx(bc-4ad)(ad+3bc)}{c\left(c+dx^2\right)(bc-ad)}}{2a(bc-ad)}+\frac{3bx(bc-3ad)}{2a(a+bx^2)(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{b^2c\left(35a^2d^2-14abcd+3b^2c^2\right)\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{4a^2d^3(7bc-ad)\int \frac{1}{dx^2+c}dx}{bc-ad}}{c(bc-ad)}+\frac{dx(bc-4ad)(ad+3bc)}{c\left(c+dx^2\right)(bc-ad)}}{2a(bc-ad)}+\frac{3bx(bc-3ad)}{2a(a+bx^2)(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 218

3.41. $\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx$

$$\frac{\frac{\frac{\frac{b^{3/2}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(35a^2d^2-14abcd+3b^2c^2\right)}{\sqrt{a}(bc-ad)}-\frac{4a^2d^{5/2}(7bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{c(bc-ad)}+\frac{dx(bc-4ad)(ad+3bc)}{c\left(c+dx^2\right)(bc-ad)}}{2a(bc-ad)}+\frac{3bx(bc-3ad)}{2a(a+bx^2)(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

output
```
(b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)) + ((3*b*(b*c - 3*a*d)*x)
/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + ((d*(b*c - 4*a*d)*(3*b*c + a*
d)*x)/(c*(b*c - a*d)*(c + d*x^2)) + ((b^(3/2)*c*(3*b^2*c^2 - 14*a*b*c*d +
35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (4*a^2*d^
(5/2)*(7*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(c
*(b*c - a*d)))/(2*a*(b*c - a*d)))/(4*a*(b*c - a*d))
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.41.4 Maple [A] (verified)

Time = 8.53 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.83

method	result
default	$\frac{b^{2}\left(\frac{\frac{b\left(11a^{2}d^{2}-14abcd+3b^{2}c^{2}\right)x^{3}}{8a^{2}}+\frac{\left(13a^{2}d^{2}-18abcd+5b^{2}c^{2}\right)x}{8a}}{\left(b\,x^{2}+a\right)^{2}}+\frac{\left(35a^{2}d^{2}-14abcd+3b^{2}c^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^{2}\sqrt{ab}}\right)}{(ad-bc)^{4}}+\frac{d^{3}\left(\frac{(ad-bc)x}{2c\left(d\,x^{2}+c\right)}+\frac{(ad-7bc)\,\mathrm{ar}}{2c}\right.}{(ad-bc)^{4}}$
risch	Expression too large to display

input
```
int(1/(b*x^2+a)^3/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/(a*d-b*c)^4*((1/8*b*(11*a^2*d^2-14*a*b*c*d+3*b^2*c^2)/a^2*x^3+1/8*(13*
a^2*d^2-18*a*b*c*d+5*b^2*c^2)/a*x)/(b*x^2+a)^2+1/8*(35*a^2*d^2-14*a*b*c*d+
3*b^2*c^2)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))+d^3/(a*d-b*c)^4*(1/2*(
a*d-b*c)/c*x/(d*x^2+c)+1/2*(a*d-7*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2
)))
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. 2(210) = 420.

Time = 2.41 (sec) , antiderivative size = 3251, normalized size of antiderivative = 13.78

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/16*(2*(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*b^3*c*d^3 + 4*a^3*b^2*d^4
)*x^5 + 2*(3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3
 + 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 35*a^4*b*c^2*d^2
 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 + (3*b^5*c^4 -
8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (6*a*b^4*c^4 -
 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*sqrt(-b/a)*l
og((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*(7*a^4*b*c^2*d^2 - a^5*
c*d^3 + (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*
b^2*c*d^3 - 2*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d
^4)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5
*a*b^4*c^4 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5
*d^4)*x)/(a^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*
d^3 + a^8*c^2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3
 - 4*a^5*b^3*c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d
 - 2*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d
^5)*x^4 + (2*a^3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2
*c^3*d^3 - 2*a^7*b*c^2*d^4 + a^8*c*d^5)*x^2), 1/16*(2*(3*b^5*c^3*d - 14*a*
b^4*c^2*d^2 + 7*a^2*b^3*c*d^3 + 4*a^3*b^2*d^4)*x^5 + 2*(3*b^5*c^4 - 9*a*b^
4*c^3*d - 7*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 - 8*(7*a^
4*b*c^2*d^2 - a^5*c*d^3 + (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*...
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**3/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(210) = 420.

Time = 0.30 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.25

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx = \frac{(3\,b^4c^2-14\,ab^3cd+35\,a^2b^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^4c^4-4\,a^3b^3c^3d+6\,a^4b^2c^2d^2-4\,a^5bcd^3+a^6d^4)\sqrt{ab}} - \frac{(7\,bcd^3-ad^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4)\sqrt{cd}} + \frac{(3\,b^4c^2d-11\,ab^3cd^2-4\,a^2b^2d^3)x^5+(3\,b^4c^3-6\,ab^3c^2d}{8\,(a^4b^3c^5-3\,a^5b^2c^4d+3\,a^6bc^3d^2-a^7c^2d^3+(a^2b^5c^4d-3\,a^3b^4c^3d^2+3\,a^4b^3c^2d^3-a^5b^2cd^4)x^6+(a^2b^5c^5}$$

input
```
integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/8*(3*b^4*c^2 - 14*a*b^3*c*d + 35*a^2*b^2*d^2)*arctan(b*x/sqrt(a*b))/((a^
2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)
*sqrt(a*b)) - 1/2*(7*b*c*d^3 - a*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^5 - 4*
a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d))
+ 1/8*((3*b^4*c^2*d - 11*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*x^5 + (3*b^4*c^3 - 6
*a*b^3*c^2*d - 13*a^2*b^2*c*d^2 - 8*a^3*b*d^3)*x^3 + (5*a*b^3*c^3 - 13*a^2
*b^2*c^2*d - 4*a^4*d^3)*x)/(a^4*b^3*c^5 - 3*a^5*b^2*c^4*d + 3*a^6*b*c^3*d^
2 - a^7*c^2*d^3 + (a^2*b^5*c^4*d - 3*a^3*b^4*c^3*d^2 + 3*a^4*b^3*c^2*d^3 -
 a^5*b^2*c*d^4)*x^6 + (a^2*b^5*c^5 - a^3*b^4*c^4*d - 3*a^4*b^3*c^3*d^2 + 5
*a^5*b^2*c^2*d^3 - 2*a^6*b*c*d^4)*x^4 + (2*a^3*b^4*c^5 - 5*a^4*b^3*c^4*d +
 3*a^5*b^2*c^3*d^2 + a^6*b*c^2*d^3 - a^7*c*d^4)*x^2)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.41

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx = -\frac{d^3x}{2\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)(dx^2+c)} + \frac{(3\,b^4c^2-14\,ab^3cd+35\,a^2b^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^4c^4-4\,a^3b^3c^3d+6\,a^4b^2c^2d^2-4\,a^5bcd^3+a^6d^4)\sqrt{ab}} - \frac{(7\,bcd^3-ad^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4)\sqrt{cd}} + \frac{3\,b^4cx^3-11\,ab^3dx^3+5\,ab^3cx-13\,a^2b^2dx}{8\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)(bx^2+a)^2}$$

input
```
integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/2*d^3*x/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(d*x^2
 + c)) + 1/8*(3*b^4*c^2 - 14*a*b^3*c*d + 35*a^2*b^2*d^2)*arctan(b*x/sqrt(a
*b))/((a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 +
 a^6*d^4)*sqrt(a*b)) - 1/2*(7*b*c*d^3 - a*d^4)*arctan(d*x/sqrt(c*d))/((b^4
*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sq
rt(c*d)) + 1/8*(3*b^4*c*x^3 - 11*a*b^3*d*x^3 + 5*a*b^3*c*x - 13*a^2*b^2*d*
x)/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*(b*x^2 + a)^
2)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 7.58 (sec) , antiderivative size = 8635, normalized size of antiderivative = 36.59

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)^3*(c + d*x^2)^2),x)
```

output
```
((x^5*(4*a^2*b^2*d^3 - 3*b^4*c^2*d + 11*a*b^3*c*d^2))/(8*a^2*c*(a^3*d^3 -
b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(4*a^3*d^3 - 5*b^3*c^3 + 13
*a*b^2*c^2*d))/(8*a*c*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*x^
3*(8*a^3*d^3 - 3*b^3*c^3 + 6*a*b^2*c^2*d + 13*a^2*b*c*d^2))/(8*a^2*c*(a*d
- b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a^2*c + x^2*(a^2*d + 2*a*b*c) +
x^4*(b^2*c + 2*a*b*d) + b^2*d*x^6) - (atan(((((x*(16*a^6*b^3*d^9 + 9*b^9*c
^6*d^3 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*
a^3*b^6*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 -
 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d
^3 + 15*a^8*b^2*c^4*d^4)) - (((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2
 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5
- 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*
a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 -
28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*
a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7
*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) -
(x*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*(256*a^4*b^11*c^
11*d^2 - 1792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*
d^5 + 3584*a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8
+ 5120*a^11*b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^1...
```

3.42 $\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx$

3.42.1 Optimal result

Integrand size = 19, antiderivative size = 315

$$\begin{aligned}\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx =& \frac{d(3b^2c^2-13abcd-2a^2d^2)\,x}{8a^2c(bc-ad)^3(c+dx^2)^2} \\ &+ \frac{bx}{4a(bc-ad)(a+bx^2)^2(c+dx^2)^2} \\ &+ \frac{b(3bc-11ad)x}{8a^2(bc-ad)^2(a+bx^2)(c+dx^2)^2} \\ &+ \frac{3d(bc+ad)(b^2c^2-6abcd+a^2d^2)\,x}{8a^2c^2(bc-ad)^4(c+dx^2)} \\ &+ \frac{3b^{5/2}(b^2c^2-6abcd+21a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^5} \\ &- \frac{3d^{5/2}(21b^2c^2-6abcd+a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^5}\end{aligned}$$

output
```
1/8*d*(-2*a^2*d^2-13*a*b*c*d+3*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(d*x^2+c)^2+1
/4*b*x/a/(-a*d+b*c)/(b*x^2+a)^2/(d*x^2+c)^2+1/8*b*(-11*a*d+3*b*c)*x/a^2/(-
a*d+b*c)^2/(b*x^2+a)/(d*x^2+c)^2+3/8*d*(a*d+b*c)*(a^2*d^2-6*a*b*c*d+b^2*c^
2)*x/a^2/c^2/(-a*d+b*c)^4/(d*x^2+c)+3/8*b^(5/2)*(21*a^2*d^2-6*a*b*c*d+b^2*
c^2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/(-a*d+b*c)^5-3/8*d^(5/2)*(a^2*d^2-6
*a*b*c*d+21*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/(-a*d+b*c)^5
```

3.42.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+bx^2)^3 (c+dx^2)^3} \, dx = \frac{1}{8} \left(-\frac{3b^{5/2}(b^2c^2-6abcd+21a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(-bc+ad)^5} + \frac{(bc-ad)x\left(\frac{3b^4c}{a^2(a+bx^2)}+\frac{3ad^4}{c^2(c+dx^2)}+\frac{b^3(2bc-17ad-15bdx^2)}{a(a+bx^2)^2}-\frac{d^3(17bc-2ad+15bdx^2)}{c(c+dx^2)^2}\right)-\frac{3d^{5/2}(21b^2c^2-6abcd+a^2d^2)\arctan}{c^{5/2}}}{(bc-ad)^5}$$

input
```
Integrate[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

output
```
((-3*b^(5/2)*(b^2*c^2 - 6*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(a^(5/2)*(-(b*c) + a*d)^5) + ((b*c - a*d)*x*((3*b^4*c)/(a^2*(a + b*x^2)
) + (3*a*d^4)/(c^2*(c + d*x^2)) + (b^3*(2*b*c - 17*a*d - 15*b*d*x^2))/(a*(
a + b*x^2)^2) - (d^3*(17*b*c - 2*a*d + 15*b*d*x^2))/(c*(c + d*x^2)^2)) - (
3*d^(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/
c^(5/2))/(b*c - a*d)^5)/8
```

3.42.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {316, 25, 402, 25, 402, 27, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^3 (c+dx^2)^3} dx$$

$$\Big\downarrow \text{316}$$

$$\frac{bx}{4a(a+bx^2)^2(c+dx^2)^2(bc-ad)} - \frac{\int -\frac{7bdx^2+3bc-4ad}{(bx^2+a)^2(dx^2+c)^3}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7bdx^2+3bc-4ad}{(bx^2+a)^2(dx^2+c)^3}dx}{4a(bc-ad)} + \frac{bx}{4a(a+bx^2)^2(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{bx(3bc-11ad)}{2a(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\int -\frac{3b^2c^2-3abdc+8a^2d^2+5bd(3bc-11ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^3}dx}{2a(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3b^2c^2-3abdc+8a^2d^2+5bd(3bc-11ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^3}dx}{2a(bc-ad)}+\frac{bx(3bc-11ad)}{2a(a+bx^2)(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{12\left(b^3c^3-3ab^2dc^2+8a^2bd^2c-2a^3d^3+bd\left(3b^2c^2-13abdc-2a^2d^2\right)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{4c(bc-ad)}+\frac{dx\left(-2a^2d^2-13abcd+3b^2c^2\right)}{c\left(c+dx^2\right)^2(bc-ad)}}{2a(bc-ad)}+\frac{bx(3bc-11ad)}{2a(a+bx^2)(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{b^3c^3-3ab^2dc^2+8a^2bd^2c-2a^3d^3+bd\left(3b^2c^2-13abdc-2a^2d^2\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{c(bc-ad)}+\frac{dx\left(-2a^2d^2-13abcd+3b^2c^2\right)}{c\left(c+dx^2\right)^2(bc-ad)}}{2a(bc-ad)}+\frac{bx(3bc-11ad)}{2a(a+bx^2)(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{3\left(\frac{\int \frac{2\left(b^4c^4-5ab^3dc^3+16a^2b^2d^2c^2-5a^3bd^3c+a^4d^4+bd(bc+ad)\left(b^2c^2-6abdc+a^2d^2\right)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{dx(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{c\left(c+dx^2\right)(bc-ad)}\right)}{c(bc-ad)}+\frac{dx\left(-2a^2d^2-13abcd+3b^2c^2\right)}{c\left(c+dx^2\right)^2(bc-ad)}}{2a(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

3.42. $\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx$

$$\frac{\frac{3\left(\frac{\frac{\int \frac{b^4c^4-5ab^3dc^3+16a^2b^2d^2c^2-5a^3bd^3c+a^4d^4+bd(bc+ad)\left(b^2c^2-6abdc+a^2d^2\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{c(bc-ad)}+\frac{dx(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{c\left(c+dx^2\right)(bc-ad)}\right)}{c(bc-ad)}+\frac{dx\left(-2a^2d^2-13abcd+3b^2c^2\right)}{c\left(c+dx^2\right)^2(bc-ad)}}{2a(bc-ad)}+}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 397

$$\frac{\frac{3\left(\frac{\frac{b^3c^2\left(21a^2d^2-6abcd+b^2c^2\right)\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{a^2d^3\left(a^2d^2-6abcd+21b^2c^2\right)\int \frac{1}{dx^2+c}dx}{bc-ad}}{c(bc-ad)}+\frac{dx(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{c\left(c+dx^2\right)(bc-ad)}\right)}{c(bc-ad)}+\frac{dx\left(-2a^2d^2-13abcd+3b^2c^2\right)}{c\left(c+dx^2\right)^2(bc-ad)}}{2a(bc-ad)}+\frac{}{2}}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 218

$$\frac{\frac{3\left(\frac{\frac{b^{5/2}c^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(21a^2d^2-6abcd+b^2c^2\right)}{\sqrt{a}(bc-ad)}-\frac{a^2d^{5/2}\left(a^2d^2-6abcd+21b^2c^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{c(bc-ad)}+\frac{dx(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{c\left(c+dx^2\right)(bc-ad)}\right)}{c(bc-ad)}+\frac{dx\left(-2a^2d^2-13abcd+3b^2\right)}{c\left(c+dx^2\right)^2(bc-ad)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

output
```
(b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)^2) + ((b*(3*b*c - 11*a*d)
*x)/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((d*(3*b^2*c^2 - 13*a*b*
c*d - 2*a^2*d^2)*x)/(c*(b*c - a*d)*(c + d*x^2)^2) + (3*((d*(b*c + a*d)*(b^
2*c^2 - 6*a*b*c*d + a^2*d^2)*x)/(c*(b*c - a*d)*(c + d*x^2)) + ((b^(5/2)*c^
2*(b^2*c^2 - 6*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]
*(b*c - a*d)) - (a^2*d^(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sq
rt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(c*(b*c - a*d))))/(c*(b*c - a*d)
))/(2*a*(b*c - a*d)))/(4*a*(b*c - a*d))
```

3.42. $\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^3}\,dx$

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.42.4 Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.82

method	result
default	$-\frac{b^3\left(\frac{\frac{3b(5a^2d^2-6abcd+b^2c^2)x^3}{8a^2}+\frac{(17a^2d^2-22abcd+5b^2c^2)x}{8a}}{(bx^2+a)^2}+\frac{3(21a^2d^2-6abcd+b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(ad-bc)^5}+\frac{d^3\left(\frac{\frac{3d(a^2d^2-6abcd+5b^2c^2)x}{8c^2}}{(d}\right.}{}$
risch	Expression too large to display

input
```
int(1/(b*x^2+a)^3/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-b^3/(a*d-b*c)^5*((3/8*b*(5*a^2*d^2-6*a*b*c*d+b^2*c^2)/a^2*x^3+1/8*(17*a^2
*d^2-22*a*b*c*d+5*b^2*c^2)/a*x)/(b*x^2+a)^2+3/8*(21*a^2*d^2-6*a*b*c*d+b^2*
c^2)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))+d^3/(a*d-b*c)^5*((3/8*d*(a^2
*d^2-6*a*b*c*d+5*b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-22*a*b*c*d+17*b^2*c^2)/c*
x)/(d*x^2+c)^2+3/8*(a^2*d^2-6*a*b*c*d+21*b^2*c^2)/c^2/(c*d)^(1/2)*arctan(d
*x/(c*d)^(1/2)))
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. 2(287) = 574.

Time = 7.15 (sec) , antiderivative size = 5070, normalized size of antiderivative = 16.10

$$\int \frac{1}{(a+bx^2)^3\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Too large to display
```

3.42.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^3 (c+dx^2)^3} dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**3/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(287) = 574$.

Time = 0.32 (sec) , antiderivative size = 820, normalized size of antiderivative = 2.60

$$\int \frac{1}{(a+bx^2)^3 (c+dx^2)^3} dx$$
$$= \frac{3 (b^5c^2 - 6 ab^4cd + 21 a^2b^3d^2) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 (a^2b^5c^5 - 5 a^3b^4c^4d + 10 a^4b^3c^3d^2 - 10 a^5b^2c^2d^3 + 5 a^6bcd^4 - a^7d^5)\sqrt{ab}}$$
$$- \frac{3 (21 b^2c^2d^3 - 6 abcd^4 + a^2d^5) \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8 (b^5c^7 - 5 ab^4c^6d + 10 a^2b^3c^5d^2 - 10 a^3b^2c^4d^3 + 5 a^4bc^3d^4 - a^5c^2d^5)\sqrt{cd}}$$
$$+ \frac{3 (b^5c^3d^2 - 5 ab^4c^2d^3 - 5 a^2b^3cd^4 + a^3b^2d^5)x^7 + (6}{8 (a^4b^4c^8 - 4 a^5b^3c^7d + 6 a^6b^2c^6d^2 - 4 a^7bc^5d^3 + a^8c^4d^4 + (a^2b^6c^6d^2 - 4 a^3b^5c^5d^3 + 6 a^4b^4c^4d^4 - 4 a^5b^3c^3}$$

input
```
integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
3/8*(b^5*c^2 - 6*a*b^4*c*d + 21*a^2*b^3*d^2)*arctan(b*x/sqrt(a*b))/((a^2*b
^5*c^5 - 5*a^3*b^4*c^4*d + 10*a^4*b^3*c^3*d^2 - 10*a^5*b^2*c^2*d^3 + 5*a^6
*b*c*d^4 - a^7*d^5)*sqrt(a*b)) - 3/8*(21*b^2*c^2*d^3 - 6*a*b*c*d^4 + a^2*d
^5)*arctan(d*x/sqrt(c*d))/((b^5*c^7 - 5*a*b^4*c^6*d + 10*a^2*b^3*c^5*d^2 -
 10*a^3*b^2*c^4*d^3 + 5*a^4*b*c^3*d^4 - a^5*c^2*d^5)*sqrt(c*d)) + 1/8*(3*(
b^5*c^3*d^2 - 5*a*b^4*c^2*d^3 - 5*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^7 + (6*b^
5*c^4*d - 25*a*b^4*c^3*d^2 - 34*a^2*b^3*c^2*d^3 - 25*a^3*b^2*c*d^4 + 6*a^4
*b*d^5)*x^5 + (3*b^5*c^5 - 5*a*b^4*c^4*d - 34*a^2*b^3*c^3*d^2 - 34*a^3*b^2
*c^2*d^3 - 5*a^4*b*c*d^4 + 3*a^5*d^5)*x^3 + (5*a*b^4*c^5 - 17*a^2*b^3*c^4*
d - 17*a^4*b*c^2*d^3 + 5*a^5*c*d^4)*x)/(a^4*b^4*c^8 - 4*a^5*b^3*c^7*d + 6*
a^6*b^2*c^6*d^2 - 4*a^7*b*c^5*d^3 + a^8*c^4*d^4 + (a^2*b^6*c^6*d^2 - 4*a^3
*b^5*c^5*d^3 + 6*a^4*b^4*c^4*d^4 - 4*a^5*b^3*c^3*d^5 + a^6*b^2*c^2*d^6)*x^
8 + 2*(a^2*b^6*c^7*d - 3*a^3*b^5*c^6*d^2 + 2*a^4*b^4*c^5*d^3 + 2*a^5*b^3*c
^4*d^4 - 3*a^6*b^2*c^3*d^5 + a^7*b*c^2*d^6)*x^6 + (a^2*b^6*c^8 - 9*a^4*b^4
*c^6*d^2 + 16*a^5*b^3*c^5*d^3 - 9*a^6*b^2*c^4*d^4 + a^8*c^2*d^6)*x^4 + 2*(
a^3*b^5*c^8 - 3*a^4*b^4*c^7*d + 2*a^5*b^3*c^6*d^2 + 2*a^6*b^2*c^5*d^3 - 3*
a^7*b*c^4*d^4 + a^8*c^3*d^5)*x^2)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.82

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx$$

$$= \frac{3\,(b^5c^2 - 6\,ab^4cd + 21\,a^2b^3d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^5c^5 - 5\,a^3b^4c^4d + 10\,a^4b^3c^3d^2 - 10\,a^5b^2c^2d^3 + 5\,a^6bcd^4 - a^7d^5)\sqrt{ab}}$$
$$- \frac{3\,(21\,b^2c^2d^3 - 6\,abcd^4 + a^2d^5)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^5c^7 - 5\,ab^4c^6d + 10\,a^2b^3c^5d^2 - 10\,a^3b^2c^4d^3 + 5\,a^4bc^3d^4 - a^5c^2d^5)\sqrt{cd}}$$
$$+ \frac{3\,b^5c^3d^2x^7 - 15\,ab^4c^2d^3x^7 - 15\,a^2b^3cd^4x^7 + 3\,a^3b^2d^5x^7 + 6\,b^5c^4dx^5 - 25\,ab^4c^3d^2x^5 - 34\,a^2b^3c^2d^3x^5 - 25}{8\,(a^2b^4c^6 - 4\,a^3}$$

input
```
integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
3/8*(b^5*c^2 - 6*a*b^4*c*d + 21*a^2*b^3*d^2)*arctan(b*x/sqrt(a*b))/((a^2*b
^5*c^5 - 5*a^3*b^4*c^4*d + 10*a^4*b^3*c^3*d^2 - 10*a^5*b^2*c^2*d^3 + 5*a^6
*b*c*d^4 - a^7*d^5)*sqrt(a*b)) - 3/8*(21*b^2*c^2*d^3 - 6*a*b*c*d^4 + a^2*d
^5)*arctan(d*x/sqrt(c*d))/((b^5*c^7 - 5*a*b^4*c^6*d + 10*a^2*b^3*c^5*d^2 -
 10*a^3*b^2*c^4*d^3 + 5*a^4*b*c^3*d^4 - a^5*c^2*d^5)*sqrt(c*d)) + 1/8*(3*b
^5*c^3*d^2*x^7 - 15*a*b^4*c^2*d^3*x^7 - 15*a^2*b^3*c*d^4*x^7 + 3*a^3*b^2*d
^5*x^7 + 6*b^5*c^4*d*x^5 - 25*a*b^4*c^3*d^2*x^5 - 34*a^2*b^3*c^2*d^3*x^5 -
 25*a^3*b^2*c*d^4*x^5 + 6*a^4*b*d^5*x^5 + 3*b^5*c^5*x^3 - 5*a*b^4*c^4*d*x^
3 - 34*a^2*b^3*c^3*d^2*x^3 - 34*a^3*b^2*c^2*d^3*x^3 - 5*a^4*b*c*d^4*x^3 +
3*a^5*d^5*x^3 + 5*a*b^4*c^5*x - 17*a^2*b^3*c^4*d*x - 17*a^4*b*c^2*d^3*x +
5*a^5*c*d^4*x)/((a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5
*b*c^3*d^3 + a^6*c^2*d^4)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)^2)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 11150, normalized size of antiderivative = 35.40

$$\int \frac{1}{(a+bx^2)^3 (c+dx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)^3*(c + d*x^2)^3),x)
```

output
```
((x*(5*a^4*d^4 + 5*b^4*c^4 - 17*a*b^3*c^3*d - 17*a^3*b*c*d^3))/(8*a*c*(a^4
*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (x^
3*(34*a^2*b^3*c^3*d^2 - 3*b^5*c^5 - 3*a^5*d^5 + 34*a^3*b^2*c^2*d^3 + 5*a*b
^4*c^4*d + 5*a^4*b*c*d^4))/(8*a^2*c^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d
^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (x^5*(25*a*b^4*c^3*d^2 - 6*b^5*c^4*
d - 6*a^4*b*d^5 + 25*a^3*b^2*c*d^4 + 34*a^2*b^3*c^2*d^3))/(8*a^2*c^2*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (3*b
*d*x^7*(a^3*b*d^4 + b^4*c^3*d - 5*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3))/(8*a^2
*c^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^
3)))/(x^4*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + x^2*(2*a*b*c^2 + 2*a^2*c*d) +
x^6*(2*a*b*d^2 + 2*b^2*c*d) + a^2*c^2 + b^2*d^2*x^8) - (atan(((((x*(9*a^8*
b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 + 702*
a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^5*b
^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^
5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3
 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - (3*((
(3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^
4)/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^1
1*c^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (1642
5*a^10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c...
```

3.42. $\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx$

3.43 $\int \frac{(-1+x^2)^3}{(1+x^2)^4} \, dx$

3.43.1 Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{(-1+x^2)^3}{(1+x^2)^4} \, dx = -\frac{x(1-x^2)^2}{3(1+x^2)^3} - \frac{2x}{3(1+x^2)}$$

output
```
-1/3*x*(-x^2+1)^2/(x^2+1)^3-2/3*x/(x^2+1)
```

3.43.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(-1+x^2)^3}{(1+x^2)^4} \, dx = -\frac{x(3+2x^2+3x^4)}{3(1+x^2)^3}$$

input
```
Integrate[(-1 + x^2)^3/(1 + x^2)^4,x]
```

output
```
-1/3*(x*(3 + 2*x^2 + 3*x^4))/(1 + x^2)^3
```

3.43.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {315, 27, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-1)^3}{(x^2+1)^4}dx$$

$$\Big\downarrow \text{ 315}$$

$$\frac{1}{6}\int -\frac{4(1-x^2)}{(x^2+1)^2}dx-\frac{x(1-x^2)^2}{3(x^2+1)^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2}{3}\int \frac{1-x^2}{(x^2+1)^2}dx-\frac{x(1-x^2)^2}{3(x^2+1)^3}$$

$$\Big\downarrow \text{ 297}$$

$$-\frac{x(1-x^2)^2}{3(x^2+1)^3}-\frac{2x}{3(x^2+1)}$$

input
```
Int[(-1 + x^2)^3/(1 + x^2)^4,x]
```

output
```
-1/3*(x*(1 - x^2)^2)/(1 + x^2)^3 - (2*x)/(3*(1 + x^2))
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.43.4 Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{x(3x^4+2x^2+3)}{3(x^2+1)^3}$	23
default	$\frac{-x^5-\frac{2}{3}x^3-x}{(x^2+1)^3}$	23
norman	$\frac{-x^5-\frac{2}{3}x^3-x}{(x^2+1)^3}$	23
risch	$\frac{-x^5-\frac{2}{3}x^3-x}{(x^2+1)^3}$	23
parallelrisch	$\frac{-3x^5-2x^3-3x}{3(x^2+1)^3}$	24
meijerg	$-\frac{x(15x^4+40x^2+33)}{48(x^2+1)^3}-\frac{x(231x^4+280x^2+105)}{336(x^2+1)^3}+\frac{x(-15x^4+40x^2+15)}{80(x^2+1)^3}-\frac{x(-3x^4-8x^2+3)}{16(x^2+1)^3}$	90

input
```
int((x^2-1)^3/(x^2+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(3*x^4+2*x^2+3)/(x^2+1)^3
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{(-1+x^2)^3}{(1+x^2)^4}\,dx = -\frac{3\,x^5+2\,x^3+3\,x}{3\,(x^6+3\,x^4+3\,x^2+1)}$$

input
```
integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="fricas")
```

output
```
-1/3*(3*x^5 + 2*x^3 + 3*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{(-1+x^2)^3}{(1+x^2)^4}\,dx = \frac{-3x^5-2x^3-3x}{3x^6+9x^4+9x^2+3}$$

input
```
integrate((x**2-1)**3/(x**2+1)**4,x)
```

output
```
(-3*x**5 - 2*x**3 - 3*x)/(3*x**6 + 9*x**4 + 9*x**2 + 3)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{(-1+x^2)^3}{(1+x^2)^4}\,dx = -\frac{3\,x^5+2\,x^3+3\,x}{3\,(x^6+3\,x^4+3\,x^2+1)}$$

input
```
integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="maxima")
```

output
```
-1/3*(3*x^5 + 2*x^3 + 3*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \frac{(-1+x^2)^3}{(1+x^2)^4}\,dx = -\frac{3\left(x+\frac{1}{x}\right)^2-4}{3\left(x+\frac{1}{x}\right)^3}$$

input
```
integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="giac")
```

output
```
-1/3*(3*(x + 1/x)^2 - 4)/(x + 1/x)^3
```

3.43.9 Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{(-1+x^2)^3}{(1+x^2)^4}\,dx = \frac{4\,x}{3\,(x^2+1)^2} - \frac{x}{x^2+1} - \frac{4\,x}{3\,(x^2+1)^3}$$

input
```
int((x^2 - 1)^3/(x^2 + 1)^4,x)
```

output
```
(4*x)/(3*(x^2 + 1)^2) - x/(x^2 + 1) - (4*x)/(3*(x^2 + 1)^3)
```

3.44 $\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx$

3.44.1 Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx = \frac{x(1-x^2)^3}{4(1+x^2)^4} + \frac{3x(1-x^2)}{8(1+x^2)^2} + \frac{3\arctan(x)}{8}$$

output
```
1/4*x*(-x^2+1)^3/(x^2+1)^4+3/8*x*(-x^2+1)/(x^2+1)^2+3/8*arctan(x)
```

3.44.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx = \frac{5x-3x^3+3x^5-5x^7+3(1+x^2)^4\arctan(x)}{8(1+x^2)^4}$$

input
```
Integrate[(-1 + x^2)^4/(1 + x^2)^5,x]
```

output
```
(5*x - 3*x^3 + 3*x^5 - 5*x^7 + 3*(1 + x^2)^4*ArcTan[x])/(8*(1 + x^2)^4)
```

3.44.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {315, 27, 315, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-1)^4}{(x^2+1)^5}\,dx$$

$$\Big\downarrow \text{ 315}$$

$$\frac{1}{8}\int \frac{6(1-x^2)^2}{(x^2+1)^3}dx+\frac{x(1-x^2)^3}{4(x^2+1)^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{4}\int \frac{(1-x^2)^2}{(x^2+1)^3}dx+\frac{x(1-x^2)^3}{4(x^2+1)^4}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{3}{4}\left(\frac{1}{4}\int \frac{2}{x^2+1}dx+\frac{x(1-x^2)}{2(x^2+1)^2}\right)+\frac{x(1-x^2)^3}{4(x^2+1)^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{x^2+1}dx+\frac{x(1-x^2)}{2(x^2+1)^2}\right)+\frac{x(1-x^2)^3}{4(x^2+1)^4}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{3}{4}\left(\frac{\arctan(x)}{2}+\frac{x(1-x^2)}{2(x^2+1)^2}\right)+\frac{x(1-x^2)^3}{4(x^2+1)^4}$$

input
```
Int[(-1 + x^2)^4/(1 + x^2)^5,x]
```

output
```
(x*(1 - x^2)^3)/(4*(1 + x^2)^4) + (3*((x*(1 - x^2))/(2*(1 + x^2)^2) + ArcT
an[x]/2))/4
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.44.4 Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

method	result
default	$\frac{-\frac{5}{8}x^7+\frac{3}{8}x^5-\frac{3}{8}x^3+\frac{5}{8}x}{(x^2+1)^4}+\frac{3\arctan(x)}{8}$
risch	$\frac{-\frac{5}{8}x^7+\frac{3}{8}x^5-\frac{3}{8}x^3+\frac{5}{8}x}{(x^2+1)^4}+\frac{3\arctan(x)}{8}$
parallelrisch	$-\frac{-36i\ln(x+i)x^2+9i\ln(x-i)+9i\ln(x-i)x^8+54i\ln(x-i)x^4+30x^7+36i\ln(x-i)x^2+36i\ln(x-i)x^6-18x^5-9i\ln(x+i)-36i\ln(x}{48(x^2+1)^4}$
meijerg	$\frac{x(105x^6+385x^4+511x^2+279)}{384(x^2+1)^4}+\frac{3\arctan(x)}{8}-\frac{x(837x^6+1533x^4+1155x^2+315)}{1152(x^2+1)^4}+\frac{x(-105x^6+511x^4+385x^2+105)}{672(x^2+1)^4}-$

input
```
int((x^2-1)^4/(x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
(-5/8*x^7+3/8*x^5-3/8*x^3+5/8*x)/(x^2+1)^4+3/8*arctan(x)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx = -\frac{5\,x^7-3\,x^5+3\,x^3-3\,(x^8+4\,x^6+6\,x^4+4\,x^2+1)\arctan(x)-5\,x}{8\,(x^8+4\,x^6+6\,x^4+4\,x^2+1)}$$

input
```
integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="fricas")
```

output
```
-1/8*(5*x^7 - 3*x^5 + 3*x^3 - 3*(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)*arctan(x
) - 5*x)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx = \frac{-5x^7+3x^5-3x^3+5x}{8x^8+32x^6+48x^4+32x^2+8}+\frac{3\,\mathrm{atan}(x)}{8}$$

input
```
integrate((x**2-1)**4/(x**2+1)**5,x)
```

output
```
(-5*x**7 + 3*x**5 - 3*x**3 + 5*x)/(8*x**8 + 32*x**6 + 48*x**4 + 32*x**2 +
8) + 3*atan(x)/8
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx = -\frac{5\,x^7-3\,x^5+3\,x^3-5\,x}{8\,(x^8+4\,x^6+6\,x^4+4\,x^2+1)}+\frac{3}{8}\arctan(x)$$

input
```
integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="maxima")
```

output
```
-1/8*(5*x^7 - 3*x^5 + 3*x^3 - 5*x)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1) + 3/8
*arctan(x)
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx = \frac{3}{32}\,\pi \mathrm{sgn}(x) - \frac{5\left(x-\frac{1}{x}\right)^3 + 12\,x - \frac{12}{x}}{8\left(\left(x-\frac{1}{x}\right)^2+4\right)^2} + \frac{3}{16}\,\arctan\left(\frac{x^2-1}{2\,x}\right)$$

input
```
integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="giac")
```

output
```
3/32*pi*sgn(x) - 1/8*(5*(x - 1/x)^3 + 12*x - 12/x)/((x - 1/x)^2 + 4)^2 + 3
/16*arctan(1/2*(x^2 - 1)/x)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx = \frac{3\,\mathrm{atan}(x)}{8} + \frac{-\frac{5\,x^7}{8} + \frac{3\,x^5}{8} - \frac{3\,x^3}{8} + \frac{5\,x}{8}}{x^8+4\,x^6+6\,x^4+4\,x^2+1}$$

input
```
int((x^2 - 1)^4/(x^2 + 1)^5,x)
```

output
```
(3*atan(x))/8 + ((5*x)/8 - (3*x^3)/8 + (3*x^5)/8 - (5*x^7)/8)/(4*x^2 + 6*x
^4 + 4*x^6 + x^8 + 1)
```

3.45 $\int \sqrt{a+bx^2}(c+dx^2)^3\,dx$

3.45.1 Optimal result

Integrand size = 21, antiderivative size = 231

$$\begin{aligned}\int \sqrt{a+bx^2}(c+dx^2)^3\,dx = {} & \frac{(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3)\,x\sqrt{a+bx^2}}{128b^3} \\ & + \frac{d(72b^2c^2-52abcd+15a^2d^2)\,x(a+bx^2)^{3/2}}{192b^3} \\ & + \frac{d(12bc-5ad)x(a+bx^2)^{3/2}\,(c+dx^2)}{48b^2} \\ & + \frac{dx(a+bx^2)^{3/2}\,(c+dx^2)^2}{8b} \\ & + \frac{a(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}\end{aligned}$$

output
```
1/192*d*(15*a^2*d^2-52*a*b*c*d+72*b^2*c^2)*x*(b*x^2+a)^(3/2)/b^3+1/48*d*(-
5*a*d+12*b*c)*x*(b*x^2+a)^(3/2)*(d*x^2+c)/b^2+1/8*d*x*(b*x^2+a)^(3/2)*(d*x
^2+c)^2/b+1/128*a*(-5*a^3*d^3+24*a^2*b*c*d^2-48*a*b^2*c^2*d+64*b^3*c^3)*ar
ctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)+1/128*(-5*a^3*d^3+24*a^2*b*c*d^2-
48*a*b^2*c^2*d+64*b^3*c^3)*x*(b*x^2+a)^(1/2)/b^3
```

3.45.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.78

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx$$
$$=\frac{\sqrt{b}x\sqrt{a+bx^2}(15a^3d^3-2a^2bd^2(36c+5dx^2)+8ab^2d(18c^2+6cdx^2+d^2x^4)+48b^3(4c^3+6c^2dx^2+4cd^2x^4}{384b^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^3*d^3 - 2*a^2*b*d^2*(36*c + 5*d*x^2) + 8*
a*b^2*d*(18*c^2 + 6*c*d*x^2 + d^2*x^4) + 48*b^3*(4*c^3 + 6*c^2*d*x^2 + 4*c
*d^2*x^4 + d^3*x^6)) + 3*a*(-64*b^3*c^3 + 48*a*b^2*c^2*d - 24*a^2*b*c*d^2
+ 5*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(384*b^(7/2))
```

3.45.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {318, 403, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx$$
$$\Big\downarrow \text{318}$$
$$\frac{\int \sqrt{bx^2+a}\left(dx^2+c\right)\left(d(12bc-5ad)x^2+c(8bc-ad)\right)dx}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$
$$\Big\downarrow \text{403}$$
$$\frac{\frac{\int \sqrt{bx^2+a}\left(d\left(72b^2c^2-52abdc+15a^2d^2\right)x^2+c\left(48b^2c^2-18abdc+5a^2d^2\right)\right)dx}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(12bc-5ad)}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$
$$\Big\downarrow \text{299}$$

$$\frac{\frac{\frac{3\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(12bc-5ad\right)}{6b}}{8b}+$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{\frac{3\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(12bc-5a\right.}{6b}}{8b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\frac{3\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}}{8b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(}{6b}}{8b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

output
```
(d*x*(a + b*x^2)^(3/2)*(c + d*x^2)^2)/(8*b) + ((d*(12*b*c - 5*a*d)*x*(a +
b*x^2)^(3/2)*(c + d*x^2))/(6*b) + ((d*(72*b^2*c^2 - 52*a*b*c*d + 15*a^2*d^
2)*x*(a + b*x^2)^(3/2))/(4*b) + (3*(64*b^3*c^3 - 48*a*b^2*c^2*d + 24*a^2*b
*c*d^2 - 5*a^3*d^3)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*Sqrt[b])))/(4*b))/(6*b))/(8*b)
```

3.45. $\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx$

3.45.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.45.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$-\frac{5\left(a\left(a^{3}d^{3}-\frac{24}{5}a^{2}bc\,d^{2}+\frac{48}{5}a\,b^{2}c^{2}d-\frac{64}{5}b^{3}c^{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-x\sqrt{b\,x^{2}+a}\left(\frac{64\left(\frac{1}{2}d^{2}x^{4}+cd\,x^{2}+c^{2}\right)\left(\frac{d\,x^{2}}{2}+c\right)b^{\frac{7}{2}}}{5}+\left(\left(\frac{8}{15}d\right.\right.\right.}{128b^{\frac{7}{2}}}$
risch	$\frac{x\left(48b^{3}d^{3}x^{6}+8a\,b^{2}d^{3}x^{4}+192b^{3}c\,d^{2}x^{4}-10x^{2}a^{2}b\,d^{3}+48x^{2}a\,b^{2}c\,d^{2}+288x^{2}b^{3}c^{2}d+15a^{3}d^{3}-72a^{2}bc\,d^{2}+144a\,b^{2}c^{2}d+192b^{3}c^{3}\right)\sqrt{}}{384b^{3}}$
default	$c^{3}\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+d^{3}\left(\frac{x^{5}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{8b}-\frac{5a\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\right.}{}\right.}{2b}\right.}{8b}\right.$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-5/128*(a*(a^3*d^3-24/5*a^2*b*c*d^2+48/5*a*b^2*c^2*d-64/5*b^3*c^3)*arctanh
((b*x^2+a)^(1/2)/x/b^(1/2))-x*(b*x^2+a)^(1/2)*(64/5*(1/2*d^2*x^4+c*d*x^2+c
^2)*(1/2*d*x^2+c)*b^(7/2)+((8/15*d^2*x^4+16/5*c*d*x^2+48/5*c^2)*b^(5/2)+d*
a*((-2/3*d*x^2-24/5*c)*b^(3/2)+a*d*b^(1/2)))*d*a))/b^(7/2)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.72

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx$$
$$=\left[-\frac{3\left(64\,ab^3c^3-48\,a^2b^2c^2d+24\,a^3bcd^2-5\,a^4d^3\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(48\,b^4d^3x^7+\right.}{}\right.$$
$$-\frac{3\left(64\,ab^3c^3-48\,a^2b^2c^2d+24\,a^3bcd^2-5\,a^4d^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(48\,b^4d^3x^7+8\left(24\,b^4cd^2+ab^3d^3\right)\right.}{38}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/768*(3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*
sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*b^4*d^3*x^
7 + 8*(24*b^4*c*d^2 + a*b^3*d^3)*x^5 + 2*(144*b^4*c^2*d + 24*a*b^3*c*d^2 -
 5*a^2*b^2*d^3)*x^3 + 3*(64*b^4*c^3 + 48*a*b^3*c^2*d - 24*a^2*b^2*c*d^2 +
5*a^3*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/384*(3*(64*a*b^3*c^3 - 48*a^2*b^2
*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (48*b^4*d^3*x^7 + 8*(24*b^4*c*d^2 + a*b^3*d^3)*x^5 + 2*(144*b^4*c
^2*d + 24*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^3 + 3*(64*b^4*c^3 + 48*a*b^3*c^2*
d - 24*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.30

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx$$
$$=\begin{cases}\sqrt{a+bx^2}\left(\frac{d^3x^7}{8}+\frac{x^5\left(\frac{ad^3}{8}+3bcd^2\right)}{6b}+\frac{x^3\cdot\left(3acd^2-\frac{5a\left(\frac{ad^3}{8}+3bcd^2\right)}{6b}+3bc^2d\right)}{4b}+\frac{x\left(3ac^2d-\frac{3a\left(3acd^2-\frac{5a\left(\frac{ad^3}{8}+3bcd^2\right)}{6b}+3bc^2d\right)}{4b}+bc^3\right.}{2b}\right. \\ \sqrt{a}\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right)\end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d**3*x**7/8 + x**5*(a*d**3/8 + 3*b*c*d**2)/(6
*b) + x**3*(3*a*c*d**2 - 5*a*(a*d**3/8 + 3*b*c*d**2)/(6*b) + 3*b*c**2*d)/(
4*b) + x*(3*a*c**2*d - 3*a*(3*a*c*d**2 - 5*a*(a*d**3/8 + 3*b*c*d**2)/(6*b)
 + 3*b*c**2*d)/(4*b) + b*c**3)/(2*b)) + (a*c**3 - a*(3*a*c**2*d - 3*a*(3*a
*c*d**2 - 5*a*(a*d**3/8 + 3*b*c*d**2)/(6*b) + 3*b*c**2*d)/(4*b) + b*c**3)/
(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0
)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(c**3*x + c**2*d*x
**3 + 3*c*d**2*x**5/5 + d**3*x**7/7), True))
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.22

$$
\begin{aligned}
\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx ={}& \frac{(bx^2+a)^{\frac{3}{2}}d^3x^5}{8\,b} + \frac{(bx^2+a)^{\frac{3}{2}}cd^2x^3}{2\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}ad^3x^3}{48\,b^2} \\
&+ \frac{1}{2}\sqrt{bx^2+a}c^3x + \frac{3\,(bx^2+a)^{\frac{3}{2}}c^2dx}{4\,b} - \frac{3\sqrt{bx^2+a}ac^2dx}{8\,b} \\
&- \frac{3\,(bx^2+a)^{\frac{3}{2}}acd^2x}{8\,b^2} + \frac{3\sqrt{bx^2+a}a^2cd^2x}{16\,b^2} + \frac{5\,(bx^2+a)^{\frac{3}{2}}a^2d^3x}{64\,b^3} \\
&- \frac{5\sqrt{bx^2+a}a^3d^3x}{128\,b^3} + \frac{ac^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} - \frac{3\,a^2c^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} \\
&+ \frac{3\,a^3cd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} - \frac{5\,a^4d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}}
\end{aligned}
$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(3/2)*d^3*x^5/b + 1/2*(b*x^2 + a)^(3/2)*c*d^2*x^3/b - 5/48
*(b*x^2 + a)^(3/2)*a*d^3*x^3/b^2 + 1/2*sqrt(b*x^2 + a)*c^3*x + 3/4*(b*x^2
+ a)^(3/2)*c^2*d*x/b - 3/8*sqrt(b*x^2 + a)*a*c^2*d*x/b - 3/8*(b*x^2 + a)^(
3/2)*a*c*d^2*x/b^2 + 3/16*sqrt(b*x^2 + a)*a^2*c*d^2*x/b^2 + 5/64*(b*x^2 +
a)^(3/2)*a^2*d^3*x/b^3 - 5/128*sqrt(b*x^2 + a)*a^3*d^3*x/b^3 + 1/2*a*c^3*a
rcsinh(b*x/sqrt(a*b))/sqrt(b) - 3/8*a^2*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(3/
2) + 3/16*a^3*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/128*a^4*d^3*arcsinh
(b*x/sqrt(a*b))/b^(7/2)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx$$
$$=\frac{1}{384}\left(2\left(4\left(6\,d^3x^2+\frac{24\,b^6cd^2+ab^5d^3}{b^6}\right)x^2+\frac{144\,b^6c^2d+24\,ab^5cd^2-5\,a^2b^4d^3}{b^6}\right)x^2+\frac{3\,(64\,b^6c^3+48\,ab^5c^2$$
$$-\frac{(64\,ab^3c^3-48\,a^2b^2c^2d+24\,a^3bcd^2-5\,a^4d^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*d^3*x^2 + (24*b^6*c*d^2 + a*b^5*d^3)/b^6)*x^2 + (144*b^6*c^
2*d + 24*a*b^5*c*d^2 - 5*a^2*b^4*d^3)/b^6)*x^2 + 3*(64*b^6*c^3 + 48*a*b^5*
c^2*d - 24*a^2*b^4*c*d^2 + 5*a^3*b^3*d^3)/b^6)*sqrt(b*x^2 + a)*x - 1/128*(
64*a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*log(abs(-sqr
t(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^3\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^3,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^3, x)
```

3.46 $\int \sqrt{a+bx^2}(c+dx^2)^2\,dx$

3.46.1 Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx=\frac{(8b^2c^2-4abcd+a^2d^2)\,x\sqrt{a+bx^2}}{16b^2}+\frac{d(8bc-3ad)x\left(a+bx^2\right)^{3/2}}{24b^2}+\frac{dx\left(a+bx^2\right)^{3/2}(c+dx^2)}{6b}+\frac{a(8b^2c^2-4abcd+a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

output
```
1/24*d*(-3*a*d+8*b*c)*x*(b*x^2+a)^(3/2)/b^2+1/6*d*x*(b*x^2+a)^(3/2)*(d*x^2
+c)/b+1/16*a*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/
2))/b^(5/2)+1/16*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2
```

3.46.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx=\frac{\sqrt{b}x\sqrt{a+bx^2}(-3a^2d^2+2abd(6c+dx^2)+8b^2(3c^2+3cdx^2+d^2x^4))-3a(8b^2c^2-4abcd+a^2d^2)\log\left(-\sqrt{\right.}}{48b^{5/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-3*a^2*d^2 + 2*a*b*d*(6*c + d*x^2) + 8*b^2*(3*
c^2 + 3*c*d*x^2 + d^2*x^4)) - 3*a*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[-(
Sqrt[b]*x) + Sqrt[a + b*x^2]])/(48*b^(5/2))
```

3.46.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {318, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx$$

$$\Big\downarrow \text{318}$$

$$\frac{\int \sqrt{bx^2+a}\left(d(8bc-3ad)x^2+c(6bc-ad)\right)dx}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(a^2d^2-4abcd+8b^2c^2)}{4b}+\frac{dx(a+bx^2)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

output
```
(d*x*(a + b*x^2)^(3/2)*(c + d*x^2))/(6*b) + ((d*(8*b*c - 3*a*d)*x*(a + b*x
^2)^(3/2))/(4*b) + (3*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*((x*Sqrt[a + b*x^2
])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/(6*b)
```

3.46.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.46.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{a\left(a^{2} d^{2}-4 a b c d+8 b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)-x \sqrt{b\,x^{2}+a}\,\left(\left(-\frac{8}{3} d^{2} x^{4}-8 c d\,x^{2}-8 c^{2}\right) b^{\frac{5}{2}}+d\left(\left(-\frac{2 d\,x^{2}}{3}-4 c\right) b^{\frac{3}{2}}+a d \sqrt{b}\right) a\right)}{16 b^{\frac{5}{2}}}$
risch	$-\frac{x\left(-8 b^{2} d^{2} x^{4}-2 x^{2} a b\,d^{2}-24 x^{2} b^{2} c d+3 a^{2} d^{2}-12 a b c d-24 b^{2} c^{2}\right) \sqrt{b\,x^{2}+a}}{48 b^{2}}+\frac{a\left(a^{2} d^{2}-4 a b c d+8 b^{2} c^{2}\right) \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{16 b^{\frac{5}{2}}}$
default	$c^{2}\left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)+d^{2} \Bigg(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6 b}-\frac{a \Big(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4 b}-\frac{a \big(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln(x \sqrt{b}+\sqrt{b\,x^{2}+a}}{2 \sqrt{b}}}{4 b}}{2 b}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(a*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x
*(b*x^2+a)^(1/2)*((-8/3*d^2*x^4-8*c*d*x^2-8*c^2)*b^(5/2)+d*((-2/3*d*x^2-4*
c)*b^(3/2)+a*d*b^(1/2))*a))/b^(5/2)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.77

$$\int \sqrt{a+bx^2}(c+dx^2)^2\,dx$$
$$= \left[\frac{3\,(8\,ab^2c^2-4\,a^2bcd+a^3d^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,b^3d^2x^5+2\,(12\,b^3cd+ab^2d^2)x^3}{96\,b^3}\right.$$
$$-\frac{3\,(8\,ab^2c^2-4\,a^2bcd+a^3d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,b^3d^2x^5+2\,(12\,b^3cd+ab^2d^2)x^3+3\,(8\,b^3c^2+4\,ab}{48\,b^3}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/96*(3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8*b^3*d^2*x^5 + 2*(12*b^3*c*d + a*b^2*d^
2)*x^3 + 3*(8*b^3*c^2 + 4*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(b*x^2 + a))/b^3,
-1/48*(3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)) - (8*b^3*d^2*x^5 + 2*(12*b^3*c*d + a*b^2*d^2)*x^3 + 3*(8*
b^3*c^2 + 4*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.27

$$\int \sqrt{a+bx^2}(c+dx^2)^2\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{d^2x^5}{6}+\frac{x^3\left(\frac{ad^2}{6}+2bcd\right)}{4b}+\frac{x\left(2acd-\frac{3a\left(\frac{ad^2}{6}+2bcd\right)}{4b}+bc^2\right)}{2b}\right)+\left(ac^2-\frac{a\left(2acd-\frac{3a\left(\frac{ad^2}{6}+2bcd\right)}{4b}+bc^2\right)}{2b}\right)\left(\begin{cases}\frac{\log}{} \\ \frac{x\,\mathrm{lo}}{\sqrt{}}\end{cases}\right. \\ \sqrt{a}\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d**2*x**5/6 + x**3*(a*d**2/6 + 2*b*c*d)/(4*b)
 + x*(2*a*c*d - 3*a*(a*d**2/6 + 2*b*c*d)/(4*b) + b*c**2)/(2*b)) + (a*c**2
- a*(2*a*c*d - 3*a*(a*d**2/6 + 2*b*c*d)/(4*b) + b*c**2)/(2*b))*Piecewise((
log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt
(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5
), True))
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.13

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx = \frac{\left(bx^2+a\right)^{\frac{3}{2}}d^2x^3}{6\,b} + \frac{1}{2}\,\sqrt{bx^2+a}c^2x + \frac{\left(bx^2+a\right)^{\frac{3}{2}}cdx}{2\,b} - \frac{\sqrt{bx^2+a}acdx}{4\,b} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}ad^2x}{8\,b^2} + \frac{\sqrt{bx^2+a}a^2d^2x}{16\,b^2} + \frac{ac^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}} - \frac{a^2cd\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{4\,b^{\frac{3}{2}}} + \frac{a^3d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(3/2)*d^2*x^3/b + 1/2*sqrt(b*x^2 + a)*c^2*x + 1/2*(b*x^2 +
 a)^(3/2)*c*d*x/b - 1/4*sqrt(b*x^2 + a)*a*c*d*x/b - 1/8*(b*x^2 + a)^(3/2)*
a*d^2*x/b^2 + 1/16*sqrt(b*x^2 + a)*a^2*d^2*x/b^2 + 1/2*a*c^2*arcsinh(b*x/s
qrt(a*b))/sqrt(b) - 1/4*a^2*c*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/16*a^3*
d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx = \frac{1}{48}\left(2\left(4\,d^2x^2+\frac{12\,b^4cd+ab^3d^2}{b^4}\right)x^2+\frac{3\left(8\,b^4c^2+4\,ab^3cd-a^2b^2d^2\right)}{b^4}\right)\sqrt{bx^2+a}x - \frac{\left(8\,ab^2c^2-4\,a^2bcd+a^3d^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/48*(2*(4*d^2*x^2 + (12*b^4*c*d + a*b^3*d^2)/b^4)*x^2 + 3*(8*b^4*c^2 + 4*
a*b^3*c*d - a^2*b^2*d^2)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(8*a*b^2*c^2 - 4*a^
2*b*c*d + a^3*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^2\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^2,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^2, x)
```

3.47 $\int \sqrt{a+bx^2}(c+dx^2)\,dx$

3.47.1 Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \sqrt{a+bx^2}(c+dx^2)\,dx = \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b}+\frac{dx(a+bx^2)^{3/2}}{4b}+\frac{a(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/4*d*x*(b*x^2+a)^(3/2)/b+1/8*a*(-a*d+4*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(
1/2))/b^(3/2)+1/8*(-a*d+4*b*c)*x*(b*x^2+a)^(1/2)/b
```

3.47.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \sqrt{a+bx^2}(c+dx^2)\,dx = \frac{x\sqrt{a+bx^2}(4bc+ad+2bdx^2)}{8b}+\frac{a(-4bc+ad)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(4*b*c + a*d + 2*b*d*x^2))/(8*b) + (a*(-4*b*c + a*d)*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

3.47.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right) \, dx$$

$$\Big\downarrow \text{299}$$

$$\frac{(4bc-ad)\int \sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(4bc-ad)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(4bc-ad)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2),x]
```

output
```
(d*x*(a + b*x^2)^(3/2))/(4*b) + ((4*b*c - a*d)*((x*Sqrt[a + b*x^2])/2 + (a
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b)
```

3.47.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.47.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{x\left(2bd\,x^{2}+ad+4bc\right)\sqrt{b\,x^{2}+a}}{8b}-\frac{a\left(ad-4bc\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{3}{2}}}$	62
pseudoelliptic	$-\frac{\left(a^{2}d-4abc\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-x\sqrt{b\,x^{2}+a}\,\left(\left(2d\,x^{2}+4c\right)b^{\frac{3}{2}}+ad\sqrt{b}\right)}{8b^{\frac{3}{2}}}$	69
default	$c\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+d\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)$	98

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*b*d*x^2+a*d+4*b*c)*(b*x^2+a)^(1/2)/b-1/8*a*(a*d-4*b*c)/b^(3/2)*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.82

$$\int \sqrt{a+bx^2}(c+dx^2)\,dx$$
$$=\left[-\frac{(4\,abc-a^2d)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(2\,b^2dx^3+(4\,b^2c+abd)x)\sqrt{bx^2+a}}{16\,b^2},\right.$$
$$\left.-\frac{(4\,abc-a^2d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,b^2dx^3+(4\,b^2c+abd)x)\sqrt{bx^2+a}}{8\,b^2}\right]$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/16*((4*a*b*c - a^2*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) - 2*(2*b^2*d*x^3 + (4*b^2*c + a*b*d)*x)*sqrt(b*x^2 + a))/b^2, -1/8
*((4*a*b*c - a^2*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*d
*x^3 + (4*b^2*c + a*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \sqrt{a+bx^2}(c+dx^2)\,dx$$
$$=\begin{cases}\sqrt{a+bx^2}\left(\frac{dx^3}{4}+\frac{x\left(\frac{ad}{4}+bc\right)}{2b}\right)+\left(ac-\frac{a\left(\frac{ad}{4}+bc\right)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right) & \text{for } b\neq 0\\ \sqrt{a}\left(cx+\frac{dx^3}{3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d*x**3/4 + x*(a*d/4 + b*c)/(2*b)) + (a*c - a*
(a*d/4 + b*c)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sq
rt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(c*x
 + d*x**3/3), True))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \sqrt{a+bx^2}\,(c+dx^2)\;dx = \frac{1}{2}\,\sqrt{bx^2+a}cx + \frac{(bx^2+a)^{\frac{3}{2}}dx}{4\,b} - \frac{\sqrt{bx^2+a}adx}{8\,b} + \frac{ac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}} - \frac{a^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*c*x + 1/4*(b*x^2 + a)^(3/2)*d*x/b - 1/8*sqrt(b*x^2 + a
)*a*d*x/b + 1/2*a*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/8*a^2*d*arcsinh(b*x
/sqrt(a*b))/b^(3/2)
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \sqrt{a+bx^2}\,(c+dx^2)\;dx = \frac{1}{8}\,\sqrt{bx^2+a}\left(2\,dx^2 + \frac{4\,b^2c+abd}{b^2}\right)x - \frac{(4\,abc-a^2d)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*(2*d*x^2 + (4*b^2*c + a*b*d)/b^2)*x - 1/8*(4*a*b*c - a
^2*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}(c+dx^2)\ dx = \int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)\ dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2), x)
```

3.48 $\int \sqrt{a+bx^2}\,dx$

3.48.1 Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}x\sqrt{a+bx^2} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

output
```
1/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+1/2*x*(b*x^2+a)^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}x\sqrt{a+bx^2} - \frac{a\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

input
```
Integrate[Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2])/2 - (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

3.48.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}$$

input
```
Int[Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b
])
```

3.48.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.48.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}$	36
risch	$\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}$	36
pseudoelliptic	$\frac{\sqrt{b\,x^{2}+a}\,x\sqrt{b}+\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a}{2\sqrt{b}}$	40

input
```
int((b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\int\sqrt{a+bx^2}\,dx$$
$$=\left[\frac{2\sqrt{bx^2+a}bx+a\sqrt{b}\log\left(-2\,bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4\,b},\frac{\sqrt{bx^2+a}bx-a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\,b}\right]$$

input
```
integrate((b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a))/b, 1/2*(sqrt(b*x^2 + a)*b*x - a*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/b]
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \sqrt{a+bx^2}\,dx = \frac{\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

input
```
integrate((b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}x + \frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*x + 1/2*a*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}x - \frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(
b)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sqrt{a+bx^2}\,dx = \frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}$$

input
```
int((a + b*x^2)^(1/2),x)
```

output
```
(x*(a + b*x^2)^(1/2))/2 + (a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2
))
```

3.49 $\int \frac{\sqrt{a+bx^2}}{c+dx^2} \, dx$

3.49.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2} \, dx = \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d} - \frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}$$

output
```
arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)/d-arctanh(x*(-a*d+b*c)^(1/2)/c^
(1/2)/(b*x^2+a)^(1/2))*(-a*d+b*c)^(1/2)/d/c^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2} \, dx = -\frac{\frac{\sqrt{-bc+ad}\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}} + \sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{d}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c + d*x^2),x]
```

output
```
-(((Sqrt[-(b*c) + a*d]*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2
))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/Sqrt[c] + Sqrt[b]*Log[-(Sqrt[b]*x) + Sqr
t[a + b*x^2]])/d)
```

3.49.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {301, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2} \, dx$$

$$\Big\downarrow \text{301}$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}$$

input
```
Int[Sqrt[a + b*x^2]/(c + d*x^2),x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*ArcTan
h[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d)
```

3.49.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

3.49.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$\frac{\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)\sqrt{(ad-bc)c}-\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right)ad+\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right)bc}{d\sqrt{(ad-bc)c}}$
default	$-\frac{\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^{2}b-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{ad-bc}{d}}-\frac{\sqrt{b}\,\sqrt{-cd}\,\ln\left(\frac{-\frac{b\sqrt{-cd}}{d}+b\left(x+\frac{\sqrt{-cd}}{d}\right)}{\sqrt{b}}+\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^{2}b-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}}\right.}{d}}{2\sqrt{-cd}}$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
(b^(1/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*((a*d-b*c)*c)^(1/2)-arctan(c*(
b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))*a*d+arctan(c*(b*x^2+a)^(1/2)/x/((a*d
-b*c)*c)^(1/2))*b*c)/d/((a*d-b*c)*c)^(1/2)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 596, normalized size of antiderivative = 7.27

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx$$
$$= \left[\frac{2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2-4\,(ac^2x+}{d^2x^4+2\,cdx^2+c^2}\right.}{4\,d}\right.$$
$$-\frac{4\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2-4\,(ac^2x+(2\,bc^2-acd)x^3)\sqrt{bx^2}}{d^2x^4+2\,cdx^2+c^2}\right.}{4\,d}$$
$$\left.-\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}\sqrt{-\frac{bc-ad}{c}}}{2\,((b^2c-abd)x^3+(abc-a^2d)x)}\right)}{2\,d}\right]$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + sqrt((b*
c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*
b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 +
a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/d, -1/4*(4*sqrt(-b)*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a
*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*
x^4 + 2*c*d*x^2 + c^2)))/d, 1/2*(sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c -
 a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3
 + (a*b*c - a^2*d)*x)) + sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a))/d, -1/2*(2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(-(b*
c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*
c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/d]
```

3.49.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx = \int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c),x)
```

output
```
Integral(sqrt(a + b*x**2)/(c + d*x**2), x)
```

3.49.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx = \int \frac{\sqrt{bx^2+a}}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(d*x^2 + c), x)
```

3.49.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx = \int \frac{\sqrt{bx^2+a}}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx$$

$$= \begin{cases} \frac{\sqrt{-b}\,\mathrm{asin}\left(x\,\sqrt{-\frac{b}{a}}\right)}{c} & \text{if } ((a+b\,c=0 \wedge d=-1) \vee a\,d=b\,c) \wedge b<0 \\ \frac{\sqrt{b}\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{d}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{a\,d-b\,c}}{\sqrt{c}\,\sqrt{b\,x^2+a}}\right)\sqrt{a\,d-b\,c}}{\sqrt{c}\,d} & \text{if } a\neq 0 \wedge (((a+b\,c\neq 0 \vee d\neq -1) \wedge a\,d\neq b\,c) \vee \neg b < \\ \int \frac{\sqrt{b\,x^2+a}}{d\,x^2+c}\,dx & \text{if } ((((a+b\,c=0 \wedge d=-1) \vee a\,d=b\,c) \wedge b<0) \vee a = \end{cases}$$

input
```
int((a + b*x^2)^(1/2)/(c + d*x^2),x)
```

output
```
piecewise((a + b*c == 0 & d == -1 | a*d == b*c) & b < 0, ((-b)^(1/2)*asin(
x*(-b/a)^(1/2)))/c, a ~= 0 & ((a + b*c ~= 0 | d ~= -1) & a*d ~= b*c | ~b <
 0), (b^(1/2)*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/d + (atan((x*(a*d -
b*c)^(1/2))/(c^(1/2)*(a + b*x^2)^(1/2)))*(a*d - b*c)^(1/2))/(c^(1/2)*d), (
(a + b*c == 0 & d == -1 | a*d == b*c) & b < 0 | a == 0) & ((a + b*c ~= 0 |
 d ~= -1) & a*d ~= b*c | ~b < 0), int((a + b*x^2)^(1/2)/(c + d*x^2), x))
```

3.50 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx$

3.50.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx=\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}+\frac{a\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}$$

output
```
1/2*a*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-a*d+b*
c)^(1/2)+1/2*x*(b*x^2+a)^(1/2)/c/(d*x^2+c)
```

3.50.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx=\frac{x\sqrt{a+bx^2}}{2c^2+2cdx^2}-\frac{a\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\left(c+dx^2\right)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{2c^{3/2}\sqrt{-bc+ad}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

output
```
(x*Sqrt[a + b*x^2])/(2*c^2 + 2*c*d*x^2) - (a*ArcTan[(-(d*x*Sqrt[a + b*x^2]
) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(2*c^(3/2)*Sqrt[-(
b*c) + a*d])
```

3.50.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {292, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}dx$$
$$\Big\downarrow \text{ 292}$$
$$\frac{a\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{2c}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}$$
$$\Big\downarrow \text{ 291}$$
$$\frac{a\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}$$
$$\Big\downarrow \text{ 221}$$
$$\frac{a\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}$$

input
```
Int[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

output
```
(x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sq
rt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*Sqrt[b*c - a*d])
```

3.50.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

3.50.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{\frac{x\sqrt{b\,x^2+a}}{d\,x^2+c}-\frac{a\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}}{2c}$	69
default	Expression too large to display	1945

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/c*(x*(b*x^2+a)^(1/2)/(d*x^2+c)-a/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a
)^(1/2)/x/((a*d-b*c)*c)^(1/2)))
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.

Time = 0.32 (sec) , antiderivative size = 369, normalized size of antiderivative = 4.50

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx$$
$$= \left[\frac{4\,(bc^2-acd)\sqrt{bx^2+a}x+(adx^2+ac)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2+4\,((2\,bc-ac}{d^2x^4+2\,cdx^2+c^2}\right.}{8\,(bc^4-ac^3d+(bc^3d-ac^2d^2)x^2)}\right.$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x + (a*d*x^2 + a*c)*sqrt(b*c^2 - a
*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2
- 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(
b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(b*c^4 - a*c^3*d + (b*c^3*d - a*
c^2*d^2)*x^2), 1/4*(2*(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x - (a*d*x^2 + a*c)*
sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 +
a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(
b*c^4 - a*c^3*d + (b*c^3*d - a*c^2*d^2)*x^2)]
```

3.50.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)/(c + d*x**2)**2, x)
```

3.50.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^2, x)
```

3.50.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(66) = 132$.

Time = 0.89 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.65

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx = -\frac{a\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}c} + \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 b^{\frac{3}{2}}c-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a\sqrt{b}d+a^2\sqrt{b}d}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 bc-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 ad+a^2d\right)cd}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/2*a*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d
)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c) + (2*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*b^(3/2)*c - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*
d + a^2*sqrt(b)*d)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)*c*d)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\left(d\,x^2+c\right)^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c + d*x^2)^2,x)
```

output
```
int((a + b*x^2)^(1/2)/(c + d*x^2)^2, x)
```

3.51 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3} dx$

3.51.1 Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3} dx = -\frac{dx(a+bx^2)^{3/2}}{4c(bc-ad)(c+dx^2)^2} + \frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)(c+dx^2)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}$$

output
```
-1/4*d*x*(b*x^2+a)^(3/2)/c/(-a*d+b*c)/(d*x^2+c)^2+1/8*a*(-3*a*d+4*b*c)*arc
tanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(3/2)+
1/8*(-3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)/(d*x^2+c)
```

3.51.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1268 vs. 2(149) = 298.

Time = 9.42 (sec) , antiderivative size = 1268, normalized size of antiderivative = 8.51

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3} dx$$

$$= \frac{\sqrt{c}\left(-a^8d^2x(5c+3dx^2)+8192b^{15/2}c^3x^{12}\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)+a^7\left(bdx(43c^2-345cdx^2-216d^2x^4)+\sqrt{b}d\sqrt{a+bx^2}(-3c^2+59cdx^2+36d^2x^4)\right)-2048c\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

output
```
((Sqrt[c]*(-(a^8*d^2*x*(5*c + 3*d*x^2)) + 8192*b^(15/2)*c^3*x^12*(-(Sqrt[b
]*x) + Sqrt[a + b*x^2]) + a^7*(b*d*x*(43*c^2 - 345*c*d*x^2 - 216*d^2*x^4)
+ Sqrt[b]*d*Sqrt[a + b*x^2]*(-3*c^2 + 59*c*d*x^2 + 36*d^2*x^4)) - 2048*a*b
^(13/2)*c*x^10*(Sqrt[a + b*x^2]*(-11*c^2 + 10*c*d*x^2 + 4*d^2*x^4) + Sqrt[
b]*(13*c^2*x - 10*c*d*x^3 - 4*d^2*x^5)) + 512*a^2*b^(11/2)*x^8*(Sqrt[a + b
*x^2]*(45*c^3 - 106*c^2*d*x^2 - 22*c*d^2*x^4 + 12*d^3*x^6) + Sqrt[b]*(-65*
c^3*x + 126*c^2*d*x^3 + 30*c*d^2*x^5 - 12*d^3*x^7)) + 256*a^3*b^(9/2)*x^6*
(Sqrt[a + b*x^2]*(42*c^3 - 207*c^2*d*x^2 + 19*c*d^2*x^4 + 60*d^3*x^6) - Sq
rt[b]*(78*c^3*x - 303*c^2*d*x^3 + c*d^2*x^5 + 72*d^3*x^7)) + 56*a^5*b^(5/2
)*x^2*(Sqrt[a + b*x^2]*(3*c^3 - 80*c^2*d*x^2 + 130*c*d^2*x^4 + 96*d^3*x^6)
 - Sqrt[b]*(13*c^3*x - 216*c^2*d*x^3 + 238*c*d^2*x^5 + 192*d^3*x^7)) + 64*
a^4*b^(7/2)*x^4*(Sqrt[a + b*x^2]*(35*c^3 - 364*c^2*d*x^2 + 220*c*d^2*x^4 +
 216*d^3*x^6) - Sqrt[b]*(91*c^3*x - 692*c^2*d*x^3 + 272*c*d^2*x^5 + 324*d^
3*x^7)) + 2*a^6*(b^(3/2)*Sqrt[a + b*x^2]*(c^3 - 150*c^2*d*x^2 + 646*c*d^2*
x^4 + 420*d^3*x^6) - b^2*(13*c^3*x - 690*c^2*d*x^3 + 1846*c*d^2*x^5 + 1260
*d^3*x^7))))/(d*(-(b*c) + a*d)*(c + d*x^2)^2*(1024*a*b^5*x^10*(13*Sqrt[b]*
x - 11*Sqrt[a + b*x^2]) + 1280*a^2*b^4*x^8*(13*Sqrt[b]*x - 9*Sqrt[a + b*x^
2]) + 768*a^3*b^3*x^6*(13*Sqrt[b]*x - 7*Sqrt[a + b*x^2]) + 224*a^4*b^2*x^4
*(13*Sqrt[b]*x - 5*Sqrt[a + b*x^2]) + 28*a^5*b*x^2*(13*Sqrt[b]*x - 3*Sqrt[
a + b*x^2]) + 4096*b^6*x^12*(Sqrt[b]*x - Sqrt[a + b*x^2]) - a^6*(-13*Sq...
```

3.51.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {296, 292, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{296}$$

$$\frac{(4bc-3ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2}dx}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{292}$$

$$\frac{(4bc-3ad)\left(\frac{a\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}\right)}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \; 291$$

$$\frac{(4bc-3ad)\left(\frac{a\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}\right)}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{(4bc-3ad)\left(\frac{a\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}\right)}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

input
```
Int[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

output
```
-1/4*(d*x*(a + b*x^2)^(3/2))/(c*(b*c - a*d)*(c + d*x^2)^2) + ((4*b*c - 3*a
*d)*((x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*ArcTanh[(Sqrt[b*c - a*d]*x
)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*Sqrt[b*c - a*d])))/(4*c*(b*c - a*
d))
```

3.51.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.51.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{a\left(-\frac{\sqrt{b\,x^{2}+a}\,\left(3a\,d^{2}x^{2}-2bcd\,x^{2}+5acd-4b\,c^{2}\right)x}{a\left(d\,x^{2}+c\right)^{2}}+\frac{(3ad-4bc)\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}\right)}{8(ad-bc)c^{2}}$	120
default	Expression too large to display	4113

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a/(a*d-b*c)/c^2*(-(b*x^2+a)^(1/2)/a*(3*a*d^2*x^2-2*b*c*d*x^2+5*a*c*d-
4*b*c^2)*x/(d*x^2+c)^2+(3*a*d-4*b*c)/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a
)^(1/2)/x/((a*d-b*c)*c)^(1/2)))
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(129) = 258.

Time = 0.35 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.68

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx$$
$$=\left[\frac{(4\,abc^3-3\,a^2c^2d+(4\,abcd^2-3\,a^2d^3)x^4+2\,(4\,abc^2d-3\,a^2cd^2)x^2)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a\cdots}{\cdots}\right.}{32\,(b^2c^7-2\,abc^6d+a^2c^5d^2+(b^2c^{\cdots}}\right.$$
$$-\frac{(4\,abc^3-3\,a^2c^2d+(4\,abcd^2-3\,a^2d^3)x^4+2\,(4\,abc^2d-3\,a^2cd^2)x^2)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}((2\,bc-\cdots}{2\,((b^2c^2-abcd)x^{\cdots}}\right.}{16\,(b^2c^7-2\,abc^6d+a^2c^5d^2+(b^2c^5d^2-2\,abc^4d^3+\cdots}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/32*((4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + 2*(4*a*b
*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d
 + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*
d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2
+ c^2)) + 4*((2*b^2*c^3*d - 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^3 + (4*b^2*c^4
- 9*a*b*c^3*d + 5*a^2*c^2*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^7 - 2*a*b*c^6*d
+ a^2*c^5*d^2 + (b^2*c^5*d^2 - 2*a*b*c^4*d^3 + a^2*c^3*d^4)*x^4 + 2*(b^2*c
^6*d - 2*a*b*c^5*d^2 + a^2*c^4*d^3)*x^2), -1/16*((4*a*b*c^3 - 3*a^2*c^2*d
+ (4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + 2*(4*a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(
-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*
sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((2
*b^2*c^3*d - 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^3 + (4*b^2*c^4 - 9*a*b*c^3*d +
 5*a^2*c^2*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^7 - 2*a*b*c^6*d + a^2*c^5*d^2 +
 (b^2*c^5*d^2 - 2*a*b*c^4*d^3 + a^2*c^3*d^4)*x^4 + 2*(b^2*c^6*d - 2*a*b*c^
5*d^2 + a^2*c^4*d^3)*x^2)]
```

3.51.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
```

output
```
Integral(sqrt(a + b*x**2)/(c + d*x**2)**3, x)
```

3.51.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^3, x)
```

3.51.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(129) = 258$.

Time = 1.83 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.27

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx = -\frac{\left(4\,ab^{\frac{3}{2}}c-3\,a^2\sqrt{b}d\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{8\,\sqrt{-b^2c^2+abcd}(bc^3-ac^2d)} - \frac{4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 ab^{\frac{3}{2}}cd^2-3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 a^2\sqrt{b}d^3-16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 b^{\frac{7}{2}}c^3+40\left(\sqrt{b}x\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/8*(4*a*b^(3/2)*c - 3*a^2*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2
 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c
*d)*(b*c^3 - a*c^2*d)) - 1/4*(4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*
c*d^2 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^3 - 16*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*b^(7/2)*c^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^
(5/2)*c^2*d - 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^2 + 9*(sq
rt(b)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d^3 - 16*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^2*b^(5/2)*c^2*d + 28*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)
*c*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d^3 - 2*a^4*b^(3/2)
*c*d^2 + 3*a^5*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^
2*d)^2*(b*c^3*d - a*c^2*d^2))
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}}{{(d\,x^2+c)}^3}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c + d*x^2)^3,x)
```

output
```
int((a + b*x^2)^(1/2)/(c + d*x^2)^3, x)
```

3.52 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4} \, dx$

3.52.1 Optimal result

Integrand size = 21, antiderivative size = 208

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4} \, dx &= \frac{x\sqrt{a+bx^2}}{6c(c+dx^2)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)(c+dx^2)^2} \\ &\quad + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2(c+dx^2)} \\ &\quad + \frac{a(8b^2c^2-12abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{5/2}}\end{aligned}$$

output
```
1/16*a*(5*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)
/(b*x^2+a)^(1/2))/c^(7/2)/(-a*d+b*c)^(5/2)+1/6*x*(b*x^2+a)^(1/2)/c/(d*x^2+
c)^3+1/24*(-5*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)/(d*x^2+c)^2+1/48
*(-5*a*d+2*b*c)*(-3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/c^3/(-a*d+b*c)^2/(d*x^2+c
)
```

3.52.2 Mathematica [A] (verified)

Time = 10.64 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx$$
$$= \frac{x\sqrt{a+bx^2}\left((bc-ad)\,(8b^2c^2(3c^2+3cdx^2+d^2x^4)-2abcd(30c^2+35cdx^2+13d^2x^4)+a^2d^2(33c^2+40cdx\right.}{48c^3(bc-ad)^3\,(c+dx^2)^3}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

output
```
(x*Sqrt[a + b*x^2]*((b*c - a*d)*(8*b^2*c^2*(3*c^2 + 3*c*d*x^2 + d^2*x^4) -
 2*a*b*c*d*(30*c^2 + 35*c*d*x^2 + 13*d^2*x^4) + a^2*d^2*(33*c^2 + 40*c*d*x
^2 + 15*d^2*x^4)) + (3*a*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Sqrt[((b*c -
 a*d)*x^2)/(c*(a + b*x^2))]*(c + d*x^2)^3*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(
c*(a + b*x^2))]])/x^2))/(48*c^3*(b*c - a*d)^3*(c + d*x^2)^3)
```

3.52.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {314, 25, 402, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx$$
$$\Big\downarrow \text{314}$$
$$\frac{x\sqrt{a+bx^2}}{6c\,(c+dx^2)^3}-\frac{\int -\frac{4bx^2+5a}{\sqrt{bx^2+a}(dx^2+c)^3}dx}{6c}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{4bx^2+5a}{\sqrt{bx^2+a}(dx^2+c)^3}dx}{6c}+\frac{x\sqrt{a+bx^2}}{6c\,(c+dx^2)^3}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{2b(4bc-5ad)x^2+a(16bc-15ad)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}+\frac{x\sqrt{a+bx^2}(4bc-5ad)}{4c(c+dx^2)^2(bc-ad)}}{6c}+\frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{3a\left(8b^2c^2-12abdc+5a^2d^2\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{x\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}+\frac{x\sqrt{a+bx^2}(4bc-5ad)}{4c(c+dx^2)^2(bc-ad)}}{6c}+\frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3a\left(5a^2d^2-12abcd+8b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{x\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}+\frac{x\sqrt{a+bx^2}(4bc-5ad)}{4c(c+dx^2)^2(bc-ad)}}{6c}+\frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{3a\left(5a^2d^2-12abcd+8b^2c^2\right)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}+\frac{x\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}+\frac{x\sqrt{a+bx^2}(4bc-5ad)}{4c(c+dx^2)^2(bc-ad)}}{6c}+\frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{3a\left(5a^2d^2-12abcd+8b^2c^2\right)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}+\frac{x\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}+\frac{x\sqrt{a+bx^2}(4bc-5ad)}{4c(c+dx^2)^2(bc-ad)}}{6c}+\frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3}$$

input
```
Int[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

output
```
(x*Sqrt[a + b*x^2])/(6*c*(c + d*x^2)^3) + (((4*b*c - 5*a*d)*x*Sqrt[a + b*x
^2])/(4*c*(b*c - a*d)*(c + d*x^2)^2) + (((2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x
*Sqrt[a + b*x^2])/(2*c*(b*c - a*d)*(c + d*x^2)) + (3*a*(8*b^2*c^2 - 12*a*b
*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/
(2*c^(3/2)*(b*c - a*d)^(3/2)))/(4*c*(b*c - a*d)))/(6*c)
```

3.52. $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.52.4 Maple [A] (verified)

Time = 8.64 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{11x\sqrt{b\,x^2+a}\left(\frac{8b^2c^4}{11}-\frac{20bd\left(-\frac{2b\,x^2}{5}+a\right)c^3}{11}+\left(-\frac{4b\,x^2}{33}+a\right)\left(-2b\,x^2+a\right)d^2c^2+\frac{40x^2d^3\left(-\frac{13b\,x^2}{20}+a\right)ac}{33}+\frac{5a^2d^4x^4}{11}\right)\sqrt{(ad-bc)c}-}{16\sqrt{(ad-bc)c}\,(d\,x^2+c)^3(ad-bc)^2c^3}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/((a*d-b*c)*c)^(1/2)*(11*x*(b*x^2+a)^(1/2)*(8/11*b^2*c^4-20/11*b*d*(-2
/5*b*x^2+a)*c^3+(-4/33*b*x^2+a)*(-2*b*x^2+a)*d^2*c^2+40/33*x^2*d^3*(-13/20
*b*x^2+a)*a*c+5/11*a^2*d^4*x^4)*((a*d-b*c)*c)^(1/2)-5*(a^2*d^2-12/5*a*b*c*
d+8/5*b^2*c^2)*(d*x^2+c)^3*a*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2
)))/(d*x^2+c)^3/(a*d-b*c)^2/c^3
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(184) = 368.

Time = 0.64 (sec) , antiderivative size = 1220, normalized size of antiderivative = 5.87

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

output
```
[1/192*(3*(8*a*b^2*c^5 - 12*a^2*b*c^4*d + 5*a^3*c^3*d^2 + (8*a*b^2*c^2*d^3
 - 12*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 3*(8*a*b^2*c^3*d^2 - 12*a^2*b*c^2*d^3
 + 5*a^3*c*d^4)*x^4 + 3*(8*a*b^2*c^4*d - 12*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)
*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2
*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(
b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((8*b^3*c
^4*d^2 - 34*a*b^2*c^3*d^3 + 41*a^2*b*c^2*d^4 - 15*a^3*c*d^5)*x^5 + 2*(12*b
^3*c^5*d - 47*a*b^2*c^4*d^2 + 55*a^2*b*c^3*d^3 - 20*a^3*c^2*d^4)*x^3 + 3*(
8*b^3*c^6 - 28*a*b^2*c^5*d + 31*a^2*b*c^4*d^2 - 11*a^3*c^3*d^3)*x)*sqrt(b*
x^2 + a))/(b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3 + (b^3
*c^7*d^3 - 3*a*b^2*c^6*d^4 + 3*a^2*b*c^5*d^5 - a^3*c^4*d^6)*x^6 + 3*(b^3*c
^8*d^2 - 3*a*b^2*c^7*d^3 + 3*a^2*b*c^6*d^4 - a^3*c^5*d^5)*x^4 + 3*(b^3*c^9
*d - 3*a*b^2*c^8*d^2 + 3*a^2*b*c^7*d^3 - a^3*c^6*d^4)*x^2), -1/96*(3*(8*a*
b^2*c^5 - 12*a^2*b*c^4*d + 5*a^3*c^3*d^2 + (8*a*b^2*c^2*d^3 - 12*a^2*b*c*d
^4 + 5*a^3*d^5)*x^6 + 3*(8*a*b^2*c^3*d^2 - 12*a^2*b*c^2*d^3 + 5*a^3*c*d^4)
*x^4 + 3*(8*a*b^2*c^4*d - 12*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(-b*c
^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt
(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((8*b^3
*c^4*d^2 - 34*a*b^2*c^3*d^3 + 41*a^2*b*c^2*d^4 - 15*a^3*c*d^5)*x^5 + 2*(12
*b^3*c^5*d - 47*a*b^2*c^4*d^2 + 55*a^2*b*c^3*d^3 - 20*a^3*c^2*d^4)*x^3 ...
```

3.52.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**4,x)
```

output
```
Timed out
```

3.52.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^4, x)
```

3.52.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 958 vs. $2(184) = 368$.

Time = 1.48 (sec) , antiderivative size = 958, normalized size of antiderivative = 4.61

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx = -\frac{\left(8\,ab^{\frac{5}{2}}c^2 - 12\,a^2b^{\frac{3}{2}}cd + 5\,a^3\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d + 2\,bc - ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{16\,(b^2c^5 - 2\,abc^4d + a^2c^3d^2)\sqrt{-b^2c^2+abcd}} - \frac{24\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}ab^{\frac{5}{2}}c^2d^3 - 36\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{3}{2}}cd^4 + 15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^3\sqrt{b}d^5 - \cdots}{\cdots}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="giac")
```

output
```
-1/16*(8*a*b^(5/2)*c^2 - 12*a^2*b^(3/2)*c*d + 5*a^3*sqrt(b)*d^2)*arctan(1/
2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*
d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) - 1/2
4*(24*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*c^2*d^3 - 36*(sqrt(b)*x -
 sqrt(b*x^2 + a))^10*a^2*b^(3/2)*c*d^4 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^
10*a^3*sqrt(b)*d^5 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c^3*d^2
 - 480*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c^2*d^3 + 330*(sqrt(b)*
x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 - 75*(sqrt(b)*x - sqrt(b*x^2 + a)
)^8*a^4*sqrt(b)*d^5 - 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*c^5 + 1
216*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d - 2016*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*a^2*b^(7/2)*c^3*d^2 + 1736*(sqrt(b)*x - sqrt(b*x^2 + a))^
6*a^3*b^(5/2)*c^2*d^3 - 800*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*c*
d^4 + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 - 384*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*a^2*b^(9/2)*c^4*d + 1392*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*a^3*b^(7/2)*c^3*d^2 - 1608*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2
)*c^2*d^3 + 780*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 - 150*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^6*sqrt(b)*d^5 - 96*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^4*b^(7/2)*c^3*d^2 + 336*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(
5/2)*c^2*d^3 - 300*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 + 75*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*sqrt(b)*d^5 - 8*a^6*b^(5/2)*c^2*d^3...
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^4}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c + d*x^2)^4,x)
```

output
```
int((a + b*x^2)^(1/2)/(c + d*x^2)^4, x)
```

3.53 $\int (a+bx^2)^{3/2} (c+dx^2)^3 \, dx$

3.53.1 Optimal result

Integrand size = 21, antiderivative size = 272

$$\int (a+bx^2)^{3/2} (c+dx^2)^3 \, dx = \frac{3a(4bc-ad)(8b^2c^2-2abcd+a^2d^2)x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)(8b^2c^2-2abcd+a^2d^2)x(a+bx^2)^{3/2}}{128b^3} + \frac{d(36b^2c^2-20abcd+5a^2d^2)x(a+bx^2)^{5/2}}{160b^3} + \frac{d(14bc-5ad)x(a+bx^2)^{5/2}(c+dx^2)}{80b^2} + \frac{dx(a+bx^2)^{5/2}(c+dx^2)^2}{10b} + \frac{3a^2(4bc-ad)(8b^2c^2-2abcd+a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}$$

output
```
1/128*(-a*d+4*b*c)*(a^2*d^2-2*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(3/2)/b^3+1/1
60*d*(5*a^2*d^2-20*a*b*c*d+36*b^2*c^2)*x*(b*x^2+a)^(5/2)/b^3+1/80*d*(-5*a*
d+14*b*c)*x*(b*x^2+a)^(5/2)*(d*x^2+c)/b^2+1/10*d*x*(b*x^2+a)^(5/2)*(d*x^2+
c)^2/b+3/256*a^2*(-a*d+4*b*c)*(a^2*d^2-2*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1
/2)/(b*x^2+a)^(1/2))/b^(7/2)+3/256*a*(-a*d+4*b*c)*(a^2*d^2-2*a*b*c*d+8*b^2
*c^2)*x*(b*x^2+a)^(1/2)/b^3
```

3.53.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.83

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3 \, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(15a^4d^3-10a^3bd^2(9c+dx^2)+4a^2b^2d(60c^2+15cdx^2+2d^2x^4)+32b^4x^2(10c^3+$$

input
```
Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^4*d^3 - 10*a^3*b*d^2*(9*c + d*x^2) + 4*a^
2*b^2*d*(60*c^2 + 15*c*d*x^2 + 2*d^2*x^4) + 32*b^4*x^2*(10*c^3 + 20*c^2*d*
x^2 + 15*c*d^2*x^4 + 4*d^3*x^6) + 16*a*b^3*(50*c^3 + 70*c^2*d*x^2 + 45*c*d
^2*x^4 + 11*d^3*x^6)) + 15*a^2*(-32*b^3*c^3 + 16*a*b^2*c^2*d - 6*a^2*b*c*d
^2 + a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(1280*b^(7/2))
```

3.53.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {318, 403, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3 \, dx$$

$$\Big\downarrow \; 318$$

$$\frac{\int \left(bx^2+a\right)^{3/2} \left(dx^2+c\right) \left(d(14bc-5ad)x^2+c(10bc-ad)\right) dx}{10b}+\frac{dx\left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^2}{10b}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\int \left(bx^2+a\right)^{3/2} \left(3d\left(36b^2c^2-20abdc+5a^2d^2\right)x^2+c\left(80b^2c^2-22abdc+5a^2d^2\right)\right) dx}{8b}+\frac{dx\left(a+bx^2\right)^{5/2} \left(c+dx^2\right)(14bc-5ad)}{8b}}{10b}+\frac{dx\left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^2}{10b}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{\frac{5(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)\int \left(bx^2+a\right)^{3/2}dx}{2b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(5a^2d^2-20abcd+36b^2c^2\right)}{2b}}{8b}+\frac{dx(a+bx^2)^{5/2}(c+dx^2)(14bc-5ad)}{8b}}{10b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}{10b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{\frac{5(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(5a^2d^2-20abcd+36b^2c^2\right)}{2b}}{8b}+\frac{dx(a+bx^2)^{5/2}(c+dx^2)(14bc-5ad)}{8b}}{10b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}{10b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{\frac{5(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(5a^2d^2-20abcd+36b^2c^2\right)}{2b}}{8b}+\frac{dx(a+bx^2)^{5/2}(c+dx^2)(14bc-5ad)}{8b}}{10b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}{10b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\frac{5(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(5a^2d^2-20abcd+36b^2c^2\right)}{2b}}{8b}+\frac{dx(a+bx^2)^{5/2}(c+dx^2)(14bc-5ad)}{8b}}{10b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}{10b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\frac{5\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)}{2b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(5a^2d^2-20abcd+36b^2c^2\right)}{2b}}{8b}+\frac{dx(a+bx^2)^{5/2}(c+dx^2)(14bc-5ad)}{8b}}{10b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}{10b}$$

input

```
Int[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```

3.53. $\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3\,dx$

output
```
(d*x*(a + b*x^2)^(5/2)*(c + d*x^2)^2)/(10*b) + ((d*(14*b*c - 5*a*d)*x*(a +
 b*x^2)^(5/2)*(c + d*x^2))/(8*b) + ((d*(36*b^2*c^2 - 20*a*b*c*d + 5*a^2*d^
2)*x*(a + b*x^2)^(5/2))/(2*b) + (5*(4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(2*b))/(8*b))/(10*b)
```

3.53.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.53.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{3\left(\left(a^5d^3-6a^4bc\,d^2+16a^3b^2c^2d-32a^2b^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-x\left(\frac{160\left(\frac{11}{50}d^3x^6+\frac{9}{10}c\,d^2x^4+\frac{7}{5}c^2d\,x^2+c^3\right)a\,b^{\frac{7}{2}}}{3}+\frac{64x^2\left(\frac{2}{5}d^3\right.}{}\right.\right.}{256b^{\frac{7}{2}}}$
risch	$\frac{x\left(128b^4d^3x^8+176a\,b^3d^3x^6+480b^4c\,d^2x^6+8a^2b^2d^3x^4+720a\,b^3c\,d^2x^4+640b^4c^2d\,x^4-10a^3b\,d^3x^2+60a^2b^2c\,d^2x^2+1120a\,b^3c^2\right.}{1280b^3}$
default	$c^3\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+d^3\left(\frac{x^5\left(b\,x^2+a\right)^{\frac{5}{2}}}{10b}-\frac{a\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^2+a\right.}{6b}\right.}{}\right)}{}\right.$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/256/b^(7/2)*((a^5*d^3-6*a^4*b*c*d^2+16*a^3*b^2*c^2*d-32*a^2*b^3*c^3)*ar
ctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*(160/3*(11/50*d^3*x^6+9/10*c*d^2*x^4+7/
5*c^2*d*x^2+c^3)*a*b^(7/2)+64/3*x^2*(2/5*d^3*x^6+3/2*c*d^2*x^4+2*c^2*d*x^2
+c^3)*b^(9/2)+((8/15*d^2*x^4+4*c*d*x^2+16*c^2)*b^(5/2)+((-2/3*d*x^2-6*c)*b
^(3/2)+a*d*b^(1/2))*d*a)*d*a^2)*(b*x^2+a)^(1/2))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.85

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3\,dx = \left[-\frac{15\left(32\,a^2b^3c^3-16\,a^3b^2c^2d+6\,a^4bcd^2-a^5d^3\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(12\cdots}{\cdots}\right.$$

$$-\frac{15\left(32\,a^2b^3c^3-16\,a^3b^2c^2d+6\,a^4bcd^2-a^5d^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(128\,b^5d^3x^9+16\,(30\,b^5cd^2+11\,ab^4d\cdots\right.}{\cdots}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/2560*(15*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)
*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(128*b^5*d^3*
x^9 + 16*(30*b^5*c*d^2 + 11*a*b^4*d^3)*x^7 + 8*(80*b^5*c^2*d + 90*a*b^4*c*
d^2 + a^2*b^3*d^3)*x^5 + 10*(32*b^5*c^3 + 112*a*b^4*c^2*d + 6*a^2*b^3*c*d^
2 - a^3*b^2*d^3)*x^3 + 5*(160*a*b^4*c^3 + 48*a^2*b^3*c^2*d - 18*a^3*b^2*c*
d^2 + 3*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/1280*(15*(32*a^2*b^3*c^3 -
16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqr
t(b*x^2 + a)) - (128*b^5*d^3*x^9 + 16*(30*b^5*c*d^2 + 11*a*b^4*d^3)*x^7 +
8*(80*b^5*c^2*d + 90*a*b^4*c*d^2 + a^2*b^3*d^3)*x^5 + 10*(32*b^5*c^3 + 112
*a*b^4*c^2*d + 6*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^3 + 5*(160*a*b^4*c^3 + 48*
a^2*b^3*c^2*d - 18*a^3*b^2*c*d^2 + 3*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.94

$$\int (a+bx^2)^{3/2}(c+dx^2)^3\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{bd^3x^9}{10}+\frac{x^7\cdot\left(\frac{11abd^3}{10}+3b^2cd^2\right)}{8b}+\frac{x^5\left(a^2d^3+6abcd^2-\frac{7a\left(\frac{11abd^3}{10}+3b^2cd^2\right)}{8b}+3b^2c^2d\right)}{6b}+\frac{x^3\cdot\left(3a^2cd^2+6abc\right.}{} \right. \\ a^{\frac{3}{2}}\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**3,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(b*d**3*x**9/10 + x**7*(11*a*b*d**3/10 + 3*b**
2*c*d**2)/(8*b) + x**5*(a**2*d**3 + 6*a*b*c*d**2 - 7*a*(11*a*b*d**3/10 + 3
*b**2*c*d**2)/(8*b) + 3*b**2*c**2*d)/(6*b) + x**3*(3*a**2*c*d**2 + 6*a*b*c
**2*d - 5*a*(a**2*d**3 + 6*a*b*c*d**2 - 7*a*(11*a*b*d**3/10 + 3*b**2*c*d**
2)/(8*b) + 3*b**2*c**2*d)/(6*b) + b**2*c**3)/(4*b) + x*(3*a**2*c**2*d + 2*
a*b*c**3 - 3*a*(3*a**2*c*d**2 + 6*a*b*c**2*d - 5*a*(a**2*d**3 + 6*a*b*c*d*
*2 - 7*a*(11*a*b*d**3/10 + 3*b**2*c*d**2)/(8*b) + 3*b**2*c**2*d)/(6*b) + b
**2*c**3)/(4*b))/(2*b)) + (a**2*c**3 - a*(3*a**2*c**2*d + 2*a*b*c**3 - 3*a
*(3*a**2*c*d**2 + 6*a*b*c**2*d - 5*a*(a**2*d**3 + 6*a*b*c*d**2 - 7*a*(11*a
*b*d**3/10 + 3*b**2*c*d**2)/(8*b) + 3*b**2*c**2*d)/(6*b) + b**2*c**3)/(4*b
))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a
, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(3/2)*(c**3*x + c**2
*d*x**3 + 3*c*d**2*x**5/5 + d**3*x**7/7), True))
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.34

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3\,dx=\frac{\left(bx^2+a\right)^{\frac{5}{2}}d^3x^5}{10\,b}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}cd^2x^3}{8\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}ad^3x^3}{16\,b^2}+\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}c^3x+\frac{3}{8}\sqrt{bx^2+a}ac^3x+\frac{\left(bx^2+a\right)^{\frac{5}{2}}c^2dx}{2\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}ac^2dx}{8\,b}-\frac{3\sqrt{bx^2+a}a^2c^2dx}{16\,b}-\frac{3\left(bx^2+a\right)^{\frac{5}{2}}acd^2x}{16\,b^2}+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}a^2cd^2x}{64\,b^2}+\frac{9\sqrt{bx^2+a}a^3cd^2x}{128\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}a^2d^3x}{32\,b^3}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}a^3d^3x}{128\,b^3}-\frac{3\sqrt{bx^2+a}a^4d^3x}{256\,b^3}+\frac{3\,a^2c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}}-\frac{3\,a^3c^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{9\,a^4cd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{3\,a^5d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(5/2)*d^3*x^5/b + 3/8*(b*x^2 + a)^(5/2)*c*d^2*x^3/b - 1/1
6*(b*x^2 + a)^(5/2)*a*d^3*x^3/b^2 + 1/4*(b*x^2 + a)^(3/2)*c^3*x + 3/8*sqrt
(b*x^2 + a)*a*c^3*x + 1/2*(b*x^2 + a)^(5/2)*c^2*d*x/b - 1/8*(b*x^2 + a)^(3
/2)*a*c^2*d*x/b - 3/16*sqrt(b*x^2 + a)*a^2*c^2*d*x/b - 3/16*(b*x^2 + a)^(5
/2)*a*c*d^2*x/b^2 + 3/64*(b*x^2 + a)^(3/2)*a^2*c*d^2*x/b^2 + 9/128*sqrt(b*
x^2 + a)*a^3*c*d^2*x/b^2 + 1/32*(b*x^2 + a)^(5/2)*a^2*d^3*x/b^3 - 1/128*(b
*x^2 + a)^(3/2)*a^3*d^3*x/b^3 - 3/256*sqrt(b*x^2 + a)*a^4*d^3*x/b^3 + 3/8*
a^2*c^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 3/16*a^3*c^2*d*arcsinh(b*x/sqrt(a
*b))/b^(3/2) + 9/128*a^4*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 3/256*a^5*
d^3*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3\,dx=\frac{1}{1280}\left(2\left(4\left(2\left(8\,bd^3x^2+\frac{30\,b^9cd^2+11\,ab^8d^3}{b^8}\right)x^2+\frac{80\,b^9c^2d+90\,ab^8cd^2+a^2b^7d^3}{b^8}\right)x^2+\frac{5\,(3\right.\right.$$

$$-\frac{3\left(32\,a^2b^3c^3-16\,a^3b^2c^2d+6\,a^4bcd^2-a^5d^3\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/1280*(2*(4*(2*(8*b*d^3*x^2 + (30*b^9*c*d^2 + 11*a*b^8*d^3)/b^8)*x^2 + (8
0*b^9*c^2*d + 90*a*b^8*c*d^2 + a^2*b^7*d^3)/b^8)*x^2 + 5*(32*b^9*c^3 + 112
*a*b^8*c^2*d + 6*a^2*b^7*c*d^2 - a^3*b^6*d^3)/b^8)*x^2 + 5*(160*a*b^8*c^3
+ 48*a^2*b^7*c^2*d - 18*a^3*b^6*c*d^2 + 3*a^4*b^5*d^3)/b^8)*sqrt(b*x^2 + a
)*x - 3/256*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3 \, dx = \int \left(b\,x^2+a\right)^{3/2} \left(d\,x^2+c\right)^3 \, dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^3,x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^3, x)
```

3.54 $\int (a+bx^2)^{3/2}(c+dx^2)^2\,dx$

3.54.1 Optimal result

Integrand size = 21, antiderivative size = 196

$$\int (a+bx^2)^{3/2}(c+dx^2)^2\,dx = \frac{a(48b^2c^2-16abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{128b^2} + \frac{(48b^2c^2-16abcd+3a^2d^2)\,x(a+bx^2)^{3/2}}{192b^2} + \frac{d(10bc-3ad)x(a+bx^2)^{5/2}}{48b^2} + \frac{dx(a+bx^2)^{5/2}(c+dx^2)}{8b} + \frac{a^2(48b^2c^2-16abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

output
```
1/192*(3*a^2*d^2-16*a*b*c*d+48*b^2*c^2)*x*(b*x^2+a)^(3/2)/b^2+1/48*d*(-3*a
*d+10*b*c)*x*(b*x^2+a)^(5/2)/b^2+1/8*d*x*(b*x^2+a)^(5/2)*(d*x^2+c)/b+1/128
*a^2*(3*a^2*d^2-16*a*b*c*d+48*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/
b^(5/2)+1/128*a*(3*a^2*d^2-16*a*b*c*d+48*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2
```

3.54.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

$$\int (a+bx^2)^{3/2}(c+dx^2)^2\,dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(-9a^3d^2+6a^2bd(8c+dx^2)+16b^3x^2(6c^2+8cdx^2+3d^2x^4)+8ab^2(30c^2+28cdx}{384b^{5/2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-9*a^3*d^2 + 6*a^2*b*d*(8*c + d*x^2) + 16*b^3*
x^2*(6*c^2 + 8*c*d*x^2 + 3*d^2*x^4) + 8*a*b^2*(30*c^2 + 28*c*d*x^2 + 9*d^2
*x^4)) - 3*a^2*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/(384*b^(5/2))
```

3.54.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {318, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\int \left(bx^2+a\right)^{3/2} \left(d(10bc-3ad)x^2+c(8bc-ad)\right)dx}{8b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{\left(3a^2d^2-16abcd+48b^2c^2\right)\int \left(bx^2+a\right)^{3/2}dx}{6b}+\frac{dx\left(a+bx^2\right)^{5/2}(10bc-3ad)}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{\left(3a^2d^2-16abcd+48b^2c^2\right)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{dx\left(a+bx^2\right)^{5/2}(10bc-3ad)}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{\left(3a^2d^2-16abcd+48b^2c^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{dx\left(a+bx^2\right)^{5/2}(10bc-3ad)}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\left(3a^2d^2-16abcd+48b^2c^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{dx\left(a+bx^2\right)^{5/2}(10bc-3ad)}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)\left(3a^2d^2-16abcd+48b^2c^2\right)}{6b}+\frac{dx\left(a+bx^2\right)^{5/2}(10bc-3ad)}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

input
```
Int[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```

output
```
(d*x*(a + b*x^2)^(5/2)*(c + d*x^2))/(8*b) + ((d*(10*b*c - 3*a*d)*x*(a + b*
x^2)^(5/2))/(6*b) + ((48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*((x*(a + b*x^2)
^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(2*Sqrt[b])))/4))/(6*b))/(8*b)
```

3.54.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.54.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{\frac{3a^2\left(a^2d^2-\frac{16}{3}abcd+16b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{128}-\frac{3x\sqrt{b\,x^2+a}\left(-\frac{80\left(\frac{3}{10}d^2x^4+\frac{14}{15}cd\,x^2+c^2\right)a\,b^{\frac{5}{2}}}{3}-\frac{32x^2\left(\frac{1}{2}d^2x^4+\frac{4}{3}cd\,x^2+c^2\right)b^{\frac{7}{2}}}{3}+\left(\right.\right.}{128}}{b^{\frac{5}{2}}}$
risch	$-\frac{x\left(-48b^3d^2x^6-72a\,b^2d^2x^4-128b^3dc\,x^4-6x^2a^2b\,d^2-224x^2a\,b^2cd-96x^2b^3c^2+9a^3d^2-48a^2bcd-240a\,b^2c^2\right)\sqrt{b\,x^2+a}}{384b^2}+\frac{a}{}$
default	$c^2\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+d^2\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)}{4}\right.}{}\right.}{}\right.$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/128/b^(5/2)*(a^2*(a^2*d^2-16/3*a*b*c*d+16*b^2*c^2)*arctanh((b*x^2+a)^(1/
2)/x/b^(1/2))-x*(b*x^2+a)^(1/2)*(-80/3*(3/10*d^2*x^4+14/15*c*d*x^2+c^2)*a*
b^(5/2)-32/3*x^2*(1/2*d^2*x^4+4/3*c*d*x^2+c^2)*b^(7/2)+(2/3*(-d*x^2-8*c)*b
^(3/2)+a*d*b^(1/2))*d*a^2))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.76

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2\,dx = \left[\frac{3\left(48\,a^2b^2c^2-16\,a^3bcd+3\,a^4d^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(48\,b^4d^2x^7+8\left(1\right.\right.}{} \right.$$

$$-\frac{3\left(48\,a^2b^2c^2-16\,a^3bcd+3\,a^4d^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(48\,b^4d^2x^7+8\left(16\,b^4cd+9\,ab^3d^2\right)x^5+2\left(48\,b^4c^2\right.\right.}{384\,b^3}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/768*(3*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*sqrt(b)*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(48*b^4*d^2*x^7 + 8*(16*b^4*c*d +
9*a*b^3*d^2)*x^5 + 2*(48*b^4*c^2 + 112*a*b^3*c*d + 3*a^2*b^2*d^2)*x^3 + 3*
(80*a*b^3*c^2 + 16*a^2*b^2*c*d - 3*a^3*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/
384*(3*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - (48*b^4*d^2*x^7 + 8*(16*b^4*c*d + 9*a*b^3*d^2)*x^5
+ 2*(48*b^4*c^2 + 112*a*b^3*c*d + 3*a^2*b^2*d^2)*x^3 + 3*(80*a*b^3*c^2 + 1
6*a^2*b^2*c*d - 3*a^3*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.68

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{bd^2x^7}{8} + \frac{x^5\cdot\left(\frac{9abd^2}{8}+2b^2cd\right)}{6b} + \frac{x^3\left(a^2d^2+4abcd-\frac{5a\left(\frac{9abd^2}{8}+2b^2cd\right)}{6b}+b^2c^2\right)}{4b} + \frac{x\left(2a^2cd+2abc^2-\frac{3a\left(a^2d^2 \right.}{}\right.}{} \right. \\ a^{\frac{3}{2}}\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(b*d**2*x**7/8 + x**5*(9*a*b*d**2/8 + 2*b**2*c
*d)/(6*b) + x**3*(a**2*d**2 + 4*a*b*c*d - 5*a*(9*a*b*d**2/8 + 2*b**2*c*d)/
(6*b) + b**2*c**2)/(4*b) + x*(2*a**2*c*d + 2*a*b*c**2 - 3*a*(a**2*d**2 + 4
*a*b*c*d - 5*a*(9*a*b*d**2/8 + 2*b**2*c*d)/(6*b) + b**2*c**2)/(4*b))/(2*b)
) + (a**2*c**2 - a*(2*a**2*c*d + 2*a*b*c**2 - 3*a*(a**2*d**2 + 4*a*b*c*d -
 5*a*(9*a*b*d**2/8 + 2*b**2*c*d)/(6*b) + b**2*c**2)/(4*b))/(2*b))*Piecewis
e((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/s
qrt(b*x**2), True)), Ne(b, 0)), (a**(3/2)*(c**2*x + 2*c*d*x**3/3 + d**2*x*
*5/5), True))
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.16

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx = \frac{\left(bx^2+a\right)^{\frac{5}{2}}d^2x^3}{8\,b} + \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}c^2x + \frac{3}{8}\sqrt{bx^2+a}ac^2x + \frac{\left(bx^2+a\right)^{\frac{5}{2}}cdx}{3\,b} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}acdx}{12\,b} - \frac{\sqrt{bx^2+a}a^2cdx}{8\,b} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}ad^2x}{16\,b^2} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}a^2d^2x}{64\,b^2} + \frac{3\sqrt{bx^2+a}a^3d^2x}{128\,b^2} + \frac{3\,a^2c^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} - \frac{a^3cd\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{3\,a^4d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(5/2)*d^2*x^3/b + 1/4*(b*x^2 + a)^(3/2)*c^2*x + 3/8*sqrt(b
*x^2 + a)*a*c^2*x + 1/3*(b*x^2 + a)^(5/2)*c*d*x/b - 1/12*(b*x^2 + a)^(3/2)
*a*c*d*x/b - 1/8*sqrt(b*x^2 + a)*a^2*c*d*x/b - 1/16*(b*x^2 + a)^(5/2)*a*d^
2*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*a^2*d^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*a^3
*d^2*x/b^2 + 3/8*a^2*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/8*a^3*c*d*arcs
inh(b*x/sqrt(a*b))/b^(3/2) + 3/128*a^4*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx = \frac{1}{384}\left(2\left(4\left(6\,bd^2x^2+\frac{16\,b^7cd+9\,ab^6d^2}{b^6}\right)x^2+\frac{48\,b^7c^2+112\,ab^6cd+3\,a^2b^5d^2}{b^6}\right)x^2+\frac{3\,(80\,ab^6c}{}\right.$$

$$-\frac{(48\,a^2b^2c^2-16\,a^3bcd+3\,a^4d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*b*d^2*x^2 + (16*b^7*c*d + 9*a*b^6*d^2)/b^6)*x^2 + (48*b^7*c
^2 + 112*a*b^6*c*d + 3*a^2*b^5*d^2)/b^6)*x^2 + 3*(80*a*b^6*c^2 + 16*a^2*b^
5*c*d - 3*a^3*b^4*d^2)/b^6)*sqrt(b*x^2 + a)*x - 1/128*(48*a^2*b^2*c^2 - 16
*a^3*b*c*d + 3*a^4*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx = \int \left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right)^2 \,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^2,x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^2, x)
```

3.55 $\int (a+bx^2)^{3/2} (c+dx^2) \, dx$

3.55.1 Optimal result

Integrand size = 19, antiderivative size = 118

$$\int (a+bx^2)^{3/2} (c+dx^2) \, dx = \frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b} + \frac{(6bc-ad)x(a+bx^2)^{3/2}}{24b} + \frac{dx(a+bx^2)^{5/2}}{6b} + \frac{a^2(6bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
1/24*(-a*d+6*b*c)*x*(b*x^2+a)^(3/2)/b+1/6*d*x*(b*x^2+a)^(5/2)/b+1/16*a^2*(
-a*d+6*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/16*a*(-a*d+6*b*c)
*x*(b*x^2+a)^(1/2)/b
```

3.55.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int (a+bx^2)^{3/2} (c+dx^2) \, dx = \frac{x\sqrt{a+bx^2}(30abc+3a^2d+12b^2cx^2+14abdx^2+8b^2dx^4)}{48b} + \frac{a^2(-6bc+ad)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(30*a*b*c + 3*a^2*d + 12*b^2*c*x^2 + 14*a*b*d*x^2 + 8*b
^2*d*x^4))/(48*b) + (a^2*(-6*b*c + a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]
])/(16*b^(3/2))
```

3.55.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right) \, dx$$

$$\Big\downarrow \ 299$$

$$\frac{(6bc-ad)\int \left(bx^2+a\right)^{3/2}dx}{6b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 211$$

$$\frac{(6bc-ad)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 211$$

$$\frac{(6bc-ad)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 224$$

$$\frac{(6bc-ad)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 219$$

$$\frac{\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)(6bc-ad)}{6b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}$$

input
```
Int[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

output
```
(d*x*(a + b*x^2)^(5/2))/(6*b) + ((6*b*c - a*d)*((x*(a + b*x^2)^(3/2))/4 +
(3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*
Sqrt[b])))/4))/(6*b)
```

3.55.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.55.4 Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x\left(8b^{2}d\,x^{4}+14x^{2}abd+12b^{2}c\,x^{2}+3a^{2}d+30abc\right)\sqrt{b\,x^{2}+a}}{48b}-\frac{a^{2}(ad-6bc)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{3}{2}}}$
pseudoelliptic	$-\frac{(a^{3}d-6a^{2}bc)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-x\sqrt{b\,x^{2}+a}\left(10\left(\frac{7d\,x^{2}}{15}+c\right)a\,b^{\frac{3}{2}}+\left(\frac{8}{3}d\,x^{4}+4c\,x^{2}\right)b^{\frac{5}{2}}+a^{2}d\sqrt{b}\right)}{16b^{\frac{3}{2}}}$
default	$c\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+d\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\cdots\right.}{\cdots}\right.}{6b}\right.$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/48/b*x*(8*b^2*d*x^4+14*a*b*d*x^2+12*b^2*c*x^2+3*a^2*d+30*a*b*c)*(b*x^2+a
)^(1/2)-1/16*a^2*(a*d-6*b*c)/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.78

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\,dx = \left[-\frac{3\left(6\,a^2bc-a^3d\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8\,b^3dx^5+2\left(6\,b^3c+7\,ab^2d\right)x^3+\cdots\right.}{96\,b^2}\right.$$
$$\left.-\frac{3\left(6\,a^2bc-a^3d\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(8\,b^3dx^5+2\left(6\,b^3c+7\,ab^2d\right)x^3+3\left(10\,ab^2c+a^2bd\right)x\right)\sqrt{bx^2+a}}{48\,b^2}\right]$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/96*(3*(6*a^2*b*c - a^3*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a) - 2*(8*b^3*d*x^5 + 2*(6*b^3*c + 7*a*b^2*d)*x^3 + 3*(10*a*b^2*c
 + a^2*b*d)*x)*sqrt(b*x^2 + a))/b^2, -1/48*(3*(6*a^2*b*c - a^3*d)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*d*x^5 + 2*(6*b^3*c + 7*a*b^2*
d)*x^3 + 3*(10*a*b^2*c + a^2*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.48

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{bdx^5}{6}+\frac{x^3\cdot\left(\frac{7abd}{6}+b^2c\right)}{4b}+\frac{x\left(a^2d+2abc-\frac{3a\left(\frac{7abd}{6}+b^2c\right)}{4b}\right)}{2b}\right)+\left(a^2c-\frac{a\left(a^2d+2abc-\frac{3a\left(\frac{7abd}{6}+b^2c\right)}{4b}\right)}{2b}\right. \\ a^{\frac{3}{2}}\left(cx+\frac{dx^3}{3}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(b*d*x**5/6 + x**3*(7*a*b*d/6 + b**2*c)/(4*b)
+ x*(a**2*d + 2*a*b*c - 3*a*(7*a*b*d/6 + b**2*c)/(4*b))/(2*b)) + (a**2*c -
 a*(a**2*d + 2*a*b*c - 3*a*(7*a*b*d/6 + b**2*c)/(4*b))/(2*b))*Piecewise((l
og(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(
b*x**2), True)), Ne(b, 0)), (a**(3/2)*(c*x + d*x**3/3), True))
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)\,dx = \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}cx+\frac{3}{8}\sqrt{bx^2+a}acx+\frac{\left(bx^2+a\right)^{\frac{5}{2}}dx}{6\,b} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}adx}{24\,b}-\frac{\sqrt{bx^2+a}a^2dx}{16\,b}+\frac{3\,a^2c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}-\frac{a^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*c*x + 3/8*sqrt(b*x^2 + a)*a*c*x + 1/6*(b*x^2 + a)^(5
/2)*d*x/b - 1/24*(b*x^2 + a)^(3/2)*a*d*x/b - 1/16*sqrt(b*x^2 + a)*a^2*d*x/
b + 3/8*a^2*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/16*a^3*d*arcsinh(b*x/sqrt
(a*b))/b^(3/2)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right) \, dx = \frac{1}{48}\left(2\left(4\,bdx^2+\frac{6\,b^5c+7\,ab^4d}{b^4}\right)x^2+\frac{3\left(10\,ab^4c+a^2b^3d\right)}{b^4}\right)\sqrt{bx^2+a}\,x - \frac{\left(6\,a^2bc-a^3d\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="giac")
```

output
```
1/48*(2*(4*b*d*x^2 + (6*b^5*c + 7*a*b^4*d)/b^4)*x^2 + 3*(10*a*b^4*c + a^2*
b^3*d)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(6*a^2*b*c - a^3*d)*log(abs(-sqrt(b)*
x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right) \, dx = \int \left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right) \,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2), x)
```

3.56 $\int (a+bx^2)^{3/2}\,dx$

3.56.1 Optimal result

Integrand size = 11, antiderivative size = 65

$$\int (a+bx^2)^{3/2}\,dx=\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x(a+bx^2)^{3/2}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

output
```
1/4*x*(b*x^2+a)^(3/2)+3/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+3
/8*a*x*(b*x^2+a)^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int (a+bx^2)^{3/2}\,dx=\frac{1}{8}x\sqrt{a+bx^2}(5a+2bx^2)-\frac{3a^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(3/2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(5*a + 2*b*x^2))/8 - (3*a^2*Log[-(Sqrt[b]*x) + Sqrt[a +
 b*x^2]])/(8*Sqrt[b])
```

3.56.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

input
```
Int[(a + b*x^2)^(3/2),x]
```

output
```
(x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b
]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4
```

3.56.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.56.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{x(2b\,x^2+5a)\sqrt{b\,x^2+a}}{8}+\frac{3a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}$	48
default	$\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}$	52
pseudoelliptic	$\frac{2b^{\frac{3}{2}}\sqrt{b\,x^2+a}\,x^3+5ax\sqrt{b}\,\sqrt{b\,x^2+a}+3\ \text{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)a^2}{8\sqrt{b}}$	62

input
```
int((b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*b*x^2+5*a)*(b*x^2+a)^(1/2)+3/8*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/
b^(1/2)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.91

$$\int (a + bx^2)^{3/2} dx = \left[\frac{3a^2\sqrt{b}\log\left(-2bx^2 - 2\sqrt{bx^2+a}\sqrt{b}x - a\right) + 2(2b^2x^3 + 5abx)\sqrt{bx^2+a}}{16b}, -\frac{3a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (2b^2x^3 + 5abx)\sqrt{bx^2+a}}{8b} \right]$$

input
```
integrate((b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b]
```

3.56.6 Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int (a + bx^2)^{3/2} dx = \frac{5a^{\frac{3}{2}}x\sqrt{1 + \frac{bx^2}{a}}}{8} + \frac{\sqrt{a}bx^3\sqrt{1 + \frac{bx^2}{a}}}{4} + \frac{3a^2 \operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}$$

input
```
integrate((b*x**2+a)**(3/2),x)
```

output
```
5*a**(3/2)*x*sqrt(1 + b*x**2/a)/8 + sqrt(a)*b*x**3*sqrt(1 + b*x**2/a)/4 +
3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b))
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \left(a+bx^2\right)^{3/2} \, dx = \frac{1}{4} \left(bx^2+a\right)^{\frac{3}{2}}x + \frac{3}{8}\sqrt{bx^2+a}ax + \frac{3\,a^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*x + 3/8*sqrt(b*x^2 + a)*a*x + 3/8*a^2*arcsinh(b*x/sq
rt(a*b))/sqrt(b)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \left(a+bx^2\right)^{3/2} \, dx = \frac{1}{8} \left(2\,bx^2+5\,a\right)\sqrt{bx^2+a}x - \frac{3\,a^2 \log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/sqrt(b)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^{3/2} \, dx = \frac{x\left(b\,x^2+a\right)^{3/2} {}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int((a + b*x^2)^(3/2),x)
```

output
```
(x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(3/2)
```

3.57 $\int \frac{(a+bx^2)^{3/2}}{c+dx^2} dx$

3.57.1 Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2} dx = \frac{bx\sqrt{a+bx^2}}{2d} - \frac{\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^2} + \frac{(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^2}$$

output
```
-1/2*(-3*a*d+2*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)/d^2+(-a*d+b
*c)^(3/2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/d^2/c^(1/2)+
1/2*b*x*(b*x^2+a)^(1/2)/d
```

3.57.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2} dx = \frac{bdx\sqrt{a+bx^2} - \frac{2(-bc+ad)^{3/2}\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}} + \sqrt{b}(2bc-3ad)\log\left(-\sqrt{b}x+\sqrt{\right.}}{2d^2}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

output
```
(b*d*x*Sqrt[a + b*x^2] - (2*(-(b*c) + a*d)^(3/2)*ArcTan[(-(d*x*Sqrt[a + b*
x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/Sqrt[c] + Sqrt
[b]*(2*b*c - 3*a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*d^2)
```

3.57.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {318, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}dx$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\int -\frac{b(2bc-3ad)x^2+a(bc-2ad)}{\sqrt{bx^2+a}(dx^2+c)}dx}{2d}+\frac{bx\sqrt{a+bx^2}}{2d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bx\sqrt{a+bx^2}}{2d}-\frac{\int \frac{b(2bc-3ad)x^2+a(bc-2ad)}{\sqrt{bx^2+a}(dx^2+c)}dx}{2d}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{bx\sqrt{a+bx^2}}{2d}-\frac{\frac{b(2bc-3ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{2(bc-ad)^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}}{2d}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{bx\sqrt{a+bx^2}}{2d}-\frac{\frac{b(2bc-3ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{2(bc-ad)^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}}{2d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{bx\sqrt{a+bx^2}}{2d}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-3ad)}{d}-\frac{2(bc-ad)^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}}{2d}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{bx\sqrt{a+bx^2}}{2d}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-3ad)}{d}-\frac{2(bc-ad)^2\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d}$$

$\Big\downarrow$ 221

$$\frac{bx\sqrt{a+bx^2}}{2d}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-3ad)}{d}-\frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}}{2d}$$

input
```
Int[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

output
```
(b*x*Sqrt[a + b*x^2])/(2*d) - ((Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/d - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(
Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d))/(2*d)
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.57.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{(ad-bc)^2 \arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)+\left(\left(b^{\frac{3}{2}} c-\frac{3ad\sqrt{b}}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-\frac{bdx\sqrt{b\,x^2+a}}{2}\right)\sqrt{(ad-bc)c}}{\sqrt{(ad-bc)c}\,d^2}$
risch	$\frac{bx\sqrt{b\,x^2+a}}{2d}+\frac{\frac{\sqrt{b}\,(3ad-2bc)\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{d}-\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\ln\left(\frac{\frac{2ad-2bc}{d}-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x+\frac{\sqrt{-}}{}\right.}}{x+\frac{\sqrt{-cd}}{d}}\right.}{\sqrt{-cd}\,d\sqrt{\frac{ad-bc}{d}}}}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-((a*d-b*c)^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+((b^(3/2)*c-
3/2*a*d*b^(1/2))*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-1/2*b*d*x*(b*x^2+a)^(1
/2))*((a*d-b*c)*c)^(1/2))/((a*d-b*c)*c)^(1/2)/d^2
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 721, normalized size of antiderivative = 6.38

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \left[\frac{2\sqrt{bx^2+a}\,bdx-(2\,bc-3\,ad)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-(bc-ad)\sqrt{b\cdots}}{4\,d\cdots}\right.$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*x^2 + a)*b*d*x - (2*b*c - 3*a*d)*sqrt(b)*log(-2*b*x^2 - 2*s
qrt(b*x^2 + a)*sqrt(b)*x - a) - (b*c - a*d)*sqrt((b*c - a*d)/c)*log(((8*b^
2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2
 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c)
)/(d^2*x^4 + 2*c*d*x^2 + c^2)))/d^2, 1/4*(2*sqrt(b*x^2 + a)*b*d*x + 2*(2*b
*c - 3*a*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*c - a*d)*sqrt
((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(
4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^
2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/d^2, 1/4*(2*sqrt
(b*x^2 + a)*b*d*x - 2*(b*c - a*d)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c
- a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^
3 + (a*b*c - a^2*d)*x)) - (2*b*c - 3*a*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a))/d^2, 1/2*(sqrt(b*x^2 + a)*b*d*x + (2*b*c - 3*a*d)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*c - a*d)*sqrt(-(b*c - a*
d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*
d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/d^2]
```

3.57.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{c+dx^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(c + d*x**2), x)
```

3.57.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(d*x^2 + c), x)
```

3.57.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{d\,x^2+c}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x^2),x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x^2), x)
```

3.58 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx$

3.58.1 Optimal result

Integrand size = 21, antiderivative size = 131

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\,(c+dx^2)} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2} - \frac{\sqrt{bc-ad}(2bc+ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}$$

output
```
b^(3/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/d^2-1/2*(a*d+2*b*c)*arctanh(x*(
-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))*(-a*d+b*c)^(1/2)/c^(3/2)/d^2-1/2*
(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/d/(d*x^2+c)
```

3.58.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = \frac{\frac{d(-bc+ad)x\sqrt{a+bx^2}}{c(c+dx^2)} - \frac{\sqrt{-bc+ad}(2bc+ad)\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{c^{3/2}} - 2b^{3/2}\log\left(-\sqrt{b}x+\sqrt{a+\right.}{2d^2}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]
```

output
```
((d*(-(b*c) + a*d)*x*Sqrt[a + b*x^2])/(c*(c + d*x^2)) - (Sqrt[-(b*c) + a*d
]*(2*b*c + a*d)*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqr
t[c]*Sqrt[-(b*c) + a*d])])/c^(3/2) - 2*b^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[a +
 b*x^2]])/(2*d^2)
```

3.58.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {315, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx$$

$$\Big\downarrow \; 315$$

$$\frac{\int \frac{2b^2cx^2+a(bc+ad)}{\sqrt{bx^2+a}(dx^2+c)}dx}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{2cd\,(c+dx^2)}$$

$$\Big\downarrow \; 398$$

$$\frac{\frac{2b^2c\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)(ad+2bc)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{2cd\,(c+dx^2)}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{2b^2c\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)(ad+2bc)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{2cd\,(c+dx^2)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{2b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)(ad+2bc)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{2cd\,(c+dx^2)}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{2b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)(ad+2bc)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{2cd\,(c+dx^2)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2b^{3/2}c\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}(ad+2bc)\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{2cd\,(c+dx^2)}$$

input
```
Int[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]
```

output
```
-1/2*((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*(c + d*x^2)) + ((2*b^(3/2)*c*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcT
anh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d))/(2*c*d)
```

3.58.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.58.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{-(d x^2+c)(ad+2bc)(ad-bc)\arctan\left(\frac{c\sqrt{b x^2+a}}{x\sqrt{(ad-bc)c}}\right)+\sqrt{(ad-bc)c}\left(2c b^{\frac{3}{2}}(d x^2+c)\operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x\sqrt{b}}\right)+dx\sqrt{b x^2+a}\,(ad-bc)\right.}{2\sqrt{(ad-bc)c}\,d^2 c(d x^2+c)}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*c)^(1/2)*(-(d*x^2+c)*(a*d+2*b*c)*(a*d-b*c)*arctan(c*(b*x^2+
a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+((a*d-b*c)*c)^(1/2)*(2*c*b^(3/2)*(d*x^2+c)
*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+d*x*(b*x^2+a)^(1/2)*(a*d-b*c)))/d^2/c/
(d*x^2+c)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 907, normalized size of antiderivative = 6.92

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = \left[-\frac{4\,(bcd-ad^2)\sqrt{bx^2+a}x - 4\,(bcdx^2+bc^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right) - \cdots}{\cdots}\right.$$

$$-\frac{4\,(bcd-ad^2)\sqrt{bx^2+a}x + 8\,(bcdx^2+bc^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (2\,bc^2+acd+(2\,bcd+ad^2)x^2)\sqrt{\frac{bc-ad}{c}}\cdots}{8\,(cd^3x^2+c^2d^2)}$$

$$-\frac{2\,(bcd-ad^2)\sqrt{bx^2+a}x - (2\,bc^2+acd+(2\,bcd+ad^2)x^2)\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}\sqrt{-\frac{bc-ad}{c}}}{2\,((b^2c-abd)x^3+(abc-a^2d)x)}\right)\cdots}{4\,(cd^3x^2+c^2d^2)}$$

$$-\frac{2\,(bcd-ad^2)\sqrt{bx^2+a}x + 4\,(bcdx^2+bc^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (2\,bc^2+acd+(2\,bcd+ad^2)x^2)\sqrt{-\frac{bc-a\cdots}{c}}\cdots}{4\,(cd^3x^2+c^2d^2)}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x - 4*(b*c*d*x^2 + b*c^2)*sqrt(b)
*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (2*b*c^2 + a*c*d + (2*b
*c*d + a*d^2)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d
^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2
- a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 +
c^2)))/(c*d^3*x^2 + c^2*d^2), -1/8*(4*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x +
8*(b*c*d*x^2 + b*c^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b*c
^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 -
8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*
c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x
^4 + 2*c*d*x^2 + c^2)))/(c*d^3*x^2 + c^2*d^2), -1/4*(2*(b*c*d - a*d^2)*sqr
t(b*x^2 + a)*x - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt(-(b*c - a*
d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*
d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) - 2*(b*c*d*x^2 + b*c^2)*s
qrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(c*d^3*x^2 + c^2*d
^2), -1/4*(2*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x + 4*(b*c*d*x^2 + b*c^2)*sqr
t(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b*c^2 + a*c*d + (2*b*c*d + a
*d^2)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(
b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x))
)/(c*d^3*x^2 + c^2*d^2)]
```

3.58.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)/(c + d*x**2)**2, x)
```

3.58.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^2, x)
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(109) = 218$.

Time = 0.30 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.42

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = -\frac{b^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\,d^2} + \frac{\left(2\,b^{\frac{5}{2}}c^2 - ab^{\frac{3}{2}}cd - a^2\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}cd^2} - \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 b^{\frac{5}{2}}c^2 - 3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 ab^{\frac{3}{2}}cd + \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a^2\sqrt{b}d^2 + a^2b^{\frac{3}{2}}cd - a^3\sqrt{b}d^2}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 d + 4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 bc - 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 ad + a^2d\right)cd^2}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/2*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d^2 + 1/2*(2*b^(5/2)*c^2
 - a*b^(3/2)*c*d - a^2*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 +
a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)
*c*d^2) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(5/2)*c^2 - 3*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a*b^(3/2)*c*d + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*sqr
t(b)*d^2 + a^2*b^(3/2)*c*d - a^3*sqrt(b)*d^2)/(((sqrt(b)*x - sqrt(b*x^2 +
a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a*d + a^2*d)*c*d^2)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^2}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x^2)^2,x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x^2)^2, x)
```

3.59 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx$

3.59.1 Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx = \frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2} + \frac{3ax\sqrt{a+bx^2}}{8c^2(c+dx^2)} + \frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}$$

output
```
1/4*x*(b*x^2+a)^(3/2)/c/(d*x^2+c)^2+3/8*a^2*arctanh(x*(-a*d+b*c)^(1/2)/c^(
1/2)/(b*x^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(1/2)+3/8*a*x*(b*x^2+a)^(1/2)/c^2
/(d*x^2+c)
```

3.59.2 Mathematica [A] (warning: unable to verify)

Time = 10.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx = \frac{x\sqrt{a+bx^2}\left(\frac{\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}(5ac+2bcx^2+3adx^2)}{(c+dx^2)\sqrt{1+\frac{dx^2}{c}}} + \frac{3a\arcsin\left(\frac{\sqrt{\left(-\frac{b}{a}+\frac{d}{c}\right)x^2}}{\sqrt{1+\frac{dx^2}{c}}}\right)}{\sqrt{\frac{(-bc+ad)x^2}{ac}}}\right)}{8c^3\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

output
```
(x*Sqrt[a + b*x^2]*((Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*(5*a*c + 2*b*c*
x^2 + 3*a*d*x^2))/((c + d*x^2)*Sqrt[1 + (d*x^2)/c]) + (3*a*ArcSin[Sqrt[(-(
b/a) + d/c)*x^2]/Sqrt[1 + (d*x^2)/c]])/Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]))/
(8*c^3*Sqrt[1 + (b*x^2)/a])
```

3.59.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {292, 292, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx$$

$$\Big\downarrow \ 292$$

$$\frac{3a\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2}dx}{4c}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}$$

$$\Big\downarrow \ 292$$

$$\frac{3a\left(\frac{a\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{2c}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}\right)}{4c}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}$$

$$\Big\downarrow \ 291$$

$$\frac{3a\left(\frac{a\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}\right)}{4c}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}$$

$$\Big\downarrow \ 221$$

$$\frac{3a\left(\frac{a\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}\right)}{4c}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}$$

input
```
Int[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

output
```
(x*(a + b*x^2)^(3/2))/(4*c*(c + d*x^2)^2) + (3*a*((x*Sqrt[a + b*x^2])/(2*c
*(c + d*x^2)) + (a*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])
/(2*c^(3/2)*Sqrt[b*c - a*d])))/(4*c)
```

3.59.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

3.59.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$-\frac{a^{2}\left(-\frac{\sqrt{b\,x^{2}+a}\,\left(3ad\,x^{2}+2cb\,x^{2}+5ac\right)x}{a^{2}\left(d\,x^{2}+c\right)^{2}}+\frac{3\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}\right)}{8c^{2}}$	94
default	Expression too large to display	6921

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^2/c^2*(-(b*x^2+a)^(1/2)*(3*a*d*x^2+2*b*c*x^2+5*a*c)*x/a^2/(d*x^2+c)
^2+3/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(93) = 186$.

Time = 0.33 (sec) , antiderivative size = 526, normalized size of antiderivative = 4.65

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx = \left[\frac{3\,(a^2d^2x^4+2\,a^2cdx^2+a^2c^2)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2+}{d^2x^4+2\,cdx^2+}\right.}{32\,(bc^6-ac^5d+(bc^4d^2}\right.$$

$$-\frac{3\,(a^2d^2x^4+2\,a^2cdx^2+a^2c^2)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}}{2\,((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)-2\,((2\,b^2c^3+abc^2d-3}{16\,(bc^6-ac^5d+(bc^4d^2-ac^3d^3)x^4+2\,(bc^5d-ac^4d^2)x^2)}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/32*(3*(a^2*d^2*x^4 + 2*a^2*c*d*x^2 + a^2*c^2)*sqrt(b*c^2 - a*c*d)*log((
(8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d
)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))
/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x
^3 + 5*(a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(b*c^6 - a*c^5*d + (b*c^4
*d^2 - a*c^3*d^3)*x^4 + 2*(b*c^5*d - a*c^4*d^2)*x^2), -1/16*(3*(a^2*d^2*x^
4 + 2*a^2*c*d*x^2 + a^2*c^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 +
 a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3
 + (a*b*c^2 - a^2*c*d)*x)) - 2*((2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3
+ 5*(a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(b*c^6 - a*c^5*d + (b*c^4*d^
2 - a*c^3*d^3)*x^4 + 2*(b*c^5*d - a*c^4*d^2)*x^2)]
```

3.59.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
```

output
```
Integral((a + b*x**2)**(3/2)/(c + d*x**2)**3, x)
```

3.59.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^3, x)
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(93) = 186$.

Time = 1.67 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.99

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx = -\frac{3\,a^2\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{8\,\sqrt{-b^2c^2+abcd}c^2} + \frac{8\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 b^{\frac{5}{2}}c^2d - 3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 a^2\sqrt{b}d^3 + 16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 b^{\frac{7}{2}}c^3 + 8\left(\sqrt{b}x-\sqrt{\right.}}{}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-3/8*a^2*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a
*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c^2) + 1/4*(8*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d - 3*(sqrt(b)*x - sqrt(b*x^2 + a)
)^6*a^2*sqrt(b)*d^3 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^3 + 8*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d - 18*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*a^2*b^(3/2)*c*d^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b
)*d^3 + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c^2*d + 16*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*a^4*sqrt(b)*d^3 + 2*a^4*b^(3/2)*c*d^2 + 3*a^5*sqrt(b)*d^3)/(((sqrt(b)*x
 - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2*c^2*d^2)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{{\left(d\,x^2+c\right)}^3} \,d x$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x^2)^3,x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x^2)^3, x)
```

3.60 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4} dx$

3.60.1 Optimal result

Integrand size = 21, antiderivative size = 199

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4} dx = -\frac{dx(a+bx^2)^{5/2}}{6c(bc-ad)(c+dx^2)^3} + \frac{(6bc-5ad)x(a+bx^2)^{3/2}}{24c^2(bc-ad)(c+dx^2)^2} + \frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)(c+dx^2)} + \frac{a^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{3/2}}$$

output
```
-1/6*d*x*(b*x^2+a)^(5/2)/c/(-a*d+b*c)/(d*x^2+c)^3+1/24*(-5*a*d+6*b*c)*x*(b
*x^2+a)^(3/2)/c^2/(-a*d+b*c)/(d*x^2+c)^2+1/16*a^2*(-5*a*d+6*b*c)*arctanh(x
*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(7/2)/(-a*d+b*c)^(3/2)+1/16*a
*(-5*a*d+6*b*c)*x*(b*x^2+a)^(1/2)/c^3/(-a*d+b*c)/(d*x^2+c)
```

3.60.2 Mathematica [A] (verified)

Time = 10.51 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4} dx = \frac{ax\left(1+\frac{bx^2}{a}\right)\left(c(a+bx^2)\left(-4b^2c^2x^2(3c+dx^2)-2abc(15c^2+11cdx^2+4d^2x^4)+a^2d(3\right.\right.}{48c^4(-bc+ad)(a+bx^2)^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```

output
```
(a*x*(1 + (b*x^2)/a)*(c*(a + b*x^2)*(-4*b^2*c^2*x^2*(3*c + d*x^2) - 2*a*b*
c*(15*c^2 + 11*c*d*x^2 + 4*d^2*x^4) + a^2*d*(33*c^2 + 40*c*d*x^2 + 15*d^2*
x^4)) + (3*a^2*(-6*b*c + 5*a*d)*(c + d*x^2)^3*ArcTanh[Sqrt[((b*c - a*d)*x^
2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(48*c^4*(-
(b*c) + a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^3)
```

3.60.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {296, 292, 292, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^4}dx$$

$$\Big\downarrow \text{296}$$

$$\frac{(6bc-5ad)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^3}dx}{6c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3(bc-ad)}$$

$$\Big\downarrow \text{292}$$

$$\frac{(6bc-5ad)\left(\frac{3a\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^2}dx}{4c}+\frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2}\right)}{6c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3(bc-ad)}$$

$$\Big\downarrow \text{292}$$

$$\frac{(6bc-5ad)\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}\right)}{4c}+\frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2}\right)}{6c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{(6bc-5ad)\left(\frac{3a\left(\frac{a\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}\right)}{4c}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}\right)}{6c(bc-ad)}-\frac{dx(a+bx^2)^{5/2}}{6c(c+dx^2)^3(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{(6bc-5ad)\left(\frac{3a\left(\frac{a\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}\right)}{4c}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}\right)}{6c(bc-ad)}-\frac{dx(a+bx^2)^{5/2}}{6c(c+dx^2)^3(bc-ad)}$$

input
```
Int[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```

output
```
-1/6*(d*x*(a + b*x^2)^(5/2))/(c*(b*c - a*d)*(c + d*x^2)^3) + ((6*b*c - 5*a
*d)*((x*(a + b*x^2)^(3/2))/(4*c*(c + d*x^2)^2) + (3*a*((x*Sqrt[a + b*x^2])
/(2*c*(c + d*x^2)) + (a*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^
2])])/(2*c^(3/2)*Sqrt[b*c - a*d])))/(4*c)))/(6*c*(b*c - a*d))
```

3.60.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.60.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{-11x\sqrt{b\,x^2+a}\left(-\frac{10\left(\frac{2b\,x^2}{5}+a\right)b\,c^3}{11}+d\left(-\frac{4}{33}b^2x^4-\frac{2}{3}ab\,x^2+a^2\right)c^2+\frac{40x^2\left(-\frac{b\,x^2}{5}+a\right)d^2ac}{33}+\frac{5a^2d^3x^4}{11}\right)\sqrt{(ad-bc)c}+5(d\,x^2+}{16\sqrt{(ad-bc)c}\,(ad-bc)c^3(d\,x^2+c)^3}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/16/((a*d-b*c)*c)^(1/2)*(-11*x*(b*x^2+a)^(1/2)*(-10/11*(2/5*b*x^2+a)*b*c
^3+d*(-4/33*b^2*x^4-2/3*a*b*x^2+a^2)*c^2+40/33*x^2*(-1/5*b*x^2+a)*d^2*a*c+
5/11*a^2*d^3*x^4)*((a*d-b*c)*c)^(1/2)+5*(d*x^2+c)^3*(a*d-6/5*b*c)*a^2*arct
an(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))/(a*d-b*c)/c^3/(d*x^2+c)^3
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(175) = 350$.

Time = 0.44 (sec) , antiderivative size = 972, normalized size of antiderivative = 4.88

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx = \left[\frac{3\,(6\,a^2bc^4-5\,a^3c^3d+(6\,a^2bcd^3-5\,a^3d^4)x^6+3\,(6\,a^2bc^2d^2-5\,a^3cd^3)x^4+3\,(6\,a^2bc^3d}{}\right.$$

$$-\frac{3\,(6\,a^2bc^4-5\,a^3c^3d+(6\,a^2bcd^3-5\,a^3d^4)x^6+3\,(6\,a^2bc^2d^2-5\,a^3cd^3)x^4+3\,(6\,a^2bc^3d-5\,a^3c^2d^2)x^2)\sqrt{-bc^2}}{96\,(b^2c^9-2\,abc^8d}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

output
```
[1/192*(3*(6*a^2*b*c^4 - 5*a^3*c^3*d + (6*a^2*b*c*d^3 - 5*a^3*d^4)*x^6 + 3
*(6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^4 + 3*(6*a^2*b*c^3*d - 5*a^3*c^2*d^2)*x
^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c
^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*
c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((4*b^3*c^4
*d + 4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^5 + 2*(6*b^3*c^5
 + 5*a*b^2*c^4*d - 31*a^2*b*c^3*d^2 + 20*a^3*c^2*d^3)*x^3 + 3*(10*a*b^2*c^
5 - 21*a^2*b*c^4*d + 11*a^3*c^3*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^9 - 2*a*b*
c^8*d + a^2*c^7*d^2 + (b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x^6 + 3*
(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4)*x^4 + 3*(b^2*c^8*d - 2*a*b*c^7
*d^2 + a^2*c^6*d^3)*x^2), -1/96*(3*(6*a^2*b*c^4 - 5*a^3*c^3*d + (6*a^2*b*c
*d^3 - 5*a^3*d^4)*x^6 + 3*(6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^4 + 3*(6*a^2*b
*c^3*d - 5*a^3*c^2*d^2)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 +
 a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3
 + (a*b*c^2 - a^2*c*d)*x)) - 2*((4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*a^2*b*
c^2*d^3 + 15*a^3*c*d^4)*x^5 + 2*(6*b^3*c^5 + 5*a*b^2*c^4*d - 31*a^2*b*c^3*
d^2 + 20*a^3*c^2*d^3)*x^3 + 3*(10*a*b^2*c^5 - 21*a^2*b*c^4*d + 11*a^3*c^3*
d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^9 - 2*a*b*c^8*d + a^2*c^7*d^2 + (b^2*c^6*d
^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x^6 + 3*(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a
^2*c^5*d^4)*x^4 + 3*(b^2*c^8*d - 2*a*b*c^7*d^2 + a^2*c^6*d^3)*x^2)]
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**4,x)
```

output
```
Timed out
```

3.60.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^4, x)
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. 2(175) = 350.

Time = 1.34 (sec) , antiderivative size = 919, normalized size of antiderivative = 4.62

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx = -\frac{\left(6\,a^2b^{\frac{3}{2}}c-5\,a^3\sqrt{b}d\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{16\,(bc^4-ac^3d)\sqrt{-b^2c^2+abcd}}$$
$$-\frac{18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{3}{2}}cd^4-15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^3\sqrt{b}d^5-96\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8b^{\frac{9}{2}}c^4d+96\,(}{}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="giac")
```

output
```
-1/16*(6*a^2*b^(3/2)*c - 5*a^3*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*
x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^4 - a*c^3*d)*
sqrt(-b^2*c^2 + a*b*c*d)) - 1/24*(18*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*
b^(3/2)*c*d^4 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*d^5 - 96*(
sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c^4*d + 96*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a*b^(7/2)*c^3*d^2 + 180*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2
)*c^2*d^3 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 + 75*(sq
rt(b)*x - sqrt(b*x^2 + a))^8*a^4*sqrt(b)*d^5 - 128*(sqrt(b)*x - sqrt(b*x^2
 + a))^6*b^(11/2)*c^5 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d
 + 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2)*c^3*d^2 - 968*(sqrt(b)*
x - sqrt(b*x^2 + a))^6*a^3*b^(5/2)*c^2*d^3 + 620*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^4*b^(3/2)*c*d^4 - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)
*d^5 - 96*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(9/2)*c^4*d - 288*(sqrt(b)
*x - sqrt(b*x^2 + a))^4*a^3*b^(7/2)*c^3*d^2 + 864*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*a^4*b^(5/2)*c^2*d^3 - 600*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3
/2)*c*d^4 + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*sqrt(b)*d^5 - 48*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7/2)*c^3*d^2 - 72*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a^5*b^(5/2)*c^2*d^3 + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^
(3/2)*c*d^4 - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*sqrt(b)*d^5 - 4*a^6*b
^(5/2)*c^2*d^3 - 8*a^7*b^(3/2)*c*d^4 + 15*a^8*sqrt(b)*d^5)/((b*c^4*d^2 ...
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^4}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x^2)^4,x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x^2)^4, x)
```

3.61 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 300

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx &= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\,(c+dx^2)^4} \\ &+ \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\,(c+dx^2)^3} + \frac{(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+bx^2}}{192c^3d(bc-ad)\,(c+dx^2)^2} \\ &+ \frac{(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3)\,x\sqrt{a+bx^2}}{384c^4d(bc-ad)^2\,(c+dx^2)} \\ &+ \frac{a^2(48b^2c^2-80abcd+35a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{5/2}}\end{aligned}$$

output
```
1/128*a^2*(35*a^2*d^2-80*a*b*c*d+48*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^
(1/2)/(b*x^2+a)^(1/2))/c^(9/2)/(-a*d+b*c)^(5/2)-1/8*(-a*d+b*c)*x*(b*x^2+a)
^(1/2)/c/d/(d*x^2+c)^4+1/48*(7*a*d+2*b*c)*x*(b*x^2+a)^(1/2)/c^2/d/(d*x^2+c
)^3+1/192*(-35*a^2*d^2+24*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/c^3/d/(-a*d
+b*c)/(d*x^2+c)^2+1/384*(105*a^3*d^3-170*a^2*b*c*d^2+40*a*b^2*c^2*d+16*b^3
*c^3)*x*(b*x^2+a)^(1/2)/c^4/d/(-a*d+b*c)^2/(d*x^2+c)
```

3.61.2 Mathematica [A] (verified)

Time = 10.91 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx = \frac{ax\left(1+\frac{bx^2}{a}\right)\left(c(a+bx^2)\left(16b^3c^3x^2(6c^2+4cdx^2+d^2x^4)+8ab^2c^2(30c^3+26c^2dx^2+19\right.\right.}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```

output
```
(a*x*(1 + (b*x^2)/a)*(c*(a + b*x^2)*(16*b^3*c^3*x^2*(6*c^2 + 4*c*d*x^2 + d
^2*x^4) + 8*a*b^2*c^2*(30*c^3 + 26*c^2*d*x^2 + 19*c*d^2*x^4 + 5*d^3*x^6) -
 2*a^2*b*c*d*(264*c^3 + 421*c^2*d*x^2 + 314*c*d^2*x^4 + 85*d^3*x^6) + a^3*
d^2*(279*c^3 + 511*c^2*d*x^2 + 385*c*d^2*x^4 + 105*d^3*x^6)) + (3*a^2*(48*
b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*(c + d*x^2)^4*ArcTanh[Sqrt[((b*c - a*d)
*x^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(384*c^
5*(b*c - a*d)^2*(a + b*x^2)^(3/2)*(c + d*x^2)^4)
```

3.61.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {315, 402, 27, 402, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx$$

$$\Big\downarrow \text{315}$$

$$\frac{\int \frac{2b(bc+3ad)x^2+a(bc+7ad)}{\sqrt{bx^2+a}(dx^2+c)^4}dx}{8cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{8cd\,(c+dx^2)^4}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{(bc-ad)\left(4b(2bc+7ad)x^2+a(4bc+35ad)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{6c(bc-ad)}+\frac{x\sqrt{a+bx^2}(7ad+2bc)}{6c(c+dx^2)^3}}{8cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{8cd\left(c+dx^2\right)^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{4b(2bc+7ad)x^2+a(4bc+35ad)}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{6c}+\frac{x\sqrt{a+bx^2}(7ad+2bc)}{6c(c+dx^2)^3}}{8cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{8cd\left(c+dx^2\right)^4}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{2b\left(8b^2c^2+24abdc-35a^2d^2\right)x^2+a\left(8b^2c^2+100abdc-105a^2d^2\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(-35a^2d^2+24abcd+8b^2c^2\right)}{4c\left(c+dx^2\right)^2(bc-ad)}}{6c}+\frac{x\sqrt{a+bx^2}(7ad+2bc)}{6c(c+dx^2)^3}}{8cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{8cd\left(c+dx^2\right)^4}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\frac{\int \frac{3a^2d\left(48b^2c^2-80abdc+35a^2d^2\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(-35a^2d^2+24abcd+8b^2c^2\right)}{4c\left(c+dx^2\right)^2(bc-ad)}}{6c}+\frac{x\sqrt{a+bx^2}(7ad+2bc)}{6c(c+dx^2)^3}}{8cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{8cd\left(c+dx^2\right)^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{3a^2d\left(35a^2d^2-80abcd+48b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(-35a^2d^2+24abcd+8b^2c^2\right)}{4c\left(c+dx^2\right)^2(bc-ad)}}{6c}+\frac{x\sqrt{a+bx^2}(7ad+2bc)}{6c(c+dx^2)^3}}{8cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{8cd\left(c+dx^2\right)^4}$$

$\Big\downarrow$ 291

3.61. $\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^5}dx$

$$\frac{\frac{\frac{\frac{3a^2d\left(35a^2d^2-80abcd+48b^2c^2\right)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(-35a^2d^2+24abcd+8b^2c^2\right)}{4c\left(c+dx^2\right)^2(bc-ad)}}{6c}+\frac{x}{}}{8cd}$$

$$\frac{x\sqrt{a+bx^2}(bc-ad)}{8cd\left(c+dx^2\right)^4}$$

$\downarrow$ 221

$$\frac{\frac{\frac{x\sqrt{a+bx^2}\left(-35a^2d^2+24abcd+8b^2c^2\right)}{4c\left(c+dx^2\right)^2(bc-ad)}+\frac{\frac{3a^2d\left(35a^2d^2-80abcd+48b^2c^2\right)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}}{6c}+\frac{x}{}}{8cd}$$

$$\frac{x\sqrt{a+bx^2}(bc-ad)}{8cd\left(c+dx^2\right)^4}$$

input
```
Int[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```

output
```
-1/8*((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*(c + d*x^2)^4) + (((2*b*c + 7*a*
d)*x*Sqrt[a + b*x^2])/(6*c*(c + d*x^2)^3) + (((8*b^2*c^2 + 24*a*b*c*d - 35
*a^2*d^2)*x*Sqrt[a + b*x^2])/(4*c*(b*c - a*d)*(c + d*x^2)^2) + (((16*b^3*c
^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*x*Sqrt[a + b*x^2])/(2
*c*(b*c - a*d)*(c + d*x^2)) + (3*a^2*d*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d
^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*
c - a*d)^(3/2)))/(4*c*(b*c - a*d)))/(6*c))/(8*c*d)
```

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.61. $\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^5}\,dx$

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.61.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{93x\left(d^2\left(\frac{35}{93}d^3x^6+\frac{385}{279}c\,d^2x^4+\frac{511}{279}c^2d\,x^2+c^3\right)a^3-\frac{176bd\left(\frac{85}{264}d^3x^6+\frac{157}{132}c\,d^2x^4+\frac{421}{264}c^2d\,x^2+c^3\right)c\,a^2}{93}+\frac{80b^2\left(\frac{1}{6}d^3x^6+\frac{19}{30}c\,d^2x^4+\frac{13}{15}\right.}{93}\right.}{128\sqrt{(ad-bc)}}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/128/((a*d-b*c)*c)^(1/2)*(93*x*(d^2*(35/93*d^3*x^6+385/279*c*d^2*x^4+511/
279*c^2*d*x^2+c^3)*a^3-176/93*b*d*(85/264*d^3*x^6+157/132*c*d^2*x^4+421/26
4*c^2*d*x^2+c^3)*c*a^2+80/93*b^2*(1/6*d^3*x^6+19/30*c*d^2*x^4+13/15*c^2*d*
x^2+c^3)*c^2*a+32/93*x^2*(1/6*d^2*x^4+2/3*c*d*x^2+c^2)*b^3*c^3)*((a*d-b*c)
*c)^(1/2)*(b*x^2+a)^(1/2)-35*(a^2*d^2-16/7*a*b*c*d+48/35*b^2*c^2)*(d*x^2+c
)^4*a^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))/(d*x^2+c)^4/(a*d-
b*c)^2/c^4
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(272) = 544$.

Time = 1.37 (sec) , antiderivative size = 1604, normalized size of antiderivative = 5.35

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="fricas")
```

output
```
[1/1536*(3*(48*a^2*b^2*c^6 - 80*a^3*b*c^5*d + 35*a^4*c^4*d^2 + (48*a^2*b^2
*c^2*d^4 - 80*a^3*b*c*d^5 + 35*a^4*d^6)*x^8 + 4*(48*a^2*b^2*c^3*d^3 - 80*a
^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^6 + 6*(48*a^2*b^2*c^4*d^2 - 80*a^3*b*c^3*d^
3 + 35*a^4*c^2*d^4)*x^4 + 4*(48*a^2*b^2*c^5*d - 80*a^3*b*c^4*d^2 + 35*a^4*
c^3*d^3)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x
^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*
x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*(
(16*b^4*c^5*d^2 + 24*a*b^3*c^4*d^3 - 210*a^2*b^2*c^3*d^4 + 275*a^3*b*c^2*d
^5 - 105*a^4*c*d^6)*x^7 + (64*b^4*c^6*d + 88*a*b^3*c^5*d^2 - 780*a^2*b^2*c
^4*d^3 + 1013*a^3*b*c^3*d^4 - 385*a^4*c^2*d^5)*x^5 + (96*b^4*c^7 + 112*a*b
^3*c^6*d - 1050*a^2*b^2*c^5*d^2 + 1353*a^3*b*c^4*d^3 - 511*a^4*c^3*d^4)*x^
3 + 3*(80*a*b^3*c^7 - 256*a^2*b^2*c^6*d + 269*a^3*b*c^5*d^2 - 93*a^4*c^4*d
^3)*x)*sqrt(b*x^2 + a))/(b^3*c^12 - 3*a*b^2*c^11*d + 3*a^2*b*c^10*d^2 - a^
3*c^9*d^3 + (b^3*c^8*d^4 - 3*a*b^2*c^7*d^5 + 3*a^2*b*c^6*d^6 - a^3*c^5*d^7
)*x^8 + 4*(b^3*c^9*d^3 - 3*a*b^2*c^8*d^4 + 3*a^2*b*c^7*d^5 - a^3*c^6*d^6)*
x^6 + 6*(b^3*c^10*d^2 - 3*a*b^2*c^9*d^3 + 3*a^2*b*c^8*d^4 - a^3*c^7*d^5)*x
^4 + 4*(b^3*c^11*d - 3*a*b^2*c^10*d^2 + 3*a^2*b*c^9*d^3 - a^3*c^8*d^4)*x^2
), -1/768*(3*(48*a^2*b^2*c^6 - 80*a^3*b*c^5*d + 35*a^4*c^4*d^2 + (48*a^2*b
^2*c^2*d^4 - 80*a^3*b*c*d^5 + 35*a^4*d^6)*x^8 + 4*(48*a^2*b^2*c^3*d^3 - 80
*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^6 + 6*(48*a^2*b^2*c^4*d^2 - 80*a^3*b*c...
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**5,x)
```

output
```
Timed out
```

3.61.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^5}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^5, x)
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1557 vs. 2(272) = 544.

Time = 4.02 (sec) , antiderivative size = 1557, normalized size of antiderivative = 5.19

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="giac")
```

output
```
-1/128*(48*a^2*b^(5/2)*c^2 - 80*a^3*b^(3/2)*c*d + 35*a^4*sqrt(b)*d^2)*arct
an(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a
*b*c*d))/((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*sqrt(-b^2*c^2 + a*b*c*d))
- 1/192*(144*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2*b^(5/2)*c^2*d^5 - 240*(s
qrt(b)*x - sqrt(b*x^2 + a))^14*a^3*b^(3/2)*c*d^6 + 105*(sqrt(b)*x - sqrt(b
*x^2 + a))^14*a^4*sqrt(b)*d^7 + 2016*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*
b^(7/2)*c^3*d^4 - 4368*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(5/2)*c^2*d^
5 + 3150*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(3/2)*c*d^6 - 735*(sqrt(b)
*x - sqrt(b*x^2 + a))^12*a^5*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 +
a))^10*b^(13/2)*c^6*d + 4096*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(11/2)*c
^5*d^2 + 7936*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(9/2)*c^4*d^3 - 26624
*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*b^(7/2)*c^3*d^4 + 26944*(sqrt(b)*x -
 sqrt(b*x^2 + a))^10*a^4*b^(5/2)*c^2*d^5 - 12320*(sqrt(b)*x - sqrt(b*x^2 +
 a))^10*a^5*b^(3/2)*c*d^6 + 2205*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^6*sqrt
(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(15/2)*c^7 - 1024*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*a*b^(13/2)*c^6*d + 27392*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^2*b^(11/2)*c^5*d^2 - 65920*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b
^(9/2)*c^4*d^3 + 81680*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*b^(7/2)*c^3*d^4
 - 58840*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^5*b^(5/2)*c^2*d^5 + 22750*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(3/2)*c*d^6 - 3675*(sqrt(b)*x - sqrt(b...
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^5}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x^2)^5,x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x^2)^5, x)
```

3.62 $\int (a+bx^2)^{5/2} (c+dx^2)^3 \, dx$

3.62.1 Optimal result

Integrand size = 21, antiderivative size = 349

$$\int (a+bx^2)^{5/2} (c+dx^2)^3 \, dx = \frac{a^2(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3)\,x\sqrt{a+bx^2}}{1024b^3} + \frac{a(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3)\,x(a+bx^2)^{3/2}}{1536b^3} + \frac{(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3)\,x(a+bx^2)^{5/2}}{1920b^3} + \frac{d(152b^2c^2-68abcd+15a^2d^2)\,x(a+bx^2)^{7/2}}{960b^3} + \frac{d(16bc-5ad)x(a+bx^2)^{7/2}(c+dx^2)}{120b^2} + \frac{dx(a+bx^2)^{7/2}(c+dx^2)^2}{12b} + \frac{a^3(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}$$

output
```
1/1536*a*(-5*a^3*d^3+36*a^2*b*c*d^2-120*a*b^2*c^2*d+320*b^3*c^3)*x*(b*x^2+
a)^(3/2)/b^3+1/1920*(-5*a^3*d^3+36*a^2*b*c*d^2-120*a*b^2*c^2*d+320*b^3*c^3
)*x*(b*x^2+a)^(5/2)/b^3+1/960*d*(15*a^2*d^2-68*a*b*c*d+152*b^2*c^2)*x*(b*x
^2+a)^(7/2)/b^3+1/120*d*(-5*a*d+16*b*c)*x*(b*x^2+a)^(7/2)*(d*x^2+c)/b^2+1/
12*d*x*(b*x^2+a)^(7/2)*(d*x^2+c)^2/b+1/1024*a^3*(-5*a^3*d^3+36*a^2*b*c*d^2
-120*a*b^2*c^2*d+320*b^3*c^3)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)+1
/1024*a^2*(-5*a^3*d^3+36*a^2*b*c*d^2-120*a*b^2*c^2*d+320*b^3*c^3)*x*(b*x^2
+a)^(1/2)/b^3
```

3.62.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.77

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(75a^5d^3-10a^4bd^2(54c+5dx^2)+40a^3b^2d(45c^2+9cdx^2+d^2x^4)+128b^5x^4(20c^3}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(75*a^5*d^3 - 10*a^4*b*d^2*(54*c + 5*d*x^2) + 4
0*a^3*b^2*d*(45*c^2 + 9*c*d*x^2 + d^2*x^4) + 128*b^5*x^4*(20*c^3 + 45*c^2*
d*x^2 + 36*c*d^2*x^4 + 10*d^3*x^6) + 48*a^2*b^3*(220*c^3 + 295*c^2*d*x^2 +
 186*c*d^2*x^4 + 45*d^3*x^6) + 64*a*b^4*x^2*(130*c^3 + 255*c^2*d*x^2 + 189
*c*d^2*x^4 + 50*d^3*x^6)) + 15*a^3*(-320*b^3*c^3 + 120*a*b^2*c^2*d - 36*a^
2*b*c*d^2 + 5*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(15360*b^(7/2)
)
```

3.62.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.74, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {318, 403, 299, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \, dx$$

$$\Big\downarrow \; 318$$

$$\frac{\int \left(bx^2+a\right)^{5/2} \left(dx^2+c\right) \left(d(16bc-5ad)x^2+c(12bc-ad)\right)dx}{12b}+\frac{dx\left(a+bx^2\right)^{7/2} \left(c+dx^2\right)^2}{12b}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\int \left(bx^2+a\right)^{5/2} \left(d\left(152b^2c^2-68abdc+15a^2d^2\right)x^2+c\left(120b^2c^2-26abdc+5a^2d^2\right)\right)dx}{10b}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)(16bc-5ad)}{10b}}{12b}+\frac{dx\left(a+bx^2\right)^{7/2} \left(c+dx^2\right)^2}{12b}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{\frac{3\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)\int\left(bx^2+a\right)^{5/2}dx}{8b}+\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2d^2-68abcd+152b^2c^2\right)}{8b}}{10b}+\frac{dx(a+bx^2)^{7/2}(c+dx^2)(16bc-5ad)}{10b}}{12b}+$$

$$\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{\frac{3\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)\left(\frac{5}{6}a\int\left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2d^2-68abcd+152b^2c^2\right)}{8b}}{10b}+\frac{dx(a+bx^2)^{7/2}(c+dx^2)}{10b}}{12b}$$

$$\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{\frac{3\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2d^2-68abcd+152b^2c^2\right)}{8b}}{10b}+dx(a}{12b}$$

$$\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{\frac{3\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2d^2-68abcd-\right.}{8b}}{10b}}{12b}$$

$$\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\frac{3\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2\right.}{8}}{10b}}{12b}$$

$$\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b}$$

$$\Big\downarrow \text{ 219}$$

3.62. $\int\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3\,dx$

$$\frac{\frac{\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2d^2-68abcd+152b^2c^2\right)}{8b}+\frac{3\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)\left(-5a^3d^3+36a^2bcd^2-1\cdots}{8b}}{10b}}{12b}$$

$$\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b}$$

input
```
Int[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

output
```
(d*x*(a + b*x^2)^(7/2)*(c + d*x^2)^2)/(12*b) + ((d*(16*b*c - 5*a*d)*x*(a +
 b*x^2)^(7/2)*(c + d*x^2))/(10*b) + ((d*(152*b^2*c^2 - 68*a*b*c*d + 15*a^2
*d^2)*x*(a + b*x^2)^(7/2))/(8*b) + (3*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*
a^2*b*c*d^2 - 5*a^3*d^3)*((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(
3/2))/4 + (3*a*((x*Sqrt[a + b*x^2]])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(2*Sqrt[b])))/4))/6))/(8*b))/(10*b))/(12*b)
```

3.62.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.62.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{5\left(a^3\left(a^3d^3-\frac{36}{5}a^2bc\,d^2+24a\,b^2c^2d-64b^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-x\left(\frac{512x^4\left(\frac{1}{2}d^3x^6+\frac{9}{5}c\,d^2x^4+\frac{9}{4}c^2d\,x^2+c^3\right)b^{\frac{11}{2}}}{15}+\left(\frac{704a\left(\cdots\right.}{\cdots}\right.\right.\right.}{\cdots}$
risch	$\frac{x\left(1280b^5d^3x^{10}+3200a\,b^4d^3x^8+4608b^5d^2c\,x^8+2160a^2b^3d^3x^6+12096a\,b^4c\,d^2x^6+5760b^5c^2d\,x^6+40a^3b^2d^3x^4+8928a^2b^3c\,d^2x^4\cdots\right.}{\cdots}$
default	$c^3\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+d^3\left(\frac{x^5\left(b\,x^2+a\right)^{\frac{7}{2}}}{12b}-\frac{5a\left(\frac{x^3\left(b\,x^2\cdots\right.}{10\cdots}\cdots\right)}{\cdots}\cdots\right.$

3.62. $\int\left(a+b x^2\right)^{5/2}\left(c+d x^2\right)^3\,dx$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-5/1024*(a^3*(a^3*d^3-36/5*a^2*b*c*d^2+24*a*b^2*c^2*d-64*b^3*c^3)*arctanh(
(b*x^2+a)^(1/2)/x/b^(1/2))-x*(512/15*x^4*(1/2*d^3*x^6+9/5*c*d^2*x^4+9/4*c^
2*d*x^2+c^3)*b^(11/2)+(704/5*a*(9/44*d^3*x^6+93/110*c*d^2*x^4+59/44*c^2*d*
x^2+c^3)*b^(7/2)+1664/15*x^2*(5/13*d^3*x^6+189/130*c*d^2*x^4+51/26*c^2*d*x
^2+c^3)*b^(9/2)+((8/15*d^2*x^4+24/5*c*d*x^2+24*c^2)*b^(5/2)+((-2/3*d*x^2-3
6/5*c)*b^(3/2)+a*d*b^(1/2))*d*a)*d*a^2)*a)*(b*x^2+a)^(1/2))/b^(7/2)
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.74

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \, dx = \left[-\frac{15 \left(320 a^3 b^3 c^3 - 120 a^4 b^2 c^2 d + 36 a^5 b c d^2 - 5 a^6 d^3\right) \sqrt{b} \log \left(-2 b x^2 + 2 \sqrt{b x^2 + a} \sqrt{b} x - a\right) - \cdots}{\cdots}\right.$$

$$-\frac{15 \left(320 a^3 b^3 c^3 - 120 a^4 b^2 c^2 d + 36 a^5 b c d^2 - 5 a^6 d^3\right) \sqrt{-b} \arctan \left(\frac{\sqrt{-b} x}{\sqrt{b x^2 + a}}\right) - \left(1280 b^6 d^3 x^{11} + 128 \left(36 b^6 c d^2 + \cdots\right.\right.}{\cdots}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/30720*(15*(320*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^
6*d^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1280*b
^6*d^3*x^11 + 128*(36*b^6*c*d^2 + 25*a*b^5*d^3)*x^9 + 144*(40*b^6*c^2*d +
84*a*b^5*c*d^2 + 15*a^2*b^4*d^3)*x^7 + 8*(320*b^6*c^3 + 2040*a*b^5*c^2*d +
 1116*a^2*b^4*c*d^2 + 5*a^3*b^3*d^3)*x^5 + 10*(832*a*b^5*c^3 + 1416*a^2*b^
4*c^2*d + 36*a^3*b^3*c*d^2 - 5*a^4*b^2*d^3)*x^3 + 15*(704*a^2*b^4*c^3 + 12
0*a^3*b^3*c^2*d - 36*a^4*b^2*c*d^2 + 5*a^5*b*d^3)*x)*sqrt(b*x^2 + a))/b^4,
 -1/15360*(15*(320*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^
6*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (1280*b^6*d^3*x^11 +
128*(36*b^6*c*d^2 + 25*a*b^5*d^3)*x^9 + 144*(40*b^6*c^2*d + 84*a*b^5*c*d^2
 + 15*a^2*b^4*d^3)*x^7 + 8*(320*b^6*c^3 + 2040*a*b^5*c^2*d + 1116*a^2*b^4*
c*d^2 + 5*a^3*b^3*d^3)*x^5 + 10*(832*a*b^5*c^3 + 1416*a^2*b^4*c^2*d + 36*a
^3*b^3*c*d^2 - 5*a^4*b^2*d^3)*x^3 + 15*(704*a^2*b^4*c^3 + 120*a^3*b^3*c^2*
d - 36*a^4*b^2*c*d^2 + 5*a^5*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

3.62.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(352) = 704$.

Time = 0.66 (sec) , antiderivative size = 823, normalized size of antiderivative = 2.36

$$\int (a+bx^2)^{5/2}(c+dx^2)^3\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{b^2d^3x^{11}}{12} + \frac{x^9\cdot\left(\frac{25ab^2d^3}{12}+3b^3cd^2\right)}{10b} + \frac{x^7\cdot\left(3a^2bd^3+9ab^2cd^2-\frac{9a\left(\frac{25ab^2d^3}{12}+3b^3cd^2\right)}{10b}+3b^3c^2d\right)}{8b} + \frac{x^5\left(a^3d^3+\right.}{}\right. \\ a^{\frac{5}{2}}\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**3,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(b**2*d**3*x**11/12 + x**9*(25*a*b**2*d**3/12
+ 3*b**3*c*d**2)/(10*b) + x**7*(3*a**2*b*d**3 + 9*a*b**2*c*d**2 - 9*a*(25*
a*b**2*d**3/12 + 3*b**3*c*d**2)/(10*b) + 3*b**3*c**2*d)/(8*b) + x**5*(a**3
*d**3 + 9*a**2*b*c*d**2 + 9*a*b**2*c**2*d - 7*a*(3*a**2*b*d**3 + 9*a*b**2*
c*d**2 - 9*a*(25*a*b**2*d**3/12 + 3*b**3*c*d**2)/(10*b) + 3*b**3*c**2*d)/(
8*b) + b**3*c**3)/(6*b) + x**3*(3*a**3*c*d**2 + 9*a**2*b*c**2*d + 3*a*b**2
*c**3 - 5*a*(a**3*d**3 + 9*a**2*b*c*d**2 + 9*a*b**2*c**2*d - 7*a*(3*a**2*b
*d**3 + 9*a*b**2*c*d**2 - 9*a*(25*a*b**2*d**3/12 + 3*b**3*c*d**2)/(10*b) +
 3*b**3*c**2*d)/(8*b) + b**3*c**3)/(6*b))/(4*b) + x*(3*a**3*c**2*d + 3*a**
2*b*c**3 - 3*a*(3*a**3*c*d**2 + 9*a**2*b*c**2*d + 3*a*b**2*c**3 - 5*a*(a**
3*d**3 + 9*a**2*b*c*d**2 + 9*a*b**2*c**2*d - 7*a*(3*a**2*b*d**3 + 9*a*b**2
*c*d**2 - 9*a*(25*a*b**2*d**3/12 + 3*b**3*c*d**2)/(10*b) + 3*b**3*c**2*d)/
(8*b) + b**3*c**3)/(6*b))/(4*b))/(2*b)) + (a**3*c**3 - a*(3*a**3*c**2*d +
3*a**2*b*c**3 - 3*a*(3*a**3*c*d**2 + 9*a**2*b*c**2*d + 3*a*b**2*c**3 - 5*a
*(a**3*d**3 + 9*a**2*b*c*d**2 + 9*a*b**2*c**2*d - 7*a*(3*a**2*b*d**3 + 9*a
*b**2*c*d**2 - 9*a*(25*a*b**2*d**3/12 + 3*b**3*c*d**2)/(10*b) + 3*b**3*c**
2*d)/(8*b) + b**3*c**3)/(6*b))/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt
(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)),
Ne(b, 0)), (a**(5/2)*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x**7/7
), True))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3\,dx &= \frac{(bx^2+a)^{\frac{7}{2}}d^3x^5}{12\,b} + \frac{3\,(bx^2+a)^{\frac{7}{2}}cd^2x^3}{10\,b} \\ &- \frac{(bx^2+a)^{\frac{7}{2}}ad^3x^3}{24\,b^2} + \frac{1}{6}\,(bx^2+a)^{\frac{5}{2}}c^3x + \frac{5}{24}\,(bx^2+a)^{\frac{3}{2}}ac^3x + \frac{5}{16}\,\sqrt{bx^2+a}a^2c^3x \\ &+ \frac{3\,(bx^2+a)^{\frac{7}{2}}c^2dx}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}}ac^2dx}{16\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}a^2c^2dx}{64\,b} \\ &- \frac{15\,\sqrt{bx^2+a}a^3c^2dx}{128\,b} - \frac{9\,(bx^2+a)^{\frac{7}{2}}acd^2x}{80\,b^2} + \frac{3\,(bx^2+a)^{\frac{5}{2}}a^2cd^2x}{160\,b^2} \\ &+ \frac{3\,(bx^2+a)^{\frac{3}{2}}a^3cd^2x}{128\,b^2} + \frac{9\,\sqrt{bx^2+a}a^4cd^2x}{256\,b^2} + \frac{(bx^2+a)^{\frac{7}{2}}a^2d^3x}{64\,b^3} - \frac{(bx^2+a)^{\frac{5}{2}}a^3d^3x}{384\,b^3} \\ &- \frac{5\,(bx^2+a)^{\frac{3}{2}}a^4d^3x}{1536\,b^3} - \frac{5\,\sqrt{bx^2+a}a^5d^3x}{1024\,b^3} + \frac{5\,a^3c^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}} \\ &- \frac{15\,a^4c^2d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}} + \frac{9\,a^5cd^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}} - \frac{5\,a^6d^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{1024\,b^{\frac{7}{2}}}\end{aligned}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/12*(b*x^2 + a)^(7/2)*d^3*x^5/b + 3/10*(b*x^2 + a)^(7/2)*c*d^2*x^3/b - 1/
24*(b*x^2 + a)^(7/2)*a*d^3*x^3/b^2 + 1/6*(b*x^2 + a)^(5/2)*c^3*x + 5/24*(b
*x^2 + a)^(3/2)*a*c^3*x + 5/16*sqrt(b*x^2 + a)*a^2*c^3*x + 3/8*(b*x^2 + a)
^(7/2)*c^2*d*x/b - 1/16*(b*x^2 + a)^(5/2)*a*c^2*d*x/b - 5/64*(b*x^2 + a)^(
3/2)*a^2*c^2*d*x/b - 15/128*sqrt(b*x^2 + a)*a^3*c^2*d*x/b - 9/80*(b*x^2 +
a)^(7/2)*a*c*d^2*x/b^2 + 3/160*(b*x^2 + a)^(5/2)*a^2*c*d^2*x/b^2 + 3/128*(
b*x^2 + a)^(3/2)*a^3*c*d^2*x/b^2 + 9/256*sqrt(b*x^2 + a)*a^4*c*d^2*x/b^2 +
 1/64*(b*x^2 + a)^(7/2)*a^2*d^3*x/b^3 - 1/384*(b*x^2 + a)^(5/2)*a^3*d^3*x/
b^3 - 5/1536*(b*x^2 + a)^(3/2)*a^4*d^3*x/b^3 - 5/1024*sqrt(b*x^2 + a)*a^5*
d^3*x/b^3 + 5/16*a^3*c^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 15/128*a^4*c^2*d
*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 9/256*a^5*c*d^2*arcsinh(b*x/sqrt(a*b))/b
^(5/2) - 5/1024*a^6*d^3*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.92

$$\int (a+bx^2)^{5/2} (c +dx^2)^3 \, dx = \frac{1}{15360} \left(2 \left(4 \left(2 \left(8 \left(10 \, b^2 d^3 x^2 + \frac{36 \, b^{12} c d^2 + 25 \, a b^{11} d^3}{b^{10}}\right) x^2 + \frac{9 \, (40 \, b^{12} c^2 d + 84 \, a b^{11} c d^2 + 15 \, a^2}{b^{10}} \right.\right.\right.\right.$$
$$- \frac{(320 \, a^3 b^3 c^3 - 120 \, a^4 b^2 c^2 d + 36 \, a^5 b c d^2 - 5 \, a^6 d^3) \log \left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{1024 \, b^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*b^2*d^3*x^2 + (36*b^12*c*d^2 + 25*a*b^11*d^3)/b^10
)*x^2 + 9*(40*b^12*c^2*d + 84*a*b^11*c*d^2 + 15*a^2*b^10*d^3)/b^10)*x^2 +
(320*b^12*c^3 + 2040*a*b^11*c^2*d + 1116*a^2*b^10*c*d^2 + 5*a^3*b^9*d^3)/b
^10)*x^2 + 5*(832*a*b^11*c^3 + 1416*a^2*b^10*c^2*d + 36*a^3*b^9*c*d^2 - 5*
a^4*b^8*d^3)/b^10)*x^2 + 15*(704*a^2*b^10*c^3 + 120*a^3*b^9*c^2*d - 36*a^4
*b^8*c*d^2 + 5*a^5*b^7*d^3)/b^10)*sqrt(b*x^2 + a)*x - 1/1024*(320*a^3*b^3*
c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6*d^3)*log(abs(-sqrt(b)*x +
 sqrt(b*x^2 + a)))/b^(7/2)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \, dx = \int \left(b\,x^2+a\right)^{5/2} \left(d\,x^2+c\right)^3 \,dx$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x^2)^3,x)
```

output
```
int((a + b*x^2)^(5/2)*(c + d*x^2)^3, x)
```

3.63 $\int (a+bx^2)^{5/2} (c+dx^2)^2 \, dx$

3.63.1 Optimal result

Integrand size = 21, antiderivative size = 241

$$\int (a+bx^2)^{5/2} (c+dx^2)^2 \, dx = \frac{a^2(80b^2c^2-20abcd+3a^2d^2)\, x\sqrt{a+bx^2}}{256b^2} + \frac{a(80b^2c^2-20abcd+3a^2d^2)\, x(a+bx^2)^{3/2}}{384b^2} + \frac{(80b^2c^2-20abcd+3a^2d^2)\, x(a+bx^2)^{5/2}}{480b^2} + \frac{3d(4bc-ad)x(a+bx^2)^{7/2}}{80b^2} + \frac{dx(a+bx^2)^{7/2}(c+dx^2)}{10b} + \frac{a^3(80b^2c^2-20abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}$$

output
```
1/384*a*(3*a^2*d^2-20*a*b*c*d+80*b^2*c^2)*x*(b*x^2+a)^(3/2)/b^2+1/480*(3*a
^2*d^2-20*a*b*c*d+80*b^2*c^2)*x*(b*x^2+a)^(5/2)/b^2+3/80*d*(-a*d+4*b*c)*x*
(b*x^2+a)^(7/2)/b^2+1/10*d*x*(b*x^2+a)^(7/2)*(d*x^2+c)/b+1/256*a^3*(3*a^2*
d^2-20*a*b*c*d+80*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/25
6*a^2*(3*a^2*d^2-20*a*b*c*d+80*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2
```

3.63.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.79

$$\int (a+bx^2)^{5/2} (c+dx^2)^2 \, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(-45a^4d^2+30a^3bd(10c+dx^2)+64b^4x^4(10c^2+15cdx^2+6d^2x^4)+16ab^3x^2(130c^2 \ldots}{\ldots}$$

input
```
Integrate[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-45*a^4*d^2 + 30*a^3*b*d*(10*c + d*x^2) + 64*b
^4*x^4*(10*c^2 + 15*c*d*x^2 + 6*d^2*x^4) + 16*a*b^3*x^2*(130*c^2 + 170*c*d
*x^2 + 63*d^2*x^4) + 8*a^2*b^2*(330*c^2 + 295*c*d*x^2 + 93*d^2*x^4)) - 15*
a^3*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/(3840*b^(5/2))
```

3.63.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.77, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {318, 299, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^{5/2} (c+dx^2)^2 \, dx$$

$$\Big\downarrow \text{318}$$

$$\frac{\int (bx^2+a)^{5/2} (3d(4bc-ad)x^2+c(10bc-ad)) \, dx}{10b} + \frac{dx(a+bx^2)^{7/2} (c+dx^2)}{10b}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{(3a^2d^2-20abcd+80b^2c^2)\int (bx^2+a)^{5/2}dx}{8b} + \frac{3dx(a+bx^2)^{7/2}(4bc-ad)}{8b}}{10b} + \frac{dx(a+bx^2)^{7/2} (c+dx^2)}{10b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\left(3a^2d^2-20abcd+80b^2c^2\right)\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{3dx\left(a+bx^2\right)^{7/2}(4bc-ad)}{8b}}{10b}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}$$

$$\Big\downarrow \textbf{211}$$

$$\frac{\frac{\left(3a^2d^2-20abcd+80b^2c^2\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{3dx\left(a+bx^2\right)^{7/2}(4bc-ad)}{8b}}{10b}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}$$

$$\Big\downarrow \textbf{211}$$

$$\frac{\frac{\left(3a^2d^2-20abcd+80b^2c^2\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{3dx\left(a+bx^2\right)^{7/2}(4bc-ad)}{8b}}{10b}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}$$

$$\Big\downarrow \textbf{224}$$

$$\frac{\frac{\left(3a^2d^2-20abcd+80b^2c^2\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{3dx\left(a+bx^2\right)^{7/2}(4bc-ad)}{8b}}{10b}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)\left(3a^2d^2-20abcd+80b^2c^2\right)}{8b}+\frac{3dx\left(a+bx^2\right)^{7/2}(4bc-ad)}{8b}}{10b}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}$$

input
```
Int[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```

3.63. $\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2\,dx$

output
```
(d*x*(a + b*x^2)^(7/2)*(c + d*x^2))/(10*b) + ((3*d*(4*b*c - a*d)*x*(a + b*
x^2)^(7/2))/(8*b) + ((80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*((x*(a + b*x^2)
^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 +
(a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))/(8*b))/(10*
b)
```

3.63.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.63.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\frac{3a^3\left(a^2d^2-\frac{20}{3}abcd+\frac{80}{3}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{256}-\frac{3x\left(-\frac{176\left(\frac{31}{110}d^2x^4+\frac{59}{66}cd\,x^2+c^2\right)a^2b^{\frac{5}{2}}}{3}-\frac{416x^2\left(\frac{63}{130}d^2x^4+\frac{17}{13}cd\,x^2+c^2\right)a\,b^{\frac{7}{2}}}{9}-\right.}{256}}{b^{\frac{5}{2}}}$
risch	$-\frac{x(-384b^4d^2x^8-1008a\,b^3d^2x^6-960b^4cd\,x^6-744a^2b^2d^2x^4-2720a\,b^3cd\,x^4-640b^4c^2x^4-30a^3b\,d^2x^2-2360a^2b^2cd\,x^2-2080}{3840b^2}$
default	$c^2\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+d^2\left(\frac{x^3(b\,x^2+a)^{\frac{7}{2}}}{10b}-\frac{3a\left(\frac{x\left(b\,x^2\right.}{8b}\right.}{}\right.$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/256*(a^3*(a^2*d^2-20/3*a*b*c*d+80/3*b^2*c^2)*arctanh((b*x^2+a)^(1/2)/x/b
^(1/2))-x*(-176/3*(31/110*d^2*x^4+59/66*c*d*x^2+c^2)*a^2*b^(5/2)-416/9*x^2
*(63/130*d^2*x^4+17/13*c*d*x^2+c^2)*a*b^(7/2)-128/9*x^4*(3/5*d^2*x^4+3/2*c
*d*x^2+c^2)*b^(9/2)+(2/3*(-d*x^2-10*c)*b^(3/2)+a*d*b^(1/2))*d*a^3)*(b*x^2+
a)^(1/2))/b^(5/2)
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.74

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2\,dx = \left[\frac{15\left(80\,a^3b^2c^2-20\,a^4bcd+3\,a^5d^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(384\,b^5d^2x^9+48\right.}{} \right.$$

$$-\frac{15\left(80\,a^3b^2c^2-20\,a^4bcd+3\,a^5d^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(384\,b^5d^2x^9+48\left(20\,b^5cd+21\,ab^4d^2\right)x^7+8\left(80\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/7680*(15*(80*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*sqrt(b)*log(-2*b*x
^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(384*b^5*d^2*x^9 + 48*(20*b^5*c*
d + 21*a*b^4*d^2)*x^7 + 8*(80*b^5*c^2 + 340*a*b^4*c*d + 93*a^2*b^3*d^2)*x^
5 + 10*(208*a*b^4*c^2 + 236*a^2*b^3*c*d + 3*a^3*b^2*d^2)*x^3 + 15*(176*a^2
*b^3*c^2 + 20*a^3*b^2*c*d - 3*a^4*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/3840*
(15*(80*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*sqrt(-b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) - (384*b^5*d^2*x^9 + 48*(20*b^5*c*d + 21*a*b^4*d^2)*x^7
+ 8*(80*b^5*c^2 + 340*a*b^4*c*d + 93*a^2*b^3*d^2)*x^5 + 10*(208*a*b^4*c^2
+ 236*a^2*b^3*c*d + 3*a^3*b^2*d^2)*x^3 + 15*(176*a^2*b^3*c^2 + 20*a^3*b^2*
c*d - 3*a^4*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

3.63.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(236) = 472$.

Time = 0.58 (sec) , antiderivative size = 520, normalized size of antiderivative = 2.16

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^2 \, dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{b^2d^2x^9}{10} + \frac{x^7\cdot\left(\frac{21ab^2d^2}{10}+2b^3cd\right)}{8b} + \frac{x^5\cdot\left(3a^2bd^2+6ab^2cd-\frac{7a\left(\frac{21ab^2d^2}{10}+2b^3cd\right)}{8b}+b^3c^2\right)}{6b} + \frac{x^3\left(a^3d^2+6a^2bcd \ldots\right.}{\ldots} \right. \\ a^{\frac{5}{2}}\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**2,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(b**2*d**2*x**9/10 + x**7*(21*a*b**2*d**2/10 +
 2*b**3*c*d)/(8*b) + x**5*(3*a**2*b*d**2 + 6*a*b**2*c*d - 7*a*(21*a*b**2*d
**2/10 + 2*b**3*c*d)/(8*b) + b**3*c**2)/(6*b) + x**3*(a**3*d**2 + 6*a**2*b
*c*d + 3*a*b**2*c**2 - 5*a*(3*a**2*b*d**2 + 6*a*b**2*c*d - 7*a*(21*a*b**2*
d**2/10 + 2*b**3*c*d)/(8*b) + b**3*c**2)/(6*b))/(4*b) + x*(2*a**3*c*d + 3*
a**2*b*c**2 - 3*a*(a**3*d**2 + 6*a**2*b*c*d + 3*a*b**2*c**2 - 5*a*(3*a**2*
b*d**2 + 6*a*b**2*c*d - 7*a*(21*a*b**2*d**2/10 + 2*b**3*c*d)/(8*b) + b**3*
c**2)/(6*b))/(4*b))/(2*b)) + (a**3*c**2 - a*(2*a**3*c*d + 3*a**2*b*c**2 -
3*a*(a**3*d**2 + 6*a**2*b*c*d + 3*a*b**2*c**2 - 5*a*(3*a**2*b*d**2 + 6*a*b
**2*c*d - 7*a*(21*a*b**2*d**2/10 + 2*b**3*c*d)/(8*b) + b**3*c**2)/(6*b))/(
4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), N
e(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(5/2)*(c**2*x + 2
*c*d*x**3/3 + d**2*x**5/5), True))
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.19

$$\int (a+bx^2)^{5/2}(c+dx^2)^2\,dx = \frac{(bx^2+a)^{\frac{7}{2}}d^2x^3}{10\,b} + \frac{1}{6}(bx^2+a)^{\frac{5}{2}}c^2x + \frac{5}{24}(bx^2+a)^{\frac{3}{2}}ac^2x + \frac{5}{16}\sqrt{bx^2+a}\,a^2c^2x + \frac{(bx^2+a)^{\frac{7}{2}}cdx}{4\,b} - \frac{(bx^2+a)^{\frac{5}{2}}acdx}{24\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}a^2cdx}{96\,b} - \frac{5\sqrt{bx^2+a}\,a^3cdx}{64\,b} - \frac{3\,(bx^2+a)^{\frac{7}{2}}ad^2x}{80\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}a^2d^2x}{160\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}a^3d^2x}{128\,b^2} + \frac{3\sqrt{bx^2+a}\,a^4d^2x}{256\,b^2} + \frac{5\,a^3c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}} - \frac{5\,a^4cd\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{64\,b^{\frac{3}{2}}} + \frac{3\,a^5d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(7/2)*d^2*x^3/b + 1/6*(b*x^2 + a)^(5/2)*c^2*x + 5/24*(b*x
^2 + a)^(3/2)*a*c^2*x + 5/16*sqrt(b*x^2 + a)*a^2*c^2*x + 1/4*(b*x^2 + a)^(
7/2)*c*d*x/b - 1/24*(b*x^2 + a)^(5/2)*a*c*d*x/b - 5/96*(b*x^2 + a)^(3/2)*a
^2*c*d*x/b - 5/64*sqrt(b*x^2 + a)*a^3*c*d*x/b - 3/80*(b*x^2 + a)^(7/2)*a*d
^2*x/b^2 + 1/160*(b*x^2 + a)^(5/2)*a^2*d^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)
*a^3*d^2*x/b^2 + 3/256*sqrt(b*x^2 + a)*a^4*d^2*x/b^2 + 5/16*a^3*c^2*arcsin
h(b*x/sqrt(a*b))/sqrt(b) - 5/64*a^4*c*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3
/256*a^5*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.92

$$\int (a+bx^2)^{5/2}(c+dx^2)^2\,dx = \frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2d^2x^2 + \frac{20\,b^{10}cd + 21\,ab^9d^2}{b^8}\right)x^2 + \frac{80\,b^{10}c^2 + 340\,ab^9cd + 93\,a^2b^8d^2}{b^8}\right)x^2 + \right.\right.\right. - \frac{(80\,a^3b^2c^2 - 20\,a^4bcd + 3\,a^5d^2)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*b^2*d^2*x^2 + (20*b^10*c*d + 21*a*b^9*d^2)/b^8)*x^2 + (
80*b^10*c^2 + 340*a*b^9*c*d + 93*a^2*b^8*d^2)/b^8)*x^2 + 5*(208*a*b^9*c^2
+ 236*a^2*b^8*c*d + 3*a^3*b^7*d^2)/b^8)*x^2 + 15*(176*a^2*b^8*c^2 + 20*a^3
*b^7*c*d - 3*a^4*b^6*d^2)/b^8)*sqrt(b*x^2 + a)*x - 1/256*(80*a^3*b^2*c^2 -
 20*a^4*b*c*d + 3*a^5*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^2 \, dx = \int \left(b\,x^2+a\right)^{5/2} \left(d\,x^2+c\right)^2 \, dx$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x^2)^2,x)
```

output
```
int((a + b*x^2)^(5/2)*(c + d*x^2)^2, x)
```

3.64 $\int (a+bx^2)^{5/2} (c+dx^2) \, dx$

3.64.1 Optimal result

Integrand size = 19, antiderivative size = 149

$$\int (a+bx^2)^{5/2} (c+dx^2) \, dx = \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b} + \frac{5a(8bc-ad)x(a+bx^2)^{3/2}}{192b} + \frac{(8bc-ad)x(a+bx^2)^{5/2}}{48b} + \frac{dx(a+bx^2)^{7/2}}{8b} + \frac{5a^3(8bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

output
```
5/192*a*(-a*d+8*b*c)*x*(b*x^2+a)^(3/2)/b+1/48*(-a*d+8*b*c)*x*(b*x^2+a)^(5/
2)/b+1/8*d*x*(b*x^2+a)^(7/2)/b+5/128*a^3*(-a*d+8*b*c)*arctanh(x*b^(1/2)/(b
*x^2+a)^(1/2))/b^(3/2)+5/128*a^2*(-a*d+8*b*c)*x*(b*x^2+a)^(1/2)/b
```

3.64.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83

$$\int (a+bx^2)^{5/2} (c+dx^2) \, dx = \frac{x\sqrt{a+bx^2}(264a^2bc+15a^3d+208ab^2cx^2+118a^2bdx^2+64b^3cx^4+136ab^2dx^4+48b^3dx^6)}{384b} + \frac{5a^3(-8bc+ad)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{128b^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(5/2)*(c + d*x^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(264*a^2*b*c + 15*a^3*d + 208*a*b^2*c*x^2 + 118*a^2*b*d
*x^2 + 64*b^3*c*x^4 + 136*a*b^2*d*x^4 + 48*b^3*d*x^6))/(384*b) + (5*a^3*(-
8*b*c + a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(3/2))
```

3.64.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {299, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right) \, dx$$

$$\Big\downarrow \text{ 299}$$

$$\frac{(8bc-ad) \int \left(bx^2+a\right)^{5/2}dx}{8b}+\frac{dx \left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{(8bc-ad) \left(\frac{5}{6} a \int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6} x \left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{dx \left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{(8bc-ad) \left(\frac{5}{6} a \left(\frac{3}{4} a \int \sqrt{bx^2+a}dx+\frac{1}{4} x \left(a+bx^2\right)^{3/2}\right)+\frac{1}{6} x \left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{dx \left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{(8bc-ad) \left(\frac{5}{6} a \left(\frac{3}{4} a \left(\frac{1}{2} a \int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2} x \sqrt{a+bx^2}\right)+\frac{1}{4} x \left(a+bx^2\right)^{3/2}\right)+\frac{1}{6} x \left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{dx \left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(8bc-ad) \left(\frac{5}{6} a \left(\frac{3}{4} a \left(\frac{1}{2} a \int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2} x \sqrt{a+bx^2}\right)+\frac{1}{4} x \left(a+bx^2\right)^{3/2}\right)+\frac{1}{6} x \left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{dx \left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow 219$$

$$\frac{\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)(8bc-ad)}{8b}+\frac{dx\left(a+bx^2\right)^{7/2}}{8b}$$

input
```
Int[(a + b*x^2)^(5/2)*(c + d*x^2),x]
```

output
```
(d*x*(a + b*x^2)^(7/2))/(8*b) + ((8*b*c - a*d)*((x*(a + b*x^2)^(5/2))/6 +
(5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))/(8*b)
```

3.64.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.64.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{5\left(\left(a^{4}d-8a^{3}bc\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-x\sqrt{b\,x^{2}+a}\,\left(\frac{88\left(\frac{59d\,x^{2}}{132}+c\right)a^{2}b^{\frac{3}{2}}}{5}+\frac{208x^{2}\left(\frac{17d\,x^{2}}{26}+c\right)a\,b^{\frac{5}{2}}}{15}+\frac{64x^{4}\left(\frac{3d\,x^{2}}{4}+c\right)b^{\frac{7}{2}}}{15}+a^{3}d\right.\right.}{128b^{\frac{3}{2}}}$
risch	$\frac{x\left(48b^{3}d\,x^{6}+136a\,b^{2}d\,x^{4}+64b^{3}c\,x^{4}+118a^{2}bd\,x^{2}+208a\,b^{2}c\,x^{2}+15a^{3}d+264a^{2}bc\right)\sqrt{b\,x^{2}+a}}{384b}-\frac{5a^{3}(ad-8bc)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+\right.}{128b^{\frac{3}{2}}}$
default	$c\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+d\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}\right.}{}\right.$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-5/128*((a^4*d-8*a^3*b*c)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*(b*x^2+a)^(
1/2)*(88/5*(59/132*d*x^2+c)*a^2*b^(3/2)+208/15*x^2*(17/26*d*x^2+c)*a*b^(5/
2)+64/15*x^4*(3/4*d*x^2+c)*b^(7/2)+a^3*d*b^(1/2)))/b^(3/2)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.74

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)\,dx=\left[-\frac{15\left(8a^3bc-a^4d\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(48b^4dx^7+8\left(8b^4c+17ab^3d\right)x\right.}{768b^2}\right.$$

$$-\frac{15\left(8a^3bc-a^4d\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(48b^4dx^7+8\left(8b^4c+17ab^3d\right)x^5+2\left(104ab^3c+59a^2b^2d\right)x^3+3\right.}{384b^2}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/768*(15*(8*a^3*b*c - a^4*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(48*b^4*d*x^7 + 8*(8*b^4*c + 17*a*b^3*d)*x^5 + 2*(104*a*
b^3*c + 59*a^2*b^2*d)*x^3 + 3*(88*a^2*b^2*c + 5*a^3*b*d)*x)*sqrt(b*x^2 + a
))/b^2, -1/384*(15*(8*a^3*b*c - a^4*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x
^2 + a)) - (48*b^4*d*x^7 + 8*(8*b^4*c + 17*a*b^3*d)*x^5 + 2*(104*a*b^3*c +
 59*a^2*b^2*d)*x^3 + 3*(88*a^2*b^2*c + 5*a^3*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

3.64.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(134) = 268$.

Time = 0.42 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.87

$$\int (a+bx^2)^{5/2}(c+dx^2)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{b^2dx^7}{8}+\frac{x^5\cdot\left(\frac{17ab^2d}{8}+b^3c\right)}{6b}+\frac{x^3\cdot\left(3a^2bd+3ab^2c-\frac{5a\left(\frac{17ab^2d}{8}+b^3c\right)}{6b}\right)}{4b}+\frac{x\left(a^3d+3a^2bc-\frac{3a\left(3a^2bd+3ab^2c-\cdots\right)}{4b}\right)}{2b}\cdots\right. \\ a^{\frac{5}{2}}\left(cx+\frac{dx^3}{3}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(b**2*d*x**7/8 + x**5*(17*a*b**2*d/8 + b**3*c)
/(6*b) + x**3*(3*a**2*b*d + 3*a*b**2*c - 5*a*(17*a*b**2*d/8 + b**3*c)/(6*b
))/(4*b) + x*(a**3*d + 3*a**2*b*c - 3*a*(3*a**2*b*d + 3*a*b**2*c - 5*a*(17
*a*b**2*d/8 + b**3*c)/(6*b))/(4*b))/(2*b)) + (a**3*c - a*(a**3*d + 3*a**2*
b*c - 3*a*(3*a**2*b*d + 3*a*b**2*c - 5*a*(17*a*b**2*d/8 + b**3*c)/(6*b))/(
4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), N
e(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(5/2)*(c*x + d*x*
*3/3), True))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)\,dx = \frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}cx + \frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}acx + \frac{5}{16}\sqrt{bx^2+a}\,a^2cx + \frac{\left(bx^2+a\right)^{\frac{7}{2}}dx}{8\,b} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}adx}{48\,b} - \frac{5\left(bx^2+a\right)^{\frac{3}{2}}a^2dx}{192\,b} - \frac{5\sqrt{bx^2+a}\,a^3dx}{128\,b} + \frac{5\,a^3c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}} - \frac{5\,a^4d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*c*x + 5/24*(b*x^2 + a)^(3/2)*a*c*x + 5/16*sqrt(b*x^2
 + a)*a^2*c*x + 1/8*(b*x^2 + a)^(7/2)*d*x/b - 1/48*(b*x^2 + a)^(5/2)*a*d*x
/b - 5/192*(b*x^2 + a)^(3/2)*a^2*d*x/b - 5/128*sqrt(b*x^2 + a)*a^3*d*x/b +
 5/16*a^3*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 5/128*a^4*d*arcsinh(b*x/sqrt(
a*b))/b^(3/2)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)\,dx = \frac{1}{384}\left(2\left(4\left(6\,b^2dx^2 + \frac{8\,b^8c+17\,ab^7d}{b^6}\right)x^2 + \frac{104\,ab^7c+59\,a^2b^6d}{b^6}\right)x^2 + \frac{3\left(88\,a^2b^6c+5\,a^3b^5d\right)}{b^6}\right) - \frac{5\left(8\,a^3bc-a^4d\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*b^2*d*x^2 + (8*b^8*c + 17*a*b^7*d)/b^6)*x^2 + (104*a*b^7*c
+ 59*a^2*b^6*d)/b^6)*x^2 + 3*(88*a^2*b^6*c + 5*a^3*b^5*d)/b^6)*sqrt(b*x^2
+ a)*x - 5/128*(8*a^3*b*c - a^4*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/
b^(3/2)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)\,dx=\int \left(b\,x^2+a\right)^{5/2}\left(d\,x^2+c\right)\,dx$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x^2),x)
```

output
```
int((a + b*x^2)^(5/2)*(c + d*x^2), x)
```

3.65 $\int (a+bx^2)^{5/2}\,dx$

3.65.1 Optimal result

Integrand size = 11, antiderivative size = 84

$$\int (a+bx^2)^{5/2}\,dx = \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax(a+bx^2)^{3/2} + \frac{1}{6}x(a+bx^2)^{5/2} + \frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

output
```
5/24*a*x*(b*x^2+a)^(3/2)+1/6*x*(b*x^2+a)^(5/2)+5/16*a^3*arctanh(x*b^(1/2)/
(b*x^2+a)^(1/2))/b^(1/2)+5/16*a^2*x*(b*x^2+a)^(1/2)
```

3.65.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int (a+bx^2)^{5/2}\,dx = \frac{1}{48}\sqrt{a+bx^2}(33a^2x+26abx^3+8b^2x^5) - \frac{5a^3\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5))/48 - (5*a^3*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

3.65.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} dx$$

$$\Big\downarrow \ 211$$

$$\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \ 211$$

$$\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \ 211$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \ 224$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \ 219$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

input
```
Int[(a + b*x^2)^(5/2),x]
```

output
```
(x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a
+ b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6
```

3.65.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.65.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{x\left(8b^{2}x^{4}+26ab\,x^{2}+33a^{2}\right)\sqrt{b\,x^{2}+a}}{48}+\frac{5a^{3}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{16\sqrt{b}}$	59
pseudoelliptic	$\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{3}}{16\sqrt{b}}+\frac{11\left(\frac{8b^{\frac{5}{2}}x^{4}}{33}+\frac{26x^{2}a\,b^{\frac{3}{2}}}{33}+\sqrt{b}\,a^{2}\right)x\sqrt{b\,x^{2}+a}}{16\sqrt{b}}$	67
default	$\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}$	68

input
```
int((b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*x*(8*b^2*x^4+26*a*b*x^2+33*a^2)*(b*x^2+a)^(1/2)+5/16*a^3*ln(x*b^(1/2)
+(b*x^2+a)^(1/2))/b^(1/2)
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.74

$$\int (a+bx^2)^{5/2}\,dx = \left[\frac{15\,a^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx)\sqrt{bx^2+a}}{96\,b}, -\frac{15\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx)\sqrt{bx^2+a}}{48\,b}\right]$$

input
```
integrate((b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/96*(15*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(8*b^3*x^5 + 26*a*b^2*x^3 + 33*a^2*b*x)*sqrt(b*x^2 + a))/b, -1/48*(15*a^3*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 26*a*b^2*x^3 +
33*a^2*b*x)*sqrt(b*x^2 + a))/b]
```

3.65.6 Sympy [A] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int (a+bx^2)^{5/2}\,dx = \frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24}+\frac{\sqrt{a}b^2x^5\sqrt{1+\frac{bx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}$$

input
```
integrate((b*x**2+a)**(5/2),x)
```

output
```
11*a**(5/2)*x*sqrt(1 + b*x**2/a)/16 + 13*a**(3/2)*b*x**3*sqrt(1 + b*x**2/a
)/24 + sqrt(a)*b**2*x**5*sqrt(1 + b*x**2/a)/6 + 5*a**3*asinh(sqrt(b)*x/sqr
t(a))/(16*sqrt(b))
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int (a+bx^2)^{5/2}\,dx = \frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}x + \frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}ax + \frac{5}{16}\sqrt{bx^2+a}a^2x + \frac{5\,a^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*x + 5/24*(b*x^2 + a)^(3/2)*a*x + 5/16*sqrt(b*x^2 + a
)*a^2*x + 5/16*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int (a+bx^2)^{5/2}\,dx = -\frac{5\,a^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}} + \frac{1}{48}\left(2\left(4\,b^2x^2+13\,ab\right)x^2+33\,a^2\right)\sqrt{bx^2+a}x$$

input
```
integrate((b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-5/16*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2*(4*b^2*
x^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

3.65.9 Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int (a+bx^2)^{5/2}\,dx = \frac{x\,\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

input
```
int((a + b*x^2)^(5/2),x)
```

output
```
(x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(5/2)
```

3.66 $\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx$

3.66.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx = -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2}+\frac{bx(a+bx^2)^{3/2}}{4d} + \frac{\sqrt{b}(8b^2c^2-20abcd+15a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^3} - \frac{(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^3}$$

output
```
1/4*b*x*(b*x^2+a)^(3/2)/d+1/8*(15*a^2*d^2-20*a*b*c*d+8*b^2*c^2)*arctanh(x*
b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)/d^3-(-a*d+b*c)^(5/2)*arctanh(x*(-a*d+b*c)
^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/d^3/c^(1/2)-1/8*b*(-7*a*d+4*b*c)*x*(b*x^2+
a)^(1/2)/d^2
```

3.66.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx = \frac{bdx\sqrt{a+bx^2}(-4bc+9ad+2bdx^2) - \frac{8(-bc+ad)^{5/2}\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}} - \sqrt{b}(8b^2c^2}{8d^3}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x^2),x]
```

output
```
(b*d*x*Sqrt[a + b*x^2]*(-4*b*c + 9*a*d + 2*b*d*x^2) - (8*(-(b*c) + a*d)^(5
/2)*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(
b*c) + a*d])])/Sqrt[c] - Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*d^3)
```

3.66.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {318, 25, 403, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2} \, dx$$

$$\Big\downarrow \text{318}$$

$$\frac{\int -\frac{\sqrt{bx^2+a}(b(4bc-7ad)x^2+a(bc-4ad))}{dx^2+c}dx}{4d}+\frac{bx(a+bx^2)^{3/2}}{4d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx(a+bx^2)^{3/2}}{4d}-\frac{\int \frac{\sqrt{bx^2+a}(b(4bc-7ad)x^2+a(bc-4ad))}{dx^2+c}dx}{4d}$$

$$\Big\downarrow \text{403}$$

$$\frac{bx(a+bx^2)^{3/2}}{4d}-\frac{\frac{\int -\frac{b(8b^2c^2-20abdc+15a^2d^2)x^2+a(4b^2c^2-9abdc+8a^2d^2)}{\sqrt{bx^2+a}(dx^2+c)}dx}{2d}+\frac{bx\sqrt{a+bx^2}(4bc-7ad)}{2d}}{4d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx(a+bx^2)^{3/2}}{4d}-\frac{\frac{bx\sqrt{a+bx^2}(4bc-7ad)}{2d}-\frac{\int \frac{b(8b^2c^2-20abdc+15a^2d^2)x^2+a(4b^2c^2-9abdc+8a^2d^2)}{\sqrt{bx^2+a}(dx^2+c)}dx}{2d}}{4d}$$

$$\Big\downarrow \text{398}$$

$$\frac{bx(a+bx^2)^{3/2}}{4d}-\frac{\frac{bx\sqrt{a+bx^2}(4bc-7ad)}{2d}-\frac{\frac{b(15a^2d^2-20abcd+8b^2c^2)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{8(bc-ad)^3\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}}{2d}}{4d}$$

$$\Big\downarrow \text{224}$$

$$\frac{bx(a+bx^2)^{3/2}}{4d}-\frac{\frac{bx\sqrt{a+bx^2}(4bc-7ad)}{2d}-\frac{\frac{b(15a^2d^2-20abcd+8b^2c^2)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{8(bc-ad)^3\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}}{2d}}{4d}$$

$\downarrow$ 219

$$\frac{bx(a+bx^2)^{3/2}}{4d}-\frac{\frac{bx\sqrt{a+bx^2}(4bc-7ad)}{2d}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(15a^2d^2-20abcd+8b^2c^2)}{d}-\frac{8(bc-ad)^3\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}}{2d}}{4d}$$

$\downarrow$ 291

$$\frac{bx(a+bx^2)^{3/2}}{4d}-\frac{\frac{bx\sqrt{a+bx^2}(4bc-7ad)}{2d}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(15a^2d^2-20abcd+8b^2c^2)}{d}-\frac{8(bc-ad)^3\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d}}{4d}$$

$\downarrow$ 221

$$\frac{bx(a+bx^2)^{3/2}}{4d}-\frac{\frac{bx\sqrt{a+bx^2}(4bc-7ad)}{2d}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(15a^2d^2-20abcd+8b^2c^2)}{d}-\frac{8(bc-ad)^{5/2}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}}{2d}}{4d}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x^2),x]
```

output
```
(b*x*(a + b*x^2)^(3/2))/(4*d) - ((b*(4*b*c - 7*a*d)*x*Sqrt[a + b*x^2])/(2*
d) - ((Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/d - (8*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d))/(2*d))/(4*d)
```

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.66. $\int \frac{(a+bx^2)^{5/2}}{c+dx^2} \, dx$

3.66.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{\frac{b\left(-d\sqrt{b\,x^2+a}\,\left(2bd\,x^2+9ad-4bc\right)x-\frac{\left(15a^2d^2-20abcd+8b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{\sqrt{b}}\right)}{4}+\frac{2(ad-bc)^3\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}}{2d^3}$
risch	$\frac{bx\left(2bd\,x^2+9ad-4bc\right)\sqrt{b\,x^2+a}}{8d^2}+\frac{\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{d}-\frac{\left(-4a^3d^3+12a^2bc\,d^2-12a\,b^2c^2d+4b^3c^3\right)\ln\left(\ldots\right.}{}}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d^3*(1/4*b*(-d*(b*x^2+a)^(1/2)*(2*b*d*x^2+9*a*d-4*b*c)*x-(15*a^2*d^2-
20*a*b*c*d+8*b^2*c^2)/b^(1/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2)))+2*(a*d-b
*c)^3/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 935, normalized size of antiderivative = 5.96

$$\int \frac{\left(a+bx^2\right)^{5/2}}{c+dx^2}\,dx = \left[\frac{\left(8\,b^2c^2-20\,abcd+15\,a^2d^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+4\left(b^2c^2-2\,abcd\ldots\right.}{}\right.$$

$$-\frac{\left(8\,b^2c^2-20\,abcd+15\,a^2d^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-2\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{\left(8\,b^2c^2-8\,abcd+a^2d^2\right)\ldots}{}\right.}{8\,d^3}$$

$$-\frac{\left(8\,b^2c^2-20\,abcd+15\,a^2d^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-4\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2\,bc-ad)x^2+\ldots}{2\left((b^2c-ab\ldots\right.}\right.}{8\,d^3}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/16*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c -
 a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c
^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*
sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 2*(2*b^2*d^2*x^3 - (4*
b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, -1/8*((8*b^2*c^2 - 20*a*b*c*
d + 15*a^2*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 2*(b^2*c^2 -
 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^
2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c
^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2
 + c^2)) - (2*b^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^
3, 1/16*(8*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2
*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c -
a*b*d)*x^3 + (a*b*c - a^2*d)*x)) + (8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*s
qrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*b^2*d^2*x^3
- (4*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, -1/8*((8*b^2*c^2 - 20*a
*b*c*d + 15*a^2*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 4*(b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*
x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*
b*c - a^2*d)*x)) - (2*b^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x...
```

3.66.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{c+dx^2}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c),x)
```

output
```
Integral((a + b*x**2)**(5/2)/(c + d*x**2), x)
```

3.66.7 Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/(d*x^2 + c), x)
```

3.66.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{d\,x^2+c}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x^2),x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x^2), x)
```

3.67 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx$

3.67.1 Optimal result

Integrand size = 21, antiderivative size = 175

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx = \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)} - \frac{b^{3/2}(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^3} + \frac{(bc-ad)^{3/2}(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^3}$$

output
```
-1/2*(-a*d+b*c)*x*(b*x^2+a)^(3/2)/c/d/(d*x^2+c)-1/2*b^(3/2)*(-5*a*d+4*b*c)
*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/d^3+1/2*(-a*d+b*c)^(3/2)*(a*d+4*b*c)*a
rctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/d^3+1/2*b*(-a*d
+2*b*c)*x*(b*x^2+a)^(1/2)/c/d^2
```

3.67.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx = \frac{\frac{dx\sqrt{a+bx^2}(-2abcd+a^2d^2+b^2c(2c+dx^2))}{c(c+dx^2)} + \frac{\sqrt{-bc+ad}(4b^2c^2-3abcd-a^2d^2)\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{c^{3/2}} + }{2d^3}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

output
```
((d*x*Sqrt[a + b*x^2]*(-2*a*b*c*d + a^2*d^2 + b^2*c*(2*c + d*x^2)))/(c*(c
+ d*x^2)) + (Sqrt[-(b*c) + a*d]*(4*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(
-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])
])/c^(3/2) + b^(3/2)*(4*b*c - 5*a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/
(2*d^3)
```

3.67.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {315, 403, 27, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx$$

$$\Big\downarrow \ 315$$

$$\frac{\int \frac{\sqrt{bx^2+a}(2b(2bc-ad)x^2+a(bc+ad))}{dx^2+c}dx}{2cd}-\frac{x(a+bx^2)^{3/2}(bc-ad)}{2cd(c+dx^2)}$$

$$\Big\downarrow \ 403$$

$$\frac{\frac{\int -\frac{2\left(b^2c(4bc-5ad)x^2+a\left(2b^2c^2-2abdc-a^2d^2\right)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2d}+\frac{bx\sqrt{a+bx^2}(2bc-ad)}{d}}{2cd}-\frac{x(a+bx^2)^{3/2}(bc-ad)}{2cd(c+dx^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{bx\sqrt{a+bx^2}(2bc-ad)}{d}-\frac{\int \frac{b^2c(4bc-5ad)x^2+a\left(2b^2c^2-2abdc-a^2d^2\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}}{2cd}-\frac{x(a+bx^2)^{3/2}(bc-ad)}{2cd(c+dx^2)}$$

$$\Big\downarrow \ 398$$

$$\frac{\frac{bx\sqrt{a+bx^2}(2bc-ad)}{d}-\frac{\frac{b^2c(4bc-5ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)^2(ad+4bc)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}}{d}}{2cd}-\frac{x(a+bx^2)^{3/2}(bc-ad)}{2cd(c+dx^2)}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{bx\sqrt{a+bx^2}(2bc-ad)}{d}-\frac{\frac{b^2c(4bc-5ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)^2(ad+4bc)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}}{d}}{2cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{bx\sqrt{a+bx^2}(2bc-ad)}{d}-\frac{\frac{b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bc-5ad)}{d}-\frac{(bc-ad)^2(ad+4bc)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}}{d}}{2cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{bx\sqrt{a+bx^2}(2bc-ad)}{d}-\frac{\frac{b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bc-5ad)}{d}-\frac{(bc-ad)^2(ad+4bc)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{d}}{2cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{bx\sqrt{a+bx^2}(2bc-ad)}{d}-\frac{\frac{b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bc-5ad)}{d}-\frac{(bc-ad)^{3/2}(ad+4bc)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}}{d}}{2cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

output
```
-1/2*((b*c - a*d)*x*(a + b*x^2)^(3/2))/(c*d*(c + d*x^2)) + ((b*(2*b*c - a*
d)*x*Sqrt[a + b*x^2])/d - ((b^(3/2)*c*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/d - ((b*c - a*d)^(3/2)*(4*b*c + a*d)*ArcTanh[(Sqrt[b*c -
 a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d))/d)/(2*c*d)
```

3.67. $\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^2}\,dx$

3.67.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.67. $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx$

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.67.4 Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{2\left(\frac{\left(d\,x^{2}+c\right)\left(ad+4bc\right)\left(ad-bc\right)^{2}\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{\left(ad-bc\right)c}}\right)}{4}+\left(\left(d\,x^{2}+c\right)\left(b^{\frac{5}{2}}c-\frac{5ad\,b^{\frac{3}{2}}}{4}\right)c\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-\frac{x\sqrt{b\,x^{2}+a}\,d\left(2b^{2}c^{2}\cdots\right.}{\cdots}\right.\right.}{\sqrt{\left(ad-bc\right)c}\,d^{3}c\left(d\,x^{2}+c\right)}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(1/4*(d*x^2+c)*(a*d+4*b*c)*(a*d-b*c)^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d
-b*c)*c)^(1/2))+((d*x^2+c)*(b^(5/2)*c-5/4*a*d*b^(3/2))*c*arctanh((b*x^2+a)
^(1/2)/x/b^(1/2))-1/4*x*(b*x^2+a)^(1/2)*d*(2*b^2*c^2-2*(-1/2*b*x^2+a)*b*d*
c+a^2*d^2))*((a*d-b*c)*c)^(1/2))/((a*d-b*c)*c)^(1/2)/d^3/c/(d*x^2+c)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 1236, normalized size of antiderivative = 7.06

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[-1/8*(2*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(
b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (4*b^2*c^3 - 3*a*b*c^
2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt((b*c - a
*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2
 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sq
rt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(b^2*c*d^2*x^3 + (2*b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3)
, 1/8*(4*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (4*b^2*c^3 - 3*a*b*c^2*d - a^2*c*
d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt((b*c - a*d)/c)*log((
(8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d
)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*
d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), -1/4*((4*b
^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x
^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 +
 a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) + (4*b
^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(b)*log(-2*b*x
^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(b^2*c*d^2*x^3 + (2*b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), 1/4*(...
```

3.67.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx=\int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
```

output
```
Integral((a + b*x**2)**(5/2)/(c + d*x**2)**2, x)
```

3.67.7 Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^2, x)
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(147) = 294$.

Time = 0.32 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.31

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx = \frac{\sqrt{bx^2+a}\,b^2x}{2\,d^2} + \frac{\left(4\,b^{\frac{5}{2}}c - 5\,ab^{\frac{3}{2}}d\right)\log\left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2\right)}{4\,d^3} - \frac{\left(4\,b^{\frac{7}{2}}c^3 - 7\,ab^{\frac{5}{2}}c^2d + 2\,a^2b^{\frac{3}{2}}cd^2 + a^3\sqrt{b}d^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}cd^3} + \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{7}{2}}c^3 - 5\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ab^{\frac{5}{2}}c^2d + 4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^2b^{\frac{3}{2}}cd^2 - \left(\sqrt{b}x-\sqrt{b\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d + 4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc - 2\left(\sqrt{b}x-\sqrt{bx^2+c\right.}}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*b^2*x/d^2 + 1/4*(4*b^(5/2)*c - 5*a*b^(3/2)*d)*log((sqr
t(b)*x - sqrt(b*x^2 + a))^2)/d^3 - 1/2*(4*b^(7/2)*c^3 - 7*a*b^(5/2)*c^2*d
+ 2*a^2*b^(3/2)*c*d^2 + a^3*sqrt(b)*d^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x
^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b
*c*d)*c*d^3) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(7/2)*c^3 - 5*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*a*b^(5/2)*c^2*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a^2*b^(3/2)*c*d^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*d^3 + a^
2*b^(5/2)*c^2*d - 2*a^3*b^(3/2)*c*d^2 + a^4*sqrt(b)*d^3)/(((sqrt(b)*x - sq
rt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x
- sqrt(b*x^2 + a))^2*a*d + a^2*d)*c*d^3)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^2}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x^2)^2,x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x^2)^2, x)
```

3.68 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3} dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 194

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3} dx = -\frac{(bc-ad)x(a+bx^2)^{3/2}}{4cd(c+dx^2)^2} - \frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2(c+dx^2)} + \frac{b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3} - \frac{\sqrt{bc-ad}(8b^2c^2+4abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}d^3}$$

output
```
-1/4*(-a*d+b*c)*x*(b*x^2+a)^(3/2)/c/d/(d*x^2+c)^2+b^(5/2)*arctanh(x*b^(1/2
)/(b*x^2+a)^(1/2))/d^3-1/8*(3*a^2*d^2+4*a*b*c*d+8*b^2*c^2)*arctanh(x*(-a*d
+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))*(-a*d+b*c)^(1/2)/c^(5/2)/d^3-1/8*(-a*
d+b*c)*(3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/c^2/d^2/(d*x^2+c)
```

3.68.2 Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3} dx =$$

$$-\frac{\frac{d(bc-ad)x\sqrt{a+bx^2}(2bc(2c+3dx^2)+ad(5c+3dx^2))}{c^2(c+dx^2)^2} + \frac{3(-4bc+ad)^2\sqrt{-bc+ad}\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{c^{5/2}} - \frac{4b(10bc-7ad)\sqrt{bc-ad}\,\mathrm{arc}}{}}{8d^3}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

output
```
-1/8*((d*(b*c - a*d)*x*Sqrt[a + b*x^2]*(2*b*c*(2*c + 3*d*x^2) + a*d*(5*c +
 3*d*x^2)))/(c^2*(c + d*x^2)^2) + (3*(-4*b*c + a*d)^2*Sqrt[-(b*c) + a*d]*A
rcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c)
+ a*d])])/c^(5/2) - (4*b*(10*b*c - 7*a*d)*Sqrt[b*c - a*d]*ArcTanh[(-(d*x*S
qrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[b*c - a*d])])/c^(3/2)
 + 8*b^(5/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/d^3
```

3.68.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {315, 401, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3} dx$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(4b^2cx^2+a(bc+3ad)\right)}{(dx^2+c)^2}dx}{4cd}-\frac{x(a+bx^2)^{3/2}(bc-ad)}{4cd(c+dx^2)^2}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{-\frac{\int -\frac{8c^2x^2b^3+a\left(4b^2c^2+ad(bc+3ad)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{2cd(c+dx^2)}}{4cd}-\frac{x(a+bx^2)^{3/2}(bc-ad)}{4cd(c+dx^2)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{8c^2x^2b^3+a\left(4b^2c^2+ad(bc+3ad)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{2cd(c+dx^2)}}{4cd}-\frac{x(a+bx^2)^{3/2}(bc-ad)}{4cd(c+dx^2)^2}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\frac{\frac{8b^3c^2\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\left(3a^2d^2+4abcd+8b^2c^2\right)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{2cd(c+dx^2)}}{4cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{8b^3c^2\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\left(3a^2d^2+4abcd+8b^2c^2\right)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{2cd(c+dx^2)}}{4cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{8b^{5/2}c^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\left(3a^2d^2+4abcd+8b^2c^2\right)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{2cd(c+dx^2)}}{4cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\frac{8b^{5/2}c^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\left(3a^2d^2+4abcd+8b^2c^2\right)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{2cd(c+dx^2)}}{4cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{8b^{5/2}c^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\left(3a^2d^2+4abcd+8b^2c^2\right)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}}{2cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{2cd(c+dx^2)}}{4cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

3.68. $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx$

output
```
-1/4*((b*c - a*d)*x*(a + b*x^2)^(3/2))/(c*d*(c + d*x^2)^2) + (-1/2*((b*c -
 a*d)*(4*b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(c*d*(c + d*x^2)) + ((8*b^(5/2)*c
^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*(8*b^2*c^2 +
 4*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^
2])])/(Sqrt[c]*d))/(2*c*d))/(4*c*d)
```

3.68.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

---

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

3.68.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{3\left((dx^2+c)^2(a^2d^2+\frac{4}{3}abcd+\frac{8}{3}b^2c^2)(ad-bc)\arctan\left(\frac{c\sqrt{bx^2+a}}{x\sqrt{(ad-bc)c}}\right)-\frac{5\sqrt{(ad-bc)c}\left(\frac{8c^2b^{\frac{5}{2}}(dx^2+c)^2\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)}{5}+xd\right.}{3}\right.}{8\sqrt{(ad-bc)c}\,d^3(dx^2+c)^2c^2}$
default	Expression too large to display

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*((d*x^2+c)^2*(a^2*d^2+4/3*a*b*c*d+8/3*b^2*c^2)*(a*d-b*c)*arctan(c*(b*
x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))-5/3*((a*d-b*c)*c)^(1/2)*(8/5*c^2*b^(5/
2)*(d*x^2+c)^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+x*d*(4/5*b*c^2+d*(6/5*b*
x^2+a)*c+3/5*a*d^2*x^2)*(b*x^2+a)^(1/2)*(a*d-b*c)))/((a*d-b*c)*c)^(1/2)/d^
3/(d*x^2+c)^2/c^2
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(168) = 336.

Time = 0.50 (sec) , antiderivative size = 1517, normalized size of antiderivative = 7.82

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/32*(16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b)*log(-2*b*x
^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c
^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d +
4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*
a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^
2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4
 + 2*c*d*x^2 + c^2)) - 4*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^3 + (4
*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2 + a))/(c^2*d^5*x^4 +
 2*c^3*d^4*x^2 + c^4*d^3), -1/32*(32*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 +
b^2*c^4)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^2*c^4 + 4*a*b*
c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*
(8*b^2*c^3*d + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt((b*c - a*d)/c)*log((
(8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d
)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*
d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^
2*d^4)*x^3 + (4*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2 + a))
/(c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3), 1/16*((8*b^2*c^4 + 4*a*b*c^3*d +
 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*
c^3*d + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*
((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c ...
```

3.68.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
```

output
```
Integral((a + b*x**2)**(5/2)/(c + d*x**2)**3, x)
```

3.68.7 Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^3, x)
```

3.68.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(168) = 336.

Time = 0.32 (sec) , antiderivative size = 659, normalized size of antiderivative = 3.40

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = -\frac{b^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\,d^3} + \frac{\left(8\,b^{\frac{7}{2}}c^3-4\,ab^{\frac{5}{2}}c^2d-a^2b^{\frac{3}{2}}cd^2-3\,a^3\sqrt{b}d^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{8\,\sqrt{-b^2c^2+abcd}c^2d^3} - \frac{16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^{\frac{7}{2}}c^3d-20\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{5}{2}}c^2d^2+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2b^{\frac{3}{2}}cd^3+3\left(\sqrt{b}x\right.}{}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d^3 + 1/8*(8*b^(7/2)*c^3
 - 4*a*b^(5/2)*c^2*d - a^2*b^(3/2)*c*d^2 - 3*a^3*sqrt(b)*d^3)*arctan(1/2*(
(sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))
/(sqrt(-b^2*c^2 + a*b*c*d)*c^2*d^3) - 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a)
)^6*b^(7/2)*c^3*d - 20*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2)*c^2*d^2 +
 (sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(3/2)*c*d^3 + 3*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a^3*sqrt(b)*d^4 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(9/2)*
c^4 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(7/2)*c^3*d + 18*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*a^2*b^(5/2)*c^2*d^2 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*a^3*b^(3/2)*c*d^3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*sqrt(b)*d^4 +
 32*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(7/2)*c^3*d - 28*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*a^3*b^(5/2)*c^2*d^2 - 13*(sqrt(b)*x - sqrt(b*x^2 + a))^2*
a^4*b^(3/2)*c*d^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*sqrt(b)*d^4 + 6*
a^4*b^(5/2)*c^2*d^2 - 3*a^5*b^(3/2)*c*d^3 - 3*a^6*sqrt(b)*d^4)/(((sqrt(b)*
x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2*c^2*d^3)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^3}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x^2)^3,x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x^2)^3, x)
```

3.69 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx$

3.69.1 Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = \frac{x(a+bx^2)^{5/2}}{6c\,(c+dx^2)^3} + \frac{5ax(a+bx^2)^{3/2}}{24c^2\,(c+dx^2)^2} + \frac{5a^2x\sqrt{a+bx^2}}{16c^3\,(c+dx^2)} + \frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}$$

output
```
1/6*x*(b*x^2+a)^(5/2)/c/(d*x^2+c)^3+5/24*a*x*(b*x^2+a)^(3/2)/c^2/(d*x^2+c)
^2+5/16*a^3*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(7/2)/(-
a*d+b*c)^(1/2)+5/16*a^2*x*(b*x^2+a)^(1/2)/c^3/(d*x^2+c)
```

3.69.2 Mathematica [A] (warning: unable to verify)

Time = 10.60 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = \frac{x\sqrt{a+bx^2}\left(\frac{\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,(8b^2c^2x^4+2abcx^2(13c+5dx^2)+a^2(33c^2+40cdx^2+15d^2x^4))}{(c+dx^2)^2\sqrt{1+\frac{dx^2}{c}}} + \frac{15a^2\arcsin\left(\frac{\sqrt{\left(-\frac{b}{a}+\frac{d}{c}\right)}}{\sqrt{1+\frac{dx^2}{c}}}\right.}{\sqrt{\frac{(-bc+ad)x^2}{ac}}}\right.}{48c^4\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

output
```
(x*Sqrt[a + b*x^2]*((Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*(8*b^2*c^2*x^4
+ 2*a*b*c*x^2*(13*c + 5*d*x^2) + a^2*(33*c^2 + 40*c*d*x^2 + 15*d^2*x^4)))/
((c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]) + (15*a^2*ArcSin[Sqrt[(-(b/a) + d/c)*x
^2]/Sqrt[1 + (d*x^2)/c]])/Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]))/(48*c^4*Sqrt[
1 + (b*x^2)/a])
```

3.69.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {292, 292, 292, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4} \, dx$$

$$\Big\downarrow \ 292$$

$$\frac{5a\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^3}dx}{6c}+\frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3}$$

$$\Big\downarrow \ 292$$

$$\frac{5a\left(\frac{3a\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2}dx}{4c}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}\right)}{6c}+\frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3}$$

$$\Big\downarrow \ 292$$

$$\frac{5a\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{2c}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}\right)}{4c}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}\right)}{6c}+\frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3}$$

$$\Big\downarrow \ 291$$

$$\frac{5a\left(\frac{3a\left(\frac{a\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}\right)}{4c}+\frac{x\left(a+bx^2\right)^{3/2}}{4c(c+dx^2)^2}\right)}{6c}+\frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3}$$

$$\Big\downarrow \; 221$$

$$\frac{5a\left(\frac{3a\left(\frac{a\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}\right)}{4c}+\frac{x\left(a+bx^2\right)^{3/2}}{4c(c+dx^2)^2}\right)}{6c}+\frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

output
```
(x*(a + b*x^2)^(5/2))/(6*c*(c + d*x^2)^3) + (5*a*((x*(a + b*x^2)^(3/2))/(4
*c*(c + d*x^2)^2) + (3*a*((x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*ArcTa
nh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*Sqrt[b*c - a
*d])))/(4*c)))/(6*c)
```

3.69.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

3.69.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{-33x\sqrt{b\,x^2+a}\left(\left(\frac{8}{33}b^2x^4+\frac{26}{33}ab\,x^2+a^2\right)c^2+\frac{40x^2\left(\frac{b\,x^2}{4}+a\right)dac}{33}+\frac{5a^2d^2x^4}{11}\right)\sqrt{(ad-bc)c}+15a^3\left(d\,x^2+c\right)^3\arctan\left(\frac{c\sqrt{b\,x^2+}}{x\sqrt{(ad-b}}\right.}{48\sqrt{(ad-bc)c}\,c^3\left(d\,x^2+c\right)^3}$
default	Expression too large to display

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48/((a*d-b*c)*c)^(1/2)*(-33*x*(b*x^2+a)^(1/2)*((8/33*b^2*x^4+26/33*a*b*
x^2+a^2)*c^2+40/33*x^2*(1/4*b*x^2+a)*d*a*c+5/11*a^2*d^2*x^4)*((a*d-b*c)*c)
^(1/2)+15*a^3*(d*x^2+c)^3*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))
/c^3/(d*x^2+c)^3
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(120) = 240$.

Time = 0.37 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.90

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = \left[\frac{15\,(a^3d^3x^6+3\,a^3cd^2x^4+3\,a^3c^2dx^2+a^3c^3)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2}{}\right.}{}\right.$$

$$-\frac{15\,(a^3d^3x^6+3\,a^3cd^2x^4+3\,a^3c^2dx^2+a^3c^3)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}}{2\,((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)-2\,((8\,b^3}{96\,(bc^8-ac^7d+(bc^5d^3-ac^4d^4)x^6+}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

output
```
[1/192*(15*(a^3*d^3*x^6 + 3*a^3*c*d^2*x^4 + 3*a^3*c^2*d*x^2 + a^3*c^3)*sqr
t(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*
(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a
*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((8*b^3*c^4 + 2*a*
b^2*c^3*d + 5*a^2*b*c^2*d^2 - 15*a^3*c*d^3)*x^5 + 2*(13*a*b^2*c^4 + 7*a^2*
b*c^3*d - 20*a^3*c^2*d^2)*x^3 + 33*(a^2*b*c^4 - a^3*c^3*d)*x)*sqrt(b*x^2 +
 a))/(b*c^8 - a*c^7*d + (b*c^5*d^3 - a*c^4*d^4)*x^6 + 3*(b*c^6*d^2 - a*c^5
*d^3)*x^4 + 3*(b*c^7*d - a*c^6*d^2)*x^2), -1/96*(15*(a^3*d^3*x^6 + 3*a^3*c
*d^2*x^4 + 3*a^3*c^2*d*x^2 + a^3*c^3)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt
(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b
*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((8*b^3*c^4 + 2*a*b^2*c^3*d + 5*a^
2*b*c^2*d^2 - 15*a^3*c*d^3)*x^5 + 2*(13*a*b^2*c^4 + 7*a^2*b*c^3*d - 20*a^3
*c^2*d^2)*x^3 + 33*(a^2*b*c^4 - a^3*c^3*d)*x)*sqrt(b*x^2 + a))/(b*c^8 - a*
c^7*d + (b*c^5*d^3 - a*c^4*d^4)*x^6 + 3*(b*c^6*d^2 - a*c^5*d^3)*x^4 + 3*(b
*c^7*d - a*c^6*d^2)*x^2)]
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**4,x)
```

output
```
Timed out
```

3.69.7 Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^4}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^4, x)
```

3.69.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. 2(120) = 240.

Time = 1.31 (sec) , antiderivative size = 846, normalized size of antiderivative = 5.88

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = -\frac{5\,a^3\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{16\,\sqrt{-b^2c^2+abcd}c^3} + \frac{48\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{7}{2}}c^3d^2-15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^3\sqrt{b}d^5+192\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8b^{\frac{9}{2}}c^4d+48\left(\cdots\right.}{}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="giac")
```

output
```
-5/16*a^3*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c -
a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c^3) + 1/24*(48*(
sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*c^3*d^2 - 15*(sqrt(b)*x - sqrt(b*x
^2 + a))^10*a^3*sqrt(b)*d^5 + 192*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*
c^4*d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c^3*d^2 - 150*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 + 75*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*a^4*sqrt(b)*d^5 + 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*c^5 -
 64*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d + 288*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*a^2*b^(7/2)*c^3*d^2 - 440*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
a^3*b^(5/2)*c^2*d^3 + 440*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*c*d^
4 - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 + 192*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*a^2*b^(9/2)*c^4*d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a^3*b^(7/2)*c^3*d^2 + 360*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*c^2
*d^3 - 420*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 + 150*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a^6*sqrt(b)*d^5 + 48*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*a^4*b^(7/2)*c^3*d^2 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(5/2)*c
^2*d^3 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 - 75*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a^7*sqrt(b)*d^5 + 8*a^6*b^(5/2)*c^2*d^3 + 10*a^7
*b^(3/2)*c*d^4 + 15*a^8*sqrt(b)*d^5)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d +
 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a)...
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b x^{2}\right)^{5/2}}{\left(c + d x^{2}\right)^{4}}\, dx = \int \frac{\left(b\,x^{2} + a\right)^{5/2}}{\left(d\,x^{2} + c\right)^{4}}\, dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x^2)^4,x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x^2)^4, x)
```

3.70 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5} \, dx$

3.70.1 Optimal result

Integrand size = 21, antiderivative size = 249

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5} \, dx = -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)(c+dx^2)^4} + \frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)(c+dx^2)^3} + \frac{5a(8bc-7ad)x(a+bx^2)^{3/2}}{192c^3(bc-ad)(c+dx^2)^2} + \frac{5a^2(8bc-7ad)x\sqrt{a+bx^2}}{128c^4(bc-ad)(c+dx^2)} + \frac{5a^3(8bc-7ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{3/2}}$$

output
```
-1/8*d*x*(b*x^2+a)^(7/2)/c/(-a*d+b*c)/(d*x^2+c)^4+1/48*(-7*a*d+8*b*c)*x*(b
*x^2+a)^(5/2)/c^2/(-a*d+b*c)/(d*x^2+c)^3+5/192*a*(-7*a*d+8*b*c)*x*(b*x^2+a
)^(3/2)/c^3/(-a*d+b*c)/(d*x^2+c)^2+5/128*a^3*(-7*a*d+8*b*c)*arctanh(x*(-a*
d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(9/2)/(-a*d+b*c)^(3/2)+5/128*a^2*(
-7*a*d+8*b*c)*x*(b*x^2+a)^(1/2)/c^4/(-a*d+b*c)/(d*x^2+c)
```

3.70.2 Mathematica [A] (verified)

Time = 10.65 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx = \frac{x\Big(c(a+bx^2)\left(-16b^3c^3x^4(4c+dx^2)-8ab^2c^2x^2(26c^2+11cdx^2+3d^2x^4)-2a^2bc(132c^3\right.}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

output
```
(x*(c*(a + b*x^2)*(-16*b^3*c^3*x^4*(4*c + d*x^2) - 8*a*b^2*c^2*x^2*(26*c^2
 + 11*c*d*x^2 + 3*d^2*x^4) - 2*a^2*b*c*(132*c^3 + 129*c^2*d*x^2 + 94*c*d^2
*x^4 + 25*d^3*x^6) + a^3*d*(279*c^3 + 511*c^2*d*x^2 + 385*c*d^2*x^4 + 105*
d^3*x^6)) + (15*a^3*(-8*b*c + 7*a*d)*(c + d*x^2)^4*ArcTanh[Sqrt[((b*c - a*
d)*x^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(384*
c^5*(-(b*c) + a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^4)
```

3.70.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {296, 292, 292, 292, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx$$

$$\Big\downarrow \; 296$$

$$\frac{(8bc-7ad)\int \frac{(bx^2+a)^{5/2}}{(dx^2+c)^4}dx}{8c(bc-ad)}-\frac{dx(a+bx^2)^{7/2}}{8c(c+dx^2)^4(bc-ad)}$$

$$\Big\downarrow \; 292$$

$$\frac{(8bc-7ad)\left(\frac{5a\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^3}dx}{6c}+\frac{x\left(a+bx^2\right)^{5/2}}{6c(c+dx^2)^3}\right)}{8c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{7/2}}{8c\left(c+dx^2\right)^4(bc-ad)}$$

$$\Big\downarrow \; 292$$

$$\frac{(8bc-7ad)\left(\frac{5a\left(\frac{3a\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^2}dx}{4c}+\frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2}\right)}{6c}+\frac{x\left(a+bx^2\right)^{5/2}}{6c(c+dx^2)^3}\right)}{8c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{7/2}}{8c\left(c+dx^2\right)^4(bc-ad)}$$

$$\Big\downarrow \; 292$$

$$\frac{(8bc-7ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}\right)}{4c}+\frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2}\right)}{6c}+\frac{x\left(a+bx^2\right)^{5/2}}{6c(c+dx^2)^3}\right)}{8c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{7/2}}{8c\left(c+dx^2\right)^4(bc-ad)}$$

$$\Big\downarrow \; 291$$

$$\frac{(8bc-7ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}\right)}{4c}+\frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2}\right)}{6c}+\frac{x\left(a+bx^2\right)^{5/2}}{6c(c+dx^2)^3}\right)}{8c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{7/2}}{8c\left(c+dx^2\right)^4(bc-ad)}$$

3.70. $\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^5}dx$

$$\downarrow 221$$

$$\frac{(8bc-7ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}\right)}{4c}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}\right)}{6c}+\frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3}\right)}{8c(bc-ad)}-\frac{dx(a+bx^2)^{7/2}}{8c(c+dx^2)^4(bc-ad)}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

output
```
-1/8*(d*x*(a + b*x^2)^(7/2))/(c*(b*c - a*d)*(c + d*x^2)^4) + ((8*b*c - 7*a
*d)*((x*(a + b*x^2)^(5/2))/(6*c*(c + d*x^2)^3) + (5*a*((x*(a + b*x^2)^(3/2
))/(4*c*(c + d*x^2)^2) + (3*a*((x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*
ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*Sqrt[b*
c - a*d])))/(4*c)))/(6*c)))/(8*c*(b*c - a*d))
```

3.70.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.70.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{93x\sqrt{(ad-bc)c}\left(d\left(\frac{35}{93}d^3x^6+\frac{385}{279}c\,d^2x^4+\frac{511}{279}c^2d\,x^2+c^3\right)a^3-\frac{88b\left(\frac{25}{132}d^3x^6+\frac{47}{66}c\,d^2x^4+\frac{43}{44}c^2d\,x^2+c^3\right)c\,a^2}{93}-\frac{208x^2b^2\left(\frac{3}{26}d^2x^4+\frac{1}{26}\right.}{279}}{128\sqrt{(ad-bc)c}\,(ad-bc)c^4(d\,x^2+}$
default	Expression too large to display

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/128/((a*d-b*c)*c)^(1/2)*(93*x*((a*d-b*c)*c)^(1/2)*(d*(35/93*d^3*x^6+385/
279*c*d^2*x^4+511/279*c^2*d*x^2+c^3)*a^3-88/93*b*(25/132*d^3*x^6+47/66*c*d
^2*x^4+43/44*c^2*d*x^2+c^3)*c*a^2-208/279*x^2*b^2*(3/26*d^2*x^4+11/26*c*d*
x^2+c^2)*c^2*a-64/279*x^4*b^3*(1/4*d*x^2+c)*c^3)*(b*x^2+a)^(1/2)-35*(d*x^2
+c)^4*(a*d-8/7*b*c)*a^3*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))/(
a*d-b*c)/c^4/(d*x^2+c)^4
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(221) = 442$.

Time = 0.79 (sec) , antiderivative size = 1258, normalized size of antiderivative = 5.05

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="fricas")
```

output
```
[1/1536*(15*(8*a^3*b*c^5 - 7*a^4*c^4*d + (8*a^3*b*c*d^4 - 7*a^4*d^5)*x^8 +
 4*(8*a^3*b*c^2*d^3 - 7*a^4*c*d^4)*x^6 + 6*(8*a^3*b*c^3*d^2 - 7*a^4*c^2*d^
3)*x^4 + 4*(8*a^3*b*c^4*d - 7*a^4*c^3*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((
8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)
*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/
(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((16*b^4*c^5*d + 8*a*b^3*c^4*d^2 + 26*a^2
*b^2*c^3*d^3 - 155*a^3*b*c^2*d^4 + 105*a^4*c*d^5)*x^7 + (64*b^4*c^6 + 24*a
*b^3*c^5*d + 100*a^2*b^2*c^4*d^2 - 573*a^3*b*c^3*d^3 + 385*a^4*c^2*d^4)*x^
5 + (208*a*b^3*c^6 + 50*a^2*b^2*c^5*d - 769*a^3*b*c^4*d^2 + 511*a^4*c^3*d^
3)*x^3 + 3*(88*a^2*b^2*c^6 - 181*a^3*b*c^5*d + 93*a^4*c^4*d^2)*x)*sqrt(b*x
^2 + a))/(b^2*c^11 - 2*a*b*c^10*d + a^2*c^9*d^2 + (b^2*c^7*d^4 - 2*a*b*c^6
*d^5 + a^2*c^5*d^6)*x^8 + 4*(b^2*c^8*d^3 - 2*a*b*c^7*d^4 + a^2*c^6*d^5)*x^
6 + 6*(b^2*c^9*d^2 - 2*a*b*c^8*d^3 + a^2*c^7*d^4)*x^4 + 4*(b^2*c^10*d - 2*
a*b*c^9*d^2 + a^2*c^8*d^3)*x^2), -1/768*(15*(8*a^3*b*c^5 - 7*a^4*c^4*d + (
8*a^3*b*c*d^4 - 7*a^4*d^5)*x^8 + 4*(8*a^3*b*c^2*d^3 - 7*a^4*c*d^4)*x^6 + 6
*(8*a^3*b*c^3*d^2 - 7*a^4*c^2*d^3)*x^4 + 4*(8*a^3*b*c^4*d - 7*a^4*c^3*d^2)
*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*
x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*
x)) - 2*((16*b^4*c^5*d + 8*a*b^3*c^4*d^2 + 26*a^2*b^2*c^3*d^3 - 155*a^3*b*
c^2*d^4 + 105*a^4*c*d^5)*x^7 + (64*b^4*c^6 + 24*a*b^3*c^5*d + 100*a^2*b...
```

3.70.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx^2)^5}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**5,x)
```

output
```
Integral((a + b*x**2)**(5/2)/(c + d*x**2)**5, x)
```

3.70.7 Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^5}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^5, x)
```

3.70.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. $2(221) = 442$.

Time = 3.77 (sec) , antiderivative size = 1448, normalized size of antiderivative = 5.82

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="giac")
```

output
```
-5/128*(8*a^3*b^(3/2)*c - 7*a^4*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b
*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^5 - a*c^4*d)
*sqrt(-b^2*c^2 + a*b*c*d)) - 1/192*(120*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a
^3*b^(3/2)*c*d^6 - 105*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^4*sqrt(b)*d^7 -
768*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(11/2)*c^5*d^2 + 768*(sqrt(b)*x - s
qrt(b*x^2 + a))^12*a*b^(9/2)*c^4*d^3 + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^
12*a^3*b^(5/2)*c^2*d^5 - 2310*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(3/2)
*c*d^6 + 735*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^5*sqrt(b)*d^7 - 2048*(sqrt
(b)*x - sqrt(b*x^2 + a))^10*b^(13/2)*c^6*d + 2048*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*a^2*b^(9/2)*c^4*d^3 + 8320*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*b
^(7/2)*c^3*d^4 - 15600*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(5/2)*c^2*d^
5 + 9800*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^5*b^(3/2)*c*d^6 - 2205*(sqrt(b
)*x - sqrt(b*x^2 + a))^10*a^6*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 +
 a))^8*b^(15/2)*c^7 + 1024*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(13/2)*c^6*
d - 4864*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(11/2)*c^5*d^2 + 21888*(sqr
t(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(9/2)*c^4*d^3 - 38000*(sqrt(b)*x - sqrt(
b*x^2 + a))^8*a^4*b^(7/2)*c^3*d^4 + 37400*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
a^5*b^(5/2)*c^2*d^5 - 18550*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(3/2)*c*
d^6 + 3675*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*sqrt(b)*d^7 - 2048*(sqrt(b)
*x - sqrt(b*x^2 + a))^6*a^2*b^(13/2)*c^6*d - 9472*(sqrt(b)*x - sqrt(b*x...
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^2)^{5/2}}{(c + dx^2)^5} \, dx = \int \frac{(b\,x^2 + a)^{5/2}}{(d\,x^2 + c)^5} \, dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x^2)^5,x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x^2)^5, x)
```

3.71 $\int \frac{\sqrt{1-x^2}}{1+x^2} \, dx$

3.71.1 Optimal result

Integrand size = 19, antiderivative size = 30

$$\int \frac{\sqrt{1-x^2}}{1+x^2} \, dx = -\arcsin(x) + \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

output
```
-arcsin(x)+arctan(x*2^(1/2)/(-x^2+1)^(1/2))*2^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{1-x^2}}{1+x^2} \, dx = \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right) + 2 \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[1 - x^2]/(1 + x^2),x]
```

output
```
Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]] + 2*ArcTan[Sqrt[1 - x^2]/(1 + x)
]
```

3.71.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {301, 223, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{x^2+1} \, dx$$
$$\Big\downarrow \text{ 301}$$
$$2\int \frac{1}{\sqrt{1-x^2}\,(x^2+1)}dx-\int \frac{1}{\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{ 223}$$
$$2\int \frac{1}{\sqrt{1-x^2}\,(x^2+1)}dx-\arcsin(x)$$
$$\Big\downarrow \text{ 291}$$
$$2\int \frac{1}{\frac{2x^2}{1-x^2}+1}d\frac{x}{\sqrt{1-x^2}}-\arcsin(x)$$
$$\Big\downarrow \text{ 216}$$
$$\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)-\arcsin(x)$$

input
```
Int[Sqrt[1 - x^2]/(1 + x^2),x]
```

output
```
-ArcSin[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

3.71.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

3.71.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result
default	$-\arcsin\left(x\right)-\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^{2}+1}\,x}{x^{2}-1}\right)$
pseudoelliptic	$\arctan\left(\frac{\sqrt{-x^{2}+1}}{x}\right)-\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^{2}+1}}{2x}\right)$
trager	$\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)-\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z \ldots\right.\right.}{\ldots}\right)}{\ldots}$

input
```
int((-x^2+1)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-arcsin(x)-2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2\,x}\right) + 2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x) + 2*arctan((sqrt(-x^2 + 1) -
 1)/x)
```

3.71.6 Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{x^2+1}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(x**2+1),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/(x**2 + 1), x)
```

3.71.7 Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx = \int \frac{\sqrt{-x^2+1}}{x^2+1}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/(x^2 + 1), x)
```

3.71.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(24) = 48.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.17

$$\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx = -\frac{1}{2}\,\pi\mathrm{sgn}(x) + \frac{1}{2}\,\sqrt{2}\left(\pi\mathrm{sgn}(x) + 2\,\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right) - \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

input
```
integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
-1/2*pi*sgn(x) + 1/2*sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-
x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - arctan(-1/2*x*((sqrt(-x^
2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.77

$$\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx = -\mathrm{asin}(x) + \frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(-1+x\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\mathrm{i}}\right)\,1\mathrm{i}}{2} - \frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(1+x\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-x^2}\,1\mathrm{i}}{x+1\mathrm{i}}\right)\,1\mathrm{i}}{2}$$

input
```
int((1 - x^2)^(1/2)/(x^2 + 1),x)
```

output
```
(2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i
)/2 - asin(x) - (2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*
1i)/(x + 1i))*1i)/2
```

3.72 $\int \frac{\sqrt{1+x^2}}{-1+x^2} \, dx$

3.72.1 Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{\sqrt{1+x^2}}{-1+x^2} \, dx = \operatorname{arcsinh}(x) - \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^2}}\right)$$

output
```
arcsinh(x)-arctanh(x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{1+x^2}}{-1+x^2} \, dx = -\sqrt{2}\operatorname{arctanh}\left(\frac{1-x^2+x\sqrt{1+x^2}}{\sqrt{2}}\right) - \log\left(-x+\sqrt{1+x^2}\right)$$

input
```
Integrate[Sqrt[1 + x^2]/(-1 + x^2),x]
```

output
```
-(Sqrt[2]*ArcTanh[(1 - x^2 + x*Sqrt[1 + x^2])/Sqrt[2]]) - Log[-x + Sqrt[1
+ x^2]]
```

3.72.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {301, 25, 222, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{x^2-1}\,dx$$

$$\Big\downarrow \text{301}$$

$$\int \frac{1}{\sqrt{x^2+1}}dx+2\int -\frac{1}{(1-x^2)\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{1}{\sqrt{x^2+1}}dx-2\int \frac{1}{(1-x^2)\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{222}$$

$$\text{arcsinh}(x)-2\int \frac{1}{(1-x^2)\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{291}$$

$$\text{arcsinh}(x)-2\int \frac{1}{1-\frac{2x^2}{x^2+1}}d\frac{x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{219}$$

$$\text{arcsinh}(x)-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+1}}\right)$$

input
```
Int[Sqrt[1 + x^2]/(-1 + x^2),x]
```

output
```
ArcSinh[x] - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^2]]
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

3.72.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(21) = 42$.

Time = 2.43 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

method	result
pseudoelliptic	$-\frac{\ln\left(\frac{-x+\sqrt{x^2+1}}{x}\right)}{2}+\frac{\ln\left(\frac{x+\sqrt{x^2+1}}{x}\right)}{2}-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^2+1}}{2x}\right)$
trager	$\ln\left(x+\sqrt{x^2+1}\right)-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{3\operatorname{RootOf}\left(\_Z^2-2\right)x^2+4\sqrt{x^2+1}\,x+\operatorname{RootOf}\left(\_Z^2-2\right)}{(-1+x)(1+x)}\right)}{2}$
default	$\frac{\sqrt{(-1+x)^2+2x}}{2}+\operatorname{arcsinh}(x)-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2+2x)\sqrt{2}}{4\sqrt{(-1+x)^2+2x}}\right)}{2}-\frac{\sqrt{(1+x)^2-2x}}{2}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2-2x)\sqrt{2}}{4\sqrt{(1+x)^2-2x}}\right)}{2}$

input
```
int((x^2+1)^(1/2)/(x^2-1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln((-x+(x^2+1)^(1/2))/x)+1/2*ln((x+(x^2+1)^(1/2))/x)-2^(1/2)*arctanh(
1/2*2^(1/2)*(x^2+1)^(1/2)/x)
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(21) = 42$.

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx = \frac{1}{2}\sqrt{2}\log\left(\frac{9\,x^2-2\,\sqrt{2}(3\,x^2+1)-2\,\sqrt{x^2+1}\big(3\,\sqrt{2}x-4\,x\big)+3}{x^2-1}\right) - \log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log((9*x^2 - 2*sqrt(2)*(3*x^2 + 1) - 2*sqrt(x^2 + 1)*(3*sqrt(2
)*x - 4*x) + 3)/(x^2 - 1)) - log(-x + sqrt(x^2 + 1))
```

3.72.6 Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx = \int \frac{\sqrt{x^2+1}}{(x-1)\,(x+1)}\,dx$$

input
```
integrate((x**2+1)**(1/2)/(x**2-1),x)
```

output
```
Integral(sqrt(x**2 + 1)/((x - 1)*(x + 1)), x)
```

3.72.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(21) = 42.

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx = -\frac{1}{2}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{2\,x}{|2\,x+2|}-\frac{2}{|2\,x+2|}\right) - \frac{1}{2}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{2\,x}{|2\,x-2|}+\frac{2}{|2\,x-2|}\right) + \mathrm{arsinh}\,(x)$$

input
```
integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arcsinh(2*x/abs(2*x + 2) - 2/abs(2*x + 2)) - 1/2*sqrt(2)*arcs
inh(2*x/abs(2*x - 2) + 2/abs(2*x - 2)) + arcsinh(x)
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(21) = 42.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

$$\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx = -\frac{1}{2}\,\sqrt{2}\log\left(\frac{\left|2\,\left(x-\sqrt{x^2+1}\right)^2-4\,\sqrt{2}-6\right|}{\left|2\,\left(x-\sqrt{x^2+1}\right)^2+4\,\sqrt{2}-6\right|}\right) - \log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(abs(2*(x - sqrt(x^2 + 1))^2 - 4*sqrt(2) - 6)/abs(2*(x - s
qrt(x^2 + 1))^2 + 4*sqrt(2) - 6)) - log(-x + sqrt(x^2 + 1))
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx = \mathrm{asinh}(x) + \frac{\sqrt{2}\,\left(\ln\left(x-1\right)-\ln\left(x+\sqrt{2}\,\sqrt{x^2+1}+1\right)\right)}{2} - \frac{\sqrt{2}\,\left(\ln\left(x+1\right)-\ln\left(\sqrt{2}\,\sqrt{x^2+1}-x+1\right)\right)}{2}$$

input
```
int((x^2 + 1)^(1/2)/(x^2 - 1),x)
```

output
```
asinh(x) + (2^(1/2)*(log(x - 1) - log(x + 2^(1/2)*(x^2 + 1)^(1/2) + 1)))/2
 - (2^(1/2)*(log(x + 1) - log(2^(1/2)*(x^2 + 1)^(1/2) - x + 1)))/2
```

3.72. $\int \frac{\sqrt{1+x^2}}{-1+x^2} \, dx$

3.73 $\int \frac{\sqrt{1-x^2}}{-1+2x^2} dx$

3.73.1 Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2} dx = -\frac{\arcsin(x)}{2} - \frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

output
```
-1/2*arcsin(x)-1/2*arctanh(x/(-x^2+1)^(1/2))
```

3.73.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2} dx = \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right) - \frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

input
```
Integrate[Sqrt[1 - x^2]/(-1 + 2*x^2),x]
```

output
```
ArcTan[Sqrt[1 - x^2]/(1 + x)] - ArcTanh[x/Sqrt[1 - x^2]]/2
```

3.73.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {301, 25, 223, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{2x^2-1} \, dx$$

$$\Big\downarrow \text{301}$$

$$\frac{1}{2} \int -\frac{1}{(1-2x^2)\sqrt{1-x^2}}dx - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2} \int \frac{1}{\sqrt{1-x^2}}dx - \frac{1}{2} \int \frac{1}{(1-2x^2)\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{223}$$

$$-\frac{1}{2} \int \frac{1}{(1-2x^2)\sqrt{1-x^2}}dx - \frac{\arcsin(x)}{2}$$

$$\Big\downarrow \text{291}$$

$$-\frac{1}{2} \int \frac{1}{1-\frac{x^2}{1-x^2}}d\frac{x}{\sqrt{1-x^2}} - \frac{\arcsin(x)}{2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arcsin(x)}{2} - \frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

input
```
Int[Sqrt[1 - x^2]/(-1 + 2*x^2),x]
```

output
```
-1/2*ArcSin[x] - ArcTanh[x/Sqrt[1 - x^2]]/2
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

3.73.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(19) = 38$.

Time = 2.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

method	result
pseudoelliptic	$\frac{\ln\left(\frac{-x+\sqrt{-x^2+1}}{x}\right)}{4}-\frac{\ln\left(\frac{x+\sqrt{-x^2+1}}{x}\right)}{4}+\frac{\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)}{2}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}-\frac{\ln\left(-\frac{2x\sqrt{-x^2+1}+1}{2x^2-1}\right)}{4}$
default	$\frac{\sqrt{2}\left(\frac{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{4}-\frac{\sqrt{2}\arcsin(x)}{4}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(1-\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}\right)\sqrt{2}}{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{4}\right)}{2}-\frac{\sqrt{2}\left(\sqrt{-4\left(x+\frac{\sqrt{2}}{\cdots}\right.}\right.}{\cdots}$

input
```
int((-x^2+1)^(1/2)/(2*x^2-1),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln((-x+(-x^2+1)^(1/2))/x)-1/4*ln((x+(-x^2+1)^(1/2))/x)+1/2*arctan((-x^
2+1)^(1/2)/x)
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(19) = 38.

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx = \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + \frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}(x+1)-x-1}{x^2}\right) - \frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}(x-1)+x-1}{x^2}\right)$$

input
```
integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="fricas")
```

output
```
arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(-(x^2 + sqrt(-x^2 + 1)*(x + 1) -
x - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1) + x - 1)/x^2)
```

3.73.6 Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{2x^2-1}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(2*x**2-1),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/(2*x**2 - 1), x)
```

3.73.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(19) = 38$.

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 4.40

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx =$$
$$-\frac{1}{8}\sqrt{2}\left(2\sqrt{2}\arcsin\left(x\right)-\sqrt{2}\log\left(\frac{1}{4}\sqrt{2}+\frac{\sqrt{2}\sqrt{-x^2+1}}{|4x+2\sqrt{2}|}+\frac{1}{|4x+2\sqrt{2}|}\right)+\sqrt{2}\log\left(-\frac{1}{4}\sqrt{2}+\frac{\sqrt{2}\sqrt{-}}{|4x-}\right.\right.$$

input
```
integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*(2*sqrt(2)*arcsin(x) - sqrt(2)*log(1/4*sqrt(2) + sqrt(2)*sqrt
(-x^2 + 1)/abs(4*x + 2*sqrt(2)) + 1/abs(4*x + 2*sqrt(2))) + sqrt(2)*log(-1
/4*sqrt(2) + sqrt(2)*sqrt(-x^2 + 1)/abs(4*x - 2*sqrt(2)) + 1/abs(4*x - 2*s
qrt(2))))
```

3.73.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(19) = 38$.

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.72

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx = -\frac{1}{4}\pi\mathrm{sgn}(x)-\frac{1}{2}\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$
$$-\frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}+2\right|\right)$$
$$+\frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}-2\right|\right)$$

input
```
integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="giac")
```

output
```
-1/4*pi*sgn(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(
-x^2 + 1) - 1)) - 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) -
1)/x + 2)) + 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x
- 2))
```

3.73.9 Mupad [B] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.40

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx = -\frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}-1\right)1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{4}+\frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}+1\right)1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{4}-\frac{\operatorname{asin}(x)}{2}$$

input
```
int((1 - x^2)^(1/2)/(2*x^2 - 1),x)
```

output
```
log((2^(1/2)*((2^(1/2)*x)/2 + 1)*1i + (1 - x^2)^(1/2)*1i)/(x + 2^(1/2)/2))
/4 - log((2^(1/2)*((2^(1/2)*x)/2 - 1)*1i - (1 - x^2)^(1/2)*1i)/(x - 2^(1/2
)/2))/4 - asin(x)/2
```

3.74 $\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx$

3.74.1 Optimal result

Integrand size = 21, antiderivative size = 169

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx = \frac{d(44b^2c^2-44abcd+15a^2d^2)\,x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}(c+dx^2)}{24b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)^2}{6b} + \frac{(2bc-ad)\,(8b^2c^2-8abcd+5a^2d^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

output
```
1/16*(-a*d+2*b*c)*(5*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2
+a)^(1/2))/b^(7/2)+1/48*d*(15*a^2*d^2-44*a*b*c*d+44*b^2*c^2)*x*(b*x^2+a)^(
1/2)/b^3+5/24*d*(-a*d+2*b*c)*x*(d*x^2+c)*(b*x^2+a)^(1/2)/b^2+1/6*d*x*(d*x^
2+c)^2*(b*x^2+a)^(1/2)/b
```

3.74.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b}dx\sqrt{a+bx^2}(15a^2d^2-2abd(27c+5dx^2)+4b^2(18c^2+9cdx^2+2d^2x^4))+(-48b^3c^3+72ab^2c^2d-54a^2b}{48b^{7/2}}$$

input
```
Integrate[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b]*d*x*Sqrt[a + b*x^2]*(15*a^2*d^2 - 2*a*b*d*(27*c + 5*d*x^2) + 4*b^
2*(18*c^2 + 9*c*d*x^2 + 2*d^2*x^4)) + (-48*b^3*c^3 + 72*a*b^2*c^2*d - 54*a
^2*b*c*d^2 + 15*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(48*b^(7/2))
```

3.74.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {318, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^3}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \ 318$$

$$\frac{\int \frac{\left(dx^2+c\right)\left(5d(2bc-ad)x^2+c(6bc-ad)\right)}{\sqrt{bx^2+a}}dx}{6b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b}$$

$$\Big\downarrow \ 403$$

$$\frac{\frac{\int \frac{d\left(44b^2c^2-44abdc+15a^2d^2\right)x^2+c\left(24b^2c^2-14abdc+5a^2d^2\right)}{\sqrt{bx^2+a}}dx}{4b}+\frac{5dx\sqrt{a+bx^2}\left(c+dx^2\right)(2bc-ad)}{4b}}{6b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{\frac{3(2bc-ad)\left(5a^2d^2-8abcd+8b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}\left(15a^2d^2-44abcd+44b^2c^2\right)}{2b}}{4b}+\frac{5dx\sqrt{a+bx^2}\left(c+dx^2\right)(2bc-ad)}{4b}}{6b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{\frac{3(2bc-ad)\left(5a^2d^2-8abcd+8b^2c^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}\left(15a^2d^2-44abcd+44b^2c^2\right)}{2b}}{4b}+\frac{5dx\sqrt{a+bx^2}\left(c+dx^2\right)(2bc-ad)}{4b}}{6b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)\left(5a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}\left(15a^2d^2-44abcd+44b^2c^2\right)}{2b}}{4b}+\frac{5dx\sqrt{a+bx^2}(c+dx^2)(2bc-ad)}{4b}}{6b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b}$$

input
```
Int[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```

output
```
(d*x*Sqrt[a + b*x^2]*(c + d*x^2)^2)/(6*b) + ((5*d*(2*b*c - a*d)*x*Sqrt[a +
 b*x^2]*(c + d*x^2))/(4*b) + ((d*(44*b^2*c^2 - 44*a*b*c*d + 15*a^2*d^2)*x*
Sqrt[a + b*x^2])/(2*b) + (3*(2*b*c - a*d)*(8*b^2*c^2 - 8*a*b*c*d + 5*a^2*d
^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/(4*b))/(6*b)
```

3.74.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.74.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$-\frac{5\left((ad-2bc)\left(a^2d^2-\frac{8}{5}abcd+\frac{8}{5}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-\left(\left(\frac{8}{15}d^2x^4+\frac{12}{5}cd\,x^2+\frac{24}{5}c^2\right)b^{\frac{5}{2}}+\left(\left(-\frac{2d\,x^2}{3}-\frac{18c}{5}\right)b^{\frac{3}{2}}+ad\sqrt{b}\right)da\right.\right.}{16b^{\frac{7}{2}}}$
risch	$\frac{dx\left(8b^2d^2x^4-10x^2ab\,d^2+36x^2b^2cd+15a^2d^2-54abcd+72b^2c^2\right)\sqrt{b\,x^2+a}}{48b^3}-\frac{\left(5a^3d^3-18a^2bc\,d^2+24a\,b^2c^2d-16b^3c^3\right)\ln\left(x\sqrt{b}\right.}{16b^{\frac{7}{2}}}$
default	$\frac{c^3\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+d^3\left(\frac{x^5\sqrt{b\,x^2+a}}{6b}-\frac{5a\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)+3c\,d^2\left(\right.$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/16*((a*d-2*b*c)*(a^2*d^2-8/5*a*b*c*d+8/5*b^2*c^2)*arctanh((b*x^2+a)^(1/
2)/x/b^(1/2))-((8/15*d^2*x^4+12/5*c*d*x^2+24/5*c^2)*b^(5/2)+((-2/3*d*x^2-1
8/5*c)*b^(3/2)+a*d*b^(1/2))*d*a)*x*(b*x^2+a)^(1/2)*d)/b^(7/2)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.78

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{3\,(16\,b^3c^3 - 24\,ab^2c^2d + 18\,a^2bcd^2 - 5\,a^3d^3)\sqrt{b}\log\left(-2\,bx^2 + 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) - 2\,(8\,b^3d^3x^5 + 2\,(}{96\,b^4}\right.$$
$$-\frac{3\,(16\,b^3c^3 - 24\,ab^2c^2d + 18\,a^2bcd^2 - 5\,a^3d^3)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (8\,b^3d^3x^5 + 2\,(18\,b^3cd^2 - 5\,ab^2d^3)x}{48\,b^4}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(
b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*b^3*d^3*x^5 + 2*
(18*b^3*c*d^2 - 5*a*b^2*d^3)*x^3 + 3*(24*b^3*c^2*d - 18*a*b^2*c*d^2 + 5*a^
2*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/48*(3*(16*b^3*c^3 - 24*a*b^2*c^2*d +
18*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
(8*b^3*d^3*x^5 + 2*(18*b^3*c*d^2 - 5*a*b^2*d^3)*x^3 + 3*(24*b^3*c^2*d - 18
*a*b^2*c*d^2 + 5*a^2*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{d^3x^5}{6b} + \frac{x^3\left(-\frac{5ad^3}{6b}+3cd^2\right)}{4b} + \frac{x\left(-\frac{3a\left(-\frac{5ad^3}{6b}+3cd^2\right)}{4b}+3c^2d\right)}{2b}\right) + \left(-\frac{a\left(-\frac{3a\left(-\frac{5ad^3}{6b}+3cd^2\right)}{4b}+3c^2d\right)}{2b}+c^3\right)\left(\left\{\right.\right. \\ \frac{c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}}{\sqrt{a}} \end{cases}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d**3*x**5/(6*b) + x**3*(-5*a*d**3/(6*b) + 3*c
*d**2)/(4*b) + x*(-3*a*(-5*a*d**3/(6*b) + 3*c*d**2)/(4*b) + 3*c**2*d)/(2*b
)) + (-a*(-3*a*(-5*a*d**3/(6*b) + 3*c*d**2)/(4*b) + 3*c**2*d)/(2*b) + c**3
)*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (
x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), ((c**3*x + c**2*d*x**3 + 3*c*d**
2*x**5/5 + d**3*x**7/7)/sqrt(a), True))
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}d^3x^5}{6\,b} + \frac{3\,\sqrt{bx^2+a}cd^2x^3}{4\,b} - \frac{5\,\sqrt{bx^2+a}ad^3x^3}{24\,b^2} + \frac{3\,\sqrt{bx^2+a}c^2dx}{2\,b} - \frac{9\,\sqrt{bx^2+a}acd^2x}{8\,b^2} + \frac{5\,\sqrt{bx^2+a}a^2d^3x}{16\,b^3} + \frac{c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{3\,ac^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{9\,a^2cd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{5\,a^3d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(b*x^2 + a)*d^3*x^5/b + 3/4*sqrt(b*x^2 + a)*c*d^2*x^3/b - 5/24*sqr
t(b*x^2 + a)*a*d^3*x^3/b^2 + 3/2*sqrt(b*x^2 + a)*c^2*d*x/b - 9/8*sqrt(b*x^
2 + a)*a*c*d^2*x/b^2 + 5/16*sqrt(b*x^2 + a)*a^2*d^3*x/b^3 + c^3*arcsinh(b*
x/sqrt(a*b))/sqrt(b) - 3/2*a*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 9/8*a^
2*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/16*a^3*d^3*arcsinh(b*x/sqrt(a*b
))/b^(7/2)
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx = \frac{1}{48}\left(2\left(\frac{4\,d^3x^2}{b} + \frac{18\,b^4cd^2 - 5\,ab^3d^3}{b^5}\right)x^2 + \frac{3\,(24\,b^4c^2d - 18\,ab^3cd^2 + 5\,a^2b^2d^3)}{b^5}\right)\sqrt{bx^2+a}x - \frac{(16\,b^3c^3 - 24\,ab^2c^2d + 18\,a^2bcd^2 - 5\,a^3d^3)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*d^3*x^2/b + (18*b^4*c*d^2 - 5*a*b^3*d^3)/b^5)*x^2 + 3*(24*b^4*c
^2*d - 18*a*b^3*c*d^2 + 5*a^2*b^2*d^3)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b
^3*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b)*x +
 sqrt(b*x^2 + a)))/b^(7/2)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx = \int \frac{(d\,x^2+c)^3}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c + d*x^2)^3/(a + b*x^2)^(1/2),x)
```

output
```
int((c + d*x^2)^3/(a + b*x^2)^(1/2), x)
```

3.75 $\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx$

3.75.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b} + \frac{(8b^2c^2-8abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
1/8*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(
5/2)+3/8*d*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/b^2+1/4*d*x*(d*x^2+c)*(b*x^2+a)^
(1/2)/b
```

3.75.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}(8bc-3ad+2bdx^2)}{8b^2} + \frac{(-8b^2c^2+8abcd-3a^2d^2)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

input
```
Integrate[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
```

output
```
(d*x*Sqrt[a + b*x^2]*(8*b*c - 3*a*d + 2*b*d*x^2))/(8*b^2) + ((-8*b^2*c^2 +
 8*a*b*c*d - 3*a^2*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2))
```

3.75.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {318, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\int \frac{3d(2bc-ad)x^2+c(4bc-ad)}{\sqrt{bx^2+a}}dx}{4b}+\frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{(3a^2d^2-8abcd+8b^2c^2)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{(3a^2d^2-8abcd+8b^2c^2)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3a^2d^2-8abcd+8b^2c^2)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b}$$

input
```
Int[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
```

output
```
(d*x*Sqrt[a + b*x^2]*(c + d*x^2))/(4*b) + ((3*d*(2*b*c - a*d)*x*Sqrt[a + b
*x^2])/(2*b) + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(2*b^(3/2)))/(4*b)
```

3.75.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.75.4 Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{dx\left(-2bd\,x^{2}+3ad-8bc\right)\sqrt{b\,x^{2}+a}}{8b^{2}}+\frac{\left(3a^{2}d^{2}-8abcd+8b^{2}c^{2}\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{5}{2}}}$
pseudoelliptic	$\frac{\frac{3\left(a^{2}d^{2}-\frac{8}{3}abcd+\frac{8}{3}b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{8}-\frac{3x\sqrt{b\,x^{2}+a}\left(\frac{2\left(-d\,x^{2}-4c\right)b^{\frac{3}{2}}}{3}+ad\sqrt{b}\right)d}{8}}{b^{\frac{5}{2}}}$
default	$\frac{c^{2}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+d^{2}\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+2cd\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\right.}{}\right.$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*d*x*(-2*b*d*x^2+3*a*d-8*b*c)*(b*x^2+a)^(1/2)/b^2+1/8*(3*a^2*d^2-8*a*b
*c*d+8*b^2*c^2)/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.78

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,b^2d^2x^3+(8\,b^2cd-3\,abd^2)x)\sqrt{bx}}{16\,b^3}\right.$$
$$\left.-\frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,b^2d^2x^3+(8\,b^2cd-3\,abd^2)x)\sqrt{bx^2+a}}{8\,b^3}\right]$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(b)*x - a) + 2*(2*b^2*d^2*x^3 + (8*b^2*c*d - 3*a*b*d^2)*x)*s
qrt(b*x^2 + a))/b^3, -1/8*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(-b)*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*d^2*x^3 + (8*b^2*c*d - 3*a*b*d^2
)*x)*sqrt(b*x^2 + a))/b^3]
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{d^2x^3}{4b}+\frac{x\left(-\frac{3ad^2}{4b}+2cd\right)}{2b}\right)+\left(-\frac{a\left(-\frac{3ad^2}{4b}+2cd\right)}{2b}+c^2\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right) & \text{for}\ b \neq \\ \frac{c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}}{\sqrt{a}} & \text{otherw} \end{cases}$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d**2*x**3/(4*b) + x*(-3*a*d**2/(4*b) + 2*c*d)
/(2*b)) + (-a*(-3*a*d**2/(4*b) + 2*c*d)/(2*b) + c**2)*Piecewise((log(2*sqr
t(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2),
 True)), Ne(b, 0)), ((c**2*x + 2*c*d*x**3/3 + d**2*x**5/5)/sqrt(a), True))
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}d^2x^3}{4\,b} + \frac{\sqrt{bx^2+a}cdx}{b} - \frac{3\,\sqrt{bx^2+a}ad^2x}{8\,b^2} + \frac{c^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{acd\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{3\,a^2d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*d^2*x^3/b + sqrt(b*x^2 + a)*c*d*x/b - 3/8*sqrt(b*x^2 +
 a)*a*d^2*x/b^2 + c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - a*c*d*arcsinh(b*x/s
qrt(a*b))/b^(3/2) + 3/8*a^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,d^2x^2}{b} + \frac{8\,b^2cd-3\,abd^2}{b^3}\right)x - \frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*(2*d^2*x^2/b + (8*b^2*c*d - 3*a*b*d^2)/b^3)*x - 1/8*(8
*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(5/2)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{{\left(d\,x^2+c\right)}^2}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c + d*x^2)^2/(a + b*x^2)^(1/2),x)
```

output
```
int((c + d*x^2)^2/(a + b*x^2)^(1/2), x)
```

3.76 $\int \frac{c+dx^2}{\sqrt{a+bx^2}} \, dx$

3.76.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}} \, dx = \frac{dx\sqrt{a+bx^2}}{2b} + \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
1/2*(-a*d+2*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*d*x*(b*x^2
+a)^(1/2)/b
```

3.76.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}} \, dx = \frac{dx\sqrt{a+bx^2}}{2b} + \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

input
```
Integrate[(c + d*x^2)/Sqrt[a + b*x^2],x]
```

output
```
(d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a]
 + Sqrt[a + b*x^2])])/b^(3/2)
```

3.76.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \ 299$$

$$\frac{(2bc-ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \ 224$$

$$\frac{(2bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \ 219$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}$$

input
```
Int[(c + d*x^2)/Sqrt[a + b*x^2],x]
```

output
```
(d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(2*b^(3/2))
```

3.76.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.76.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{dx\sqrt{b\,x^{2}+a}}{2b}-\frac{(ad-2bc)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}$	47
default	$\frac{c\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+d\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)$	63
pseudoelliptic	$\frac{\sqrt{b\,x^{2}+a}\,dx\sqrt{b}-\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)ad+2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)bc}{2b^{\frac{3}{2}}}$	64

input
```
int((d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*x*(b*x^2+a)^(1/2)/b-1/2*(a*d-2*b*c)/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(
1/2))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.95

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{2\sqrt{bx^2+a}bdx-(2\,bc-ad)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4\,b^2},\frac{\sqrt{bx^2+a}bdx-(2\,bc-ad)\sqrt{-b}\operatorname{ar}}{2\,b^2}\right.$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*x^2 + a)*b*d*x - (2*b*c - a*d)*sqrt(b)*log(-2*b*x^2 + 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*d*x - (2*b*c - a*
d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/b^2]
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx = \begin{cases} \left(-\frac{ad}{2b}+c\right)\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)+\frac{dx\sqrt{a+bx^2}}{2b} & \text{for } b\neq 0 \\ \frac{cx+\frac{dx^3}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise(((-a*d/(2*b) + c)*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*
b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)) + d*x*sqrt(a + b*x
**2)/(2*b), Ne(b, 0)), ((c*x + d*x**3/3)/sqrt(a), True))
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}\,dx}{2\,b}+\frac{c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{ad\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*d*x/b + c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*d*arc
sinh(b*x/sqrt(a*b))/b^(3/2)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}\,dx}{2\,b}-\frac{(2\,bc-ad)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*d*x/b - 1/2*(2*b*c - a*d)*log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(3/2)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{d\,x^3+3\,c\,x}{3\,\sqrt{a}} & \text{if } b=0 \\ \frac{c\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{a\,d\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} + \frac{d\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int((c + d*x^2)/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, (3*c*x + d*x^3)/(3*a^(1/2)), b ~= 0, (c*log(b^(1/2)*x +
(a + b*x^2)^(1/2)))/b^(1/2) - (a*d*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))
/(2*b^(3/2)) + (d*x*(a + b*x^2)^(1/2))/(2*b))
```

3.77 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

3.77.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

output
```
arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[a + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

3.77.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{1-\frac{bx^2}{a+bx^2}}d\frac{x}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[a + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

3.77.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.77.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$	21
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{\sqrt{b}}$	22

input
```
int(1/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x**2+a)**(1/2),x)
```

output
```
asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx=\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx=\frac{1}{2}\sqrt{bx^2+a}x-\frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(
b)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx=\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

input
```
int(1/(a + b*x^2)^(1/2),x)
```

output
```
log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

3.78 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)} dx$

3.78.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

output
```
arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-a*d+b*c)^(1/
2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = -\frac{\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}\sqrt{-bc+ad}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
```

output
```
-(ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*
c) + a*d])]/(Sqrt[c]*Sqrt[-(b*c) + a*d]))
```

3.78.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx$$

$$\Big\downarrow \ 291$$

$$\int \frac{1}{c-\frac{x^2(bc-ad)}{a+bx^2}}d\frac{x}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 221$$

$$\frac{\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
```

output
```
ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])]/(Sqrt[c]*Sqrt[b*c -
 a*d])
```

3.78.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.78.4 Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}$
default	$\frac{\ln\left(\frac{\frac{2ad-2bc}{d}-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 b-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{ad-bc}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\sqrt{\frac{ad-bc}{d}}}-\frac{\ln\left(\frac{\frac{2ad-2bc}{d}+\frac{2b\sqrt{-cd}\left(x-\right.}{d}}{}\right)}{}$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))
```

3.78.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(39) = 78.

Time = 0.30 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.92

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx$$
$$= \left[\frac{\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2+4\,((2\,bc-ad)x^3+acx)\sqrt{bc^2-acd}\sqrt{bx^2+a}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\sqrt{bc^2-acd}},\right.$$
$$\left.-\frac{\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}}{2\,((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)}{2\,(bc^2-acd)}\right]$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/4*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 -
 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b
*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2))/sqrt(b*c^2 - a*c*d), -1/2*sqrt(-b*
c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqr
t(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x))/(b*c^2 - a
*c*d)]
```

3.78.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)), x)
```

3.78.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)), x)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
-sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt
(-b^2*c^2 + a*b*c*d))/sqrt(-b^2*c^2 + a*b*c*d)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{x\sqrt{a\,d-b\,c}}{\sqrt{c}\,\sqrt{b\,x^2+a}}\right)}{\sqrt{c\,(a\,d-b\,c)}} & \text{if } 0<a\,d-b\,c \\ \frac{\ln\left(\frac{\sqrt{c\,\left(b\,x^2+a\right)}+x\,\sqrt{b\,c-a\,d}}{\sqrt{c\,\left(b\,x^2+a\right)}-x\,\sqrt{b\,c-a\,d}}\right)}{2\,\sqrt{-c\,(a\,d-b\,c)}} & \text{if } a\,d-b\,c<0 \\ \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)}\,dx & \text{if } a\,d-b\,c\notin\mathbb{R}\vee a\,d=b\,c \end{cases}$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)),x)
```

output
```
piecewise(0 < a*d - b*c, atan((x*(a*d - b*c)^(1/2))/(c^(1/2)*(a + b*x^2)^(
1/2)))/(c*(a*d - b*c))^(1/2), a*d - b*c < 0, log(((c*(a + b*x^2))^(1/2) +
x*(- a*d + b*c)^(1/2))/((c*(a + b*x^2))^(1/2) - x*(- a*d + b*c)^(1/2)))/(2
*(-c*(a*d - b*c))^(1/2)), ~in(a*d - b*c, 'real') | a*d == b*c, int(1/((a +
 b*x^2)^(1/2)*(c + d*x^2)), x))
```

3.79 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx$

3.79.1 Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx=-\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}$$

output
```
1/2*(-a*d+2*b*c)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/
2)/(-a*d+b*c)^(3/2)-1/2*d*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)
```

3.79.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx$$
$$=-\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{(2bc-ad)\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\left(c+dx^2\right)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{2c^{3/2}(-bc+ad)^{3/2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

output
```
-1/2*(d*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)) + ((2*b*c - a*d)*Ar
cTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) +
 a*d])])/(2*c^(3/2)*(-(b*c) + a*d)^(3/2))
```

3.79.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {296, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{296}$$

$$\frac{(2bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{2c\,(c+dx^2)\,(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{(2bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{2c\,(c+dx^2)\,(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{(2bc-ad)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}}{2c\,(c+dx^2)\,(bc-ad)}$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

output
```
-1/2*(d*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)) + ((2*b*c - a*d)*Ar
cTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*
d)^(3/2))
```

3.79.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.79.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{\frac{d\sqrt{b\,x^2+a}\,x}{d\,x^2+c}-\frac{(ad-2bc)\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}}{2(ad-bc)c}$
default	$-\frac{-\frac{d\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+\frac{2b\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{ad-bc}{d}}}{(ad-bc)\left(x-\frac{\sqrt{-cd}}{d}\right)}+\frac{b\sqrt{-cd}\,\ln\left(\frac{\frac{2ad-2bc}{d}+\frac{2b\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+\frac{2b}{\cdots}}}{x-\frac{\sqrt{-cd}}{d}}\cdots\right.}{(ad-bc)\sqrt{\frac{ad-bc}{d}}}\cdots}{4dc}$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*d-b*c)/c*(d*(b*x^2+a)^(1/2)*x/(d*x^2+c)-(a*d-2*b*c)/((a*d-b*c)*c)^(
1/2)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))
```

3.79.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(85) = 170$.

Time = 0.36 (sec) , antiderivative size = 463, normalized size of antiderivative = 4.58

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx$$

$$= \left[-\frac{4\,(bc^2d-acd^2)\sqrt{bx^2+a}x-(2\,bc^2-acd+(2\,bcd-ad^2)x^2)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\cdots}{\cdots}\right.}{8\,(b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)x\cdots}\right.$$

$$-\frac{2\,(bc^2d-acd^2)\sqrt{bx^2+a}x+(2\,bc^2-acd+(2\,bcd-ad^2)x^2)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}((2\,bc-ad)x^2+\cdots}{2\,((b^2c^2-abcd)x^3+(abc^2-\cdots}\right.}{4\,(b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)x^2)}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a)*x - (2*b*c^2 - a*c*d + (2*b*c
*d - a*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2
)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a
*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(
b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2
*d^3)*x^2), -1/4*(2*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a)*x + (2*b*c^2 - a*c
*d + (2*b*c*d - a*d^2)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 +
a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3
+ (a*b*c^2 - a^2*c*d)*x)))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^4
*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)]
```

3.79.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**2), x)
```

3.79.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^2), x)
```

3.79.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(85) = 170$.

Time = 0.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx$$
$$= \frac{1}{2}b^{\frac{3}{2}}\left(\frac{(2\,bc-ad)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{(b^2c^2-abcd)\sqrt{-b^2c^2+abcd}} - \frac{2\left(2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 bc - \right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 b\right.}\right.$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*b^(3/2)*((2*b*c - a*d)*arctan(-1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d
+ 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2 - a*b*c*d)*sqrt(-b^2*c^
2 + a*b*c*d)) - 2*(2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - (sqrt(b)*x - sq
rt(b*x^2 + a))^2*a*d + a^2*d)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d +
a^2*d)*(b^2*c^2 - a*b*c*d)))
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^2), x)
```

3.80 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx$

3.80.1 Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx = -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{5/2}}$$

output
```
1/8*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*
x^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(5/2)-1/4*d*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c
)/(d*x^2+c)^2-3/8*d*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)^2/(d*x^2
+c)
```

3.80.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx = \frac{dx\sqrt{a+bx^2}\left(-2bc\left(4c+3dx^2\right)+ad\left(5c+3dx^2\right)\right)}{8c^2(bc-ad)^2\left(c+dx^2\right)^2}-\frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\left(c+dx^2\right)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{8c^{5/2}(-bc+ad)^{5/2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

output
```
(d*x*Sqrt[a + b*x^2]*(-2*b*c*(4*c + 3*d*x^2) + a*d*(5*c + 3*d*x^2)))/(8*c^
2*(b*c - a*d)^2*(c + d*x^2)^2) - ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcT
an[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a
*d])])/(8*c^(5/2)*(-(b*c) + a*d)^(5/2))
```

3.80.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {316, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{-2bdx^2+4bc-3ad}{\sqrt{bx^2+a}(dx^2+c)^2}dx}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{8b^2c^2-8abdc+3a^2d^2}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2c(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2c(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}-\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2c(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}-\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2c(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

output
```
-1/4*(d*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)^2) + ((-3*d*(2*b*c -
 a*d)*x*Sqrt[a + b*x^2])/(2*c*(b*c - a*d)*(c + d*x^2)) + ((8*b^2*c^2 - 8*a
*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])]
)/(2*c^(3/2)*(b*c - a*d)^(3/2)))/(4*c*(b*c - a*d))
```

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.80.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$\frac{-\frac{3\left(d\,x^2+c\right)^2\left(a^2d^2-\frac{8}{3}abcd+\frac{8}{3}b^2c^2\right)\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{8}+\frac{5x\left(-\frac{8b\,c^2}{5}+d\left(-\frac{6b\,x^2}{5}+a\right)c+\frac{3a\,d^2x^2}{5}\right)\sqrt{b\,x^2+a}\,d\sqrt{(ad-bc)c}}{8}}{\sqrt{(ad-bc)c}\,(ad-bc)^2c^2(d\,x^2+c)^2}$	149
default	Expression too large to display	184

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8/((a*d-b*c)*c)^(1/2)*(-3/5*(d*x^2+c)^2*(a^2*d^2-8/3*a*b*c*d+8/3*b^2*c^2
)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+x*(-8/5*b*c^2+d*(-6/5*b*
x^2+a)*c+3/5*a*d^2*x^2)*(b*x^2+a)^(1/2)*d*((a*d-b*c)*c)^(1/2))/(a*d-b*c)^2
/c^2/(d*x^2+c)^2
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(143) = 286.

Time = 0.49 (sec) , antiderivative size = 864, normalized size of antiderivative = 5.30

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3}\,dx$$

$$= \left[\frac{(8\,b^2c^4 - 8\,abc^3d + 3\,a^2c^2d^2 + (8\,b^2c^2d^2 - 8\,abcd^3 + 3\,a^2d^4)x^4 + 2\,(8\,b^2c^3d - 8\,abc^2d^2 + 3\,a^2cd^3)x^2)\sqrt{bc^2}}{32\,(b^3c^8 - 3\,ab^2c^7d + 3\,a^2bc^6d^2 - a^3}\right.$$

$$-\frac{(8\,b^2c^4 - 8\,abc^3d + 3\,a^2c^2d^2 + (8\,b^2c^2d^2 - 8\,abcd^3 + 3\,a^2d^4)x^4 + 2\,(8\,b^2c^3d - 8\,abc^2d^2 + 3\,a^2cd^3)x^2)\sqrt{-b}}{16\,(b^3c^8 - 3\,ab^2c^7d + 3\,a^2bc^6d^2 - a^3c^5d^3 + (b^3c^6d^2 -}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/32*((8*b^2*c^4 - 8*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 8*a*b*c
*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d - 8*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)
*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2
+ 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2
 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(3*(2*b^2*c^3*
d^2 - 3*a*b*c^2*d^3 + a^2*c*d^4)*x^3 + (8*b^2*c^4*d - 13*a*b*c^3*d^2 + 5*a
^2*c^2*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2
 - a^3*c^5*d^3 + (b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^
3*d^5)*x^4 + 2*(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^
4)*x^2), -1/16*((8*b^2*c^4 - 8*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2
- 8*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d - 8*a*b*c^2*d^2 + 3*a^2*c*
d^3)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a
*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c
*d)*x)) + 2*(3*(2*b^2*c^3*d^2 - 3*a*b*c^2*d^3 + a^2*c*d^4)*x^3 + (8*b^2*c^
4*d - 13*a*b*c^3*d^2 + 5*a^2*c^2*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^8 - 3*a*b
^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c^5*d^3 + (b^3*c^6*d^2 - 3*a*b^2*c^5*d^3
+ 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x^4 + 2*(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*
a^2*b*c^5*d^3 - a^3*c^4*d^4)*x^2)]
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.80.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^3), x)
```

3.80.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(143) = 286$.

Time = 1.74 (sec) , antiderivative size = 538, normalized size of antiderivative = 3.30

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx = -\frac{1}{8}\,b^{\frac{5}{2}}\left(\frac{\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\left(b^4c^4-2\,ab^3c^3d+a^2b^2c^2d^2\right)\sqrt{-b^2c^2+abcd}}+\frac{2\left(8\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^2c^2d-8\right.}{}\right.$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/8*b^(5/2)*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*arctan(1/2*((sqrt(b)*x -
 sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^4*c^4 -
 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) + 2*(8*(sqrt(b
)*x - sqrt(b*x^2 + a))^6*b^2*c^2*d - 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b
*c*d^2 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*d^3 + 48*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*b^3*c^3 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^2*c^2*d + 4
2*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b*c*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*a^3*d^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^2*c^2*d - 40*(s
qrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b*c*d^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*a^4*d^3 + 6*a^4*b*c*d^2 - 3*a^5*d^3)/((b^4*c^4 - 2*a*b^3*c^3*d + a^2*b
^2*c^2*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2))
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^3}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^3), x)
```

3.81 $\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx$

3.81.1 Optimal result

Integrand size = 21, antiderivative size = 257

$$\begin{aligned}\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = &-\frac{d(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3)\,x\sqrt{a+bx^2}}{48ab^4}\\ &-\frac{d(24b^2c^2-64abcd+35a^2d^2)\,x\sqrt{a+bx^2}(c+dx^2)}{24ab^3}\\ &-\frac{d(6bc-7ad)x\sqrt{a+bx^2}(c+dx^2)^2}{6ab^2}+\frac{(bc-ad)x(c+dx^2)^3}{ab\sqrt{a+bx^2}}\\ &+\frac{d(64b^3c^3-144ab^2c^2d+120a^2bcd^2-35a^3d^3)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}\end{aligned}$$

output
```
1/16*d*(-35*a^3*d^3+120*a^2*b*c*d^2-144*a*b^2*c^2*d+64*b^3*c^3)*arctanh(x*
b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)+(-a*d+b*c)*x*(d*x^2+c)^3/a/b/(b*x^2+a)^(1
/2)-1/48*d*(-105*a^3*d^3+290*a^2*b*c*d^2-248*a*b^2*c^2*d+48*b^3*c^3)*x*(b*
x^2+a)^(1/2)/a/b^4-1/24*d*(35*a^2*d^2-64*a*b*c*d+24*b^2*c^2)*x*(d*x^2+c)*(
b*x^2+a)^(1/2)/a/b^3-1/6*d*(-7*a*d+6*b*c)*x*(d*x^2+c)^2*(b*x^2+a)^(1/2)/a/
b^2
```

3.81.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.77

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \frac{\frac{\sqrt{b}x\left(48b^4c^4+105a^4d^4+5a^3bd^3\left(-72c+7dx^2\right)-2a^2b^2d^2\left(-216c^2+60cdx^2+7d^2x^4\right)+8ab^3d\left(-24c^3+18c^2dx^2+6cd^2x^4+d\right.\right.}{a\sqrt{a+bx^2}}}{48b^{9/2}}$$

input
```
Integrate[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*x*(48*b^4*c^4 + 105*a^4*d^4 + 5*a^3*b*d^3*(-72*c + 7*d*x^2) - 2*
a^2*b^2*d^2*(-216*c^2 + 60*c*d*x^2 + 7*d^2*x^4) + 8*a*b^3*d*(-24*c^3 + 18*
c^2*d*x^2 + 6*c*d^2*x^4 + d^3*x^6)))/(a*Sqrt[a + b*x^2]) + 3*d*(-64*b^3*c^
3 + 144*a*b^2*c^2*d - 120*a^2*b*c*d^2 + 35*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqr
t[a + b*x^2]])/(48*b^(9/2))
```

3.81.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {315, 27, 403, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \; 315$$

$$\frac{\int \frac{d(dx^2+c)^2(ac-(6bc-7ad)x^2)}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(c+dx^2)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{(dx^2+c)^2(ac-(6bc-7ad)x^2)}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(c+dx^2)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 403$$

$$\frac{d\left(\frac{\int \frac{\left(dx^2+c\right)\left(ac(12bc-7ad)-\left(24b^2c^2-64abdc+35a^2d^2\right)x^2\right)}{\sqrt{bx^2+a}}dx}{6b}-\frac{x\sqrt{a+bx^2}\left(c+dx^2\right)^2(6bc-7ad)}{6b}\right)}{ab}+$$

$$\frac{x\left(c+dx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{403}$$

$$\frac{d\Bigg(\frac{\frac{\int \frac{ac\left(72b^2c^2-92abdc+35a^2d^2\right)-\left(48b^3c^3-248ab^2dc^2+290a^2bd^2c-105a^3d^3\right)x^2}{\sqrt{bx^2+a}}dx}{4b}-\frac{x\sqrt{a+bx^2}\left(c+dx^2\right)\left(35a^2d^2-64abcd+24b^2c^2\right)}{4b}}{6b}-\frac{x\sqrt{a+bx^2}\left(c+dx^2\right)}{6b}}{ab}$$

$$\frac{x\left(c+dx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{d\Bigg(\frac{\frac{\frac{3a\left(-35a^3d^3+120a^2bcd^2-144ab^2c^2d+64b^3c^3\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{x\sqrt{a+bx^2}\left(-105a^3d^3+290a^2bcd^2-248ab^2c^2d+48b^3c^3\right)}{2b}}{4b}-\frac{x\sqrt{a+bx^2}\left(c+dx^2\right)\left(35a^2d^2-64abc}{4b}}{6b}}{ab}$$

$$\frac{x\left(c+dx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\Bigg(\frac{\frac{\frac{3a\left(-35a^3d^3+120a^2bcd^2-144ab^2c^2d+64b^3c^3\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{x\sqrt{a+bx^2}\left(-105a^3d^3+290a^2bcd^2-248ab^2c^2d+48b^3c^3\right)}{2b}}{4b}-\frac{x\sqrt{a+bx^2}\left(c+dx^2\right)\left(35a^2}{4b}}{6b}}{ab}$$

$$\frac{x\left(c+dx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

3.81. $\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^{3/2}}dx$

$$\frac{d\left(\frac{\frac{\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-35a^3d^3+120a^2bcd^2-144ab^2c^2d+64b^3c^3\right)}{2b^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-105a^3d^3+290a^2bcd^2-248ab^2c^2d+48b^3c^3\right)}{2b}}{4b}}{6b}-\frac{x\sqrt{a+bx^2}\left(c+dx^2\right)\left(35a^2\right.}{4b}\right.}{ab}$$

$$\frac{x\left(c+dx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

output
```
((b*c - a*d)*x*(c + d*x^2)^3)/(a*b*Sqrt[a + b*x^2]) + (d*(-1/6*((6*b*c - 7
*a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2)^2)/b + (-1/4*((24*b^2*c^2 - 64*a*b*c*d
 + 35*a^2*d^2)*x*Sqrt[a + b*x^2]*(c + d*x^2))/b + (-1/2*((48*b^3*c^3 - 248
*a*b^2*c^2*d + 290*a^2*b*c*d^2 - 105*a^3*d^3)*x*Sqrt[a + b*x^2])/b + (3*a*
(64*b^3*c^3 - 144*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 35*a^3*d^3)*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/(4*b))/(6*b)))/(a*b)
```

3.81.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.81.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{35\left(\sqrt{b\,x^{2}+a}\,a d\left(a^{3} d^{3}-\frac{24}{7} a^{2} b c\,d^{2}+\frac{144}{35} a\,b^{2} c^{2} d-\frac{64}{35} b^{3} c^{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)-x\left(-\frac{64 d\left(-\frac{1}{24} d^{3} x^{6}-\frac{1}{4} c\,d^{2} x^{4}-\frac{3}{4} c^{2} d\,x^{2}+c^{3}\right)\cdots}{35}\right.\right.}{16 b^{\frac{9}{2}} \sqrt{b\,x^{2}+a}\,a}$
risch	$\frac{x\,d^{2}\left(8 b^{2} d^{2} x^{4}-22 x^{2} a b\,d^{2}+48 x^{2} b^{2} c d+57 a^{2} d^{2}-168 a b c d+144 b^{2} c^{2}\right) \sqrt{b\,x^{2}+a}}{48 b^{4}}-\frac{\frac{19 a^{3} d^{4} x}{\sqrt{b\,x^{2}+a}}-\frac{16 b^{4} c^{4} x}{a \sqrt{b\,x^{2}+a}}-\frac{56 a^{2} b c\,d^{3} x}{\sqrt{b\,x^{2}+a}}+\frac{48 a\,b^{2} c^{2} d \cdots}{\sqrt{b\,x^{2}+\cdots}}}{\cdots}$
default	$\frac{c^{4} x}{a \sqrt{b\,x^{2}+a}}+d^{4}\left(\frac{x^{7}}{6 b \sqrt{b\,x^{2}+a}}-\frac{7 a\left(\frac{x^{5}}{4 b \sqrt{b\,x^{2}+a}}-\frac{5 a\left(\frac{x^{3}}{2 b \sqrt{b\,x^{2}+a}}-\frac{3 a\left(-\frac{x}{b \sqrt{b\,x^{2}+a}}+\frac{\ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2 b}\right)}{4 b}\right)}{6 b}\right)+4 c\,a \cdots$

input
```
int((d*x^2+c)^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-35/16*((b*x^2+a)^(1/2)*a*d*(a^3*d^3-24/7*a^2*b*c*d^2+144/35*a*b^2*c^2*d-6
4/35*b^3*c^3)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*(-64/35*d*(-1/24*d^3*x^
6-1/4*c*d^2*x^4-3/4*c^2*d*x^2+c^3)*a*b^(7/2)+144/35*d^2*(-7/216*d^2*x^4-5/
18*c*d*x^2+c^2)*a^2*b^(5/2)-24/7*d^3*(-7/72*d*x^2+c)*a^3*b^(3/2)+b^(1/2)*a
^4*d^4+16/35*b^(9/2)*c^4))/b^(9/2)/(b*x^2+a)^(1/2)/a
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.27

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \Bigg[-\frac{3\,(64\,a^2b^3c^3d - 144\,a^3b^2c^2d^2 + 120\,a^4bcd^3 - 35\,a^5d^4 + (64\,ab^4c^3d - 144\,a^2b^3c^2d^2 + 1}{}$$

$$-\frac{3\,(64\,a^2b^3c^3d - 144\,a^3b^2c^2d^2 + 120\,a^4bcd^3 - 35\,a^5d^4 + (64\,ab^4c^3d - 144\,a^2b^3c^2d^2 + 120\,a^3b^2cd^3 - 35\,a^4bd^4)}{}$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(64*a^2*b^3*c^3*d - 144*a^3*b^2*c^2*d^2 + 120*a^4*b*c*d^3 - 35*a
^5*d^4 + (64*a*b^4*c^3*d - 144*a^2*b^3*c^2*d^2 + 120*a^3*b^2*c*d^3 - 35*a^
4*b*d^4)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*
(8*a*b^4*d^4*x^7 + 2*(24*a*b^4*c*d^3 - 7*a^2*b^3*d^4)*x^5 + (144*a*b^4*c^2
*d^2 - 120*a^2*b^3*c*d^3 + 35*a^3*b^2*d^4)*x^3 + 3*(16*b^5*c^4 - 64*a*b^4*
c^3*d + 144*a^2*b^3*c^2*d^2 - 120*a^3*b^2*c*d^3 + 35*a^4*b*d^4)*x)*sqrt(b*
x^2 + a))/(a*b^6*x^2 + a^2*b^5), -1/48*(3*(64*a^2*b^3*c^3*d - 144*a^3*b^2*
c^2*d^2 + 120*a^4*b*c*d^3 - 35*a^5*d^4 + (64*a*b^4*c^3*d - 144*a^2*b^3*c^2
*d^2 + 120*a^3*b^2*c*d^3 - 35*a^4*b*d^4)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) - (8*a*b^4*d^4*x^7 + 2*(24*a*b^4*c*d^3 - 7*a^2*b^3*d^4)*x^
5 + (144*a*b^4*c^2*d^2 - 120*a^2*b^3*c*d^3 + 35*a^3*b^2*d^4)*x^3 + 3*(16*b
^5*c^4 - 64*a*b^4*c^3*d + 144*a^2*b^3*c^2*d^2 - 120*a^3*b^2*c*d^3 + 35*a^4
*b*d^4)*x)*sqrt(b*x^2 + a))/(a*b^6*x^2 + a^2*b^5)]
```

3.81.6 Sympy [F]

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^4}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**4/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**4/(a + b*x**2)**(3/2), x)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \frac{d^4x^7}{6\sqrt{bx^2+a}b} + \frac{cd^3x^5}{\sqrt{bx^2+a}b} - \frac{7\,ad^4x^5}{24\sqrt{bx^2+a}b^2} + \frac{3\,c^2d^2x^3}{\sqrt{bx^2+a}b} - \frac{5\,acd^3x^3}{2\sqrt{bx^2+a}b^2} + \frac{35\,a^2d^4x^3}{48\sqrt{bx^2+a}b^3} + \frac{c^4x}{\sqrt{bx^2+a}a} - \frac{4\,c^3dx}{\sqrt{bx^2+a}b} + \frac{9\,ac^2d^2x}{\sqrt{bx^2+a}b^2} - \frac{15\,a^2cd^3x}{2\sqrt{bx^2+a}b^3} + \frac{35\,a^3d^4x}{16\sqrt{bx^2+a}b^4} + \frac{4\,c^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{9\,ac^2d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} + \frac{15\,a^2cd^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}} - \frac{35\,a^3d^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/6*d^4*x^7/(sqrt(b*x^2 + a)*b) + c*d^3*x^5/(sqrt(b*x^2 + a)*b) - 7/24*a*d
^4*x^5/(sqrt(b*x^2 + a)*b^2) + 3*c^2*d^2*x^3/(sqrt(b*x^2 + a)*b) - 5/2*a*c
*d^3*x^3/(sqrt(b*x^2 + a)*b^2) + 35/48*a^2*d^4*x^3/(sqrt(b*x^2 + a)*b^3) +
 c^4*x/(sqrt(b*x^2 + a)*a) - 4*c^3*d*x/(sqrt(b*x^2 + a)*b) + 9*a*c^2*d^2*x
/(sqrt(b*x^2 + a)*b^2) - 15/2*a^2*c*d^3*x/(sqrt(b*x^2 + a)*b^3) + 35/16*a^
3*d^4*x/(sqrt(b*x^2 + a)*b^4) + 4*c^3*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 9
*a*c^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 15/2*a^2*c*d^3*arcsinh(b*x/sqr
t(a*b))/b^(7/2) - 35/16*a^3*d^4*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(2\left(\frac{4\,d^4x^2}{b}+\frac{24\,ab^6cd^3-7\,a^2b^5d^4}{ab^7}\right)x^2+\frac{144\,ab^6c^2d^2-120\,a^2b^5cd^3+35\,a^3b^4d^4}{ab^7}\right)x^2+\frac{3\,(16\,b^7c^4-64\,ab^6c^3d+}{}\right.}{48\,\sqrt{bx^2+a}}$$
$$-\frac{(64\,b^3c^3d-144\,ab^2c^2d^2+120\,a^2bcd^3-35\,a^3d^4)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/48*((2*(4*d^4*x^2/b + (24*a*b^6*c*d^3 - 7*a^2*b^5*d^4)/(a*b^7))*x^2 + (1
44*a*b^6*c^2*d^2 - 120*a^2*b^5*c*d^3 + 35*a^3*b^4*d^4)/(a*b^7))*x^2 + 3*(1
6*b^7*c^4 - 64*a*b^6*c^3*d + 144*a^2*b^5*c^2*d^2 - 120*a^3*b^4*c*d^3 + 35*
a^4*b^3*d^4)/(a*b^7))*x/sqrt(b*x^2 + a) - 1/16*(64*b^3*c^3*d - 144*a*b^2*c
^2*d^2 + 120*a^2*b*c*d^3 - 35*a^3*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a
)))/b^(9/2)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^4}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c + d*x^2)^4/(a + b*x^2)^(3/2),x)
```

output
```
int((c + d*x^2)^4/(a + b*x^2)^(3/2), x)
```

3.82 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx$

3.82.1 Optimal result

Integrand size = 21, antiderivative size = 169

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3} - \frac{d(4bc-5ad)x\sqrt{a+bx^2}(c+dx^2)}{4ab^2} + \frac{(bc-ad)x(c+dx^2)^2}{ab\sqrt{a+bx^2}} + \frac{3d(8b^2c^2-12abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

output
```
3/8*d*(5*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/
b^(7/2)+(-a*d+b*c)*x*(d*x^2+c)^2/a/b/(b*x^2+a)^(1/2)-1/8*d*(-5*a*d+2*b*c)*
(-3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/a/b^3-1/4*d*(-5*a*d+4*b*c)*x*(d*x^2+c)*(b
*x^2+a)^(1/2)/a/b^2
```

3.82.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{x(8b^3c^3-15a^3d^3+a^2bd^2(36c-5dx^2)+2ab^2d(-12c^2+6cdx^2+d^2x^4))}{8ab^3\sqrt{a+bx^2}} - \frac{3d(8b^2c^2-12abcd+5a^2d^2)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{7/2}}$$

input
```
Integrate[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

output
```
(x*(8*b^3*c^3 - 15*a^3*d^3 + a^2*b*d^2*(36*c - 5*d*x^2) + 2*a*b^2*d*(-12*c
^2 + 6*c*d*x^2 + d^2*x^4)))/(8*a*b^3*Sqrt[a + b*x^2]) - (3*d*(8*b^2*c^2 -
12*a*b*c*d + 5*a^2*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(7/2))
```

3.82.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {315, 27, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \; 315$$

$$\frac{\int \frac{d(dx^2+c)(ac-(4bc-5ad)x^2)}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(c+dx^2)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{(dx^2+c)(ac-(4bc-5ad)x^2)}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(c+dx^2)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 403$$

$$\frac{d\left(\frac{\int \frac{ac(8bc-5ad)-(2bc-5ad)(4bc-3ad)x^2}{\sqrt{bx^2+a}}dx}{4b}-\frac{x\sqrt{a+bx^2}(c+dx^2)(4bc-5ad)}{4b}\right)}{ab}+\frac{x(c+dx^2)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 299$$

$$\frac{d\left(\frac{\frac{3a(5a^2d^2-12abcd+8b^2c^2)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{x\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{2b}}{4b}-\frac{x\sqrt{a+bx^2}(c+dx^2)(4bc-5ad)}{4b}\right)}{ab}+\frac{x(c+dx^2)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 224$$

$$\frac{d\left(\frac{\frac{3a\left(5a^2d^2-12abcd+8b^2c^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{x\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{2b}}{4b}-\frac{x\sqrt{a+bx^2}(c+dx^2)(4bc-5ad)}{4b}\right)}{ab}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{d\left(\frac{\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(5a^2d^2-12abcd+8b^2c^2\right)}{2b^{3/2}}-\frac{x\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{2b}}{4b}-\frac{x\sqrt{a+bx^2}(c+dx^2)(4bc-5ad)}{4b}\right)}{ab}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

output
```
((b*c - a*d)*x*(c + d*x^2)^2)/(a*b*Sqrt[a + b*x^2]) + (d*(-1/4*((4*b*c - 5
*a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/b + (-1/2*((2*b*c - 5*a*d)*(4*b*c - 3
*a*d)*x*Sqrt[a + b*x^2])/b + (3*a*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/(4*b)))/(a*b)
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.82.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{\frac{15\sqrt{b x^2+a}\, a d\left(a^2 d^2-\frac{12}{5} abcd+\frac{8}{5} b^2 c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x\sqrt{b}}\right)}{8}-\frac{15x\left(\frac{8d\left(-\frac{1}{12} d^2 x^4-\frac{1}{2} c d x^2+c^2\right) a b^{\frac{5}{2}}}{5}-\frac{12\left(-\frac{5 d x^2}{36}+c\right) d^2 a^2 b^{\frac{3}{2}}}{5}+\sqrt{b}\, a^3 d\right.}{8}}{a b^{\frac{7}{2}}\sqrt{b x^2+a}}$
risch	$-\frac{x d^2\left(-2bd x^2+7ad-12bc\right)\sqrt{b x^2+a}}{8b^3}+\frac{\frac{7a^2 d^3 x}{\sqrt{b x^2+a}}+\frac{8b^3 c^3 x}{a\sqrt{b x^2+a}}-\frac{12abc d^2 x}{\sqrt{b x^2+a}}+\left(15a^2 b d^3-36a b^2 c d^2+24b^3 c^2 d\right)\left(-\frac{x}{b\sqrt{b x^2+a}}+\right.}{8b^3}$
default	$\frac{c^3 x}{a\sqrt{b x^2+a}}+d^3\left(\frac{x^5}{4b\sqrt{b x^2+a}}-\frac{5a\left(\frac{x^3}{2b\sqrt{b x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)}{4b}\right)+3c d^2\left(\frac{x^3}{2b\sqrt{b x^2+a}}-\right.$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
15/8/(b*x^2+a)^(1/2)*((b*x^2+a)^(1/2)*a*d*(a^2*d^2-12/5*a*b*c*d+8/5*b^2*c^
2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*(8/5*d*(-1/12*d^2*x^4-1/2*c*d*x^2+
c^2)*a*b^(5/2)-12/5*(-5/36*d*x^2+c)*d^2*a^2*b^(3/2)+b^(1/2)*a^3*d^3-8/15*b
^(7/2)*c^3))/b^(7/2)/a
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.46

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \Bigg[\frac{3\,(8\,a^2b^2c^2d - 12\,a^3bcd^2 + 5\,a^4d^3 + (8\,ab^3c^2d - 12\,a^2b^2cd^2 + 5\,a^3bd^3)x^2)\sqrt{b}\log\Big(-2\,bx}{}$$

$$-\frac{3\,(8\,a^2b^2c^2d - 12\,a^3bcd^2 + 5\,a^4d^3 + (8\,ab^3c^2d - 12\,a^2b^2cd^2 + 5\,a^3bd^3)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (2\,ab^3d^3x}{8\,(ab^5x^2 + a^2b^4)}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*(8*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 5*a^4*d^3 + (8*a*b^3*c^2*d -
12*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(b)*x - a) + 2*(2*a*b^3*d^3*x^5 + (12*a*b^3*c*d^2 - 5*a^2*b^2*d^3)
*x^3 + (8*b^4*c^3 - 24*a*b^3*c^2*d + 36*a^2*b^2*c*d^2 - 15*a^3*b*d^3)*x)*s
qrt(b*x^2 + a))/(a*b^5*x^2 + a^2*b^4), -1/8*(3*(8*a^2*b^2*c^2*d - 12*a^3*b
*c*d^2 + 5*a^4*d^3 + (8*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*a*b^3*d^3*x^5 + (12*a*b^
3*c*d^2 - 5*a^2*b^2*d^3)*x^3 + (8*b^4*c^3 - 24*a*b^3*c^2*d + 36*a^2*b^2*c*
d^2 - 15*a^3*b*d^3)*x)*sqrt(b*x^2 + a))/(a*b^5*x^2 + a^2*b^4)]
```

3.82.6 Sympy [F]

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^3}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**3/(a + b*x**2)**(3/2), x)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{d^3x^5}{4\sqrt{bx^2+a}b} + \frac{3\,cd^2x^3}{2\sqrt{bx^2+a}b} - \frac{5\,ad^3x^3}{8\sqrt{bx^2+a}b^2} + \frac{c^3x}{\sqrt{bx^2+a}a} - \frac{3\,c^2dx}{\sqrt{bx^2+a}b} + \frac{9\,acd^2x}{2\sqrt{bx^2+a}b^2} - \frac{15\,a^2d^3x}{8\sqrt{bx^2+a}b^3} + \frac{3\,c^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{9\,acd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{15\,a^2d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*d^3*x^5/(sqrt(b*x^2 + a)*b) + 3/2*c*d^2*x^3/(sqrt(b*x^2 + a)*b) - 5/8*
a*d^3*x^3/(sqrt(b*x^2 + a)*b^2) + c^3*x/(sqrt(b*x^2 + a)*a) - 3*c^2*d*x/(s
qrt(b*x^2 + a)*b) + 9/2*a*c*d^2*x/(sqrt(b*x^2 + a)*b^2) - 15/8*a^2*d^3*x/(
sqrt(b*x^2 + a)*b^3) + 3*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 9/2*a*c*d^
2*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 15/8*a^2*d^3*arcsinh(b*x/sqrt(a*b))/b^(
7/2)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,d^3x^2}{b} + \frac{12\,ab^4cd^2-5\,a^2b^3d^3}{ab^5}\right)x^2 + \frac{8\,b^5c^3-24\,ab^4c^2d+36\,a^2b^3cd^2-15\,a^3b^2d^3}{ab^5}\right)x}{8\sqrt{bx^2+a}} - \frac{3\,(8\,b^2c^2d-12\,abcd^2+5\,a^2d^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/8*((2*d^3*x^2/b + (12*a*b^4*c*d^2 - 5*a^2*b^3*d^3)/(a*b^5))*x^2 + (8*b^5
*c^3 - 24*a*b^4*c^2*d + 36*a^2*b^3*c*d^2 - 15*a^3*b^2*d^3)/(a*b^5))*x/sqrt
(b*x^2 + a) - 3/8*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*log(abs(-sqrt(b
)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^3}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c + d*x^2)^3/(a + b*x^2)^(3/2),x)
```

output
```
int((c + d*x^2)^3/(a + b*x^2)^(3/2), x)
```

3.83 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}} \, dx$

3.83.1 Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}} \, dx = \frac{(bc-ad)^2 x}{ab^2\sqrt{a+bx^2}} + \frac{d^2 x\sqrt{a+bx^2}}{2b^2} + \frac{d(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output
```
1/2*d*(-3*a*d+4*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+(-a*d+b*c)
^2*x/a/b^2/(b*x^2+a)^(1/2)+1/2*d^2*x*(b*x^2+a)^(1/2)/b^2
```

3.83.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}} \, dx = \frac{x(2b^2c^2-4abcd+3a^2d^2+abd^2x^2)}{2ab^2\sqrt{a+bx^2}} - \frac{d(4bc-3ad)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

input
```
Integrate[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]
```

output
```
(x*(2*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 + a*b*d^2*x^2))/(2*a*b^2*Sqrt[a + b*
x^2]) - (d*(4*b*c - 3*a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2)
)
```

3.83.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {315, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 315$$

$$\frac{\int \frac{d(ac-(2bc-3ad)x^2)}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(c+dx^2)(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{ac-(2bc-3ad)x^2}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(c+dx^2)(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 299$$

$$\frac{d\left(\frac{a(4bc-3ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{x\sqrt{a+bx^2}(2bc-3ad)}{2b}\right)}{ab}+\frac{x(c+dx^2)(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 224$$

$$\frac{d\left(\frac{a(4bc-3ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{x\sqrt{a+bx^2}(2bc-3ad)}{2b}\right)}{ab}+\frac{x(c+dx^2)(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{d\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bc-3ad)}{2b^{3/2}}-\frac{x\sqrt{a+bx^2}(2bc-3ad)}{2b}\right)}{ab}+\frac{x(c+dx^2)(bc-ad)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]
```

output
```
((b*c - a*d)*x*(c + d*x^2))/(a*b*Sqrt[a + b*x^2]) + (d*(-1/2*((2*b*c - 3*a
*d)*x*Sqrt[a + b*x^2])/b + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/(2*b^(3/2))))/(a*b)
```

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.83.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{3\left(\sqrt{b x^2+a}\, a d \left(a d-\frac{4 b c}{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x\sqrt{b}}\right)-x\left(-\frac{4\left(-\frac{d x^2}{4}+c\right) d a b^{\frac{3}{2}}}{3}+\sqrt{b}\, a^2 d^2+\frac{2 b^{\frac{5}{2}} c^2}{3}\right)\right)}{2\sqrt{b x^2+a}\, b^{\frac{5}{2}} a}$
risch	$\frac{d^2 x\sqrt{b x^2+a}}{2b^2}-\frac{\frac{a d^2 x}{\sqrt{b x^2+a}}-\frac{2b^2c^2x}{a\sqrt{bx^2+a}}+(3abd^2-4b^2cd)\left(-\frac{x}{b\sqrt{bx^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b^2}$
default	$\frac{c^2x}{a\sqrt{bx^2+a}}+d^2\left(\frac{x^3}{2b\sqrt{bx^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{bx^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+2cd\left(-\frac{x}{b\sqrt{bx^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{bx^2+\right.}{b^{\frac{3}{2}}}\right.$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3/2/(b*x^2+a)^(1/2)/b^(5/2)*((b*x^2+a)^(1/2)*a*d*(a*d-4/3*b*c)*arctanh((b
*x^2+a)^(1/2)/x/b^(1/2))-x*(-4/3*(-1/4*d*x^2+c)*d*a*b^(3/2)+b^(1/2)*a^2*d^
2+2/3*b^(5/2)*c^2))/a
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.07

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{(4\,a^2bcd-3\,a^3d^2+(4\,ab^2cd-3\,a^2bd^2)x^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-}{4\,(ab^4x^2+a^2b^3)}\right.$$

$$-\frac{(4\,a^2bcd-3\,a^3d^2+(4\,ab^2cd-3\,a^2bd^2)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(ab^2d^2x^3+(2\,b^3c^2-4\,ab^2cd+3\,a^2bd^2)}{2\,(ab^4x^2+a^2b^3)}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(b)
*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(a*b^2*d^2*x^3 + (2*b
^3*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b
^3), -1/2*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqr
t(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (a*b^2*d^2*x^3 + (2*b^3*c^2 - 4
*a*b^2*c*d + 3*a^2*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

3.83.6 Sympy [F]

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^2}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**2/(a + b*x**2)**(3/2), x)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{d^2x^3}{2\sqrt{bx^2+ab}} + \frac{c^2x}{\sqrt{bx^2+aa}} - \frac{2\,cdx}{\sqrt{bx^2+ab}} + \frac{3\,ad^2x}{2\sqrt{bx^2+ab^2}} + \frac{2\,cd\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{3\,ad^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*d^2*x^3/(sqrt(b*x^2 + a)*b) + c^2*x/(sqrt(b*x^2 + a)*a) - 2*c*d*x/(sqr
t(b*x^2 + a)*b) + 3/2*a*d^2*x/(sqrt(b*x^2 + a)*b^2) + 2*c*d*arcsinh(b*x/sq
rt(a*b))/b^(3/2) - 3/2*a*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\frac{d^2x^2}{b} + \frac{2\,b^3c^2-4\,ab^2cd+3\,a^2bd^2}{ab^3}\right)x}{2\,\sqrt{bx^2+a}} - \frac{(4\,bcd-3\,ad^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(d^2*x^2/b + (2*b^3*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)/(a*b^3))*x/sqrt(b
*x^2 + a) - 1/2*(4*b*c*d - 3*a*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))
/b^(5/2)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^2}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c + d*x^2)^2/(a + b*x^2)^(3/2),x)
```

output
```
int((c + d*x^2)^2/(a + b*x^2)^(3/2), x)
```

3.84 $\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx$

3.84.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx=\frac{(bc-ad)x}{ab\sqrt{a+bx^2}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
d*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+(-a*d+b*c)*x/a/b/(b*x^2+a)^(1
/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx=\frac{bcx-adx}{ab\sqrt{a+bx^2}}-\frac{d\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[(c + d*x^2)/(a + b*x^2)^(3/2),x]
```

output
```
(b*c*x - a*d*x)/(a*b*Sqrt[a + b*x^2]) - (d*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/b^(3/2)
```

3.84.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 298}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}}dx}{b}+\frac{x(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{d\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}+\frac{x(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}+\frac{x(bc-ad)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(c + d*x^2)/(a + b*x^2)^(3/2),x]
```

output
```
((b*c - a*d)*x)/(a*b*Sqrt[a + b*x^2]) + (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/b^(3/2)
```

3.84.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.84.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{cx}{a\sqrt{b\,x^2+a}}+d\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)$	55
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)ad\sqrt{b\,x^2+a}-adx\sqrt{b}+b^{\frac{3}{2}}cx}{b^{\frac{3}{2}}\sqrt{b\,x^2+a}\,a}$	61

input
```
int((d*x^2+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
c*x/a/(b*x^2+a)^(1/2)+d*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^
2+a)^(1/2)))
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.09

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx = \left[\frac{2\,(b^2c-abd)\sqrt{bx^2+a}x+(abdx^2+a^2d)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\,(ab^3x^2+a^2b^2)}, \frac{(b^2c$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(2*(b^2*c - a*b*d)*sqrt(b*x^2 + a)*x + (a*b*d*x^2 + a^2*d)*sqrt(b)*lo
g(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(a*b^3*x^2 + a^2*b^2), ((b^
2*c - a*b*d)*sqrt(b*x^2 + a)*x - (a*b*d*x^2 + a^2*d)*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2)]
```

3.84.6 Sympy [A] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx = d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + \frac{cx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(3/2),x)
```

output
```
d*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
 c*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx = \frac{cx}{\sqrt{bx^2+a}a} - \frac{dx}{\sqrt{bx^2+a}b} + \frac{d\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
c*x/(sqrt(b*x^2 + a)*a) - d*x/(sqrt(b*x^2 + a)*b) + d*arcsinh(b*x/sqrt(a*b
))/b^(3/2)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx = -\frac{d\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}} + \frac{(bc-ad)x}{\sqrt{bx^2+a}ab}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-d*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + (b*c - a*d)*x/(sqrt(b*
x^2 + a)*a*b)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx = \frac{d\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} + \frac{c\,x}{a\,\sqrt{b\,x^2+a}} - \frac{d\,x}{b\,\sqrt{b\,x^2+a}}$$

input
```
int((c + d*x^2)/(a + b*x^2)^(3/2),x)
```

output
```
(d*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) + (c*x)/(a*(a + b*x^2)^(1/2
)) - (d*x)/(b*(a + b*x^2)^(1/2))
```

3.85 $\int \frac{1}{(a+bx^2)^{3/2}} dx$

3.85.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{(a+bx^2)^{3/2}} dx = \frac{x}{a\sqrt{a+bx^2}}$$

output
```
x/a/(b*x^2+a)^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{3/2}} dx = \frac{x}{a\sqrt{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-3/2),x]
```

output
```
x/(a*Sqrt[a + b*x^2])
```

3.85.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx$$
$$\Big\downarrow \ 208$$
$$\frac{x}{a\sqrt{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-3/2),x]
```

output
```
x/(a*Sqrt[a + b*x^2])
```

3.85.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

3.85.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
default	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
trager	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
pseudoelliptic	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15

input
```
int(1/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/a/(b*x^2+a)^(1/2)
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{bx^2+a}\,x}{abx^2+a^2}$$

input
```
integrate(1/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(b*x^2 + a)*x/(a*b*x^2 + a^2)
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(1/(b*x**2+a)**(3/2),x)
```

output
```
x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{\sqrt{bx^2+a}\,a}$$

input
```
integrate(1/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
x/(sqrt(b*x^2 + a)*a)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{\sqrt{bx^2+a}a}$$

input
```
integrate(1/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
x/(sqrt(b*x^2 + a)*a)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{a\sqrt{b\,x^2+a}}$$

input
```
int(1/(a + b*x^2)^(3/2),x)
```

output
```
x/(a*(a + b*x^2)^(1/2))
```

3.86 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx$

3.86.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = \frac{bx}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}$$

output
```
-d*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/(-a*d+b*c)^(3/2)/c^
(1/2)+b*x/a/(-a*d+b*c)/(b*x^2+a)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = \frac{bx}{(abc-a^2d)\sqrt{a+bx^2}} - \frac{d\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}(-bc+ad)^{3/2}}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
```

output
```
(b*x)/((a*b*c - a^2*d)*Sqrt[a + b*x^2]) - (d*ArcTan[(-(d*x*Sqrt[a + b*x^2]
) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*(-(b*c) +
 a*d)^(3/2))
```

3.86.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {296, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx$$

$$\Big\downarrow \text{ 296}$$

$$\frac{bx}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{bc-ad}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{bx}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{bc-ad}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{bx}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]) - (d*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sq
rt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(3/2))
```

3.86.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.86.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$-\frac{d \arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right) a\sqrt{b\,x^{2}+a}+bx\sqrt{(ad-bc)c}}{a(ad-bc)\sqrt{b\,x^{2}+a}\,\sqrt{(ad-bc)c}}$
default	$-\frac{\frac{d}{(ad-bc)\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^{2} b-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{ad-bc}{d}}}+\frac{2b\sqrt{-cd}\left(2b\left(x+\frac{\sqrt{-cd}}{d}\right)-\frac{2b\sqrt{-cd}}{d}\right)}{(ad-bc)\left(\frac{4b(ad-bc)}{d}+\frac{4b^{2}c}{d}\right)\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^{2} b-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+a}}}{2\sqrt{-cd}}$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-(d*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))*a*(b*x^2+a)^(1/2)+b*x*
((a*d-b*c)*c)^(1/2))/a/(a*d-b*c)/(b*x^2+a)^(1/2)/((a*d-b*c)*c)^(1/2)
```

3.86.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(67) = 134.

Time = 0.32 (sec) , antiderivative size = 441, normalized size of antiderivative = 5.58

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = \left[\frac{4\,(b^2c^2-abcd)\sqrt{bx^2+a}x-(abdx^2+a^2d)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2}{}\right.}{4\,(a^2b^2c^3-2\,a^3bc^2d+a^4cd^2+(ab^3c^3-2}\right.$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/4*(4*(b^2*c^2 - a*b*c*d)*sqrt(b*x^2 + a)*x - (a*b*d*x^2 + a^2*d)*sqrt(b
*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*
a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*
d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(a^2*b^2*c^3 - 2*a^3*b*c
^2*d + a^4*c*d^2 + (a*b^3*c^3 - 2*a^2*b^2*c^2*d + a^3*b*c*d^2)*x^2), 1/2*(
2*(b^2*c^2 - a*b*c*d)*sqrt(b*x^2 + a)*x + (a*b*d*x^2 + a^2*d)*sqrt(-b*c^2
+ a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*
x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a^2*b^2*c^3
- 2*a^3*b*c^2*d + a^4*c*d^2 + (a*b^3*c^3 - 2*a^2*b^2*c^2*d + a^3*b*c*d^2)*
x^2)]
```

3.86.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)), x)
```

3.86.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)), x)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = -\frac{\sqrt{b}d\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}(bc-ad)}+\frac{bx}{(abc-a^2d)\sqrt{bx^2+a}}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
-sqrt(b)*d*arctan(-1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/s
qrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*(b*c - a*d)) + b*x/((a*
b*c - a^2*d)*sqrt(b*x^2 + a))
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)), x)
```

3.87 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx$

3.87.1 Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx = \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)} - \frac{d(4bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}}$$

output
```
-1/2*d*(-a*d+4*b*c)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^
(3/2)/(-a*d+b*c)^(5/2)+1/2*b*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(1/2
)-1/2*d*x/c/(-a*d+b*c)/(d*x^2+c)/(b*x^2+a)^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx = \frac{\frac{\sqrt{c}x(a^2d^2+abd^2x^2+2b^2c(c+dx^2))}{a(bc-ad)^2\sqrt{a+bx^2}(c+dx^2)} + \frac{d(4bc-ad)\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}}{2c^{3/2}}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```

output
```
((Sqrt[c]*x*(a^2*d^2 + a*b*d^2*x^2 + 2*b^2*c*(c + d*x^2)))/(a*(b*c - a*d)^
2*Sqrt[a + b*x^2]*(c + d*x^2)) + (d*(4*b*c - a*d)*ArcTan[(-(d*x*Sqrt[a + b
*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(-(b*c) + a*d
)^(5/2))/(2*c^(3/2))
```

3.87.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {316, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{-2bdx^2+2bc-ad}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx}{2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{bx(ad+2bc)}{a\sqrt{a+bx^2}(bc-ad)}-\frac{\int \frac{ad(4bc-ad)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{a(bc-ad)}}{2c(bc-ad)}-\frac{dx}{2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{bx(ad+2bc)}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d(4bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{bc-ad}}{2c(bc-ad)}-\frac{dx}{2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{bx(ad+2bc)}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d(4bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{bc-ad}}{2c(bc-ad)}-\frac{dx}{2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{bx(ad+2bc)}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d(4bc-ad)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}}{2c(bc-ad)}-\frac{dx}{2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```

output
```
-1/2*(d*x)/(c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)) + ((b*(2*b*c + a*d)
*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]) - (d*(4*b*c - a*d)*ArcTanh[(Sqrt[b*c -
 a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(3/2)))/(2*c*(b*
c - a*d))
```

3.87.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.87.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.05

method	result	size
pseudoelliptic	$\frac{-ad\sqrt{b\,x^{2}+a}\,(d\,x^{2}+c)\,(ad-4bc)\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right)+x\sqrt{(ad-bc)c}\,(2b^{2}c^{2}+2x^{2}b^{2}cd+a\,d^{2}\,(b\,x^{2}+a))}{2\sqrt{b\,x^{2}+a}\,\sqrt{(ad-bc)c}\,c(d\,x^{2}+c)(ad-bc)^{2}a}$	150
default	Expression too large to display	1906

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a*d*(b*x^2+a)^(1/2)*(d*x^2+c)*(a*d-4*b*c)*arctan(c*(b*x^2+a)^(1/2)/x
/((a*d-b*c)*c)^(1/2))+x*((a*d-b*c)*c)^(1/2)*(2*b^2*c^2+2*x^2*b^2*c*d+a*d^2
*(b*x^2+a)))/(b*x^2+a)^(1/2)/((a*d-b*c)*c)^(1/2)/c/(d*x^2+c)/(a*d-b*c)^2/a
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(123) = 246$.

Time = 0.49 (sec) , antiderivative size = 864, normalized size of antiderivative = 6.04

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^2}\,dx = \left[-\frac{(4\,a^2bc^2d - a^3cd^2 + (4\,ab^2cd^2 - a^2bd^3)x^4 + (4\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x}{8\,(a^2b^3c^6 - 3\,a^3b}\right.$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[-1/8*((4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (4*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2
*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2
+ 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*
x^4 + 2*c*d*x^2 + c^2)) - 4*((2*b^3*c^3*d - a*b^2*c^2*d^2 - a^2*b*c*d^3)*x
^3 + (2*b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 - a^3*c*d^3)*x)*sqrt(b*x^2
 + a))/(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 + (a
*b^4*c^5*d - 3*a^2*b^3*c^4*d^2 + 3*a^3*b^2*c^3*d^3 - a^4*b*c^2*d^4)*x^4 +
(a*b^4*c^6 - 2*a^2*b^3*c^5*d + 2*a^4*b*c^3*d^3 - a^5*c^2*d^4)*x^2), 1/4*((
4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (4*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b
*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*
d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*((2*b^3*c^3*d - a*b^2*c^2*d^2 - a^2*b
*c*d^3)*x^3 + (2*b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 - a^3*c*d^3)*x)*s
qrt(b*x^2 + a))/(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3
*d^3 + (a*b^4*c^5*d - 3*a^2*b^3*c^4*d^2 + 3*a^3*b^2*c^3*d^3 - a^4*b*c^2*d^
4)*x^4 + (a*b^4*c^6 - 2*a^2*b^3*c^5*d + 2*a^4*b*c^3*d^3 - a^5*c^2*d^4)*x^2
)]
```

3.87.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**2), x)
```

3.87.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^2), x)
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(123) = 246$.

Time = 0.87 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.22

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx = \frac{b^2x}{(ab^2c^2-2a^2bcd+a^3d^2)\sqrt{bx^2+a}} + \frac{\left(4b^{\frac{3}{2}}cd-a\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{2(b^2c^3-2abc^2d+a^2cd^2)\sqrt{-b^2c^2+abcd}} + \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}cd-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}d^2+a^2\sqrt{b}d^2}{(b^2c^3-2abc^2d+a^2cd^2)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ad+\cdots\right.}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
b^2*x/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(b*x^2 + a)) + 1/2*(4*b^(3/
2)*c*d - a*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*
b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*
sqrt(-b^2*c^2 + a*b*c*d)) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*d
 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d^2 + a^2*sqrt(b)*d^2)/((b^2*
c^3 - 2*a*b*c^2*d + a^2*c*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d +
a^2*d))
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^2), x)
```

3.88 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\,dx$

3.88.1 Optimal result

Integrand size = 21, antiderivative size = 225

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\,dx = -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)} + \frac{d(4bc-ad)(2bc+3ad)x\sqrt{a+bx^2}}{8ac^2(bc-ad)^3\,(c+dx^2)} - \frac{3d(8b^2c^2-4abcd+a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{7/2}}$$

output
```
-3/8*d*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b
*x^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(7/2)-1/4*d*x/c/(-a*d+b*c)/(d*x^2+c)^2/(
b*x^2+a)^(1/2)+1/4*b*(a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)/(b*x^2+a)^(1
/2)+1/8*d*(-a*d+4*b*c)*(3*a*d+2*b*c)*x*(b*x^2+a)^(1/2)/a/c^2/(-a*d+b*c)^3/
(d*x^2+c)
```

3.88.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 13.40 (sec) , antiderivative size = 1392, normalized size of antiderivative = 6.19

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]
```

output
```
(x*(-108045*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - (324135*d*x^2*Sqrt[(
(b*c - a*d)*x^2)/(c*(a + b*x^2))])/c - (324135*d^2*x^4*Sqrt[((b*c - a*d)*x
^2)/(c*(a + b*x^2))])/c^2 - (103320*d^3*x^6*Sqrt[((b*c - a*d)*x^2)/(c*(a +
 b*x^2))])/c^3 + 42735*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2) + (128205
*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c + (139545*d^2*x^4*(((b
*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c^2 + (46200*d^3*x^6*(((b*c - a*d)*
x^2)/(c*(a + b*x^2)))^(3/2))/c^3 - 3864*(((b*c - a*d)*x^2)/(c*(a + b*x^2))
)^(5/2) - (4032*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2))/c - (4032
*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2))/c^2 - (1344*d^3*x^6*((
(b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2))/c^3 + 108045*ArcTanh[Sqrt[((b*c -
 a*d)*x^2)/(c*(a + b*x^2))]] + (324135*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2
)/(c*(a + b*x^2))]])/c + (324135*d^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c
*(a + b*x^2))]])/c^2 + (103320*d^3*x^6*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(
a + b*x^2))]])/c^3 + (8505*(b*c - a*d)^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2
)/(c*(a + b*x^2))]])/(c^2*(a + b*x^2)^2) + (17955*d*(b*c - a*d)^2*x^6*ArcT
anh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^3*(a + b*x^2)^2) + (21735
*d^2*(b*c - a*d)^2*x^8*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(
c^4*(a + b*x^2)^2) + (7560*d^3*(b*c - a*d)^2*x^10*ArcTanh[Sqrt[((b*c - a*d
)*x^2)/(c*(a + b*x^2))]])/(c^5*(a + b*x^2)^2) - (78750*(b*c - a*d)*x^2*Arc
Tanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c*(a + b*x^2)) + (23625...
```

3.88.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {316, 402, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}dx$$

$$\Big\downarrow \text{ 316}$$

$$\frac{\int \frac{-4bdx^2+4bc-3ad}{(bx^2+a)^{3/2}(dx^2+c)^2}dx}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a+bx^2}\,(c+dx^2)^2\,(bc-ad)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt{a+bx^2}(c+dx^2)(bc-ad)}-\frac{\int \frac{d\left(a(8bc-3ad)-2b(4bc+ad)x^2\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a+bx^2}\,(c+dx^2)^2\,(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt{a+bx^2}(c+dx^2)(bc-ad)}-\frac{d\int \frac{a(8bc-3ad)-2b(4bc+ad)x^2}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a+bx^2}\,(c+dx^2)^2\,(bc-ad)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt{a+bx^2}(c+dx^2)(bc-ad)}-\frac{d\left(\frac{\int \frac{3a\left(8b^2c^2-4abdc+a^2d^2\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{x\sqrt{a+bx^2}(4bc-ad)(3ad+2bc)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a+bx^2}\,(c+dx^2)^2\,(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt{a+bx^2}(c+dx^2)(bc-ad)}-\frac{d\left(\frac{3a\left(a^2d^2-4abcd+8b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{x\sqrt{a+bx^2}(4bc-ad)(3ad+2bc)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a+bx^2}\,(c+dx^2)^2\,(bc-ad)}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt{a+bx^2}(c+dx^2)(bc-ad)}-\frac{d\left(\frac{3a\left(a^2d^2-4abcd+8b^2c^2\right)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}-\frac{x\sqrt{a+bx^2}(4bc-ad)(3ad+2bc)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a+bx^2}\,(c+dx^2)^2\,(bc-ad)}$$

3.88. $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}dx$

$$\Big\downarrow \; 221$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt{a+bx^2}(c+dx^2)(bc-ad)}-\frac{d\left(\frac{3a\left(a^2d^2-4abcd+8b^2c^2\right)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}-\frac{x\sqrt{a+bx^2}(4bc-ad)(3ad+2bc)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a+bx^2}\left(c+dx^2\right)^2(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]
```

output
```
-1/4*(d*x)/(c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^2) + ((b*(4*b*c + a*
d)*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)) - (d*(-1/2*((4*b*c - a*d
)*(2*b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)) + (3*a*(8
*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[
a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(3/2))))/(a*(b*c - a*d)))/(4*c*(b*c -
 a*d))
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.88.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{3\left(\sqrt{b x^2+a}\, a d \left(d x^2+c\right)^2 \left(a^2 d^2-4abcd+8b^2c^2\right) \arctan\left(\frac{c\sqrt{b x^2+a}}{x\sqrt{(ad-bc)c}}\right)-\frac{5x\left(d^3\left(\frac{3d x^2}{5}+c\right)a^3-\frac{12\left(-\frac{d x^2}{3}+c\right)b d^2\left(\frac{3d x^2}{4}+c\right)c}{5}\cdots\right.}{\cdots}\right.}{8\sqrt{b x^2+a}\,\sqrt{(ad-bc)c}\,\left(d x^2+c\right)^2 c^2 (ad-bc)^3 a}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*((b*x^2+a)^(1/2)*a*d*(d*x^2+c)^2*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctan
(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))-5/3*x*(d^3*(3/5*d*x^2+c)*a^3-12/
5*(-1/3*d*x^2+c)*b*d^2*(3/4*d*x^2+c)*a^2-12/5*x^2*(5/6*d*x^2+c)*b^2*d^2*c*
a-8/5*b^3*c^2*(d*x^2+c)^2)*((a*d-b*c)*c)^(1/2))/(b*x^2+a)^(1/2)/((a*d-b*c)
*c)^(1/2)/(d*x^2+c)^2/c^2/(a*d-b*c)^3/a
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. 2(201) = 402.

Time = 0.88 (sec) , antiderivative size = 1482, normalized size of antiderivative = 6.59

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/32*(3*(8*a^2*b^2*c^4*d - 4*a^3*b*c^3*d^2 + a^4*c^2*d^3 + (8*a*b^3*c^2*
d^3 - 4*a^2*b^2*c*d^4 + a^3*b*d^5)*x^6 + (16*a*b^3*c^3*d^2 - 2*a^3*b*c*d^4
 + a^4*d^5)*x^4 + (8*a*b^3*c^4*d + 12*a^2*b^2*c^3*d^2 - 7*a^3*b*c^2*d^3 +
2*a^4*c*d^4)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^
2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 +
a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) -
 4*((8*b^4*c^4*d^2 + 2*a*b^3*c^3*d^3 - 13*a^2*b^2*c^2*d^4 + 3*a^3*b*c*d^5)
*x^5 + (16*b^4*c^5*d - 4*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3 - 8*a^3*b*c^2*d
^4 + 3*a^4*c*d^5)*x^3 + (8*b^4*c^6 - 8*a*b^3*c^5*d + 12*a^2*b^2*c^4*d^2 -
17*a^3*b*c^3*d^3 + 5*a^4*c^2*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^4*c^9 - 4*a^3
*b^3*c^8*d + 6*a^4*b^2*c^7*d^2 - 4*a^5*b*c^6*d^3 + a^6*c^5*d^4 + (a*b^5*c^
7*d^2 - 4*a^2*b^4*c^6*d^3 + 6*a^3*b^3*c^5*d^4 - 4*a^4*b^2*c^4*d^5 + a^5*b*
c^3*d^6)*x^6 + (2*a*b^5*c^8*d - 7*a^2*b^4*c^7*d^2 + 8*a^3*b^3*c^6*d^3 - 2*
a^4*b^2*c^5*d^4 - 2*a^5*b*c^4*d^5 + a^6*c^3*d^6)*x^4 + (a*b^5*c^9 - 2*a^2*
b^4*c^8*d - 2*a^3*b^3*c^7*d^2 + 8*a^4*b^2*c^6*d^3 - 7*a^5*b*c^5*d^4 + 2*a^
6*c^4*d^5)*x^2), 1/16*(3*(8*a^2*b^2*c^4*d - 4*a^3*b*c^3*d^2 + a^4*c^2*d^3
+ (8*a*b^3*c^2*d^3 - 4*a^2*b^2*c*d^4 + a^3*b*d^5)*x^6 + (16*a*b^3*c^3*d^2
- 2*a^3*b*c*d^4 + a^4*d^5)*x^4 + (8*a*b^3*c^4*d + 12*a^2*b^2*c^3*d^2 - 7*a
^3*b*c^2*d^3 + 2*a^4*c*d^4)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c
^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c...
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.88.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^3), x)
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(201) = 402$.

Time = 1.21 (sec) , antiderivative size = 643, normalized size of antiderivative = 2.86

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\,dx = \frac{b^3x}{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{bx^2+a}}$$
$$+\frac{3\left(8\,b^{\frac{5}{2}}c^2d-4\,ab^{\frac{3}{2}}cd^2+a^2\sqrt{b}d^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{8\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\sqrt{-b^2c^2+abcd}}$$
$$+\frac{16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^{\frac{5}{2}}c^2d^2-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{3}{2}}cd^3+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2\sqrt{b}d^4+80\left(\sqrt{b}\right.}{}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(b*x^2
+ a)) + 3/8*(8*b^(5/2)*c^2*d - 4*a*b^(3/2)*c*d^2 + a^2*sqrt(b)*d^3)*arctan
(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b
*c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(-b^
2*c^2 + a*b*c*d)) + 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d^
2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d^3 + 3*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*a^2*sqrt(b)*d^4 + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7
/2)*c^3*d - 104*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d^2 + 54*(sq
rt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^3 - 9*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^3*sqrt(b)*d^4 + 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*
c^2*d^2 - 52*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^3 + 9*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d^4 + 10*a^4*b^(3/2)*c*d^3 - 3*a^5*sq
rt(b)*d^4)/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*((sq
rt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\,dx=\int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^3), x)
```

3.89 $\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx$

3.89.1 Optimal result

Integrand size = 21, antiderivative size = 255

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = -\frac{d(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3)\,x\sqrt{a+bx^2}}{24a^2b^4} - \frac{d(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+bx^2}(c+dx^2)}{12a^2b^3} + \frac{(bc-ad)(2bc+7ad)x(c+dx^2)^2}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x(c+dx^2)^3}{3ab\,(a+bx^2)^{3/2}} + \frac{d^2(48b^2c^2-80abcd+35a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}$$

output
```
1/3*(-a*d+b*c)*x*(d*x^2+c)^3/a/b/(b*x^2+a)^(3/2)+1/8*d^2*(35*a^2*d^2-80*a*
b*c*d+48*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)+1/3*(-a*d+b*c
)*(7*a*d+2*b*c)*x*(d*x^2+c)^2/a^2/b^2/(b*x^2+a)^(1/2)-1/24*d*(105*a^3*d^3-
170*a^2*b*c*d^2+40*a*b^2*c^2*d+16*b^3*c^3)*x*(b*x^2+a)^(1/2)/a^2/b^4-1/12*
d*(-35*a^2*d^2+24*a*b*c*d+8*b^2*c^2)*x*(d*x^2+c)*(b*x^2+a)^(1/2)/a^2/b^3
```

3.89.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = \frac{x(-105a^5d^4+16b^5c^4x^2+20a^4bd^3(12c-7dx^2)+8ab^4c^3(3c+4dx^2)+a^3b^2d^2(-144c^2 \cdots}{24a^2b^4(a+bx^2)^{3/2}}$$
$$-\frac{d^2(48b^2c^2-80abcd+35a^2d^2)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{9/2}}$$

input
```
Integrate[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```

output
```
(x*(-105*a^5*d^4 + 16*b^5*c^4*x^2 + 20*a^4*b*d^3*(12*c - 7*d*x^2) + 8*a*b^
4*c^3*(3*c + 4*d*x^2) + a^3*b^2*d^2*(-144*c^2 + 320*c*d*x^2 - 21*d^2*x^4)
+ 6*a^2*b^3*d^2*x^2*(-32*c^2 + 8*c*d*x^2 + d^2*x^4)))/(24*a^2*b^4*(a + b*x
^2)^(3/2)) - (d^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/(8*b^(9/2))
```

3.89.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {315, 401, 27, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\int \frac{(dx^2+c)^2(c(2bc+ad)-d(4bc-7ad)x^2)}{(bx^2+a)^{3/2}}dx}{3ab}+\frac{x(c+dx^2)^3(bc-ad)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{\frac{x(c+dx^2)^2\left(\frac{2bc^2}{a}-\frac{7ad^2}{b}+5cd\right)}{\sqrt{a+bx^2}}-\frac{\int \frac{d(dx^2+c)\left(\left(8b^2c^2+24abdc-35a^2d^2\right)x^2+ac(4bc-7ad)\right)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x(c+dx^2)^3(bc-ad)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{x(c+dx^2)^2\left(\frac{2bc^2}{a}-\frac{7ad^2}{b}+5cd\right)}{\sqrt{a+bx^2}}-\frac{d\int\frac{(dx^2+c)\left((8b^2c^2+24abdc-35a^2d^2)x^2+ac(4bc-7ad)\right)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^3(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 403

$$\frac{\frac{x(c+dx^2)^2\left(\frac{2bc^2}{a}-\frac{7ad^2}{b}+5cd\right)}{\sqrt{a+bx^2}}-\frac{d\left(\frac{\int\frac{\left(16b^3c^3+40ab^2dc^2-170a^2bd^2c+105a^3d^3\right)x^2+ac\left(8b^2c^2-52abdc+35a^2d^2\right)}{\sqrt{bx^2+a}}dx}{4b}+\frac{x\sqrt{a+bx^2}\left(c+dx^2\right)\left(-35a^2d^2+24abc\ldots\right.}{4b}\right.}{ab}}{3ab}$$

$$\frac{x\left(c+dx^2\right)^3(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 299

$$\frac{\frac{x(c+dx^2)^2\left(\frac{2bc^2}{a}-\frac{7ad^2}{b}+5cd\right)}{\sqrt{a+bx^2}}-\frac{d\left(\frac{\frac{x\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{2b}-\frac{3a^2d\left(35a^2d^2-80abcd+48b^2c^2\right)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}}{4b}+\frac{x\sqrt{a+bx^2}\left(c+\ldots\right.}{\ldots}\right.}{ab}}{3ab}$$

$$\frac{x\left(c+dx^2\right)^3(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 224

$$\frac{\frac{x(c+dx^2)^2\left(\frac{2bc^2}{a}-\frac{7ad^2}{b}+5cd\right)}{\sqrt{a+bx^2}}-\frac{d\left(\frac{\frac{x\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{2b}-\frac{3a^2d\left(35a^2d^2-80abcd+48b^2c^2\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}}{4b}+\frac{x\sqrt{a\ldots}}{\ldots}\right.}{ab}}{3ab}$$

$$\frac{x\left(c+dx^2\right)^3(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 219

$$\frac{\frac{x(c+dx^2)^2\left(\frac{2bc^2}{a}-\frac{7ad^2}{b}+5cd\right)}{\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(c+dx^2\right)\left(-35a^2d^2+24abcd+8b^2c^2\right)}{4b}+\frac{\frac{x\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{2b}-\frac{3a^2d\,\text{arctan}\ldots}{\ldots}}{4b}\right.}{ab}}{3ab}$$

$$\frac{x\left(c+dx^2\right)^3(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

3.89. $\int\frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^{5/2}}dx$

input
```
Int[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```

output
```
((b*c - a*d)*x*(c + d*x^2)^3)/(3*a*b*(a + b*x^2)^(3/2)) + ((((2*b*c^2)/a +
 5*c*d - (7*a*d^2)/b)*x*(c + d*x^2)^2)/Sqrt[a + b*x^2] - (d*(((8*b^2*c^2 +
 24*a*b*c*d - 35*a^2*d^2)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(4*b) + (((16*b^3
*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*x*Sqrt[a + b*x^2])/
(2*b) - (3*a^2*d*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/(4*b)))/(a*b))/(3*a*b)
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.89.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\frac{35\left(b\,x^2+a\right)^{\frac{3}{2}} d^2\left(a^2 d^2-\frac{16}{7} abcd+\frac{48}{35} b^2 c^2\right) a^2 \operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{8}-35x\left(\frac{48d^2a^3\left(\frac{7}{48}d^2x^4-\frac{20}{9}cd\,x^2+c^2\right)b^{\frac{5}{2}}}{35}+\frac{64x^2\left(-\frac{1}{32}d^2x^4-\frac{1}{4}cd\,x^2\right.}{35}\right.}{a^2 b^{\frac{9}{2}}\left(b\,x^2+a\right)^{\frac{3}{2}}}$
default	$c^4\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+d^4\left(\frac{x^7}{4b(b\,x^2+a)^{\frac{3}{2}}}-\frac{7a\left(\frac{x^5}{2b\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\right.}{b}}{}\right.}{2b}\right.}{4b}\right.$
risch	$-\frac{d^3x\left(-2bd\,x^2+11ad-16bc\right)\sqrt{b\,x^2+a}}{8b^4}+\frac{\frac{d^2\left(35a^2d^2-80abcd+48b^2c^2\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{2\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3\right.}{}}{}$

input
```
int((d*x^2+c)^4/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
35/8*((b*x^2+a)^(3/2)*d^2*(a^2*d^2-16/7*a*b*c*d+48/35*b^2*c^2)*a^2*arctanh
((b*x^2+a)^(1/2)/x/b^(1/2))-x*(48/35*d^2*a^3*(7/48*d^2*x^4-20/9*c*d*x^2+c^
2)*b^(5/2)+64/35*x^2*(-1/32*d^2*x^4-1/4*c*d*x^2+c^2)*d^2*a^2*b^(7/2)-16/7*
(-7/12*d*x^2+c)*d^3*a^4*b^(3/2)-8/35*(4/3*d*x^2+c)*c^3*a*b^(9/2)+b^(1/2)*a
^5*d^4-16/105*b^(11/2)*c^4*x^2))/b^(9/2)/(b*x^2+a)^(3/2)/a^2
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 684, normalized size of antiderivative = 2.68

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(48\,a^4b^2c^2d^2 - 80\,a^5bcd^3 + 35\,a^6d^4 + (48\,a^2b^4c^2d^2 - 80\,a^3b^3cd^3 + 35\,a^4b^2d^4)x^4 + 2\,(4}{}\right.$$

$$-\frac{3\,(48\,a^4b^2c^2d^2 - 80\,a^5bcd^3 + 35\,a^6d^4 + (48\,a^2b^4c^2d^2 - 80\,a^3b^3cd^3 + 35\,a^4b^2d^4)x^4 + 2\,(48\,a^3b^3c^2d^2 - 80\,a^4b^2}{}$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(48*a^4*b^2*c^2*d^2 - 80*a^5*b*c*d^3 + 35*a^6*d^4 + (48*a^2*b^4*c
^2*d^2 - 80*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(48*a^3*b^3*c^2*d^2 -
80*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) + 2*(6*a^2*b^4*d^4*x^7 + 3*(16*a^2*b^4*c*d^3 - 7*a^3*b
^3*d^4)*x^5 + 4*(4*b^6*c^4 + 8*a*b^5*c^3*d - 48*a^2*b^4*c^2*d^2 + 80*a^3*b
^3*c*d^3 - 35*a^4*b^2*d^4)*x^3 + 3*(8*a*b^5*c^4 - 48*a^3*b^3*c^2*d^2 + 80*
a^4*b^2*c*d^3 - 35*a^5*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^7*x^4 + 2*a^3*b^6
*x^2 + a^4*b^5), -1/24*(3*(48*a^4*b^2*c^2*d^2 - 80*a^5*b*c*d^3 + 35*a^6*d^
4 + (48*a^2*b^4*c^2*d^2 - 80*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(48*a
^3*b^3*c^2*d^2 - 80*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) - (6*a^2*b^4*d^4*x^7 + 3*(16*a^2*b^4*c*d^3 - 7*a^
3*b^3*d^4)*x^5 + 4*(4*b^6*c^4 + 8*a*b^5*c^3*d - 48*a^2*b^4*c^2*d^2 + 80*a^
3*b^3*c*d^3 - 35*a^4*b^2*d^4)*x^3 + 3*(8*a*b^5*c^4 - 48*a^3*b^3*c^2*d^2 +
80*a^4*b^2*c*d^3 - 35*a^5*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^7*x^4 + 2*a^3*
b^6*x^2 + a^4*b^5)]
```

3.89.6 Sympy [F]

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^4}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**4/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**4/(a + b*x**2)**(5/2), x)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.54

$$\begin{aligned}\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx &= \frac{d^4x^7}{4\,(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,cd^3x^5}{(bx^2+a)^{\frac{3}{2}}b} - \frac{7\,ad^4x^5}{8\,(bx^2+a)^{\frac{3}{2}}b^2} \\ &\quad - 2\,c^2d^2x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) + \frac{10\,acd^3x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b} \\ &\quad - \frac{35\,a^2d^4x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{24\,b^2} + \frac{2\,c^4x}{3\,\sqrt{bx^2+a}a^2} + \frac{c^4x}{3\,(bx^2+a)^{\frac{3}{2}}a} \\ &\quad - \frac{4\,c^3dx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{4\,c^3dx}{3\,\sqrt{bx^2+a}ab} - \frac{2\,c^2d^2x}{\sqrt{bx^2+a}b^2} + \frac{10\,acd^3x}{3\,\sqrt{bx^2+a}b^3} - \frac{35\,a^2d^4x}{24\,\sqrt{bx^2+a}b^4} \\ &\quad + \frac{6\,c^2d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} - \frac{10\,acd^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{7}{2}}} + \frac{35\,a^2d^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{9}{2}}}\end{aligned}$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/4*d^4*x^7/((b*x^2 + a)^(3/2)*b) + 2*c*d^3*x^5/((b*x^2 + a)^(3/2)*b) - 7/
8*a*d^4*x^5/((b*x^2 + a)^(3/2)*b^2) - 2*c^2*d^2*x*(3*x^2/((b*x^2 + a)^(3/2
)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 10/3*a*c*d^3*x*(3*x^2/((b*x^2 + a)^(
3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b - 35/24*a^2*d^4*x*(3*x^2/((b*x^2
+ a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 2/3*c^4*x/(sqrt(b*x^2 +
 a)*a^2) + 1/3*c^4*x/((b*x^2 + a)^(3/2)*a) - 4/3*c^3*d*x/((b*x^2 + a)^(3/2
)*b) + 4/3*c^3*d*x/(sqrt(b*x^2 + a)*a*b) - 2*c^2*d^2*x/(sqrt(b*x^2 + a)*b^
2) + 10/3*a*c*d^3*x/(sqrt(b*x^2 + a)*b^3) - 35/24*a^2*d^4*x/(sqrt(b*x^2 +
a)*b^4) + 6*c^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 10*a*c*d^3*arcsinh(b*
x/sqrt(a*b))/b^(7/2) + 35/8*a^2*d^4*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(3\left(\frac{2d^4x^2}{b}+\frac{16a^2b^6cd^3-7a^3b^5d^4}{a^2b^7}\right)x^2+\frac{4\left(4b^8c^4+8ab^7c^3d-48a^2b^6c^2d^2+80a^3b^5cd^3-35a^4b^4d^4\right)}{a^2b^7}\right)x^2+\frac{3}{}\cdots}{24\,(bx^2+a)^{\frac{3}{2}}} - \frac{(48\,b^2c^2d^2-80\,abcd^3+35\,a^2d^4)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/24*((3*(2*d^4*x^2/b + (16*a^2*b^6*c*d^3 - 7*a^3*b^5*d^4)/(a^2*b^7))*x^2
+ 4*(4*b^8*c^4 + 8*a*b^7*c^3*d - 48*a^2*b^6*c^2*d^2 + 80*a^3*b^5*c*d^3 - 3
5*a^4*b^4*d^4)/(a^2*b^7))*x^2 + 3*(8*a*b^7*c^4 - 48*a^3*b^5*c^2*d^2 + 80*a
^4*b^4*c*d^3 - 35*a^5*b^3*d^4)/(a^2*b^7))*x/(b*x^2 + a)^(3/2) - 1/8*(48*b^
2*c^2*d^2 - 80*a*b*c*d^3 + 35*a^2*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a
)))/b^(9/2)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^4}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c + d*x^2)^4/(a + b*x^2)^(5/2),x)
```

output
```
int((c + d*x^2)^4/(a + b*x^2)^(5/2), x)
```

3.89. $\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx$

3.90 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx$

3.90.1 Optimal result

Integrand size = 21, antiderivative size = 172

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx = -\frac{d(4b^2c^2+8abcd-15a^2d^2)\,x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x(c+dx^2)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x(c+dx^2)^2}{3ab\,(a+bx^2)^{3/2}} + \frac{d^2(6bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

output
```
1/3*(-a*d+b*c)*x*(d*x^2+c)^2/a/b/(b*x^2+a)^(3/2)+1/2*d^2*(-5*a*d+6*b*c)*ar
ctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)+1/3*(-a*d+b*c)*(5*a*d+2*b*c)*x*(d
*x^2+c)/a^2/b^2/(b*x^2+a)^(1/2)-1/6*d*(-15*a^2*d^2+8*a*b*c*d+4*b^2*c^2)*x*
(b*x^2+a)^(1/2)/a^2/b^3
```

3.90.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{x(15a^4d^3+4b^4c^3x^2+3a^2b^2d^2x^2(-8c+dx^2)+6ab^3c^2(c+dx^2)+2a^3bd^2(-9c+10dx^2}{6a^2b^3\,(a+bx^2)^{3/2}} + \frac{d^2(-6bc+5ad)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{7/2}}$$

input
```
Integrate[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```

output
```
(x*(15*a^4*d^3 + 4*b^4*c^3*x^2 + 3*a^2*b^2*d^2*x^2*(-8*c + d*x^2) + 6*a*b^
3*c^2*(c + d*x^2) + 2*a^3*b*d^2*(-9*c + 10*d*x^2)))/(6*a^2*b^3*(a + b*x^2)
^(3/2)) + (d^2*(-6*b*c + 5*a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^
(7/2))
```

3.90.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {315, 401, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{315}$$

$$\frac{\int \frac{(dx^2+c)\left(c(2bc+ad)-d(2bc-5ad)x^2\right)}{(bx^2+a)^{3/2}}dx}{3ab}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{401}$$

$$\frac{\frac{x(c+dx^2)\left(\frac{2bc^2}{a}-\frac{5ad^2}{b}+3cd\right)}{\sqrt{a+bx^2}}-\frac{\int \frac{d\left(\left(4b^2c^2+8abdc-15a^2d^2\right)x^2+ac(2bc-5ad)\right)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x(c+dx^2)\left(\frac{2bc^2}{a}-\frac{5ad^2}{b}+3cd\right)}{\sqrt{a+bx^2}}-\frac{d\int \frac{\left(4b^2c^2+8abdc-15a^2d^2\right)x^2+ac(2bc-5ad)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{x(c+dx^2)\left(\frac{2bc^2}{a}-\frac{5ad^2}{b}+3cd\right)}{\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(-15a^2d^2+8abcd+4b^2c^2\right)}{2b}-\frac{3a^2d(6bc-5ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{x(c+dx^2)\left(\frac{2bc^2}{a}-\frac{5ad^2}{b}+3cd\right)}{\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(-15a^2d^2+8abcd+4b^2c^2\right)}{2b}-\frac{3a^2d(6bc-5ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{x(c+dx^2)\left(\frac{2bc^2}{a}-\frac{5ad^2}{b}+3cd\right)}{\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(-15a^2d^2+8abcd+4b^2c^2\right)}{2b}-\frac{3a^2d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(6bc-5ad)}{2b^{3/2}}\right)}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```

output
```
((b*c - a*d)*x*(c + d*x^2)^2)/(3*a*b*(a + b*x^2)^(3/2)) + ((((2*b*c^2)/a +
 3*c*d - (5*a*d^2)/b)*x*(c + d*x^2))/Sqrt[a + b*x^2] - (d*(((4*b^2*c^2 + 8
*a*b*c*d - 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(2*b) - (3*a^2*d*(6*b*c - 5*a*d)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(a*b))/(3*a*b)
```

3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.90. $\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{5/2}}\,dx$

rule 299

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

3.90.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{5\left(\left(b\,x^{2}+a\right)^{\frac{3}{2}} d^{2}\left(a d-\frac{6 b c}{5}\right) a^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)-x\left(-\frac{6\left(-\frac{10 d\,x^{2}}{9}+c\right) d^{2} a^{3} b^{\frac{3}{2}}}{5}-\frac{8 x^{2}\left(-\frac{d\,x^{2}}{8}+c\right) d^{2} a^{2} b^{\frac{5}{2}}}{5}+\frac{2 a\,c^{2}\left(d\,x^{2}+c\right) b^{\cdots}}{5} \cdots\right.\right.}{2 b^{\frac{7}{2}}\left(b\,x^{2}+a\right)^{\frac{3}{2}} a^{2}}$
default	$c^{3}\left(\frac{x}{3 a(b\,x^{2}+a)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b\,x^{2}+a}}\right)+d^{3}\left(\frac{x^{5}}{2 b(b\,x^{2}+a)^{\frac{3}{2}}}-\frac{5 a\left(-\frac{x^{3}}{3 b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b \sqrt{b\,x^{2}+a}}+\frac{\ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)}{2 b}\right)$
risch	$\frac{d^{3} x \sqrt{b\,x^{2}+a}}{2 b^{3}}-\frac{\frac{d^{2}(5 a d-6 b c) \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{\left(a^{3} d^{3}-3 a^{2} b c\,d^{2}+3 a\,b^{2} c^{2} d-b^{3} c^{3}\right)\left(-\frac{\sqrt{\left(x-\frac{\sqrt{-a b}}{b}\right)^{2} b+2 \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}}{3 \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}}-\cdots\right.}{2 b a}}{\cdots}$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-5/2*((b*x^2+a)^(3/2)*d^2*(a*d-6/5*b*c)*a^2*arctanh((b*x^2+a)^(1/2)/x/b^(1
/2))-x*(-6/5*(-10/9*d*x^2+c)*d^2*a^3*b^(3/2)-8/5*x^2*(-1/8*d*x^2+c)*d^2*a^
2*b^(5/2)+2/5*a*c^2*(d*x^2+c)*b^(7/2)+b^(1/2)*a^4*d^3+4/15*b^(9/2)*c^3*x^2
))/b^(7/2)/(b*x^2+a)^(3/2)/a^2
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.83

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx = \left[-\frac{3\,(6\,a^4bcd^2-5\,a^5d^3+(6\,a^2b^3cd^2-5\,a^3b^2d^3)x^4+2\,(6\,a^3b^2cd^2-5\,a^4bd^3)x^2)\sqrt{b}\log\left(\cdots\right.}{\cdots}\right.$$

$$-\frac{3\,(6\,a^4bcd^2-5\,a^5d^3+(6\,a^2b^3cd^2-5\,a^3b^2d^3)x^4+2\,(6\,a^3b^2cd^2-5\,a^4bd^3)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(3\,a^2b^3\cdots}{6\,(a^2b^6x^4+2\,a^3\cdots}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(6*a^4*b*c*d^2 - 5*a^5*d^3 + (6*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x
^4 + 2*(6*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(
b*x^2 + a)*sqrt(b)*x - a) - 2*(3*a^2*b^3*d^3*x^5 + 2*(2*b^5*c^3 + 3*a*b^4*
c^2*d - 12*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^3 + 3*(2*a*b^4*c^3 - 6*a^3*b^
2*c*d^2 + 5*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 +
a^4*b^4), -1/6*(3*(6*a^4*b*c*d^2 - 5*a^5*d^3 + (6*a^2*b^3*c*d^2 - 5*a^3*b^
2*d^3)*x^4 + 2*(6*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt(-b)*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)) - (3*a^2*b^3*d^3*x^5 + 2*(2*b^5*c^3 + 3*a*b^4*c^2*d
- 12*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^3 + 3*(2*a*b^4*c^3 - 6*a^3*b^2*c*d^
2 + 5*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^
4)]
```

3.90.6 Sympy [F]

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^3}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**3/(a + b*x**2)**(5/2), x)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{d^3x^5}{2\,(bx^2+a)^{\frac{3}{2}}b} - cd^2x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) + \frac{5\,ad^3x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6\,b} + \frac{2\,c^3x}{3\,\sqrt{bx^2+a}a^2} + \frac{c^3x}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{c^2dx}{(bx^2+a)^{\frac{3}{2}}b} + \frac{c^2dx}{\sqrt{bx^2+a}ab} - \frac{cd^2x}{\sqrt{bx^2+a}b^2} + \frac{5\,ad^3x}{6\,\sqrt{bx^2+a}b^3} + \frac{3\,cd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} - \frac{5\,ad^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/2*d^3*x^5/((b*x^2 + a)^(3/2)*b) - c*d^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) +
 2*a/((b*x^2 + a)^(3/2)*b^2)) + 5/6*a*d^3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) +
 2*a/((b*x^2 + a)^(3/2)*b^2))/b + 2/3*c^3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c^
3*x/((b*x^2 + a)^(3/2)*a) - c^2*d*x/((b*x^2 + a)^(3/2)*b) + c^2*d*x/(sqrt(
b*x^2 + a)*a*b) - c*d^2*x/(sqrt(b*x^2 + a)*b^2) + 5/6*a*d^3*x/(sqrt(b*x^2
+ a)*b^3) + 3*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/2*a*d^3*arcsinh(b*x
/sqrt(a*b))/b^(7/2)
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\frac{3\,d^3x^2}{b} + \frac{2\,(2\,b^6c^3+3\,ab^5c^2d-12\,a^2b^4cd^2+10\,a^3b^3d^3)}{a^2b^5}\right)x^2 + \frac{3\,(2\,ab^5c^3-6\,a^3b^3cd^2+5\,a^4b^2d^3)}{a^2b^5}\right)x}{6\,(bx^2+a)^{\frac{3}{2}}} - \frac{(6\,bcd^2-5\,ad^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/6*((3*d^3*x^2/b + 2*(2*b^6*c^3 + 3*a*b^5*c^2*d - 12*a^2*b^4*c*d^2 + 10*a
^3*b^3*d^3)/(a^2*b^5))*x^2 + 3*(2*a*b^5*c^3 - 6*a^3*b^3*c*d^2 + 5*a^4*b^2*
d^3)/(a^2*b^5))*x/(b*x^2 + a)^(3/2) - 1/2*(6*b*c*d^2 - 5*a*d^3)*log(abs(-s
qrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^3}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c + d*x^2)^3/(a + b*x^2)^(5/2),x)
```

output
```
int((c + d*x^2)^3/(a + b*x^2)^(5/2), x)
```

3.91 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx$

3.91.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x(c+dx^2)}{3ab\,(a+bx^2)^{3/2}} + \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

output
```
1/3*(-a*d+b*c)*x*(d*x^2+c)/a/b/(b*x^2+a)^(3/2)+d^2*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))/b^(5/2)+1/3*(-a*d+b*c)*(3*a*d+2*b*c)*x/a^2/b^2/(b*x^2+a)^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = -\frac{(-bc+ad)x(3abc+3a^2d+2b^2cx^2+4abdx^2)}{3a^2b^2\,(a+bx^2)^{3/2}} - \frac{d^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{5/2}}$$

input
```
Integrate[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*((-(b*c) + a*d)*x*(3*a*b*c + 3*a^2*d + 2*b^2*c*x^2 + 4*a*b*d*x^2))/(a
^2*b^2*(a + b*x^2)^(3/2)) - (d^2*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5
/2)
```

3.91.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {315, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 315$$

$$\frac{\int \frac{3ad^2x^2+c(2bc+ad)}{(bx^2+a)^{3/2}}dx}{3ab}+\frac{x(c+dx^2)(bc-ad)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \ 298$$

$$\frac{\frac{3ad^2\int \frac{1}{\sqrt{bx^2+a}}dx}{b}+\frac{x(bc-ad)(3ad+2bc)}{ab\sqrt{a+bx^2}}}{3ab}+\frac{x(c+dx^2)(bc-ad)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{3ad^2\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}+\frac{x(bc-ad)(3ad+2bc)}{ab\sqrt{a+bx^2}}}{3ab}+\frac{x(c+dx^2)(bc-ad)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{3ad^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}+\frac{x(bc-ad)(3ad+2bc)}{ab\sqrt{a+bx^2}}}{3ab}+\frac{x(c+dx^2)(bc-ad)}{3ab(a+bx^2)^{3/2}}$$

input
```
Int[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

output
```
((b*c - a*d)*x*(c + d*x^2))/(3*a*b*(a + b*x^2)^(3/2)) + (((b*c - a*d)*(2*b
*c + 3*a*d)*x)/(a*b*Sqrt[a + b*x^2]) + (3*a*d^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/b^(3/2))/(3*a*b)
```

3.91.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.91.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{a^{2} \left(b \,x^{2}+a \right)^{\frac{3}{2}} d^{2} \operatorname{arctanh}\left(\frac{\sqrt{b \,x^{2}+a}}{x \sqrt{b}}\right)-x \left(-c \left(\frac{2 d \,x^{2}}{3}+c \right) a \,b^{\frac{5}{2}}+\frac{4 b^{\frac{3}{2}} a^{2} d^{2} x^{2}}{3}+\sqrt{b}\, a^{3} d^{2}-\frac{2 b^{\frac{7}{2}} c^{2} x^{2}}{3}\right)}{\left(b \,x^{2}+a \right)^{\frac{3}{2}} b^{\frac{5}{2}} a^{2}}$
default	$c^{2} \left(\frac{x}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b \,x^{2}+a}}\right)+d^{2} \left(-\frac{x^{3}}{3 b \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{-\frac{x}{b \sqrt{b \,x^{2}+a}}+\frac{\ln\left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+2 c d \left(-\frac{}{2 b(b x}\right.$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(3/2)*(a^2*(b*x^2+a)^(3/2)*d^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/
2))-x*(-c*(2/3*d*x^2+c)*a*b^(5/2)+4/3*b^(3/2)*a^2*d^2*x^2+b^(1/2)*a^3*d^2-
2/3*b^(7/2)*c^2*x^2))/b^(5/2)/a^2
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.03

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(a^2b^2d^2x^4 + 2\,a^3bd^2x^2 + a^4d^2)\sqrt{b}\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) + 2\,(2\,(b^4c^2 +}{6\,(a^2b^5x^4 + 2\,a^3b^4x^2 + a^4b^3)}\right.$$

$$-\frac{3\,(a^2b^2d^2x^4 + 2\,a^3bd^2x^2 + a^4d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (2\,(b^4c^2 + ab^3cd - 2\,a^2b^2d^2)x^3 + 3\,(ab^3c^2 - a^3bd^2)}{3\,(a^2b^5x^4 + 2\,a^3b^4x^2 + a^4b^3)}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(b)*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^
2*d^2)*x^3 + 3*(a*b^3*c^2 - a^3*b*d^2)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 +
2*a^3*b^4*x^2 + a^4*b^3), -1/3*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4
*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*(b^4*c^2 + a*b^3*c*
d - 2*a^2*b^2*d^2)*x^3 + 3*(a*b^3*c^2 - a^3*b*d^2)*x)*sqrt(b*x^2 + a))/(a^
2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3)]
```

3.91.6 Sympy [F]

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^2}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**2/(a + b*x**2)**(5/2), x)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3}\,d^2x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)+\frac{2\,c^2x}{3\,\sqrt{bx^2+a}a^2} + \frac{c^2x}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{2\,cdx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,cdx}{3\,\sqrt{bx^2+a}ab}-\frac{d^2x}{3\,\sqrt{bx^2+a}b^2}+\frac{d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*d^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 2
/3*c^2*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c^2*x/((b*x^2 + a)^(3/2)*a) - 2/3*c*d
*x/((b*x^2 + a)^(3/2)*b) + 2/3*c*d*x/(sqrt(b*x^2 + a)*a*b) - 1/3*d^2*x/(sq
rt(b*x^2 + a)*b^2) + d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{x\left(\frac{2\,(b^4c^2+ab^3cd-2\,a^2b^2d^2)x^2}{a^2b^3}+\frac{3\,(ab^3c^2-a^3bd^2)}{a^2b^3}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{d^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*d^2)*x^2/(a^2*b^3) + 3*(a*b^3*c^
2 - a^3*b*d^2)/(a^2*b^3))/(b*x^2 + a)^(3/2) - d^2*log(abs(-sqrt(b)*x + sqr
t(b*x^2 + a)))/b^(5/2)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^2}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c + d*x^2)^2/(a + b*x^2)^(5/2),x)
```

output
```
int((c + d*x^2)^2/(a + b*x^2)^(5/2), x)
```

3.92 $\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx$

3.92.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = \frac{2cx}{3a^2\sqrt{a+bx^2}} + \frac{x(c+dx^2)}{3a\,(a+bx^2)^{3/2}}$$

output
```
1/3*x*(d*x^2+c)/a/(b*x^2+a)^(3/2)+2/3*c*x/a^2/(b*x^2+a)^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = \frac{x(3ac+2bcx^2+adx^2)}{3a^2\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(c + d*x^2)/(a + b*x^2)^(5/2),x]
```

output
```
(x*(3*a*c + 2*b*c*x^2 + a*d*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

3.92.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{292}$$

$$\frac{2c\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}+\frac{x(c+dx^2)}{3a(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x(c+dx^2)}{3a(a+bx^2)^{3/2}}$$

input
```
Int[(c + d*x^2)/(a + b*x^2)^(5/2),x]
```

output
```
(2*c*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(c + d*x^2))/(3*a*(a + b*x^2)^(3/2))
```

3.92.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

3.92.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x\left(a d\,x^{2}+2 c b\,x^{2}+3 a c \right)}{3 \left(b\,x^{2}+a \right)^{\frac{3}{2}} a^{2}}$	34
trager	$\frac{x\left(a d\,x^{2}+2 c b\,x^{2}+3 a c \right)}{3 \left(b\,x^{2}+a \right)^{\frac{3}{2}} a^{2}}$	34
pseudoelliptic	$\frac{x\left(a d\,x^{2}+2 c b\,x^{2}+3 a c \right)}{3 \left(b\,x^{2}+a \right)^{\frac{3}{2}} a^{2}}$	34
default	$c \left(\frac{x}{3 a \left(b\,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b\,x^{2}+a}}\right)+d \left(-\frac{x}{2 b \left(b\,x^{2}+a \right)^{\frac{3}{2}}}+\frac{a \left(\frac{x}{3 a \left(b\,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b\,x^{2}+a}}\right)}{2 b}\right)$	90

input
```
int((d*x^2+c)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(a*d*x^2+2*b*c*x^2+3*a*c)/(b*x^2+a)^(3/2)/a^2
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = \frac{((2\,bc+ad)x^3+3\,acx)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*b*c + a*d)*x^3 + 3*a*c*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x
^2 + a^4)
```

3.92.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(41) = 82$.

Time = 4.02 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.06

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = c\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + \frac{dx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(5/2),x)
```

output
```
c*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + d*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2
*sqrt(1 + b*x**2/a))
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = \frac{2\,cx}{3\,\sqrt{bx^2+a}a^2} + \frac{cx}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{dx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{dx}{3\,\sqrt{bx^2+a}ab}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*c*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c*x/((b*x^2 + a)^(3/2)*a) - 1/3*d*x/((
b*x^2 + a)^(3/2)*b) + 1/3*d*x/(sqrt(b*x^2 + a)*a*b)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = \frac{x\left(\frac{3c}{a}+\frac{(2b^2c+abd)x^2}{a^2b}\right)}{3(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(3*c/a + (2*b^2*c + a*b*d)*x^2/(a^2*b))/(b*x^2 + a)^(3/2)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = \frac{3\,a\,c\,x+a\,d\,x^3+2\,b\,c\,x^3}{3\,a^2\,(b\,x^2+a)^{3/2}}$$

input
```
int((c + d*x^2)/(a + b*x^2)^(5/2),x)
```

output
```
(3*a*c*x + a*d*x^3 + 2*b*c*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

3.93 $\int \frac{1}{(a+bx^2)^{5/2}}\,dx$

3.93.1 Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{x}{3a\,(a+bx^2)^{3/2}} + \frac{2x}{3a^2\sqrt{a+bx^2}}$$

output
```
1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{3ax+2bx^3}{3a^2\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(-5/2),x]
```

output
```
(3*a*x + 2*b*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

3.93.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{209}$$

$$\frac{2\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(a + b*x^2)^(-5/2),x]
```

output
```
x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

3.93.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

3.93.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x(2b\,x^2+3a)}{3(b\,x^2+a)^{\frac{3}{2}}a^2}$	26
trager	$\frac{x(2b\,x^2+3a)}{3(b\,x^2+a)^{\frac{3}{2}}a^2}$	26
pseudoelliptic	$\frac{x(2b\,x^2+3a)}{3(b\,x^2+a)^{\frac{3}{2}}a^2}$	26
default	$\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}$	32

input
```
int(1/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(2*b*x^2+3*a)/(b*x^2+a)^(3/2)/a^2
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{(2\,bx^3+3\,ax)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

input
```
integrate(1/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*b*x^3 + 3*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

3.93.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(32) = 64.

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(1/(b*x**2+a)**(5/2),x)
```

output
```
3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a
)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
b*x**2/a))
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{2\,x}{3\,\sqrt{bx^2+a}a^2} + \frac{x}{3\,(bx^2+a)^{\frac{3}{2}}a}$$

input
```
integrate(1/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*x/((b*x^2 + a)^(3/2)*a)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{x\left(\frac{2\,bx^2}{a^2}+\frac{3}{a}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate(1/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(2*b*x^2/a^2 + 3/a)/(b*x^2 + a)^(3/2)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx=\frac{2\,x\,\left(b\,x^2+a\right)+a\,x}{3\,a^2\,{\left(b\,x^2+a\right)}^{3/2}}$$

input
```
int(1/(a + b*x^2)^(5/2),x)
```

output
```
(2*x*(a + b*x^2) + a*x)/(3*a^2*(a + b*x^2)^(3/2))
```

3.94 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx$

3.94.1 Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx = \frac{bx}{3a(bc-ad)(a+bx^2)^{3/2}} + \frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{5/2}}$$

output
```
1/3*b*x/a/(-a*d+b*c)/(b*x^2+a)^(3/2)+d^2*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2
)/(b*x^2+a)^(1/2))/(-a*d+b*c)^(5/2)/c^(1/2)+1/3*b*(-5*a*d+2*b*c)*x/a^2/(-a
*d+b*c)^2/(b*x^2+a)^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx = \frac{bx(-6a^2d+2b^2cx^2+ab(3c-5dx^2))}{3a^2(bc-ad)^2(a+bx^2)^{3/2}} - \frac{d^2\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}(-bc+ad)^{5/2}}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

output
```
(b*x*(-6*a^2*d + 2*b^2*c*x^2 + a*b*(3*c - 5*d*x^2)))/(3*a^2*(b*c - a*d)^2*
(a + b*x^2)^(3/2)) - (d^2*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*
x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*(-(b*c) + a*d)^(5/2))
```

3.94.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {316, 25, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{bx}{3a(a+bx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{2bdx^2+2bc-3ad}{(bx^2+a)^{3/2}(dx^2+c)}dx}{3a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2bdx^2+2bc-3ad}{(bx^2+a)^{3/2}(dx^2+c)}dx}{3a(bc-ad)}+\frac{bx}{3a(a+bx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{bx(2bc-5ad)}{a\sqrt{a+bx^2}(bc-ad)}-\frac{\int -\frac{3a^2d^2}{\sqrt{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a(a+bx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3ad^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{bc-ad}+\frac{bx(2bc-5ad)}{a\sqrt{a+bx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a(a+bx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{3ad^2\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{bc-ad}+\frac{bx(2bc-5ad)}{a\sqrt{a+bx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a(a+bx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{3ad^2\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}+\frac{bx(2bc-5ad)}{a\sqrt{a+bx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

output
```
(b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + ((b*(2*b*c - 5*a*d)*x)/(a*(b*c
 - a*d)*Sqrt[a + b*x^2]) + (3*a*d^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*S
qrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(3/2)))/(3*a*(b*c - a*d))
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.94.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

method	result	size
pseudoelliptic	$-\frac{a^{2} d^{2} \arctan\left(\frac{c \sqrt{b\,x^{2}+a}}{x \sqrt{(a d-b c) c}}\right) \left(b\,x^{2}+a\right)^{\frac{3}{2}}+2 x \sqrt{(a d-b c) c}\, b \left(a^{2} d-\frac{\left(-\frac{5 d\,x^{2}}{3}+c\right) b a}{2}-\frac{b^{2} c\,x^{2}}{3}\right)}{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \sqrt{(a d-b c) c}\,(a d-b c)^{2} a^{2}}$	124
default	Expression too large to display	1378

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/(b*x^2+a)^(3/2)*(a^2*d^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)
)*(b*x^2+a)^(3/2)+2*x*((a*d-b*c)*c)^(1/2)*b*(a^2*d-1/2*(-5/3*d*x^2+c)*b*a-
1/3*b^2*c*x^2))/((a*d-b*c)*c)^(1/2)/(a*d-b*c)^2/a^2
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(104) = 208.

Time = 0.48 (sec) , antiderivative size = 764, normalized size of antiderivative = 6.26

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx = \left[\frac{3\left(a^2b^2d^2x^4+2\,a^3bd^2x^2+a^4d^2\right)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4}{}\right.}{12\left(a^4b^3c^4-3\,a^5b^2c^3d+3\,a^6bc^2d^2-a^7cd^3+\right.}\right.$$

$$-\frac{3\left(a^2b^2d^2x^4+2\,a^3bd^2x^2+a^4d^2\right)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\left((2\,bc-ad)x^2+ac\right)\sqrt{bx^2+a}}{2\left((b^2c^2-abcd)x^3+(abc^2-a^2cd)x\right)}\right)-2\left((2\,b^4c^3-7\,ab^3c^2\right.}{6\left(a^4b^3c^4-3\,a^5b^2c^3d+3\,a^6bc^2d^2-a^7cd^3+\left(a^2b^5c^4-3\,a^3b^4c^3d+3\,a^4b^3c^2d^2-a^5b^2cd^3\right)x^4+\right.}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/12*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(b*c^2 - a*c*d)
*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a
^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2
 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((2*b^4*c^3 - 7*a*b^3*c^2*d + 5*a^
2*b^2*c*d^2)*x^3 + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 2*a^3*b*c*d^2)*x)*sqrt
(b*x^2 + a))/(a^4*b^3*c^4 - 3*a^5*b^2*c^3*d + 3*a^6*b*c^2*d^2 - a^7*c*d^3
+ (a^2*b^5*c^4 - 3*a^3*b^4*c^3*d + 3*a^4*b^3*c^2*d^2 - a^5*b^2*c*d^3)*x^4
+ 2*(a^3*b^4*c^4 - 3*a^4*b^3*c^3*d + 3*a^5*b^2*c^2*d^2 - a^6*b*c*d^3)*x^2)
, -1/6*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(-b*c^2 + a*c*
d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 +
a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((2*b^4*c^3 - 7*
a*b^3*c^2*d + 5*a^2*b^2*c*d^2)*x^3 + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 2*a^
3*b*c*d^2)*x)*sqrt(b*x^2 + a))/(a^4*b^3*c^4 - 3*a^5*b^2*c^3*d + 3*a^6*b*c^
2*d^2 - a^7*c*d^3 + (a^2*b^5*c^4 - 3*a^3*b^4*c^3*d + 3*a^4*b^3*c^2*d^2 - a
^5*b^2*c*d^3)*x^4 + 2*(a^3*b^4*c^4 - 3*a^4*b^3*c^3*d + 3*a^5*b^2*c^2*d^2 -
 a^6*b*c*d^3)*x^2)]
```

3.94.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}(c+dx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)), x)
```

3.94.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)), x)
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(104) = 208$.

Time = 0.28 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.62

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx = -\frac{\sqrt{b}d^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c^2+abcd}} + \frac{\left(\frac{(2\,b^6c^3-9\,ab^5c^2d+12\,a^2b^4cd^2-5\,a^3b^3d^3)x^2}{a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4}+\frac{3\,(ab^5c^3-4\,a^2b^4c^2d+5\,a^3b^3cd^2-2\,a^4b^2d^3)}{a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4}\right)x}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
-sqrt(b)*d^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/
sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c^2 +
 a*b*c*d)) + 1/3*((2*b^6*c^3 - 9*a*b^5*c^2*d + 12*a^2*b^4*c*d^2 - 5*a^3*b^
3*d^3)*x^2/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*
c*d^3 + a^6*b*d^4) + 3*(a*b^5*c^3 - 4*a^2*b^4*c^2*d + 5*a^3*b^3*c*d^2 - 2*
a^4*b^2*d^3)/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^
2*c*d^3 + a^6*b*d^4))*x/(b*x^2 + a)^(3/2)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)), x)
```

3.95 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx$

3.95.1 Optimal result

Integrand size = 21, antiderivative size = 202

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx = \frac{b(2bc+3ad)x}{6ac(bc-ad)^2(a+bx^2)^{3/2}} + \frac{b(4b^2c^2-16abcd-3a^2d^2)\,x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\,(a+bx^2)^{3/2}\,(c+dx^2)} + \frac{d^2(6bc-ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}}$$

output
```
1/6*b*(3*a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(3/2)-1/2*d*x/c/(-a*d+b*c
)/(b*x^2+a)^(3/2)/(d*x^2+c)+1/2*d^2*(-a*d+6*b*c)*arctanh(x*(-a*d+b*c)^(1/2
)/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-a*d+b*c)^(7/2)+1/6*b*(-3*a^2*d^2-16*a
*b*c*d+4*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^2+a)^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx = \frac{x(3a^4d^3+6a^3bd^3x^2-4b^4c^2x^2(c+dx^2)+3a^2b^2d(6c^2+6cdx^2+d^2x^4)+2ab}{6a^2c(-bc+ad)^3\,(a+bx^2)^{3/2}\,(c+dx^2)} + \frac{d^2(6bc-ad)\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{2c^{3/2}(-bc+ad)^{7/2}}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```

output
```
(x*(3*a^4*d^3 + 6*a^3*b*d^3*x^2 - 4*b^4*c^2*x^2*(c + d*x^2) + 3*a^2*b^2*d*
(6*c^2 + 6*c*d*x^2 + d^2*x^4) + 2*a*b^3*c*(-3*c^2 + 5*c*d*x^2 + 8*d^2*x^4)
))/(6*a^2*c*(-(b*c) + a*d)^3*(a + b*x^2)^(3/2)*(c + d*x^2)) + (d^2*(6*b*c
- a*d)*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt
[-(b*c) + a*d])])/(2*c^(3/2)*(-(b*c) + a*d)^(7/2))
```

3.95.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {316, 402, 25, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}dx$$

$$\Big\downarrow \; 316$$

$$\frac{\int \frac{-4bdx^2+2bc-ad}{(bx^2+a)^{5/2}(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx}{2c(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{bx(3ad+2bc)}{3a(a+bx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{4b^2c^2-12abdc+3a^2d^2+2bd(2bc+3ad)x^2}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)}dx}{3a(bc-ad)}}{2c(bc-ad)}-\frac{dx}{2c(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{4b^2c^2-12abdc+3a^2d^2+2bd(2bc+3ad)x^2}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)}dx}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a+bx^2)^{3/2}(bc-ad)}}{2c(bc-ad)}-\frac{dx}{2c(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{bx\left(-3a^2d^2-16abcd+4b^2c^2\right)}{a\sqrt{a+bx^2}(bc-ad)}-\frac{\int -\frac{3a^2d^2(6bc-ad)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{a(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a+bx^2)^{3/2}(bc-ad)}}{2c(bc-ad)}-\frac{dx}{2c(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3ad^2(6bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{bc-ad}+\frac{bx(-3a^2d^2-16abcd+4b^2c^2)}{a\sqrt{a+bx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a+bx^2)^{3/2}(bc-ad)}}{2c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{3ad^2(6bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{bc-ad}+\frac{bx(-3a^2d^2-16abcd+4b^2c^2)}{a\sqrt{a+bx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a+bx^2)^{3/2}(bc-ad)}}{2c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{bx(-3a^2d^2-16abcd+4b^2c^2)}{a\sqrt{a+bx^2}(bc-ad)}+\frac{3ad^2(6bc-ad)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a+bx^2)^{3/2}(bc-ad)}}{2c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```

output
```
-1/2*(d*x)/(c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)) + ((b*(2*b*c + 3*
a*d)*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + ((b*(4*b^2*c^2 - 16*a*b*c*d
- 3*a^2*d^2)*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]) + (3*a*d^2*(6*b*c - a*d)*A
rcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d
)^(3/2)))/(3*a*(b*c - a*d)))/(2*c*(b*c - a*d))
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.95.4 Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-(ad-6bc)\left(d\,x^{2}+c\right)d^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right)+x\sqrt{(ad-bc)c}\left(\left(-\frac{4}{3}b^{4}x^{2}-2a\,b^{3}\right)c^{3}+6b^{2}d\left(-\frac{2}{9}b^{2}x^{4}+\frac{5}{9}ab\,x^{2}+a^{2}\right)\right.}{2\sqrt{(ad-bc)c}\,\left(b\,x^{2}+a\right)^{\frac{3}{2}}c\left(d\,x^{2}+c\right)(ad-bc)^{3}a^{2}}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*c)^(1/2)/(b*x^2+a)^(3/2)*(-(a*d-6*b*c)*(d*x^2+c)*d^2*(b*x^2
+a)^(3/2)*a^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+x*((a*d-b*c)
*c)^(1/2)*((-4/3*b^4*x^2-2*a*b^3)*c^3+6*b^2*d*(-2/9*b^2*x^4+5/9*a*b*x^2+a^
2)*c^2+6*x^2*b^2*d^2*a*(8/9*b*x^2+a)*c+a^2*d^3*(b*x^2+a)^2))/c/(d*x^2+c)/(
a*d-b*c)^3/a^2
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(178) = 356$.

Time = 0.95 (sec) , antiderivative size = 1440, normalized size of antiderivative = 7.13

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/24*(3*(6*a^4*b*c^2*d^2 - a^5*c*d^3 + (6*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^
6 + (6*a^2*b^3*c^2*d^2 + 11*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^4 + (12*a^3*b^2
*c^2*d^2 + 4*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c
^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 +
4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^
4 + 2*c*d*x^2 + c^2)) + 4*((4*b^5*c^4*d - 20*a*b^4*c^3*d^2 + 13*a^2*b^3*c^
2*d^3 + 3*a^3*b^2*c*d^4)*x^5 + 2*(2*b^5*c^5 - 7*a*b^4*c^4*d - 4*a^2*b^3*c^
3*d^2 + 6*a^3*b^2*c^2*d^3 + 3*a^4*b*c*d^4)*x^3 + 3*(2*a*b^4*c^5 - 8*a^2*b^
3*c^4*d + 6*a^3*b^2*c^3*d^2 - a^4*b*c^2*d^3 + a^5*c*d^4)*x)*sqrt(b*x^2 + a
))/(a^4*b^4*c^7 - 4*a^5*b^3*c^6*d + 6*a^6*b^2*c^5*d^2 - 4*a^7*b*c^4*d^3 +
a^8*c^3*d^4 + (a^2*b^6*c^6*d - 4*a^3*b^5*c^5*d^2 + 6*a^4*b^4*c^4*d^3 - 4*a
^5*b^3*c^3*d^4 + a^6*b^2*c^2*d^5)*x^6 + (a^2*b^6*c^7 - 2*a^3*b^5*c^6*d - 2
*a^4*b^4*c^5*d^2 + 8*a^5*b^3*c^4*d^3 - 7*a^6*b^2*c^3*d^4 + 2*a^7*b*c^2*d^5
)*x^4 + (2*a^3*b^5*c^7 - 7*a^4*b^4*c^6*d + 8*a^5*b^3*c^5*d^2 - 2*a^6*b^2*c
^4*d^3 - 2*a^7*b*c^3*d^4 + a^8*c^2*d^5)*x^2), -1/12*(3*(6*a^4*b*c^2*d^2 -
a^5*c*d^3 + (6*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (6*a^2*b^3*c^2*d^2 + 11*
a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^4 + (12*a^3*b^2*c^2*d^2 + 4*a^4*b*c*d^3 - a
^5*d^4)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c
- a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^
2*c*d)*x)) - 2*((4*b^5*c^4*d - 20*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 + ...
```

3.95.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}(c+dx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**2), x)
```

3.95.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^2), x)
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(178) = 356$.

Time = 0.89 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.07

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx = \frac{\left(\frac{2\,(b^8c^4-7\,ab^7c^3d+15\,a^2b^6c^2d^2-13\,a^3b^5cd^3+4\,a^4b^4d^4)x^2}{a^2b^7c^6-6\,a^3b^6c^5d+15\,a^4b^5c^4d^2-20\,a^5b^4c^3d^3+15\,a^6b^3c^2d^4-6\,a^7b^2cd^5+a^8bd^6}+\frac{3\,(ab^7c^4-}{a^2b^7c^6-6\,a^3b^6c^5d+}\right.}{3\,(bx^2+a)^{\frac{3}{2}}}$$

$$+\frac{\left(6\,b^{\frac{3}{2}}cd^2-a\sqrt{b}d^3\right)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\sqrt{-b^2c^2+abcd}}$$

$$-\frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}cd^2-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}d^3+a^2\sqrt{b}d^3}{(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}x-\sqrt{bx\right.\right.}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/3*(2*(b^8*c^4 - 7*a*b^7*c^3*d + 15*a^2*b^6*c^2*d^2 - 13*a^3*b^5*c*d^3 +
4*a^4*b^4*d^4)*x^2/(a^2*b^7*c^6 - 6*a^3*b^6*c^5*d + 15*a^4*b^5*c^4*d^2 - 2
0*a^5*b^4*c^3*d^3 + 15*a^6*b^3*c^2*d^4 - 6*a^7*b^2*c*d^5 + a^8*b*d^6) + 3*
(a*b^7*c^4 - 6*a^2*b^6*c^3*d + 12*a^3*b^5*c^2*d^2 - 10*a^4*b^4*c*d^3 + 3*a
^5*b^3*d^4)/(a^2*b^7*c^6 - 6*a^3*b^6*c^5*d + 15*a^4*b^5*c^4*d^2 - 20*a^5*b
^4*c^3*d^3 + 15*a^6*b^3*c^2*d^4 - 6*a^7*b^2*c*d^5 + a^8*b*d^6))*x/(b*x^2 +
 a)^(3/2) + 1/2*(6*b^(3/2)*c*d^2 - a*sqrt(b)*d^3)*arctan(-1/2*((sqrt(b)*x
- sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^3*c^4
- 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(-b^2*c^2 + a*b*c*d)) -
 (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*d^2 - (sqrt(b)*x - sqrt(b*x^
2 + a))^2*a*sqrt(b)*d^3 + a^2*sqrt(b)*d^3)/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a
^2*b*c^2*d^2 - a^3*c*d^3)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d
))
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^2), x)
```

3.96 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3} dx$

3.96.1 Optimal result

Integrand size = 21, antiderivative size = 313

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3} dx = -\frac{dx}{4c(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^2)^{3/2}(c+dx^2)} + \frac{b(8b^2c^2-40abcd-3a^2d^2)x}{12a^2c(bc-ad)^3\sqrt{a+bx^2}(c+dx^2)} + \frac{d(16b^3c^3-88ab^2c^2d-42a^2bcd^2+9a^3d^3)x\sqrt{a+bx^2}}{24a^2c^2(bc-ad)^4(c+dx^2)} + \frac{d^2(48b^2c^2-16abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{9/2}}$$

output
```
-1/4*d*x/c/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^2+1/12*b*(3*a*d+4*b*c)*x/a
/c/(-a*d+b*c)^2/(b*x^2+a)^(3/2)/(d*x^2+c)+1/8*d^2*(3*a^2*d^2-16*a*b*c*d+48
*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(5/2)/(-a*
d+b*c)^(9/2)+1/12*b*(-3*a^2*d^2-40*a*b*c*d+8*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3
/(d*x^2+c)/(b*x^2+a)^(1/2)+1/24*d*(9*a^3*d^3-42*a^2*b*c*d^2-88*a*b^2*c^2*d
+16*b^3*c^3)*x*(b*x^2+a)^(1/2)/a^2/c^2/(-a*d+b*c)^4/(d*x^2+c)
```

3.96.2 Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx = \frac{\frac{\sqrt{c}x\left(16b^5c^3x^2(c+dx^2)^2+8ab^4c^2(3c-11dx^2)(c+dx^2)^2+3a^5d^4(5c+3dx^2)+3a^3b^2d^3x^2(-32c^2-23cdx^2+\right.}{a^2(bc-ad)^4(a+bx^2)^{3/2}(c-}}{}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

output
```
((Sqrt[c]*x*(16*b^5*c^3*x^2*(c + d*x^2)^2 + 8*a*b^4*c^2*(3*c - 11*d*x^2)*(
c + d*x^2)^2 + 3*a^5*d^4*(5*c + 3*d*x^2) + 3*a^3*b^2*d^3*x^2*(-32*c^2 - 23
*c*d*x^2 + 3*d^2*x^4) + 6*a^4*b*d^3*(-8*c^2 - 2*c*d*x^2 + 3*d^2*x^4) - 6*a
^2*b^3*c*d*(16*c^3 + 32*c^2*d*x^2 + 24*c*d^2*x^4 + 7*d^3*x^6)))/(a^2*(b*c
- a*d)^4*(a + b*x^2)^(3/2)*(c + d*x^2)^2) - (9*d^2*(-4*b*c + a*d)^2*ArcTan
[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d
])])/(-(b*c) + a*d)^(9/2) + (24*a*b*c*d^3*ArcTanh[(-(d*x*Sqrt[a + b*x^2])
+ Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[b*c - a*d])])/(b*c - a*d)^(9/2))/(24*
c^(5/2))
```

3.96.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {316, 402, 25, 402, 25, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}dx$$

$$\Big\downarrow \; 316$$

$$\frac{\int \frac{-6bdx^2+4bc-3ad}{(bx^2+a)^{5/2}(dx^2+c)^2}dx}{4c(bc-ad)} - \frac{dx}{4c(a+bx^2)^{3/2}(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{bx(3ad+4bc)}{3a(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}-\frac{\int -\frac{8b^2c^2-24abdc+9a^2d^2+4bd(4bc+3ad)x^2}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2}dx}{3a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{8b^2c^2-24abdc+9a^2d^2+4bd(4bc+3ad)x^2}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2}dx}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}-\frac{\int -\frac{d\left(2b\left(8b^2c^2-40abdc-3a^2d^2\right)x^2+a\left(8b^2c^2+36abdc-9a^2d^2\right)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{a(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{d\left(2b\left(8b^2c^2-40abdc-3a^2d^2\right)x^2+a\left(8b^2c^2+36abdc-9a^2d^2\right)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{d\int \frac{2b\left(8b^2c^2-40abdc-3a^2d^2\right)x^2+a\left(8b^2c^2+36abdc-9a^2d^2\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \; 402$$

3.96. $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx$

$$\frac{\frac{\frac{d\left(\frac{\int \frac{3a^2d\left(48b^2c^2-16abdc+3a^2d^2\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(9a^3d^3-42a^2bcd^2-88ab^2c^2d+16b^3c^3\right)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}}{4c(bc-ad)}+\frac{dx}{4c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{d\left(\frac{3a^2d\left(3a^2d^2-16abcd+48b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(9a^3d^3-42a^2bcd^2-88ab^2c^2d+16b^3c^3\right)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}}{4c(bc-ad)}+\frac{dx}{4c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

$\downarrow$ 291

$$\frac{\frac{\frac{d\left(\frac{3a^2d\left(3a^2d^2-16abcd+48b^2c^2\right)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(9a^3d^3-42a^2bcd^2-88ab^2c^2d+16b^3c^3\right)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}}{4c(bc-ad)}+\frac{dx}{4c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

$\downarrow$ 221

$$\frac{\frac{\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}+\frac{d\left(\frac{3a^2d\left(3a^2d^2-16abcd+48b^2c^2\right)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(9a^3d^3-42a^2bcd^2-88ab^2c^2d+16b^3c^3\right)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{a(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}}{4c(bc-ad)}+\frac{dx}{4c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

3.96. $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx$

output
```
-1/4*(d*x)/(c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^2) + ((b*(4*b*c +
3*a*d)*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)) + ((b*(8*b^2*c^2
 - 40*a*b*c*d - 3*a^2*d^2)*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2))
+ (d*(((16*b^3*c^3 - 88*a*b^2*c^2*d - 42*a^2*b*c*d^2 + 9*a^3*d^3)*x*Sqrt[a
 + b*x^2])/(2*c*(b*c - a*d)*(c + d*x^2)) + (3*a^2*d*(48*b^2*c^2 - 16*a*b*c
*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2
*c^(3/2)*(b*c - a*d)^(3/2))))/(a*(b*c - a*d)))/(3*a*(b*c - a*d)))/(4*c*(b*
c - a*d))
```

3.96.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.96.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{3\left(\left(b\,x^2+a\right)^{\frac{3}{2}}\left(d\,x^2+c\right)^2\left(a^2d^2-\frac{16}{3}abcd+16b^2c^2\right)d^2a^2\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)-\frac{5x\sqrt{(ad-bc)c}\left(\frac{3a^3x^2\left(b\,x^2+a\right)^2d^5}{5}+\left(b\,x^2+a\right)\right.}{\ldots}\right.}{8\left(b\,x^2+\ldots\right.}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8/(b*x^2+a)^(3/2)*((b*x^2+a)^(3/2)*(d*x^2+c)^2*(a^2*d^2-16/3*a*b*c*d+16
*b^2*c^2)*d^2*a^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))-5/3*x*((
a*d-b*c)*c)^(1/2)*(3/5*a^3*x^2*(b*x^2+a)^2*d^5+(b*x^2+a)^2*(-14/5*b*x^2+a)
*c*a^2*d^4-16/5*(11/6*b^3*x^6+3*a*b^2*x^4+2*a^2*b*x^2+a^3)*b*c^2*a*d^3-64/
5*x^2*b^3*(-1/12*b^2*x^4+19/24*a*b*x^2+a^2)*c^3*d^2-32/5*b^3*(-1/3*b^2*x^4
+5/12*a*b*x^2+a^2)*c^4*d+8/5*b^4*(2/3*b*x^2+a)*c^5))/((a*d-b*c)*c)^(1/2)/(
d*x^2+c)^2/c^2/(a*d-b*c)^4/a^2
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. 2(285) = 570.

Time = 2.76 (sec) , antiderivative size = 2250, normalized size of antiderivative = 7.19

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/96*(3*(48*a^4*b^2*c^4*d^2 - 16*a^5*b*c^3*d^3 + 3*a^6*c^2*d^4 + (48*a^2*
b^4*c^2*d^4 - 16*a^3*b^3*c*d^5 + 3*a^4*b^2*d^6)*x^8 + 2*(48*a^2*b^4*c^3*d^
3 + 32*a^3*b^3*c^2*d^4 - 13*a^4*b^2*c*d^5 + 3*a^5*b*d^6)*x^6 + (48*a^2*b^4
*c^4*d^2 + 176*a^3*b^3*c^3*d^3 - 13*a^4*b^2*c^2*d^4 - 4*a^5*b*c*d^5 + 3*a^
6*d^6)*x^4 + 2*(48*a^3*b^3*c^4*d^2 + 32*a^4*b^2*c^3*d^3 - 13*a^5*b*c^2*d^4
 + 3*a^6*c*d^5)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2
*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3
 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)
) + 4*((16*b^6*c^5*d^2 - 104*a*b^5*c^4*d^3 + 46*a^2*b^4*c^3*d^4 + 51*a^3*b
^3*c^2*d^5 - 9*a^4*b^2*c*d^6)*x^7 + (32*b^6*c^6*d - 184*a*b^5*c^5*d^2 + 8*
a^2*b^4*c^4*d^3 + 75*a^3*b^3*c^3*d^4 + 87*a^4*b^2*c^2*d^5 - 18*a^5*b*c*d^6
)*x^5 + (16*b^6*c^7 - 56*a*b^5*c^6*d - 152*a^2*b^4*c^5*d^2 + 96*a^3*b^3*c^
4*d^3 + 84*a^4*b^2*c^3*d^4 + 21*a^5*b*c^2*d^5 - 9*a^6*c*d^6)*x^3 + 3*(8*a*
b^5*c^7 - 40*a^2*b^4*c^6*d + 32*a^3*b^3*c^5*d^2 - 16*a^4*b^2*c^4*d^3 + 21*
a^5*b*c^3*d^4 - 5*a^6*c^2*d^5)*x)*sqrt(b*x^2 + a))/(a^4*b^5*c^10 - 5*a^5*b
^4*c^9*d + 10*a^6*b^3*c^8*d^2 - 10*a^7*b^2*c^7*d^3 + 5*a^8*b*c^6*d^4 - a^9
*c^5*d^5 + (a^2*b^7*c^8*d^2 - 5*a^3*b^6*c^7*d^3 + 10*a^4*b^5*c^6*d^4 - 10*
a^5*b^4*c^5*d^5 + 5*a^6*b^3*c^4*d^6 - a^7*b^2*c^3*d^7)*x^8 + 2*(a^2*b^7*c^
9*d - 4*a^3*b^6*c^8*d^2 + 5*a^4*b^5*c^7*d^3 - 5*a^6*b^3*c^5*d^5 + 4*a^7*b^
2*c^4*d^6 - a^8*b*c^3*d^7)*x^6 + (a^2*b^7*c^10 - a^3*b^6*c^9*d - 9*a^4*...
```

3.96.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.96.7 Maxima [F]

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\left(dx^2+c\right)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^3), x)
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. $2(285) = 570$.

Time = 1.72 (sec) , antiderivative size = 1010, normalized size of antiderivative = 3.23

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3}\,dx = \frac{\left(\frac{\left(2\,b^{10}c^5-19\,ab^9c^4d+56\,a^2b^8c^3d^2-74\,a^3b^7c^2d^3+46\,a^4b^6cd^4-11\,a^5b^5d^5\right)x^2}{a^2b^9c^8-8\,a^3b^8c^7d+28\,a^4b^7c^6d^2-56\,a^5b^6c^5d^3+70\,a^6b^5c^4d^4-56\,a^7b^4c^3d^5+28\,a^8b^3c^2d^6-8\,a^9b^2cd^7+a^{10}\cdots}\cdots}{3\cdots}$$

$$-\frac{\left(48\,b^{\frac{5}{2}}c^2d^2-16\,ab^{\frac{3}{2}}cd^3+3\,a^2\sqrt{b}d^4\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{8\left(b^4c^6-4\,ab^3c^5d+6\,a^2b^2c^4d^2-4\,a^3bc^3d^3+a^4c^2d^4\right)\sqrt{-b^2c^2+abcd}}$$

$$-\frac{24\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^{\frac{5}{2}}c^2d^3-16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{3}{2}}cd^4+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2\sqrt{b}d^5+112\left(\sqrt{\cdots}\right.\cdots}{\cdots}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/3*((2*b^10*c^5 - 19*a*b^9*c^4*d + 56*a^2*b^8*c^3*d^2 - 74*a^3*b^7*c^2*d^
3 + 46*a^4*b^6*c*d^4 - 11*a^5*b^5*d^5)*x^2/(a^2*b^9*c^8 - 8*a^3*b^8*c^7*d
+ 28*a^4*b^7*c^6*d^2 - 56*a^5*b^6*c^5*d^3 + 70*a^6*b^5*c^4*d^4 - 56*a^7*b^
4*c^3*d^5 + 28*a^8*b^3*c^2*d^6 - 8*a^9*b^2*c*d^7 + a^10*b*d^8) + 3*(a*b^9*
c^5 - 8*a^2*b^8*c^4*d + 22*a^3*b^7*c^3*d^2 - 28*a^4*b^6*c^2*d^3 + 17*a^5*b
^5*c*d^4 - 4*a^6*b^4*d^5)/(a^2*b^9*c^8 - 8*a^3*b^8*c^7*d + 28*a^4*b^7*c^6*
d^2 - 56*a^5*b^6*c^5*d^3 + 70*a^6*b^5*c^4*d^4 - 56*a^7*b^4*c^3*d^5 + 28*a^
8*b^3*c^2*d^6 - 8*a^9*b^2*c*d^7 + a^10*b*d^8))*x/(b*x^2 + a)^(3/2) - 1/8*(
48*b^(5/2)*c^2*d^2 - 16*a*b^(3/2)*c*d^3 + 3*a^2*sqrt(b)*d^4)*arctan(1/2*((
sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/
((b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*
d^4)*sqrt(-b^2*c^2 + a*b*c*d)) - 1/4*(24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b
^(5/2)*c^2*d^3 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d^4 + 3*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^5 + 112*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*b^(7/2)*c^3*d^2 - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*
c^2*d^3 + 66*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^4 - 9*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d^5 + 88*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*a^2*b^(5/2)*c^2*d^3 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c
*d^4 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d^5 + 14*a^4*b^(3/2)*
c*d^4 - 3*a^5*sqrt(b)*d^5)/((b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^...
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^3), x)
```

3.97 $\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^{11/2}}\,dx$

3.97.1 Optimal result

Integrand size = 21, antiderivative size = 224

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^{11/2}}\,dx=-\frac{dx\left(a+bx^2\right)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}}+\frac{(9bc-8ad)x\left(a+bx^2\right)^3}{63c^2(bc-ad)\left(c+dx^2\right)^{7/2}}+\frac{2a(9bc-8ad)x\left(a+bx^2\right)^2}{105c^3(bc-ad)\left(c+dx^2\right)^{5/2}}+\frac{8a^2(9bc-8ad)x\left(a+bx^2\right)}{315c^4(bc-ad)\left(c+dx^2\right)^{3/2}}+\frac{16a^3(9bc-8ad)x}{315c^5(bc-ad)\sqrt{c+dx^2}}$$

output
```
-1/9*d*x*(b*x^2+a)^4/c/(-a*d+b*c)/(d*x^2+c)^(9/2)+1/63*(-8*a*d+9*b*c)*x*(b
*x^2+a)^3/c^2/(-a*d+b*c)/(d*x^2+c)^(7/2)+2/105*a*(-8*a*d+9*b*c)*x*(b*x^2+a
)^2/c^3/(-a*d+b*c)/(d*x^2+c)^(5/2)+8/315*a^2*(-8*a*d+9*b*c)*x*(b*x^2+a)/c^
4/(-a*d+b*c)/(d*x^2+c)^(3/2)+16/315*a^3*(-8*a*d+9*b*c)*x/c^5/(-a*d+b*c)/(d
*x^2+c)^(1/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.73

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^{11/2}}\,dx=\frac{5b^3c^3x^7(9c+2dx^2)+3ab^2c^2x^5(63c^2+36cdx^2+8d^2x^4)+3a^2bcx^3(105c^3+126c^2dx^2+\cdots}{315c^5\,(c+\cdots}$$

input
```
Integrate[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```

output
```
(5*b^3*c^3*x^7*(9*c + 2*d*x^2) + 3*a*b^2*c^2*x^5*(63*c^2 + 36*c*d*x^2 + 8*
d^2*x^4) + 3*a^2*b*c*x^3*(105*c^3 + 126*c^2*d*x^2 + 72*c*d^2*x^4 + 16*d^3*
x^6) + a^3*(315*c^4*x + 840*c^3*d*x^3 + 1008*c^2*d^2*x^5 + 576*c*d^3*x^7 +
 128*d^4*x^9))/(315*c^5*(c + d*x^2)^(9/2))
```

3.97.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {296, 292, 292, 292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^{11/2}}dx$$

$$\Big\downarrow \text{ 296}$$

$$\frac{(9bc-8ad)\int \frac{\left(bx^2+a\right)^3}{\left(dx^2+c\right)^{9/2}}dx}{9c(bc-ad)}-\frac{dx\left(a+bx^2\right)^4}{9c\left(c+dx^2\right)^{9/2}(bc-ad)}$$

$$\Big\downarrow \text{ 292}$$

$$\frac{(9bc-8ad)\left(\frac{6a\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{7/2}}dx}{7c}+\frac{x\left(a+bx^2\right)^3}{7c(c+dx^2)^{7/2}}\right)}{9c(bc-ad)}-\frac{dx\left(a+bx^2\right)^4}{9c\left(c+dx^2\right)^{9/2}(bc-ad)}$$

$$\Big\downarrow \text{ 292}$$

$$\frac{(9bc-8ad)\left(\frac{6a\left(\frac{4a\int \frac{bx^2+a}{\left(dx^2+c\right)^{5/2}}dx}{5c}+\frac{x\left(a+bx^2\right)^2}{5c\left(c+dx^2\right)^{5/2}}\right)}{7c}+\frac{x\left(a+bx^2\right)^3}{7c(c+dx^2)^{7/2}}\right)}{9c(bc-ad)}-\frac{dx\left(a+bx^2\right)^4}{9c\left(c+dx^2\right)^{9/2}(bc-ad)}$$

$$\Big\downarrow \text{ 292}$$

$$\frac{(9bc-8ad)\left(\frac{6a\left(\frac{4a\left(\frac{2a\int \frac{1}{\left(dx^2+c\right)^{3/2}}dx}{3c}+\frac{x\left(a+bx^2\right)}{3c\left(c+dx^2\right)^{3/2}}\right)}{5c}+\frac{x\left(a+bx^2\right)^2}{5c\left(c+dx^2\right)^{5/2}}\right)}{7c}+\frac{x\left(a+bx^2\right)^3}{7c(c+dx^2)^{7/2}}\right)}{9c(bc-ad)}-\frac{dx\left(a+bx^2\right)^4}{9c\left(c+dx^2\right)^{9/2}(bc-ad)}$$

$$\Big\downarrow \text{208}$$

$$\frac{(9bc-8ad)\left(\frac{6a\left(\frac{4a\left(\frac{x\left(a+bx^2\right)}{3c\left(c+dx^2\right)^{3/2}}+\frac{2ax}{3c^2\sqrt{c+dx^2}}\right)}{5c}+\frac{x\left(a+bx^2\right)^2}{5c\left(c+dx^2\right)^{5/2}}\right)}{7c}+\frac{x\left(a+bx^2\right)^3}{7c(c+dx^2)^{7/2}}\right)}{9c(bc-ad)}-\frac{dx\left(a+bx^2\right)^4}{9c\left(c+dx^2\right)^{9/2}(bc-ad)}$$

input
```
Int[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```

output
```
-1/9*(d*x*(a + b*x^2)^4)/(c*(b*c - a*d)*(c + d*x^2)^(9/2)) + ((9*b*c - 8*a
*d)*((x*(a + b*x^2)^3)/(7*c*(c + d*x^2)^(7/2)) + (6*a*((x*(a + b*x^2)^2)/(
5*c*(c + d*x^2)^(5/2)) + (4*a*((x*(a + b*x^2))/(3*c*(c + d*x^2)^(3/2)) + (
2*a*x)/(3*c^2*Sqrt[c + d*x^2])))/(5*c)))/(7*c)))/(9*c*(b*c - a*d))
```

3.97. $\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^{11/2}}\,dx$

3.97.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.97.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{7}b^3x^6+\frac{3}{5}a\,b^2x^4+a^2b\,x^2+a^3\right)c^4+\frac{8x^2\left(\frac{1}{84}b^3x^6+\frac{9}{70}a\,b^2x^4+\frac{9}{20}a^2b\,x^2+a^3\right)d\,c^3}{3}+\frac{16x^4\left(\frac{1}{42}b^2x^4+\frac{3}{14}ab\,x^2+a^2\right)d^2a\,c^2}{5}+\frac{64x^6\left(\frac{b}{\cdots}\right.}{\cdots}\right.}{(d\,x^2+c)^{\frac{9}{2}}c^5}$
gosper	$\frac{x(128a^3d^4x^8+48a^2bc\,d^3x^8+24a\,b^2c^2d^2x^8+10b^3c^3d\,x^8+576a^3c\,d^3x^6+216a^2b\,c^2d^2x^6+108a\,b^2c^3d\,x^6+45b^3c^4x^6+1008a^3c^2\cdots}{315(d\,x^2+c)^{\frac{9}{2}}c^5}$
trager	$\frac{x(128a^3d^4x^8+48a^2bc\,d^3x^8+24a\,b^2c^2d^2x^8+10b^3c^3d\,x^8+576a^3c\,d^3x^6+216a^2b\,c^2d^2x^6+108a\,b^2c^3d\,x^6+45b^3c^4x^6+1008a^3c^2\cdots}{315(d\,x^2+c)^{\frac{9}{2}}c^5}$

3.97. $\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}dx$

$$\left(\ 8\left(\frac{6x}{35c\left(d\,x^2+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(d\,x^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{d\,x^2+c}}\right)}{7c}\right)\right)$$

input
```
int((b*x^2+a)^3/(d*x^2+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
x/(d*x^2+c)^(9/2)*((1/7*b^3*x^6+3/5*a*b^2*x^4+a^2*b*x^2+a^3)*c^4+8/3*x^2*(
1/84*b^3*x^6+9/70*a*b^2*x^4+9/20*a^2*b*x^2+a^3)*d*c^3+16/5*x^4*(1/42*b^2*x
^4+3/14*a*b*x^2+a^2)*d^2*a*c^2+64/35*x^6*(1/12*b*x^2+a)*d^3*a^2*c+128/315*
a^3*d^4*x^8)/c^5
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = \frac{(2\,(5\,b^3c^3d+12\,ab^2c^2d^2+24\,a^2bcd^3+64\,a^3d^4)x^9+315\,a^3c^4x+9\,(5\,b^3c^4+12\,ab^2c^3d+}{315\,(c^5d^5x^{10}+5\,c^6d^4x^8}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="fricas")
```

output
```
1/315*(2*(5*b^3*c^3*d + 12*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 64*a^3*d^4)*x^
9 + 315*a^3*c^4*x + 9*(5*b^3*c^4 + 12*a*b^2*c^3*d + 24*a^2*b*c^2*d^2 + 64*
a^3*c*d^3)*x^7 + 63*(3*a*b^2*c^4 + 6*a^2*b*c^3*d + 16*a^3*c^2*d^2)*x^5 + 1
05*(3*a^2*b*c^4 + 8*a^3*c^3*d)*x^3)*sqrt(d*x^2 + c)/(c^5*d^5*x^10 + 5*c^6*
d^4*x^8 + 10*c^7*d^3*x^6 + 10*c^8*d^2*x^4 + 5*c^9*d*x^2 + c^10)
```

3.97.6 Sympy [F]

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = \int \frac{(a+bx^2)^3}{(c+dx^2)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**3/(d*x**2+c)**(11/2),x)
```

output
```
Integral((a + b*x**2)**3/(c + d*x**2)**(11/2), x)
```

3.97.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(204) = 408$.

Time = 0.22 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.08

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = -\frac{b^3x^5}{4\,(dx^2+c)^{\frac{9}{2}}d} - \frac{5\,b^3cx^3}{24\,(dx^2+c)^{\frac{9}{2}}d^2} - \frac{ab^2x^3}{2\,(dx^2+c)^{\frac{9}{2}}d} + \frac{128\,a^3x}{315\,\sqrt{dx^2+c}c^5} + \frac{64\,a^3x}{315\,(dx^2+c)^{\frac{3}{2}}c^4} + \frac{16\,a^3x}{105\,(dx^2+c)^{\frac{5}{2}}c^3} + \frac{8\,a^3x}{63\,(dx^2+c)^{\frac{7}{2}}c^2} + \frac{a^3x}{9\,(dx^2+c)^{\frac{9}{2}}c} + \frac{b^3x}{84\,(dx^2+c)^{\frac{5}{2}}d^3} + \frac{2\,b^3x}{63\,\sqrt{dx^2+c}c^2d^3} + \frac{b^3x}{63\,(dx^2+c)^{\frac{3}{2}}cd^3} + \frac{5\,b^3cx}{504\,(dx^2+c)^{\frac{7}{2}}d^3} - \frac{5\,b^3c^2x}{72\,(dx^2+c)^{\frac{9}{2}}d^3} + \frac{ab^2x}{42\,(dx^2+c)^{\frac{7}{2}}d^2} + \frac{8\,ab^2x}{105\,\sqrt{dx^2+c}c^3d^2} + \frac{4\,ab^2x}{105\,(dx^2+c)^{\frac{3}{2}}c^2d^2} + \frac{ab^2x}{35\,(dx^2+c)^{\frac{5}{2}}cd^2} - \frac{ab^2cx}{6\,(dx^2+c)^{\frac{9}{2}}d^2} - \frac{a^2bx}{3\,(dx^2+c)^{\frac{9}{2}}d} + \frac{16\,a^2bx}{105\,\sqrt{dx^2+c}c^4d} + \frac{8\,a^2bx}{105\,(dx^2+c)^{\frac{3}{2}}c^3d} + \frac{2\,a^2bx}{35\,(dx^2+c)^{\frac{5}{2}}c^2d} + \frac{a^2bx}{21\,(dx^2+c)^{\frac{7}{2}}cd}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="maxima")
```

output
```
-1/4*b^3*x^5/((d*x^2 + c)^(9/2)*d) - 5/24*b^3*c*x^3/((d*x^2 + c)^(9/2)*d^2
) - 1/2*a*b^2*x^3/((d*x^2 + c)^(9/2)*d) + 128/315*a^3*x/(sqrt(d*x^2 + c)*c
^5) + 64/315*a^3*x/((d*x^2 + c)^(3/2)*c^4) + 16/105*a^3*x/((d*x^2 + c)^(5/
2)*c^3) + 8/63*a^3*x/((d*x^2 + c)^(7/2)*c^2) + 1/9*a^3*x/((d*x^2 + c)^(9/2
)*c) + 1/84*b^3*x/((d*x^2 + c)^(5/2)*d^3) + 2/63*b^3*x/(sqrt(d*x^2 + c)*c^
2*d^3) + 1/63*b^3*x/((d*x^2 + c)^(3/2)*c*d^3) + 5/504*b^3*c*x/((d*x^2 + c)
^(7/2)*d^3) - 5/72*b^3*c^2*x/((d*x^2 + c)^(9/2)*d^3) + 1/42*a*b^2*x/((d*x^
2 + c)^(7/2)*d^2) + 8/105*a*b^2*x/(sqrt(d*x^2 + c)*c^3*d^2) + 4/105*a*b^2*
x/((d*x^2 + c)^(3/2)*c^2*d^2) + 1/35*a*b^2*x/((d*x^2 + c)^(5/2)*c*d^2) - 1
/6*a*b^2*c*x/((d*x^2 + c)^(9/2)*d^2) - 1/3*a^2*b*x/((d*x^2 + c)^(9/2)*d) +
 16/105*a^2*b*x/(sqrt(d*x^2 + c)*c^4*d) + 8/105*a^2*b*x/((d*x^2 + c)^(3/2)
*c^3*d) + 2/35*a^2*b*x/((d*x^2 + c)^(5/2)*c^2*d) + 1/21*a^2*b*x/((d*x^2 +
c)^(7/2)*c*d)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = \frac{\left(\left(\left(\left(x^2\left(\frac{2\,(5\,b^3c^3d^5+12\,ab^2c^2d^6+24\,a^2bcd^7+64\,a^3d^8)x^2}{c^5d^4}+\frac{9\,(5\,b^3c^4d^4+12\,ab^2c^3d^5+24\,a^2bc^2d^6+64\,a^3cd^7)}{c^5d^4}\right)+\right.\right.\right.\right.}{315\,(dx^2+c)^{\frac{9}{2}}}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="giac")
```

output
```
1/315*(((x^2*(2*(5*b^3*c^3*d^5 + 12*a*b^2*c^2*d^6 + 24*a^2*b*c*d^7 + 64*a^
3*d^8)*x^2/(c^5*d^4) + 9*(5*b^3*c^4*d^4 + 12*a*b^2*c^3*d^5 + 24*a^2*b*c^2*
d^6 + 64*a^3*c*d^7)/(c^5*d^4)) + 63*(3*a*b^2*c^4*d^4 + 6*a^2*b*c^3*d^5 + 1
6*a^3*c^2*d^6)/(c^5*d^4))*x^2 + 105*(3*a^2*b*c^4*d^4 + 8*a^3*c^3*d^5)/(c^5
*d^4))*x^2 + 315*a^3/c)*x/(d*x^2 + c)^(9/2)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 5.04 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = \frac{x\left(\frac{a^3}{9\,c}-\frac{c\left(\frac{c\left(\frac{b^3}{9\,d}-\frac{a\,b^2}{3\,c}\right)}{d}+\frac{a^2\,b}{3\,c}\right)}{d}\right)}{(d\,x^2+c)^{9/2}} - \frac{x\left(\frac{b^3}{5\,d^3}-\frac{16\,a^3\,d^3+6\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-4\,b^3\,c^3}{105\,c^3\,d^3}\right)}{(d\,x^2+c)^{5/2}} + \frac{x\left(\frac{c\left(\frac{b^3}{7\,d^2}-\frac{b^2\,(3\,a\,d-b\,c)}{7\,c\,d^2}\right)}{d}+\frac{8\,a^3\,d^3+3\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+b^3\,c^3}{63\,c^2\,d^3}\right)}{(d\,x^2+c)^{7/2}} + \frac{x\,(64\,a^3\,d^3+24\,a^2\,b\,c\,d^2+12\,a\,b^2\,c^2\,d+5\,b^3\,c^3)}{315\,c^4\,d^3\,(d\,x^2+c)^{3/2}} + \frac{x\,(128\,a^3\,d^3+48\,a^2\,b\,c\,d^2+24\,a\,b^2\,c^2\,d+10\,b^3\,c^3)}{315\,c^5\,d^3\,\sqrt{d\,x^2+c}}$$

input
```
int((a + b*x^2)^3/(c + d*x^2)^(11/2),x)
```

output
```
(x*(a^3/(9*c) - (c*((c*(b^3/(9*d) - (a*b^2)/(3*c)))/d + (a^2*b)/(3*c)))/d)
)/(c + d*x^2)^(9/2) - (x*(b^3/(5*d^3) - (16*a^3*d^3 - 4*b^3*c^3 + 3*a*b^2*
c^2*d + 6*a^2*b*c*d^2)/(105*c^3*d^3)))/(c + d*x^2)^(5/2) + (x*((c*(b^3/(7*
d^2) - (b^2*(3*a*d - b*c))/(7*c*d^2)))/d + (8*a^3*d^3 + b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2)/(63*c^2*d^3)))/(c + d*x^2)^(7/2) + (x*(64*a^3*d^3 +
 5*b^3*c^3 + 12*a*b^2*c^2*d + 24*a^2*b*c*d^2))/(315*c^4*d^3*(c + d*x^2)^(3
/2)) + (x*(128*a^3*d^3 + 10*b^3*c^3 + 24*a*b^2*c^2*d + 48*a^2*b*c*d^2))/(3
15*c^5*d^3*(c + d*x^2)^(1/2))
```

3.97. $\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx$

3.98 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx$

3.98.1 Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = -\frac{dx(a+bx^2)^3}{7c(bc-ad)\,(c+dx^2)^{7/2}} + \frac{(7bc-6ad)x(a+bx^2)^2}{35c^2(bc-ad)\,(c+dx^2)^{5/2}} + \frac{4a(7bc-6ad)x(a+bx^2)}{105c^3(bc-ad)\,(c+dx^2)^{3/2}} + \frac{8a^2(7bc-6ad)x}{105c^4(bc-ad)\sqrt{c+dx^2}}$$

output
```
-1/7*d*x*(b*x^2+a)^3/c/(-a*d+b*c)/(d*x^2+c)^(7/2)+1/35*(-6*a*d+7*b*c)*x*(b
*x^2+a)^2/c^2/(-a*d+b*c)/(d*x^2+c)^(5/2)+4/105*a*(-6*a*d+7*b*c)*x*(b*x^2+a
)/c^3/(-a*d+b*c)/(d*x^2+c)^(3/2)+8/105*a^2*(-6*a*d+7*b*c)*x/c^4/(-a*d+b*c)
/(d*x^2+c)^(1/2)
```

3.98.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{3b^2c^2x^5(7c+2dx^2)+2abcx^3(35c^2+28cdx^2+8d^2x^4)+3a^2(35c^3x+70c^2dx^3+56cd^2x}{105c^4\,(c+dx^2)^{7/2}}$$

input
```
Integrate[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
```

output
```
(3*b^2*c^2*x^5*(7*c + 2*d*x^2) + 2*a*b*c*x^3*(35*c^2 + 28*c*d*x^2 + 8*d^2*
x^4) + 3*a^2*(35*c^3*x + 70*c^2*d*x^3 + 56*c*d^2*x^5 + 16*d^3*x^7))/(105*c
^4*(c + d*x^2)^(7/2))
```

3.98.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {296, 292, 292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{296}$$

$$\frac{(7bc-6ad)\int \frac{(bx^2+a)^2}{(dx^2+c)^{7/2}}dx}{7c(bc-ad)}-\frac{dx(a+bx^2)^3}{7c(c+dx^2)^{7/2}(bc-ad)}$$

$$\Big\downarrow \text{292}$$

$$\frac{(7bc-6ad)\left(\frac{4a\int \frac{bx^2+a}{(dx^2+c)^{5/2}}dx}{5c}+\frac{x(a+bx^2)^2}{5c(c+dx^2)^{5/2}}\right)}{7c(bc-ad)}-\frac{dx(a+bx^2)^3}{7c(c+dx^2)^{7/2}(bc-ad)}$$

$$\Big\downarrow \text{292}$$

$$\frac{(7bc-6ad)\left(\frac{4a\left(\frac{2a\int \frac{1}{(dx^2+c)^{3/2}}dx}{3c}+\frac{x(a+bx^2)}{3c(c+dx^2)^{3/2}}\right)}{5c}+\frac{x(a+bx^2)^2}{5c(c+dx^2)^{5/2}}\right)}{7c(bc-ad)}-\frac{dx(a+bx^2)^3}{7c(c+dx^2)^{7/2}(bc-ad)}$$

$$\Big\downarrow \text{208}$$

$$\frac{(7bc-6ad)\left(\frac{4a\left(\frac{x(a+bx^2)}{3c(c+dx^2)^{3/2}}+\frac{2ax}{3c^2\sqrt{c+dx^2}}\right)}{5c}+\frac{x(a+bx^2)^2}{5c(c+dx^2)^{5/2}}\right)}{7c(bc-ad)}-\frac{dx(a+bx^2)^3}{7c(c+dx^2)^{7/2}(bc-ad)}$$

3.98. $\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx$

input
```
Int[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
```

output
```
-1/7*(d*x*(a + b*x^2)^3)/(c*(b*c - a*d)*(c + d*x^2)^(7/2)) + ((7*b*c - 6*a
*d)*((x*(a + b*x^2)^2)/(5*c*(c + d*x^2)^(5/2)) + (4*a*((x*(a + b*x^2))/(3*
c*(c + d*x^2)^(3/2)) + (2*a*x)/(3*c^2*Sqrt[c + d*x^2])))/(5*c)))/(7*c*(b*c
 - a*d))
```

3.98.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.98.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{5}b^2x^4+\frac{2}{3}ab\,x^2+a^2\right)c^3+2x^2d\left(\frac{1}{35}b^2x^4+\frac{4}{15}ab\,x^2+a^2\right)c^2+\frac{8x^4d^2a\left(\frac{2b\,x^2}{21}+a\right)c}{5}+\frac{16a^2d^3x^6}{35}\right)}{\left(d\,x^2+c\right)^{\frac{7}{2}}c^4}$
gosper	$\frac{x\left(48a^2d^3x^6+16abc\,d^2x^6+6b^2c^2d\,x^6+168a^2c\,d^2x^4+56ab\,c^2d\,x^4+21b^2c^3x^4+210a^2c^2d\,x^2+70ab\,c^3x^2+105a^2c^3\right)}{105\left(d\,x^2+c\right)^{\frac{7}{2}}c^4}$
trager	$\frac{x\left(48a^2d^3x^6+16abc\,d^2x^6+6b^2c^2d\,x^6+168a^2c\,d^2x^4+56ab\,c^2d\,x^4+21b^2c^3x^4+210a^2c^2d\,x^2+70ab\,c^3x^2+105a^2c^3\right)}{105\left(d\,x^2+c\right)^{\frac{7}{2}}c^4}$
default	$a^2\left(\frac{x}{7c\left(d\,x^2+c\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35c\left(d\,x^2+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(d\,x^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{d\,x^2+c}}\right)}{7c}}{c}\right)+b^2\left(-\frac{x^3}{4d\left(d\,x^2+c\right)^{\frac{7}{2}}}+\frac{3c\left(-\frac{x}{6d\left(d\,x^2+c\right)}\right.}{}\right.$

input
```
int((b*x^2+a)^2/(d*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
x*((1/5*b^2*x^4+2/3*a*b*x^2+a^2)*c^3+2*x^2*d*(1/35*b^2*x^4+4/15*a*b*x^2+a^
2)*c^2+8/5*x^4*d^2*a*(2/21*b*x^2+a)*c+16/35*a^2*d^3*x^6)/(d*x^2+c)^(7/2)/c
^4
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{(2\,(3\,b^2c^2d+8\,abcd^2+24\,a^2d^3)x^7+105\,a^2c^3x+7\,(3\,b^2c^3+8\,abc^2d+24\,a^2cd^2)x^5+70}{105\,(c^4d^4x^8+4\,c^5d^3x^6+6\,c^6d^2x^4+4\,c^7dx^2+c^8)}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(2*(3*b^2*c^2*d + 8*a*b*c*d^2 + 24*a^2*d^3)*x^7 + 105*a^2*c^3*x + 7*
(3*b^2*c^3 + 8*a*b*c^2*d + 24*a^2*c*d^2)*x^5 + 70*(a*b*c^3 + 3*a^2*c^2*d)*
x^3)*sqrt(d*x^2 + c)/(c^4*d^4*x^8 + 4*c^5*d^3*x^6 + 6*c^6*d^2*x^4 + 4*c^7*
d*x^2 + c^8)
```

3.98.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = \int \frac{(a+bx^2)^2}{(c+dx^2)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**(9/2),x)
```

output
```
Integral((a + b*x**2)**2/(c + d*x**2)**(9/2), x)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = -\frac{b^2x^3}{4\,(dx^2+c)^{\frac{7}{2}}d} + \frac{16\,a^2x}{35\,\sqrt{dx^2+c}c^4} + \frac{8\,a^2x}{35\,(dx^2+c)^{\frac{3}{2}}c^3} + \frac{6\,a^2x}{35\,(dx^2+c)^{\frac{5}{2}}c^2} + \frac{a^2x}{7\,(dx^2+c)^{\frac{7}{2}}c} + \frac{3\,b^2x}{140\,(dx^2+c)^{\frac{5}{2}}d^2} + \frac{2\,b^2x}{35\,\sqrt{dx^2+c}c^2d^2} + \frac{b^2x}{35\,(dx^2+c)^{\frac{3}{2}}cd^2} - \frac{3\,b^2cx}{28\,(dx^2+c)^{\frac{7}{2}}d^2} - \frac{2\,abx}{7\,(dx^2+c)^{\frac{7}{2}}d} + \frac{16\,abx}{105\,\sqrt{dx^2+c}c^3d} + \frac{8\,abx}{105\,(dx^2+c)^{\frac{3}{2}}c^2d} + \frac{2\,abx}{35\,(dx^2+c)^{\frac{5}{2}}cd}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
-1/4*b^2*x^3/((d*x^2 + c)^(7/2)*d) + 16/35*a^2*x/(sqrt(d*x^2 + c)*c^4) + 8
/35*a^2*x/((d*x^2 + c)^(3/2)*c^3) + 6/35*a^2*x/((d*x^2 + c)^(5/2)*c^2) + 1
/7*a^2*x/((d*x^2 + c)^(7/2)*c) + 3/140*b^2*x/((d*x^2 + c)^(5/2)*d^2) + 2/3
5*b^2*x/(sqrt(d*x^2 + c)*c^2*d^2) + 1/35*b^2*x/((d*x^2 + c)^(3/2)*c*d^2) -
 3/28*b^2*c*x/((d*x^2 + c)^(7/2)*d^2) - 2/7*a*b*x/((d*x^2 + c)^(7/2)*d) +
16/105*a*b*x/(sqrt(d*x^2 + c)*c^3*d) + 8/105*a*b*x/((d*x^2 + c)^(3/2)*c^2*
d) + 2/35*a*b*x/((d*x^2 + c)^(5/2)*c*d)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{2\,(3\,b^2c^2d^4+8\,abcd^5+24\,a^2d^6)x^2}{c^4d^3} + \frac{7\,(3\,b^2c^3d^3+8\,abc^2d^4+24\,a^2cd^5)}{c^4d^3}\right) + \frac{70\,(abc^3d^3+3\,a^2c^2d^4)}{c^4d^3}\right)x^2 + 1\right.}{105\,(dx^2+c)^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*(2*(3*b^2*c^2*d^4 + 8*a*b*c*d^5 + 24*a^2*d^6)*x^2/(c^4*d^3) +
7*(3*b^2*c^3*d^3 + 8*a*b*c^2*d^4 + 24*a^2*c*d^5)/(c^4*d^3)) + 70*(a*b*c^3*
d^3 + 3*a^2*c^2*d^4)/(c^4*d^3))*x^2 + 105*a^2/c)*x/(d*x^2 + c)^(7/2)
```

3.98.9 Mupad [B] (verification not implemented)

Time = 5.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{x\left(\frac{a^2}{7c}+\frac{c\left(\frac{b^2}{7d}-\frac{2ab}{7c}\right)}{d}\right)}{(dx^2+c)^{7/2}} - \frac{x\left(\frac{b^2}{5d^2}-\frac{6a^2d^2+2abcd-b^2c^2}{35c^2d^2}\right)}{(dx^2+c)^{5/2}} + \frac{x\,(24a^2d^2+8abcd+3b^2c^2)}{105c^3d^2\,(dx^2+c)^{3/2}} + \frac{x\,(48a^2d^2+16abcd+6b^2c^2)}{105c^4d^2\sqrt{dx^2+c}}$$

input

```
int((a + b*x^2)^2/(c + d*x^2)^(9/2),x)
```

output

```
(x*(a^2/(7*c) + (c*(b^2/(7*d) - (2*a*b)/(7*c)))/d))/(c + d*x^2)^(7/2) - (x
*(b^2/(5*d^2) - (6*a^2*d^2 - b^2*c^2 + 2*a*b*c*d)/(35*c^2*d^2)))/(c + d*x^
2)^(5/2) + (x*(24*a^2*d^2 + 3*b^2*c^2 + 8*a*b*c*d))/(105*c^3*d^2*(c + d*x^
2)^(3/2)) + (x*(48*a^2*d^2 + 6*b^2*c^2 + 16*a*b*c*d))/(105*c^4*d^2*(c + d*
x^2)^(1/2))
```

3.99 $\int \frac{a+bx^2}{(c+dx^2)^{7/2}} dx$

3.99.1 Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}} dx = -\frac{(bc-ad)x}{5cd(c+dx^2)^{5/2}} + \frac{(bc+4ad)x}{15c^2d(c+dx^2)^{3/2}} + \frac{2(bc+4ad)x}{15c^3d\sqrt{c+dx^2}}$$

output
```
-1/5*(-a*d+b*c)*x/c/d/(d*x^2+c)^(5/2)+1/15*(4*a*d+b*c)*x/c^2/d/(d*x^2+c)^(
3/2)+2/15*(4*a*d+b*c)*x/c^3/d/(d*x^2+c)^(1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}} dx = \frac{15ac^2x + 5bc^2x^3 + 20acdx^3 + 2bcdx^5 + 8ad^2x^5}{15c^3(c+dx^2)^{5/2}}$$

input
```
Integrate[(a + b*x^2)/(c + d*x^2)^(7/2),x]
```

output
```
(15*a*c^2*x + 5*b*c^2*x^3 + 20*a*c*d*x^3 + 2*b*c*d*x^5 + 8*a*d^2*x^5)/(15*
c^3*(c + d*x^2)^(5/2))
```

3.99.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {298, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx$$
$$\Big\downarrow \text{ 298}$$
$$\frac{(4ad+bc)\int \frac{1}{(dx^2+c)^{5/2}}dx}{5cd}-\frac{x(bc-ad)}{5cd\,(c+dx^2)^{5/2}}$$
$$\Big\downarrow \text{ 209}$$
$$\frac{(4ad+bc)\left(\frac{2\int \frac{1}{(dx^2+c)^{3/2}}dx}{3c}+\frac{x}{3c(c+dx^2)^{3/2}}\right)}{5cd}-\frac{x(bc-ad)}{5cd\,(c+dx^2)^{5/2}}$$
$$\Big\downarrow \text{ 208}$$
$$\frac{\left(\frac{2x}{3c^2\sqrt{c+dx^2}}+\frac{x}{3c(c+dx^2)^{3/2}}\right)(4ad+bc)}{5cd}-\frac{x(bc-ad)}{5cd\,(c+dx^2)^{5/2}}$$

input
```
Int[(a + b*x^2)/(c + d*x^2)^(7/2),x]
```

output
```
-1/5*((b*c - a*d)*x)/(c*d*(c + d*x^2)^(5/2)) + ((b*c + 4*a*d)*(x/(3*c*(c +
 d*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + d*x^2])))/(5*c*d)
```

3.99.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.99.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{b\,x^{2}}{3}+a\right)c^{2}+\frac{4x^{2}d\left(\frac{b\,x^{2}}{10}+a\right)c}{3}+\frac{8a\,d^{2}x^{4}}{15}\right)}{\left(d\,x^{2}+c\right)^{\frac{5}{2}}c^{3}}$
gosper	$\frac{x\left(8a\,d^{2}x^{4}+2bcd\,x^{4}+20acd\,x^{2}+5b\,c^{2}x^{2}+15c^{2}a\right)}{15\left(d\,x^{2}+c\right)^{\frac{5}{2}}c^{3}}$
trager	$\frac{x\left(8a\,d^{2}x^{4}+2bcd\,x^{4}+20acd\,x^{2}+5b\,c^{2}x^{2}+15c^{2}a\right)}{15\left(d\,x^{2}+c\right)^{\frac{5}{2}}c^{3}}$
default	$a\left(\frac{x}{5c\left(d\,x^{2}+c\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15c\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^{2}\sqrt{d\,x^{2}+c}}}{c}\right)+b\left(-\frac{x}{4d\left(d\,x^{2}+c\right)^{\frac{5}{2}}}+\frac{c\left(\frac{x}{5c\left(d\,x^{2}+c\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15c\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{8\ldots}{15c^{2}\sqrt{\ldots}}}{c}\ldots\right.}{4d}\ldots\right.$

input
```
int((b*x^2+a)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
x/(d*x^2+c)^(5/2)*((1/3*b*x^2+a)*c^2+4/3*x^2*d*(1/10*b*x^2+a)*c+8/15*a*d^2
*x^4)/c^3
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{7/2}}\,dx=\frac{\left(2\,(bcd+4\,ad^2)x^5+15\,ac^2x+5\,(bc^2+4\,acd)x^3\right)\sqrt{dx^2+c}}{15\,(c^3d^3x^6+3\,c^4d^2x^4+3\,c^5dx^2+c^6)}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(2*(b*c*d + 4*a*d^2)*x^5 + 15*a*c^2*x + 5*(b*c^2 + 4*a*c*d)*x^3)*sqrt
(d*x^2 + c)/(c^3*d^3*x^6 + 3*c^4*d^2*x^4 + 3*c^5*d*x^2 + c^6)
```

3.99.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(83) = 166$.

Time = 10.37 (sec) , antiderivative size = 566, normalized size of antiderivative = 6.22

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx = a\left(\frac{15c^5x}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}} + \frac{35c^4dx^3}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}} + \frac{28c^3d^2x^5}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}} + \frac{8c^2d^3x^7}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}\right) + b\left(\frac{5cx^3}{15c^{\frac{9}{2}}\sqrt{1+\frac{dx^2}{c}}+30c^{\frac{7}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{5}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}} + \frac{2dx^5}{15c^{\frac{9}{2}}\sqrt{1+\frac{dx^2}{c}}+30c^{\frac{7}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{5}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}}\right)$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**(7/2),x)
```

output
```
a*(15*c**5*x/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x**2*sqrt(1
 + d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)*d*
*3*x**6*sqrt(1 + d*x**2/c)) + 35*c**4*d*x**3/(15*c**(17/2)*sqrt(1 + d*x**2
/c) + 45*c**(15/2)*d*x**2*sqrt(1 + d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt
(1 + d*x**2/c) + 15*c**(11/2)*d**3*x**6*sqrt(1 + d*x**2/c)) + 28*c**3*d**2
*x**5/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x**2*sqrt(1 + d*x*
*2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)*d**3*x**6
*sqrt(1 + d*x**2/c)) + 8*c**2*d**3*x**7/(15*c**(17/2)*sqrt(1 + d*x**2/c) +
 45*c**(15/2)*d*x**2*sqrt(1 + d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 +
d*x**2/c) + 15*c**(11/2)*d**3*x**6*sqrt(1 + d*x**2/c))) + b*(5*c*x**3/(15*
c**(9/2)*sqrt(1 + d*x**2/c) + 30*c**(7/2)*d*x**2*sqrt(1 + d*x**2/c) + 15*c
**(5/2)*d**2*x**4*sqrt(1 + d*x**2/c)) + 2*d*x**5/(15*c**(9/2)*sqrt(1 + d*x
**2/c) + 30*c**(7/2)*d*x**2*sqrt(1 + d*x**2/c) + 15*c**(5/2)*d**2*x**4*sqr
t(1 + d*x**2/c)))
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx = \frac{8\,ax}{15\,\sqrt{dx^2+c}c^3} + \frac{4\,ax}{15\,(dx^2+c)^{\frac{3}{2}}c^2} + \frac{ax}{5\,(dx^2+c)^{\frac{5}{2}}c} - \frac{bx}{5\,(dx^2+c)^{\frac{5}{2}}d} + \frac{2\,bx}{15\,\sqrt{dx^2+c}c^2d} + \frac{bx}{15\,(dx^2+c)^{\frac{3}{2}}cd}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
8/15*a*x/(sqrt(d*x^2 + c)*c^3) + 4/15*a*x/((d*x^2 + c)^(3/2)*c^2) + 1/5*a*
x/((d*x^2 + c)^(5/2)*c) - 1/5*b*x/((d*x^2 + c)^(5/2)*d) + 2/15*b*x/(sqrt(d
*x^2 + c)*c^2*d) + 1/15*b*x/((d*x^2 + c)^(3/2)*c*d)
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx = \frac{\left(x^2\left(\frac{2\,(bcd^3+4\,ad^4)x^2}{c^3d^2} + \frac{5\,(bc^2d^2+4\,acd^3)}{c^3d^2}\right) + \frac{15\,a}{c}\right)x}{15\,(dx^2+c)^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
1/15*(x^2*(2*(b*c*d^3 + 4*a*d^4)*x^2/(c^3*d^2) + 5*(b*c^2*d^2 + 4*a*c*d^3)
/(c^3*d^2)) + 15*a/c)*x/(d*x^2 + c)^(5/2)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx = \frac{8\,a\,d\,x\,(d\,x^2+c)^2-3\,b\,c^3\,x+2\,b\,c\,x\,(d\,x^2+c)^2+b\,c^2\,x\,(d\,x^2+c)+3\,a\,c^2\,d\,x+4\,a\,c\,d\,x}{15\,c^3\,d\,(d\,x^2+c)^{5/2}}$$

input
```
int((a + b*x^2)/(c + d*x^2)^(7/2),x)
```

output
```
(8*a*d*x*(c + d*x^2)^2 - 3*b*c^3*x + 2*b*c*x*(c + d*x^2)^2 + b*c^2*x*(c +
d*x^2) + 3*a*c^2*d*x + 4*a*c*d*x*(c + d*x^2))/(15*c^3*d*(c + d*x^2)^(5/2))
```

3.100 $\int \frac{1}{(c+dx^2)^{5/2}}\,dx$

3.100.1 Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{(c+dx^2)^{5/2}}\,dx=\frac{x}{3c(c+dx^2)^{3/2}}+\frac{2x}{3c^2\sqrt{c+dx^2}}$$

output
```
1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2)
```

3.100.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{(c+dx^2)^{5/2}}\,dx=\frac{3cx+2dx^3}{3c^2(c+dx^2)^{3/2}}$$

input
```
Integrate[(c + d*x^2)^(-5/2),x]
```

output
```
(3*c*x + 2*d*x^3)/(3*c^2*(c + d*x^2)^(3/2))
```

3.100.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 209}$$

$$\frac{2\int \frac{1}{(dx^2+c)^{3/2}}dx}{3c}+\frac{x}{3c\,(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{2x}{3c^2\sqrt{c+dx^2}}+\frac{x}{3c\,(c+dx^2)^{3/2}}$$

input
```
Int[(c + d*x^2)^(-5/2),x]
```

output
```
x/(3*c*(c + d*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + d*x^2])
```

3.100.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

3.100.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x(2d\,x^{2}+3c)}{3(d\,x^{2}+c)^{\frac{3}{2}}c^{2}}$	26
trager	$\frac{x(2d\,x^{2}+3c)}{3(d\,x^{2}+c)^{\frac{3}{2}}c^{2}}$	26
pseudoelliptic	$\frac{x(2d\,x^{2}+3c)}{3(d\,x^{2}+c)^{\frac{3}{2}}c^{2}}$	26
default	$\frac{x}{3c(d\,x^{2}+c)^{\frac{3}{2}}}+\frac{2x}{3c^{2}\sqrt{d\,x^{2}+c}}$	32

input
```
int(1/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(2*d*x^2+3*c)/(d*x^2+c)^(3/2)/c^2
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{1}{(c+dx^2)^{5/2}}\,dx=\frac{(2\,dx^3+3\,cx)\sqrt{dx^2+c}}{3\,(c^2d^2x^4+2\,c^3dx^2+c^4)}$$

input
```
integrate(1/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*d*x^3 + 3*c*x)*sqrt(d*x^2 + c)/(c^2*d^2*x^4 + 2*c^3*d*x^2 + c^4)
```

3.100.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(32) = 64.

Time = 0.52 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \frac{1}{(c+dx^2)^{5/2}}\,dx=\frac{3cx}{3c^{\frac{7}{2}}\sqrt{1+\frac{dx^2}{c}}+3c^{\frac{5}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}}+\frac{2dx^3}{3c^{\frac{7}{2}}\sqrt{1+\frac{dx^2}{c}}+3c^{\frac{5}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}}$$

input
```
integrate(1/(d*x**2+c)**(5/2),x)
```

output
```
3*c*x/(3*c**(7/2)*sqrt(1 + d*x**2/c) + 3*c**(5/2)*d*x**2*sqrt(1 + d*x**2/c
)) + 2*d*x**3/(3*c**(7/2)*sqrt(1 + d*x**2/c) + 3*c**(5/2)*d*x**2*sqrt(1 +
d*x**2/c))
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(c+dx^2\right)^{5/2}}\,dx=\frac{2\,x}{3\,\sqrt{dx^2+c}c^2}+\frac{x}{3\,\left(dx^2+c\right)^{\frac{3}{2}}c}$$

input
```
integrate(1/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x/(sqrt(d*x^2 + c)*c^2) + 1/3*x/((d*x^2 + c)^(3/2)*c)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(c+dx^2\right)^{5/2}}\,dx=\frac{x\left(\frac{2\,dx^2}{c^2}+\frac{3}{c}\right)}{3\,\left(dx^2+c\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(2*d*x^2/c^2 + 3/c)/(d*x^2 + c)^(3/2)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{1}{(c+dx^2)^{5/2}}\,dx = \frac{2\,x\,(d\,x^2+c)+c\,x}{3\,c^2\,(d\,x^2+c)^{3/2}}$$

input
```
int(1/(c + d*x^2)^(5/2),x)
```

output
```
(2*x*(c + d*x^2) + c*x)/(3*c^2*(c + d*x^2)^(3/2))
```

3.101 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

3.101.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}$$

output
```
b*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/(-a*d+b*c)^(3/2)/a^(1
/2)-d*x/c/(-a*d+b*c)/(d*x^2+c)^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{dx}{c(-bc+ad)\sqrt{c+dx^2}}-\frac{b\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
(d*x)/(c*(-(b*c) + a*d)*Sqrt[c + d*x^2]) - (b*ArcTan[(a*Sqrt[d] + b*x*(Sqr
t[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a]*(b*c - a*d
)^(3/2))
```

3.101.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{296}$$
$$\frac{b\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$
$$\Big\downarrow \text{291}$$
$$\frac{b\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{bc-ad}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$
$$\Big\downarrow \text{218}$$
$$\frac{b\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
-((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c - a*d]*x)/(
Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

3.101.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.101.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{-bc\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{\left(ad-bc\right)a}}\right)\sqrt{d\,x^{2}+c}+dx\sqrt{\left(ad-bc\right)a}}{\left(ad-bc\right)\sqrt{\left(ad-bc\right)a}\,\sqrt{d\,x^{2}+c}\,c}$
default	$-\frac{b}{\left(ad-bc\right)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\left(2d\left(x-\frac{\sqrt{-ab}}{b}\right)+\frac{2d\sqrt{-ab}}{b}\right)}{\left(ad-bc\right)\left(-\frac{4d\left(ad-bc\right)}{b}+\frac{4d^{2}a}{b}\right)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-}}$ … $2\sqrt{-ab}$

input
```
int(1/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*c*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))*(d*x^2+c)^(1/2)+d*x
*((a*d-b*c)*a)^(1/2))/(a*d-b*c)/((a*d-b*c)*a)^(1/2)/(d*x^2+c)^(1/2)/c
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(67) = 134.

Time = 0.32 (sec) , antiderivative size = 442, normalized size of antiderivative = 5.59

$$\int \frac{1}{\left(a+bx^{2}\right)\left(c+dx^{2}\right)^{3/2}}\,dx = \left[-\frac{4\,(abcd-a^{2}d^{2})\sqrt{dx^{2}+c}\,x-(bcdx^{2}+bc^{2})\sqrt{-abc+a^{2}d}\log\left(\frac{(b^{2}c^{2}-8\,abcd+8}{\cdots}\right.}{4\,(ab^{2}c^{4}-2\,a^{2}bc^{3}d+a^{3}c^{2}d^{2}+(ab^{2}c^{3}d\cdots}\right.$$

$$\left.-\frac{2\,(abcd-a^{2}d^{2})\sqrt{dx^{2}+c}\,x-(bcdx^{2}+bc^{2})\sqrt{abc-a^{2}d}\arctan\left(\frac{\sqrt{abc-a^{2}d}\,((bc-2\,ad)x^{2}-ac)\sqrt{dx^{2}+c}}{2\,((abcd-a^{2}d^{2})x^{3}+(abc^{2}-a^{2}cd)x)}\right)}{2\,(ab^{2}c^{4}-2\,a^{2}bc^{3}d+a^{3}c^{2}d^{2}+(ab^{2}c^{3}d-2\,a^{2}bc^{2}d^{2}+a^{3}cd^{3})x^{2})}\right]$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(
-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(
3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a
^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^2*c^4 - 2*a^2*b*
c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2), -1
/2*(2*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(a*b
*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(
d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a*b^2*c^4
- 2*a^2*b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3
)*x^2)]
```

3.101.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.101.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{b\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}(bc-ad)} - \frac{dx}{(bc^2-acd)\sqrt{dx^2+c}}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
b*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) - d*x/((b*c^2
 - a*c*d)*sqrt(d*x^2 + c))
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.102 $\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

3.102.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\,(a+bx^2)} + \frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}$$

output
```
1/2*(-2*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2
)/(-a*d+b*c)^(3/2)+1/2*b*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)
```

3.102.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = -\frac{bx\sqrt{c+dx^2}}{2a(-bc+ad)\,(a+bx^2)} + \frac{(-bc+2ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^2-bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
-1/2*(b*x*Sqrt[c + d*x^2])/(a*(-(b*c) + a*d)*(a + b*x^2)) + ((-(b*c) + 2*a
*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt
[b*c - a*d])])/(2*a^(3/2)*(b*c - a*d)^(3/2))
```

3.102.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{296}$$

$$\frac{(bc-2ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{2a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{2a(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{(bc-2ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{2a(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(bc-2ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a(a+bx^2)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
(b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTa
n[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(
3/2))
```

3.102.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.102.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{-\frac{b\sqrt{d\,x^2+c}\,x}{b\,x^2+a}+\frac{(2ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{2(ad-bc)a}$
default	$-\frac{\frac{b\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2 -}}{x-\frac{\sqrt{-ab}}{b}}\right.}{(ad-bc)\sqrt{-\frac{ad-bc}{b}}}}{4ba}$

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*d-b*c)/a*(-b*(d*x^2+c)^(1/2)*x/(b*x^2+a)+(2*a*d-b*c)/((a*d-b*c)*a)^
(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(84) = 168$.

Time = 0.35 (sec) , antiderivative size = 459, normalized size of antiderivative = 4.59

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{4\,(ab^2c-a^2bd)\sqrt{dx^2+c}x-(abc-2\,a^2d+(b^2c-2\,abd)x^2)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2}{}\right.}{8\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x}\right.$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - (a*b*c - 2*a^2*d + (b^2*c
- 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a
*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/
(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*
b*d^2)*x^2), 1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + sqrt(a*b*c - a
^2*d)*(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^
2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 +
(a*b*c^2 - a^2*c*d)*x)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c
^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^2)]
```

3.102.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.102.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

3.102.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.25

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = -\frac{1}{2}d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}} + \frac{2\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc-\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right.}\right.$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b
- b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x - sqrt(d*x^2 + c))^2*b*c - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d -
 b*c^2)/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 +
c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b*c*d - a^2*
d^2)))
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^2\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.103 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3} dx$

3.103.1 Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3} dx = \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)(a+bx^2)} + \frac{bx(c+dx^2)^{3/2}}{4a(bc-ad)(a+bx^2)^2} + \frac{c(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

output
```
1/4*b*x*(d*x^2+c)^(3/2)/a/(-a*d+b*c)/(b*x^2+a)^2+1/8*c*(-4*a*d+3*b*c)*arct
an(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2)/(-a*d+b*c)^(3/2)+1/
8*(-4*a*d+3*b*c)*x*(d*x^2+c)^(1/2)/a^2/(-a*d+b*c)/(b*x^2+a)
```

3.103.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3} dx = \frac{x\sqrt{c+dx^2}(-5abc+4a^2d-3b^2cx^2+2abdx^2)}{8a^2(-bc+ad)(a+bx^2)^2} - \frac{c(3bc-4ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^2-bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

input
```
Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]
```

output
```
(x*Sqrt[c + d*x^2]*(-5*a*b*c + 4*a^2*d - 3*b^2*c*x^2 + 2*a*b*d*x^2))/(8*a^
2*(-(b*c) + a*d)*(a + b*x^2)^2) - (c*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b
*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^(5/2)
*(b*c - a*d)^(3/2))
```

3.103.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {296, 292, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{296}$$

$$\frac{(3bc-4ad)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^2}dx}{4a(bc-ad)}+\frac{bx(c+dx^2)^{3/2}}{4a(a+bx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{292}$$

$$\frac{(3bc-4ad)\left(\frac{c\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{2a}+\frac{x\sqrt{c+dx^2}}{2a(a+bx^2)}\right)}{4a(bc-ad)}+\frac{bx(c+dx^2)^{3/2}}{4a(a+bx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{(3bc-4ad)\left(\frac{c\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2a}+\frac{x\sqrt{c+dx^2}}{2a(a+bx^2)}\right)}{4a(bc-ad)}+\frac{bx(c+dx^2)^{3/2}}{4a(a+bx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(3bc-4ad)\left(\frac{c\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{c+dx^2}}{2a(a+bx^2)}\right)}{4a(bc-ad)}+\frac{bx(c+dx^2)^{3/2}}{4a(a+bx^2)^2(bc-ad)}$$

input
```
Int[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]
```

output
```
(b*x*(c + d*x^2)^(3/2))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + ((3*b*c - 4*a*d)
*((x*Sqrt[c + d*x^2])/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*Sqrt[b*c - a*d])))/(4*a*(b*c - a*d))
```

3.103.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.103.4 Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d\,x^{2}+c}\,\left(2x^{2}abd-3b^{2}c\,x^{2}+4a^{2}d-5abc\right)x}{c\left(b\,x^{2}+a\right)^{2}}-\frac{(4ad-3bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{8(ad-bc)a^{2}}$	121
default	Expression too large to display	4155

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*c/(a*d-b*c)/a^2*(-(d*x^2+c)^(1/2)/c*(2*a*b*d*x^2-3*b^2*c*x^2+4*a^2*d-
5*a*b*c)*x/(b*x^2+a)^2-(4*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)
^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(129) = 258$.

Time = 0.38 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.68

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx = \left[-\frac{(3a^2bc^2-4a^3cd+(3b^3c^2-4ab^2cd)x^4+2(3ab^2c^2-4a^2bcd)x^2)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x}{\cdots}\right.}{32(a^5b^2c^2-2a^6bcd+a^7d^2+(\cdots}\right.$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/32*((3*a^2*b*c^2 - 4*a^3*c*d + (3*b^3*c^2 - 4*a*b^2*c*d)*x^4 + 2*(3*a*
b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d
 + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*
a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x
^2 + a^2)) - 4*((3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^3 + (5*a^2*b
^2*c^2 - 9*a^3*b*c*d + 4*a^4*d^2)*x)*sqrt(d*x^2 + c))/(a^5*b^2*c^2 - 2*a^6
*b*c*d + a^7*d^2 + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*x^4 + 2*(a^
4*b^3*c^2 - 2*a^5*b^2*c*d + a^6*b*d^2)*x^2), 1/16*((3*a^2*b*c^2 - 4*a^3*c*
d + (3*b^3*c^2 - 4*a*b^2*c*d)*x^4 + 2*(3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqr
t(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*
sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*((3
*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^3 + (5*a^2*b^2*c^2 - 9*a^3*b*c
*d + 4*a^4*d^2)*x)*sqrt(d*x^2 + c))/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2 +
 (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*x^4 + 2*(a^4*b^3*c^2 - 2*a^5*
b^2*c*d + a^6*b*d^2)*x^2)]
```

3.103.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)**3,x)
```

output
```
Integral(sqrt(c + d*x**2)/(a + b*x**2)**3, x)
```

3.103.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^3}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^3, x)
```

3.103.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(129) = 258$.

Time = 1.66 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.27

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx = -\frac{\left(3\,bc^2\sqrt{d}-4\,acd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{8\,(a^2bc-a^3d)\sqrt{abcd-a^2d^2}}$$
$$-\frac{3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b^3c^2\sqrt{d}-4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6ab^2cd^{\frac{3}{2}}-9\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^3c^3\sqrt{d}+30\left(\sqrt{\right.}\right.}{}$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/8*(3*b*c^2*sqrt(d) - 4*a*c*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c - a^3*d)*sqrt
(a*b*c*d - a^2*d^2)) - 1/4*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^3*c^2*sqrt
(d) - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b^2*c*d^(3/2) - 9*(sqrt(d)*x - s
qrt(d*x^2 + c))^4*b^3*c^3*sqrt(d) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b
^2*c^2*d^(3/2) - 40*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*b*c*d^(5/2) + 16*(
sqrt(d)*x - sqrt(d*x^2 + c))^4*a^3*d^(7/2) + 9*(sqrt(d)*x - sqrt(d*x^2 + c
))^2*b^3*c^4*sqrt(d) - 28*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^3*d^(3/2
) + 16*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b*c^2*d^(5/2) - 3*b^3*c^5*sqrt(
d) + 2*a*b^2*c^4*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^
2)^2*(a^2*b^2*c - a^3*b*d))
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx=\int \frac{\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^3}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^3,x)
```

output
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^3, x)
```

3.104 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4} dx$

3.104.1 Optimal result

Integrand size = 21, antiderivative size = 199

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4} dx = \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)(a+bx^2)} + \frac{(5bc-6ad)x(c+dx^2)^{3/2}}{24a^2(bc-ad)(a+bx^2)^2} + \frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)(a+bx^2)^3} + \frac{c^2(5bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{16a^{7/2}(bc-ad)^{3/2}}$$

output
```
1/24*(-6*a*d+5*b*c)*x*(d*x^2+c)^(3/2)/a^2/(-a*d+b*c)/(b*x^2+a)^2+1/6*b*x*(
d*x^2+c)^(5/2)/a/(-a*d+b*c)/(b*x^2+a)^3+1/16*c^2*(-6*a*d+5*b*c)*arctan(x*(
-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(7/2)/(-a*d+b*c)^(3/2)+1/16*c*(
-6*a*d+5*b*c)*x*(d*x^2+c)^(1/2)/a^3/(-a*d+b*c)/(b*x^2+a)
```

3.104.2 Mathematica [A] (verified)

Time = 15.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4} dx = \frac{-\frac{\sqrt{a}x\sqrt{c+dx^2}(15b^3c^2x^4+8ab^2cx^2(5c-dx^2)-6a^3d(5c+2dx^2)+a^2b(33c^2-22cdx^2-4d^2x^4))}{(-bc+ad)(a+bx^2)^3} + \frac{3c^2(5bc-6ad)\arctan}{(bc-ad)^3}}{48a^{7/2}}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```

output
```
(-((Sqrt[a]*x*Sqrt[c + d*x^2]*(15*b^3*c^2*x^4 + 8*a*b^2*c*x^2*(5*c - d*x^2
) - 6*a^3*d*(5*c + 2*d*x^2) + a^2*b*(33*c^2 - 22*c*d*x^2 - 4*d^2*x^4)))/((
-(b*c) + a*d)*(a + b*x^2)^3)) + (3*c^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2))/(48*a^(7/2))
```

3.104.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {296, 292, 292, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^4}dx$$

$$\Big\downarrow \text{296}$$

$$\frac{(5bc-6ad)\int \frac{\left(dx^2+c\right)^{3/2}}{\left(bx^2+a\right)^3}dx}{6a(bc-ad)}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a\left(a+bx^2\right)^3(bc-ad)}$$

$$\Big\downarrow \text{292}$$

$$\frac{(5bc-6ad)\left(\frac{3c\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2}dx}{4a}+\frac{x\left(c+dx^2\right)^{3/2}}{4a\left(a+bx^2\right)^2}\right)}{6a(bc-ad)}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a\left(a+bx^2\right)^3(bc-ad)}$$

$$\Big\downarrow \text{292}$$

$$\frac{(5bc-6ad)\left(\frac{3c\left(\frac{c\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2a}+\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x\left(c+dx^2\right)^{3/2}}{4a\left(a+bx^2\right)^2}\right)}{6a(bc-ad)}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a\left(a+bx^2\right)^3(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{(5bc-6ad)\left(\frac{3c\left(\frac{c\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2a}+\frac{x\sqrt{c+dx^2}}{2a(a+bx^2)}\right)}{4a}+\frac{x(c+dx^2)^{3/2}}{4a(a+bx^2)^2}\right)}{6a(bc-ad)}+\frac{bx(c+dx^2)^{5/2}}{6a(a+bx^2)^3(bc-ad)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(5bc-6ad)\left(\frac{3c\left(\frac{c\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{c+dx^2}}{2a(a+bx^2)}\right)}{4a}+\frac{x(c+dx^2)^{3/2}}{4a(a+bx^2)^2}\right)}{6a(bc-ad)}+\frac{bx(c+dx^2)^{5/2}}{6a(a+bx^2)^3(bc-ad)}$$

input
```
Int[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```

output
```
(b*x*(c + d*x^2)^(5/2))/(6*a*(b*c - a*d)*(a + b*x^2)^3) + ((5*b*c - 6*a*d)
*((x*(c + d*x^2)^(3/2))/(4*a*(a + b*x^2)^2) + (3*c*((x*Sqrt[c + d*x^2])/(2
*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]
)/(2*a^(3/2)*Sqrt[b*c - a*d])))/(4*a)))/(6*a*(b*c - a*d))
```

3.104.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.104.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{5\left(d\left(\frac{2d x^2}{5}+c\right)a^3-\frac{11\left(-\frac{4}{33}d^2x^4-\frac{2}{3}cd x^2+c^2\right)b a^2}{10}-\frac{4\left(-\frac{d x^2}{5}+c\right)x^2b^2ca}{3}-\frac{c^2b^3x^4}{2}\right)x\sqrt{d x^2+c}\,\sqrt{(ad-bc)a}+3\left(b x^2+a\right)^3\left(ad-5\right.}{8\sqrt{(ad-bc)a}\,(ad-bc)a^3\left(b x^2+a\right)^3}$
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)/(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*(5*(d*(2/5*d*x^2+c)*a^3-11/10*(-4/33*d^2*x^4-2/3*c*d*x^2+c^2)*b*a^2-4/
3*(-1/5*d*x^2+c)*x^2*b^2*c*a-1/2*c^2*b^3*x^4)*x*(d*x^2+c)^(1/2)*((a*d-b*c)
*a)^(1/2)+3*(b*x^2+a)^3*(a*d-5/6*b*c)*c^2*arctanh((d*x^2+c)^(1/2)/x*a/((a*
d-b*c)*a)^(1/2)))/((a*d-b*c)*a)^(1/2)/(a*d-b*c)/a^3/(b*x^2+a)^3
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(175) = 350.

Time = 0.48 (sec) , antiderivative size = 972, normalized size of antiderivative = 4.88

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx = \left[-\frac{3\left(5a^3bc^3-6a^4c^2d+(5b^4c^3-6ab^3c^2d)x^6+3(5ab^3c^3-6a^2b^2c^2d)x^4+3(5a^2b^2c^3\right.}{}\right.$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/192*(3*(5*a^3*b*c^3 - 6*a^4*c^2*d + (5*b^4*c^3 - 6*a*b^3*c^2*d)*x^6 +
3*(5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^4 + 3*(5*a^2*b^2*c^3 - 6*a^3*b*c^2*d)*
x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2
*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(
-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*a*b
^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^5 + 2*(20*a^2
*b^3*c^3 - 31*a^3*b^2*c^2*d + 5*a^4*b*c*d^2 + 6*a^5*d^3)*x^3 + 3*(11*a^3*b
^2*c^3 - 21*a^4*b*c^2*d + 10*a^5*c*d^2)*x)*sqrt(d*x^2 + c))/(a^7*b^2*c^2 -
 2*a^8*b*c*d + a^9*d^2 + (a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x^6 +
 3*(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2)*x^4 + 3*(a^6*b^3*c^2 - 2*a^
7*b^2*c*d + a^8*b*d^2)*x^2), 1/96*(3*(5*a^3*b*c^3 - 6*a^4*c^2*d + (5*b^4*c
^3 - 6*a*b^3*c^2*d)*x^6 + 3*(5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^4 + 3*(5*a^2
*b^2*c^3 - 6*a^3*b*c^2*d)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c -
 a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3
 + (a*b*c^2 - a^2*c*d)*x)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b
^2*c*d^2 + 4*a^4*b*d^3)*x^5 + 2*(20*a^2*b^3*c^3 - 31*a^3*b^2*c^2*d + 5*a^4
*b*c*d^2 + 6*a^5*d^3)*x^3 + 3*(11*a^3*b^2*c^3 - 21*a^4*b*c^2*d + 10*a^5*c*
d^2)*x)*sqrt(d*x^2 + c))/(a^7*b^2*c^2 - 2*a^8*b*c*d + a^9*d^2 + (a^4*b^5*c
^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x^6 + 3*(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a
^7*b^2*d^2)*x^4 + 3*(a^6*b^3*c^2 - 2*a^7*b^2*c*d + a^8*b*d^2)*x^2)]
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**(3/2)/(b*x**2+a)**4,x)
```

output
```
Timed out
```

3.104.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^4}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^4, x)
```

3.104.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(175) = 350$.

Time = 1.26 (sec) , antiderivative size = 919, normalized size of antiderivative = 4.62

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx = -\frac{\left(5\,bc^3\sqrt{d}-6\,ac^2d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{16\,(a^3bc-a^4d)\sqrt{abcd-a^2d^2}} - \frac{15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}b^5c^3\sqrt{d}-18\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}ab^4c^2d^{\frac{3}{2}}-75\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8b^5c^4\sqrt{d}+24}{}$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="giac")
```

output
```
-1/16*(5*b*c^3*sqrt(d) - 6*a*c^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*
x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c - a^4*d)*s
qrt(a*b*c*d - a^2*d^2)) - 1/24*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^5*c^
3*sqrt(d) - 18*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b^4*c^2*d^(3/2) - 75*(sq
rt(d)*x - sqrt(d*x^2 + c))^8*b^5*c^4*sqrt(d) + 240*(sqrt(d)*x - sqrt(d*x^2
 + c))^8*a*b^4*c^3*d^(3/2) - 180*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*b^3*c
^2*d^(5/2) - 96*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^3*b^2*c*d^(7/2) + 96*(sq
rt(d)*x - sqrt(d*x^2 + c))^8*a^4*b*d^(9/2) + 150*(sqrt(d)*x - sqrt(d*x^2 +
 c))^6*b^5*c^5*sqrt(d) - 620*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b^4*c^4*d^(
3/2) + 968*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*b^3*c^3*d^(5/2) - 720*(sqrt
(d)*x - sqrt(d*x^2 + c))^6*a^3*b^2*c^2*d^(7/2) + 64*(sqrt(d)*x - sqrt(d*x^
2 + c))^6*a^4*b*c*d^(9/2) + 128*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^5*d^(11/
2) - 150*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^5*c^6*sqrt(d) + 600*(sqrt(d)*x
- sqrt(d*x^2 + c))^4*a*b^4*c^5*d^(3/2) - 864*(sqrt(d)*x - sqrt(d*x^2 + c))
^4*a^2*b^3*c^4*d^(5/2) + 288*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^3*b^2*c^3*d
^(7/2) + 96*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^4*b*c^2*d^(9/2) + 75*(sqrt(d
)*x - sqrt(d*x^2 + c))^2*b^5*c^7*sqrt(d) - 210*(sqrt(d)*x - sqrt(d*x^2 + c
))^2*a*b^4*c^6*d^(3/2) + 72*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b^3*c^5*d^
(5/2) + 48*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*b^2*c^4*d^(7/2) - 15*b^5*c^
8*sqrt(d) + 8*a*b^4*c^7*d^(3/2) + 4*a^2*b^3*c^6*d^(5/2))/((a^3*b^3*c - ...
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)^4}\,dx$$

input
```
int((c + d*x^2)^(3/2)/(a + b*x^2)^4,x)
```

output
```
int((c + d*x^2)^(3/2)/(a + b*x^2)^4, x)
```

3.105 $\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx$

3.105.1 Optimal result

Integrand size = 26, antiderivative size = 20

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx=\frac{dx}{bc\sqrt{c+dx^2}}$$

output
```
d*x/b/c/(d*x^2+c)^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx=\frac{dx}{bc\sqrt{c+dx^2}}$$

input
```
Integrate[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
(d*x)/(b*c*Sqrt[c + d*x^2])
```

3.105.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {281, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c+dx^2}\left(\frac{bc}{d}+bx^2\right)}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{d\int \frac{1}{(dx^2+c)^{3/2}}dx}{b}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

input
```
Int[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
(d*x)/(b*c*Sqrt[c + d*x^2])
```

3.105.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.105.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{dx}{bc\sqrt{d\,x^{2}+c}}$	19
default	$\frac{dx}{bc\sqrt{d\,x^{2}+c}}$	19
trager	$\frac{dx}{bc\sqrt{d\,x^{2}+c}}$	19
pseudoelliptic	$\frac{dx}{bc\sqrt{d\,x^{2}+c}}$	19

input
```
int(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d*x/b/c/(d*x^2+c)^(1/2)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx=\frac{\sqrt{dx^2+c}dx}{bcdx^2+bc^2}$$

input
```
integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^2 + c)*d*x/(b*c*d*x^2 + b*c^2)
```

3.105.6 Sympy [F]

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx=\frac{d\int \frac{1}{c\sqrt{c+dx^2}+dx^2\sqrt{c+dx^2}}\,dx}{b}$$

input
```
integrate(1/(b*c/d+b*x**2)/(d*x**2+c)**(1/2),x)
```

output
```
d*Integral(1/(c*sqrt(c + d*x**2) + d*x**2*sqrt(c + d*x**2)), x)/b
```

3.105.7 Maxima [F]

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx=\int \frac{1}{\left(bx^2+\frac{bc}{d}\right)\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + b*c/d)*sqrt(d*x^2 + c)), x)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx=\frac{dx}{\sqrt{dx^2+c}bc}$$

input
```
integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
d*x/(sqrt(d*x^2 + c)*b*c)
```

3.105.9 Mupad [B] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx=\frac{d\,x}{b\,c\,\sqrt{d\,x^2+c}}$$

input
```
int(1/((c + d*x^2)^(1/2)*(b*x^2 + (b*c)/d)),x)
```

output
```
(d*x)/(b*c*(c + d*x^2)^(1/2))
```

3.106 $\int \frac{1}{\sqrt{1-x^2}(1+x^2)}\,dx$

3.106.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(x*2^(1/2)/(-x^2+1)^(1/2))*2^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[1 - x^2]*(1 + x^2)),x]
```

output
```
ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

3.106.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\,(x^2+1)}\,dx$$

$$\Big\downarrow \; 291$$

$$\int \frac{1}{\frac{2x^2}{1-x^2}+1}d\frac{x}{\sqrt{1-x^2}}$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[1 - x^2]*(1 + x^2)),x]
```

output
```
ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

3.106.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.106.4 Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
pseudoelliptic	$-\frac{\sqrt{2}\, \arctan\left(\frac{\sqrt{2}\,\sqrt{-x^{2}+1}}{2x}\right)}{2}$	24
default	$-\frac{\sqrt{2}\, \arctan\left(\frac{\sqrt{2}\,\sqrt{-x^{2}+1}\,x}{x^{2}-1}\right)}{2}$	28
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right) \ln\left(\frac{3\operatorname{RootOf}\left(\_Z^{2}+2\right) x^{2}+4x\sqrt{-x^{2}+1}-\operatorname{RootOf}\left(\_Z^{2}+2\right)}{x^{2}+1}\right)}{4}$	50

input
```
int(1/(x^2+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)*arctan(1/2/x*2^(1/2)*(-x^2+1)^(1/2))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = -\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2\,x}\right)$$

input
```
integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x)
```

3.106.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \int \frac{1}{\sqrt{-(x-1)\,(x+1)}\,(x^2+1)}\,dx$$

input
```
integrate(1/(x**2+1)/(-x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 1)), x)
```

3.106.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \int \frac{1}{(x^2+1)\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 1)*sqrt(-x^2 + 1)), x)
```

3.106.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(20) = 40$.

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \frac{1}{4}\sqrt{2}\left(\pi\mathrm{sgn}(x) + 2\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$

input
```
integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1) - 1)^2/x
^2 - 1)/(sqrt(-x^2 + 1) - 1)))
```

3.106.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \frac{\sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(-1+x\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\mathrm{i}}\right)1\mathrm{i}}{4} - \frac{\sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(1+x\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-x^2}\,1\mathrm{i}}{x+1\mathrm{i}}\right)1\mathrm{i}}{4}$$

input
```
int(1/((1 - x^2)^(1/2)*(x^2 + 1)),x)
```

output
```
(2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i
)/4 - (2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*1i)/(x + 1
i))*1i)/4
```

3.107 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.107.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

output
```
arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(1/2)/(-a*d+b*c)^(1/2
)
```

3.107.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx=-\frac{\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

input
```
Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
-(ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c
 - a*d])]/(Sqrt[a]*Sqrt[b*c - a*d]))
```

3.107.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \; 291$$

$$\int \frac{1}{a-\frac{x^2(ad-bc)}{c+dx^2}}d\frac{x}{\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

input
```
Int[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c -
a*d])
```

3.107.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.107.4 Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}$
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}+\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d}{}}{}\right)}{}$

input
```
int(1/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))
```

3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(39) = 78.

Time = 0.31 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.92

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,(abc-a^2d)}, \arctan\left(\right.\right.$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^
2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt
(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*b*c - a^
2*d), 1/2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*
x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x))/sqrt(a*b*c - a
^2*d)]
```

3.107.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.107.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt
(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{x\sqrt{b\,c-a\,d}}{\sqrt{a}\,\sqrt{d\,x^2+c}}\right)}{\sqrt{-a\,(a\,d-b\,c)}} & \text{if } 0 < b\,c-a\,d \\ \frac{\ln\left(\frac{\sqrt{a\,\left(d\,x^2+c\right)}+x\,\sqrt{a\,d-b\,c}}{\sqrt{a\,\left(d\,x^2+c\right)}-x\,\sqrt{a\,d-b\,c}}\right)}{2\,\sqrt{a\,(a\,d-b\,c)}} & \text{if } b\,c-a\,d < 0 \\ \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^2+c}}\,dx & \text{if } b\,c-a\,d \notin \mathbb{R} \vee a\,d = b\,c \end{cases}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
piecewise(0 < - a*d + b*c, atan((x*(- a*d + b*c)^(1/2))/(a^(1/2)*(c + d*x^
2)^(1/2)))/(-a*(a*d - b*c))^(1/2), - a*d + b*c < 0, log(((a*(c + d*x^2))^(
1/2) + x*(a*d - b*c)^(1/2))/((a*(c + d*x^2))^(1/2) - x*(a*d - b*c)^(1/2)))
/(2*(a*(a*d - b*c))^(1/2)), ~in(- a*d + b*c, 'real') | a*d == b*c, int(1/(
(a + b*x^2)*(c + d*x^2)^(1/2)), x))
```

3.108 $\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx$

3.108.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = -\frac{2x}{\sqrt{1+x^2}} + \operatorname{arcsinh}(x)$$

output
```
arcsinh(x)-2*x/(x^2+1)^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = -\frac{2x}{\sqrt{1+x^2}} - \log\left(-x+\sqrt{1+x^2}\right)$$

input
```
Integrate[(-1 + x^2)/(1 + x^2)^(3/2),x]
```

output
```
(-2*x)/Sqrt[1 + x^2] - Log[-x + Sqrt[1 + x^2]]
```

3.108.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {298, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{(x^2+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 298}$$

$$\int \frac{1}{\sqrt{x^2+1}}dx - \frac{2x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 222}$$

$$\text{arcsinh}(x) - \frac{2x}{\sqrt{x^2+1}}$$

input
```
Int[(-1 + x^2)/(1 + x^2)^(3/2),x]
```

output
```
(-2*x)/Sqrt[1 + x^2] + ArcSinh[x]
```

3.108.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.108.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\operatorname{arcsinh}(x)-\frac{2x}{\sqrt{x^{2}+1}}$	14
risch	$\operatorname{arcsinh}(x)-\frac{2x}{\sqrt{x^{2}+1}}$	14
trager	$-\frac{2x}{\sqrt{x^{2}+1}}+\ln\left(x+\sqrt{x^{2}+1}\right)$	22
meijerg	$-\frac{x}{\sqrt{x^{2}+1}}+\frac{-\frac{\sqrt{\pi}\,x}{\sqrt{x^{2}+1}}+\sqrt{\pi}\,\operatorname{arcsinh}(x)}{\sqrt{\pi}}$	36
pseudoelliptic	$\frac{-\ln\left(\frac{-x+\sqrt{x^{2}+1}}{x}\right)\sqrt{x^{2}+1}+\ln\left(\frac{x+\sqrt{x^{2}+1}}{x}\right)\sqrt{x^{2}+1}-4x}{2\sqrt{x^{2}+1}}$	61

input
```
int((x^2-1)/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(x)-2*x/(x^2+1)^(1/2)
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

$$\int \frac{-1+x^2}{\left(1+x^2\right)^{3/2}}\,dx=-\frac{2\,x^{2}+\left(x^{2}+1\right)\log\left(-x+\sqrt{x^{2}+1}\right)+2\,\sqrt{x^{2}+1}x+2}{x^{2}+1}$$

input
```
integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-(2*x^2 + (x^2 + 1)*log(-x + sqrt(x^2 + 1)) + 2*sqrt(x^2 + 1)*x + 2)/(x^2
+ 1)
```

3.108.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 2.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = \frac{x^2\operatorname{asinh}(x)}{x^2+1} - \frac{2x}{\sqrt{x^2+1}} + \frac{\operatorname{asinh}(x)}{x^2+1}$$

input
```
integrate((x**2-1)/(x**2+1)**(3/2),x)
```

output
```
x**2*asinh(x)/(x**2 + 1) - 2*x/sqrt(x**2 + 1) + asinh(x)/(x**2 + 1)
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = -\frac{2\,x}{\sqrt{x^2+1}} + \operatorname{arsinh}(x)$$

input
```
integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
-2*x/sqrt(x^2 + 1) + arcsinh(x)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = -\frac{2\,x}{\sqrt{x^2+1}} - \log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="giac")
```

output
```
-2*x/sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

3.108.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = \frac{\operatorname{asinh}(x) + x^2 \operatorname{asinh}(x) - 2\,x\,\sqrt{x^2+1}}{x^2+1}$$

input
```
int((x^2 - 1)/(x^2 + 1)^(3/2),x)
```

output
```
(asinh(x) + x^2*asinh(x) - 2*x*(x^2 + 1)^(1/2))/(x^2 + 1)
```

3.109 $\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^3 \, dx$

3.109.1 Optimal result

Integrand size = 24, antiderivative size = 648

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^3 \, dx = \frac{18144a^3x(a-bx^2)^{2/3}}{1235} - \frac{23544a^2x(a-bx^2)^{5/3}}{6175}$$

$$-\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right) - \frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2 - \frac{72576a^4x}{1235\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{36288\sqrt[4]{3}}{}$$

output
```
18144/1235*a^3*x*(-b*x^2+a)^(2/3)-23544/6175*a^2*x*(-b*x^2+a)^(5/3)-378/47
5*a*x*(-b*x^2+a)^(5/3)*(b*x^2+3*a)-3/25*x*(-b*x^2+a)^(5/3)*(b*x^2+3*a)^2-7
2576/1235*a^4*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+24192/1235*3^(3/4)
*a^(13/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*
(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2
)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+
a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(
-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-36288/1235*3^(1/4)*a^(13/3)*
(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)
))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3
)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)
))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1
/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.109.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.15

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3\,dx =$$

$$-\frac{3\left(-15255a^4x+3390a^3bx^3+8992a^2b^2x^5+2626ab^3x^7+247b^4x^9-40320a^4x\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric}\right.}{6175\sqrt[3]{a-bx^2}}$$

input
```
Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x]
```

output
```
(-3*(-15255*a^4*x + 3390*a^3*b*x^3 + 8992*a^2*b^2*x^5 + 2626*a*b^3*x^7 + 2
47*b^4*x^9 - 40320*a^4*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2,
 3/2, (b*x^2)/a]))/(6175*(a - b*x^2)^(1/3))
```

3.109.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {318, 27, 403, 27, 299, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3\,dx$$

$$\Big\downarrow \text{318}$$

$$-\frac{3\int -6ab\left(a-bx^2\right)^{2/3}\left(bx^2+3a\right)\left(7bx^2+13a\right)dx}{25b}-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{18}{25}a\int \left(a-bx^2\right)^{2/3}\left(bx^2+3a\right)\left(7bx^2+13a\right)dx-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{403}$$

$$\frac{18}{25}a\left(-\frac{3\int -\frac{4}{3}ab\left(a-bx^2\right)^{2/3}\left(109bx^2+201a\right)dx}{19b}-\frac{21}{19}x\left(3a+bx^2\right)\left(a-bx^2\right)^{5/3}\right)-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \ 27$$

$$\frac{18}{25}a\left(\frac{4}{19}a\int\left(a-bx^2\right)^{2/3}\left(109bx^2+201a\right)dx-\frac{21}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\right)-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \ 299$$

$$\frac{18}{25}a\left(\frac{4}{19}a\left(\frac{2940}{13}a\int\left(a-bx^2\right)^{2/3}dx-\frac{327}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{21}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\right)-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \ 211$$

$$\frac{18}{25}a\left(\frac{4}{19}a\left(\frac{2940}{13}a\left(\frac{4}{7}a\int\frac{1}{\sqrt[3]{a-bx^2}}dx+\frac{3}{7}x\left(a-bx^2\right)^{2/3}\right)-\frac{327}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{21}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\right)$$
$$\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \ 233$$

$$\frac{18}{25}a\left(\frac{4}{19}a\left(\frac{2940}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\int\frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}\right)-\frac{327}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{21}{19}x\left(a-bx^2\right.\right.$$
$$\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \ 833$$

$$\frac{18}{25}a\left(\frac{4}{19}a\left(\frac{2940}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{\right.}}{7bx}\right.\right.\right.$$
$$\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \ 760$$

3.109. $\int\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3\,dx$

$$\frac{18}{25}a\left(\frac{4}{19}a\left(\frac{2940}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{\right.}}{}\right.}{}\right.\right.\right.$$

$$\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

$\downarrow$ 2418

$$\frac{18}{25}a\left(\frac{4}{19}a\left(\frac{2940}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right.}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{\right.}}}}\right.}{}\right.\right.\right.$$

$$\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

input
```
Int[(a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x]
```

3.109. $\int\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3\,dx$

output
```
(-3*x*(a - b*x^2)^(5/3)*(3*a + b*x^2)^2)/25 + (18*a*((-21*x*(a - b*x^2)^(5
/3)*(3*a + b*x^2))/19 + (4*a*((-327*x*(a - b*x^2)^(5/3))/13 + (2940*a*((3*
x*(a - b*x^2)^(2/3))/7 - (6*a*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (
a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^
2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(
1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(7*b*x))
)/13))/19))/25
```

3.109.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.109. $\int (a-bx^2)^{2/3} (3a+bx^2)^3 \, dx$

3.109.4 Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{\frac{2}{3}} \left(b\,x^{2}+3a\right)^{3}dx$$

input
```
int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x)
```

output
```
int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x)
```

3.109.5 Fricas [F]

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^3\,dx=\int \left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x, algorithm="fricas")
```

output
```
integral((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3
), x)
```

3.109.6 Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.21

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^3\,dx=27a^{\frac{11}{3}}x\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)+9a^{\frac{8}{3}}bx^3\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{9a^{\frac{5}{3}}b^2x^5\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{5}+\frac{a^{\frac{2}{3}}b^3x^7\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{7}$$

input
```
integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a)**3,x)
```

output
```
27*a**(11/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*
a**(8/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 9
*a**(5/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)
/5 + a**(2/3)*b**3*x**7*hyper((-2/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi
)/a)/7
```

3.109.7 Maxima [F]

$$\int (a-bx^2)^{2/3} (3a+bx^2)^3 \, dx = \int (bx^2+3a)^3 (-bx^2+a)^{\frac{2}{3}} \, dx$$

input
```
integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3), x)
```

3.109.8 Giac [F]

$$\int (a-bx^2)^{2/3} (3a+bx^2)^3 \, dx = \int (bx^2+3a)^3 (-bx^2+a)^{\frac{2}{3}} \, dx$$

input
```
integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int (a-bx^2)^{2/3} (3a+bx^2)^3 \, dx = \int (a-b\,x^2)^{2/3} (b\,x^2+3\,a)^3 \, dx$$

input
```
int((a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x)
```

output
```
int((a - b*x^2)^(2/3)*(3*a + b*x^2)^3, x)
```

3.110 $\int (a-bx^2)^{2/3} (3a+bx^2)^2 \, dx$

3.110.1 Optimal result

Integrand size = 24, antiderivative size = 617

$$\int (a-bx^2)^{2/3} (3a+bx^2)^2 \, dx = \frac{7776a^2x(a-bx^2)^{2/3}}{1729} - \frac{252}{247}ax(a-bx^2)^{5/3} - \frac{3}{19}x(a-bx^2)^{5/3}(3a+bx^2) - \frac{31104a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{15552\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right.}{}$$

output
```
7776/1729*a^2*x*(-b*x^2+a)^(2/3)-252/247*a*x*(-b*x^2+a)^(5/3)-3/19*x*(-b*x
^2+a)^(5/3)*(b*x^2+3*a)-31104/1729*a^3*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(
1/2)))+10368/1729*3^(3/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-
(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2
))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(
2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1
/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-155
52/1729*3^(1/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)
^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*
3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)
^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/
3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(
1/2)
```

3.110.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.29

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^2 \, dx = \frac{x(a-bx^2)^{2/3}\left(21a(45a^2+10abx^2+b^2x^4)\,\text{Gamma}\left(-\frac{2}{3}\right)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{7}{2},\frac{bx^2}{a}\right)+\right.}{}$$

input
```
Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x]
```

output
```
(x*(a - b*x^2)^(2/3)*(21*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*Gamma[-2/3]*Hyp
ergeometric2F1[-2/3, 1/2, 7/2, (b*x^2)/a] + 8*b*x^2*(18*a^2 + 9*a*b*x^2 +
b^2*x^4)*Gamma[1/3]*Hypergeometric2F1[1/3, 3/2, 9/2, (b*x^2)/a] + 4*b*(3*a
*x + b*x^3)^2*Gamma[1/3]*HypergeometricPFQ[{1/3, 3/2, 2}, {1, 9/2}, (b*x^2
)/a]))/(105*a*(1 - (b*x^2)/a)^(2/3)*Gamma[-2/3])
```

3.110.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {318, 27, 299, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^2 \, dx$$

$$\Big\downarrow \; 318$$

$$-\frac{3\int -4ab\left(a-bx^2\right)^{2/3}\left(7bx^2+15a\right)dx}{19b}-\frac{3}{19}x\left(3a+bx^2\right)\left(a-bx^2\right)^{5/3}$$

$$\Big\downarrow \; 27$$

$$\frac{12}{19}a\int \left(a-bx^2\right)^{2/3}\left(7bx^2+15a\right)dx-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)$$

$$\Big\downarrow \; 299$$

$$\frac{12}{19}a\left(\frac{216}{13}a\int \left(a-bx^2\right)^{2/3}dx-\frac{21}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{12}{19}a\left(\frac{216}{13}a\left(\frac{4}{7}a\int \frac{1}{\sqrt[3]{a-bx^2}}dx+\frac{3}{7}x\left(a-bx^2\right)^{2/3}\right)-\frac{21}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)$$

$$\Big\downarrow \text{233}$$

$$\frac{12}{19}a\left(\frac{216}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}\right)-\frac{21}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)$$

$$\Big\downarrow \text{833}$$

$$\frac{12}{19}a\left(\frac{216}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right.}{7bx}\right.\right.$$

$$\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)$$

$$\Big\downarrow \text{760}$$

$$\frac{12}{19}a\left(\frac{216}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a}\right.}{}\right.}{}\right.\right.$$

$$\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)$$

3.110. $\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\,dx$

$$\Big\downarrow \ 2418$$

$$\frac{12}{19}a\left(\frac{216}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\mathrm{E}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}\right.}}}\right.}{}\right.\right.$$

$$\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)$$

input
```
Int[(a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x]
```

output
```
(-3*x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/19 + (12*a*((-21*x*(a - b*x^2)^(5/3
))/13 + (216*a*((3*x*(a - b*x^2)^(2/3))/7 - (6*a*Sqrt[-(b*x^2)]*((-2*Sqrt[
-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 +
 Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^
(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)
]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1
/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a -
b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/
3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2)])))/(7*b*x)))/13))/19
```

3.110. $\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\,dx$

3.110.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.110.4 Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{\frac{2}{3}} \left(b\,x^{2}+3a\right)^{2}dx$$

input
```
int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x)
```

output
```
int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x)
```

3.110.5 Fricas [F]

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^2 \,dx=\int \left(bx^{2}+3\,a\right)^{2}\left(-bx^{2}+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3), x)
```

3.110.6 Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.16

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^2 \,dx=9a^{\frac{8}{3}}x\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)+2a^{\frac{5}{3}}bx^3\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{a^{\frac{2}{3}}b^2x^5\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{5}$$

input
```
integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a)**2,x)
```

output
```
9*a**(8/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 2*a*
*(5/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + a**
(2/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

3.110.7 Maxima [F]

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^2 \, dx = \int \left(bx^2+3\,a\right)^2 \left(-bx^2+a\right)^{\frac{2}{3}} \, dx$$

input
```
integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3), x)
```

3.110.8 Giac [F]

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^2 \, dx = \int \left(bx^2+3\,a\right)^2 \left(-bx^2+a\right)^{\frac{2}{3}} \, dx$$

input
```
integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^2 \, dx = \int \left(a-b\,x^2\right)^{2/3} \left(b\,x^2+3\,a\right)^2 \, dx$$

input
```
int((a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x)
```

output
```
int((a - b*x^2)^(2/3)*(3*a + b*x^2)^2, x)
```

3.111 $\int (a-bx^2)^{2/3}(3a+bx^2)\,dx$

3.111.1 Optimal result

Integrand size = 22, antiderivative size = 588

$$\int (a-bx^2)^{2/3}(3a+bx^2)\,dx = \frac{18}{13}ax(a-bx^2)^{2/3}$$

$$-\frac{3}{13}x(a-bx^2)^{5/3}-\frac{72a^2x}{13\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{24\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
18/13*a*x*(-b*x^2+a)^(2/3)-3/13*x*(-b*x^2+a)^(5/3)-72/13*a^2*x/(-(-b*x^2+a
)^(1/3)+a^(1/3)*(1-3^(1/2)))+24/13*3^(3/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/
3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a
^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1
/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x
/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2
)))^2)^(1/2)-36/13*3^(1/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-
(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2
))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-
(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b
/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1
/2)))^2)^(1/2)
```

3.111.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.66 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.11

$$\int (a-bx^2)^{2/3}(3a+bx^2)\,dx=\frac{3}{13}x(a-bx^2)^{2/3}\left(-a+bx^2+\frac{14a\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{2/3}}\right)$$

input
```
Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2),x]
```

output
```
(3*x*(a - b*x^2)^(2/3)*(-a + b*x^2 + (14*a*Hypergeometric2F1[-2/3, 1/2, 3/
2, (b*x^2)/a])/(1 - (b*x^2)/a)^(2/3)))/13
```

3.111.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {299, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right) \, dx$$

$$\Big\downarrow \; 299$$

$$\frac{42}{13}a\int \left(a-bx^2\right)^{2/3}dx-\frac{3}{13}x\left(a-bx^2\right)^{5/3}$$

$$\Big\downarrow \; 211$$

$$\frac{42}{13}a\left(\frac{4}{7}a\int \frac{1}{\sqrt[3]{a-bx^2}}dx+\frac{3}{7}x\left(a-bx^2\right)^{2/3}\right)-\frac{3}{13}x\left(a-bx^2\right)^{5/3}$$

$$\Big\downarrow \; 233$$

$$\frac{42}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}\right)-\frac{3}{13}x\left(a-bx^2\right)^{5/3}$$

$$\Big\downarrow \; 833$$

$$\frac{42}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{7bx}\right)-$$
$$\frac{3}{13}x\left(a-bx^2\right)^{5/3}$$

$$\Big\downarrow \; 760$$

$$\frac{42}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\cdots}\right.}{7bx}\right.$$
$$\frac{3}{13}x\left(a-bx^2\right)^{5/3}$$

3.111. $\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right) \, dx$

$$\Big\downarrow \text{2418}$$

$$\frac{42}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\Big(}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\right.}{}\right.$$

$$\frac{3}{13}x\left(a-bx^2\right)^{5/3}$$

input
```
Int[(a - b*x^2)^(2/3)*(3*a + b*x^2),x]
```

output
```
(-3*x*(a - b*x^2)^(5/3))/13 + (42*a*((3*x*(a - b*x^2)^(2/3))/7 - (6*a*Sqrt
[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[
(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqr
t[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + S
qrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a -
 b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(
b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2)])))/(7*b*x)))/13
```

3.111. $\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\,dx$

3.111.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.111.4 Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{\frac{2}{3}} \left(b\,x^{2}+3a\right)dx$$

input
```
int((-b*x^2+a)^(2/3)*(b*x^2+3*a),x)
```

output
```
int((-b*x^2+a)^(2/3)*(b*x^2+3*a),x)
```

3.111.5 Fricas [F]

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)\,dx = \int \left(bx^{2}+3\,a\right)\left(-bx^{2}+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a),x, algorithm="fricas")
```

output
```
integral((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

3.111.6 Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.11

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)\,dx = 3a^{\frac{5}{3}}x\,{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\,\frac{bx^{2}e^{2i\pi}}{a}\right) + \frac{a^{\frac{2}{3}}bx^{3}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3},\frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle|\,\frac{bx^{2}e^{2i\pi}}{a}\right)}{3}$$

input
```
integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a),x)
```

output
```
3*a**(5/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + a**(
2/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

3.111.7 Maxima [F]

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right) \, dx = \int \left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{2}{3}} \, dx$$

input
```
integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

3.111.8 Giac [F]

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right) \, dx = \int \left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{2}{3}} \, dx$$

input
```
integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right) \, dx = \int \left(a-b\,x^2\right)^{2/3} \left(b\,x^2+3\,a\right) \, dx$$

input
```
int((a - b*x^2)^(2/3)*(3*a + b*x^2),x)
```

output
```
int((a - b*x^2)^(2/3)*(3*a + b*x^2), x)
```

3.112 $\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx$

3.112.1 Optimal result

Integrand size = 24, antiderivative size = 740

$$
\begin{aligned}
\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx &= \frac{3x}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}} \\
&+ \frac{\sqrt[3]{2}\sqrt[6]{a}\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} \\
&- \frac{\sqrt[3]{2}\sqrt[6]{a}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}} \\
&+ \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} \\
&- \frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}
\end{aligned}
$$

output
```
3*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+2^(1/3)*a^(1/6)*arctanh(x*b^(1
/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))/b^(1/2)-1/3*2^(1/3)*a^(1/6
)*arctanh(x*b^(1/2)/a^(1/2))/b^(1/2)+1/3*2^(1/3)*a^(1/6)*arctan(a^(1/6)*(a
^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*3^(1/2)/b^(1/2)+1/3*2^
(1/3)*a^(1/6)*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*3^(1/2)/b^(1/2)-3^(3/4)*a^
(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3
^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((
a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x
^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+3/2*3^(1/4)*a^(1/3)*(a^(1/3)-(-b
*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)
^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*
(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x
^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.112.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.22

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx = \frac{9ax(a-bx^2)^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{2}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{(3a+bx^2)\left(9a\operatorname{AppellF1}\left(\frac{1}{2},-\frac{2}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-2bx^2\left(\operatorname{AppellF1}\left(\frac{3}{2},-\frac{2}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right.\right.\right.}$$

input
```
Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2),x]
```

output
```
(9*a*x*(a - b*x^2)^(2/3)*AppellF1[1/2, -2/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^
2)/a])/((3*a + b*x^2)*(9*a*AppellF1[1/2, -2/3, 1, 3/2, (b*x^2)/a, -1/3*(b*
x^2)/a] - 2*b*x^2*(AppellF1[3/2, -2/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]
+ 2*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

3.112.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {301, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{2/3}}{3a+bx^2}dx$$

$$\Big\downarrow \text{301}$$

$$4a\int \frac{1}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx-\int \frac{1}{\sqrt[3]{a-bx^2}}dx$$

$$\Big\downarrow \text{233}$$

$$\frac{3\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}+4a\int \frac{1}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx$$

$$\Big\downarrow \text{305}$$

$$\frac{3\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}+4a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{bx}}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\text{arctanl}}{6\ 2^{2/3}}\right.$$

$$\Big\downarrow \text{833}$$

$$\frac{3\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{2bx}+4a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{bx}}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\text{arctanl}}{6\ 2^{2/3}}\right.$$

$$\Big\downarrow \text{760}$$

$$\frac{3\sqrt{-bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right.}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}}}\right.}{2bx}$$

$$4a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanl}}{6\ 2^{2/3}}\right.$$

$$\Big\downarrow \color{blue}{2418}$$

$$4a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-\right.}\right.}}{2\ 2^{2/3}a^{5/6}\sqrt{b}}\right.$$

$$\frac{3\sqrt{-bx^2}\left(\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\right.}{}$$

input
```
Int[(a - b*x^2)^(2/3)/(3*a + b*x^2),x]
```

3.112. $\int \frac{\left(a-bx^2\right)^{2/3}}{3a+bx^2}\,dx$

output
```
4*a*(ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt
[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqr
t[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]
]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^
(1/3)*(a - b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])) + (3*Sqrt[-(b*x^2)
]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(S
qrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a
^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(
1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*S
qrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))^2)])))/(2*b*x)
```

3.112.3.1 Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.112.4 Maple [F]

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{2}{3}}}{b\,x^{2}+3a}dx$$

input
```
int((-b*x^2+a)^(2/3)/(b*x^2+3*a),x)
```

output
```
int((-b*x^2+a)^(2/3)/(b*x^2+3*a),x)
```

3.112.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a),x, algorithm="fricas")
```

output
```
Timed out
```

3.112.6 Sympy [F]

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx = \int \frac{(a-bx^2)^{\frac{2}{3}}}{3a+bx^2}\,dx$$

input
```
integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a),x)
```

output
```
Integral((a - b*x**2)**(2/3)/(3*a + b*x**2), x)
```

3.112.7 Maxima [F]

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx = \int \frac{(-bx^2+a)^{\frac{2}{3}}}{bx^2+3\,a}\,dx$$

input
```
integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a), x)
```

3.112.8 Giac [F]

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx=\int \frac{(-bx^2+a)^{\frac{2}{3}}}{bx^2+3\,a}\,dx$$

input
```
integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx=\int \frac{(a-b\,x^2)^{2/3}}{b\,x^2+3\,a}\,dx$$

input
```
int((a - b*x^2)^(2/3)/(3*a + b*x^2),x)
```

output
```
int((a - b*x^2)^(2/3)/(3*a + b*x^2), x)
```

3.113 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx$

3.113.1 Optimal result

Integrand size = 24, antiderivative size = 584

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx = \frac{x(a-bx^2)^{2/3}}{6a\,(3a+bx^2)} - \frac{x}{6a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{}\right.}{4\,3^{3/4}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right.}{3\sqrt{2}\sqrt[4]{3}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
1/6*x*(-b*x^2+a)^(2/3)/a/(b*x^2+3*a)-1/6*x/a/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1
-3^(1/2)))+1/18*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^
(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*
((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^
(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)
-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1/12*(
a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2))
)/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)
*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3
)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.113.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.15

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx=\frac{x(a-bx^2)^{2/3}}{6a\,(3a+bx^2)}+\frac{x\sqrt[3]{\frac{a-bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{18a\sqrt[3]{a-bx^2}}$$

input
```
Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x]
```

output
```
(x*(a - b*x^2)^(2/3))/(6*a*(3*a + b*x^2)) + (x*((a - b*x^2)/a)^(1/3)*Hyper
geometric2F1[1/3, 1/2, 3/2, (b*x^2)/a])/(18*a*(a - b*x^2)^(1/3))
```

3.113.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {314, 27, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 314$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)}-\frac{\int -\frac{1}{3\sqrt[3]{a-bx^2}}dx}{6a}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{1}{\sqrt[3]{a-bx^2}}dx}{18a}+\frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)}$$

$\Big\downarrow$ 233

$$\frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)}-\frac{\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{12abx}$$

$\Big\downarrow$ 833

$$\frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)}-\frac{\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{12abx}$$

$\Big\downarrow$ 760

$$\frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)}-$$

$$\frac{\sqrt{-bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\right.}}}\right.}{12abx}$$

$\Big\downarrow$ 2418

$$\frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)}-$$

$$\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx}}\right.\right.}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\right.$$

3.113. $\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^2}dx$

input
```
Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x]
```

output
```
(x*(a - b*x^2)^(2/3))/(6*a*(3*a + b*x^2)) - (Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*
x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqr
t[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b
*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sq
rt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^
2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^
2)])))/(12*a*b*x)
```

3.113.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.113.4 Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{2}{3}}}{(b\,x^2+3a)^2}dx$$

input
```
int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x)
```

output
```
int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x)
```

3.113.5 Fricas [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx=\int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^2}\,dx$$

input
```
integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(2/3)/(b^2*x^4 + 6*a*b*x^2 + 9*a^2), x)
```

3.113.6 Sympy [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx=\int \frac{(a-bx^2)^{\frac{2}{3}}}{(3a+bx^2)^2}\,dx$$

input
```
integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**2,x)
```

output
```
Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**2, x)
```

3.113.7 Maxima [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx=\int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^2}\,dx$$

input
```
integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2, x)
```

3.113.8 Giac [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx=\int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^2}\,dx$$

input
```
integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2, x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx=\int \frac{(a-b\,x^2)^{2/3}}{(b\,x^2+3\,a)^2}\,dx$$

input
```
int((a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x)
```

output
```
int((a - b*x^2)^(2/3)/(3*a + b*x^2)^2, x)
```

3.114 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx$

3.114.1 Optimal result

Integrand size = 24, antiderivative size = 818

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \frac{x(a-bx^2)^{2/3}}{12a\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{36a^2\,(3a+bx^2)} - \frac{x}{36a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+ \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$

$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{216\ 2^{2/3}a^{11/6}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{72\ 2^{2/3}a^{11/6}\sqrt{b}}$$

$$- \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{}\right.}{24\ 3^{3/4}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+ \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right.}{18\sqrt{2}\sqrt[4]{3}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
1/12*x*(-b*x^2+a)^(2/3)/a/(b*x^2+3*a)^2+1/36*x*(-b*x^2+a)^(2/3)/a^2/(b*x^2
+3*a)-1/36*x/a^2/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/144*arctanh(x*b
^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(11/6)/b^(1/2
)-1/432*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(11/6)/b^(1/2)+1/432*arctan(a
^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(11
/6)*3^(1/2)/b^(1/2)+1/432*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(11/
6)*3^(1/2)/b^(1/2)+1/108*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)
^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*
3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)
^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b/x*2^(1/2)/(-a^(1/3)
*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)-1/72*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1
+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3
)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1
-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(5/3)/b/x/(-a^(1/3
)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)
```

3.114.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.31

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \frac{\frac{bx^3\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^3}+\frac{27x\left(6-\frac{5bx^2}{a}-\frac{b^2x^4}{a^2}+\frac{18(3a+bx^2)\cdots}{9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-\cdots\right)}\right)}{(3a+\cdots}}{972\sqrt[3]{a-bx^2}}$$

input
```
Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x]
```

output
```
((b*x^3*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*(
b*x^2)/a])/a^3 + (27*x*(6 - (5*b*x^2)/a - (b^2*x^4)/a^2 + (18*(3*a + b*x^2
)*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2
, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2
, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a,
-1/3*(b*x^2)/a]))))/(3*a + b*x^2)^2)/(972*(a - b*x^2)^(1/3))
```

3.114.3 Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {314, 27, 402, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}dx$$

$$\Big\downarrow \text{314}$$

$$\frac{x(a-bx^2)^{2/3}}{12a(3a+bx^2)^2}-\frac{\int -\frac{9a-5bx^2}{3\sqrt[3]{a-bx^2}(bx^2+3a)^2}dx}{12a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{9a-5bx^2}{\sqrt[3]{a-bx^2}(bx^2+3a)^2}dx}{36a}+\frac{x(a-bx^2)^{2/3}}{12a(3a+bx^2)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{x(a-bx^2)^{2/3}}{a(3a+bx^2)}-\frac{\int -\frac{8ab(bx^2+6a)}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{24a^2b}}{36a}+\frac{x(a-bx^2)^{2/3}}{12a(3a+bx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{bx^2+6a}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{3a}+\frac{x(a-bx^2)^{2/3}}{a(3a+bx^2)}}{36a}+\frac{x(a-bx^2)^{2/3}}{12a(3a+bx^2)^2}$$

$$\Big\downarrow \text{405}$$

$$\frac{\frac{\int \frac{1}{\sqrt[3]{a-bx^2}}dx+3a\int \frac{1}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{3a}+\frac{x(a-bx^2)^{2/3}}{a(3a+bx^2)}}{36a}+\frac{x(a-bx^2)^{2/3}}{12a(3a+bx^2)^2}$$

$$\Big\downarrow \text{233}$$

$$\frac{\frac{3a\int \frac{1}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx-\frac{3\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}}{3a}+\frac{x(a-bx^2)^{2/3}}{a(3a+bx^2)}}{36a}+\frac{x(a-bx^2)^{2/3}}{12a(3a+bx^2)^2}$$

$\downarrow$ 305

$$\frac{3a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}\right)-\frac{3\sqrt{-bx}\cdots}{\cdots}}{3a}}{36a}$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{12a\left(3a+bx^2\right)^2}$$

$\downarrow$ 833

$$\frac{3a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}\right)-\frac{3\sqrt{-bx}\cdots}{\cdots}}{3a}}{36a}$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{12a\left(3a+bx^2\right)^2}$$

$\downarrow$ 760

$$\frac{3a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}\right)-\frac{3\sqrt{-bx}\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{12a\left(3a+bx^2\right)^2}$$

$\downarrow$ 2418

3.114. $\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^3}\,dx$

$$\frac{\left(a-bx^2\right)^{2/3}x}{12a\left(bx^2+3a\right)^2}+$$

$$\frac{\frac{\left(a-bx^2\right)^{2/3}x}{a(bx^2+3a)}+\frac{3a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-b\ldots}\right.}\right.}{2\ 2^{2/3}a^{5/6}\sqrt{b}}\right.}{}$$

input
```
Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x]
```

output
```
(x*(a - b*x^2)^(2/3))/(12*a*(3*a + b*x^2)^2) + ((x*(a - b*x^2)^(2/3))/(a*(
3*a + b*x^2)) + (3*a*(ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqr
t[3]*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*
x^2)^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(S
qrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1
/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])) -
 (3*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3
)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]
]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1
/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)
*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(2*b*x))/(3*a))/(36*a)
```

3.114. $\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^3}\,dx$

3.114.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

3.114. $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3} \, dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.114.4 Maple [F]

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{2}{3}}}{\left(b\,x^{2}+3a\right)^{3}}dx$$

input
```
int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x)
```

output
```
int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x)
```

3.114.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.114.6 Sympy [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \int \frac{(a-bx^2)^{\frac{2}{3}}}{(3a+bx^2)^3}\,dx$$

input
```
integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**3,x)
```

output
```
Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**3, x)
```

3.114.7 Maxima [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^3}\,dx$$

input
```
integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3, x)
```

3.114.8 Giac [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^3}\,dx$$

input
```
integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3, x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \int \frac{(a-b\,x^2)^{2/3}}{(b\,x^2+3\,a)^3}\,dx$$

input
```
int((a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x)
```

output
```
int((a - b*x^2)^(2/3)/(3*a + b*x^2)^3, x)
```

3.115 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4} \, dx$

3.115.1 Optimal result

Integrand size = 24, antiderivative size = 849

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)^3} + \frac{x(a-bx^2)^{2/3}}{54a^2(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{144a^3(3a+bx^2)} - \frac{x}{144a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{7\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{7\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} - \frac{7\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3888\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{7\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{1296\ 2^{2/3}a^{17/6}\sqrt{b}} - \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{}\right.}{96\ 3^{3/4}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} + \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right)}{72\sqrt{2}\sqrt[4]{3}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
1/18*x*(-b*x^2+a)^(2/3)/a/(b*x^2+3*a)^3+1/54*x*(-b*x^2+a)^(2/3)/a^2/(b*x^2
+3*a)^2+1/144*x*(-b*x^2+a)^(2/3)/a^3/(b*x^2+3*a)-1/144*x/a^3/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))+7/2592*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/
3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(17/6)/b^(1/2)-7/7776*arctanh(x*b^(1/2)/a^
(1/2))*2^(1/3)/a^(17/6)/b^(1/2)+7/7776*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b
*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)+7/7776*
arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)+1/432*(
a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2))
)/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)
*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)*3^(3/4)/a^(8/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3)
)/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1/288*(a^(1/3)-(-b*x^2+
a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(
1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3
)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*
6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(8/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/
3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.115.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.31

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \frac{x\left(\frac{9a(a-bx^2)(75a^2+26abx^2+3b^2x^4)}{(3a+bx^2)^3} + bx^2\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right) + \frac{\cdots}{(3a+bx^2\cdots}\right.}{3888a^4\sqrt[3]{\cdots}}$$

input
```
Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x]
```

output
```
(x*((9*a*(a - b*x^2)*(75*a^2 + 26*a*b*x^2 + 3*b^2*x^4))/(3*a + b*x^2)^3 +
b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*(b*
x^2)/a] + (621*a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/
((3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a]
+ 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + Appell
F1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))))/(3888*a^4*(a - b*x^2)
^(1/3))
```

3.115.3 Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 917, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {314, 27, 402, 27, 402, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}dx$$

$$\Big\downarrow \text{314}$$

$$\frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)^3}-\frac{\int -\frac{15a-11bx^2}{3\sqrt[3]{a-bx^2}(bx^2+3a)^3}dx}{18a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{15a-11bx^2}{\sqrt[3]{a-bx^2}(bx^2+3a)^3}dx}{54a}+\frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)^3}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{x(a-bx^2)^{2/3}}{a(3a+bx^2)^2}-\frac{\int -\frac{16ab(12a-5bx^2)}{\sqrt[3]{a-bx^2}(bx^2+3a)^2}dx}{48a^2b}}{54a}+\frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{12a-5bx^2}{\sqrt[3]{a-bx^2}(bx^2+3a)^2}dx}{3a}+\frac{x(a-bx^2)^{2/3}}{a(3a+bx^2)^2}}{54a}+\frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)^3}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{9x(a-bx^2)^{2/3}}{8a(3a+bx^2)}-\frac{\int -\frac{3ab(3bx^2+23a)}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{24a^2b}}{3a}+\frac{x(a-bx^2)^{2/3}}{a(3a+bx^2)^2}}{54a}+\frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{3bx^2+23a}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx}{8a}+\frac{9x\left(a-bx^2\right)^{2/3}}{8a\left(3a+bx^2\right)}}{3a}+\frac{x\left(a-bx^2\right)^{2/3}}{a(3a+bx^2)^2}}{54a}+\frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3}$$

$$\Big\downarrow \; 405$$

$$\frac{\frac{\frac{3\int \frac{1}{\sqrt[3]{a-bx^2}}dx+14a\int \frac{1}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx}{8a}+\frac{9x\left(a-bx^2\right)^{2/3}}{8a\left(3a+bx^2\right)}}{3a}+\frac{x\left(a-bx^2\right)^{2/3}}{a(3a+bx^2)^2}}{54a}+\frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3}$$

$$\Big\downarrow \; 233$$

$$\frac{\frac{\frac{14a\int \frac{1}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx-\frac{9\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}}{8a}+\frac{9x\left(a-bx^2\right)^{2/3}}{8a\left(3a+bx^2\right)}}{3a}+\frac{x\left(a-bx^2\right)^{2/3}}{a(3a+bx^2)^2}}{54a}+\frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3}$$

$$\Big\downarrow \; 305$$

$$\frac{\frac{\frac{14a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}\right)-\frac{9\sqrt{-bx^2}\cdots}{\cdots}}{8a}\cdots}{3a}\cdots}{54a}\cdots$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3}$$

$$\Big\downarrow \; 833$$

3.115. $\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^4}dx$

$$14a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}\right)-\frac{9\sqrt{-bx^2}}{}$$

$$8a$$

$$3a$$

$$54a$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3}$$

$$\downarrow 760$$

$$14a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}\right)-\ \ 9\sqrt{-bx^2}$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3}$$

$$\downarrow 2418$$

3.115. $\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^4}\,dx$

$$\frac{\left(a-bx^2\right)^{2/3}x}{18a\left(bx^2+3a\right)^3}+$$

$$\frac{\frac{\left(a-bx^2\right)^{2/3}x}{a(bx^2+3a)^2}+\frac{\frac{9\left(a-bx^2\right)^{2/3}x}{8a\left(bx^2+3a\right)}+\frac{14a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\cdots}{\sqrt[6]{a}\left(\sqrt[3]{a}\cdots\right.}\right.}{2\ 2^{2/3}\cdots}\right.}{\cdots}}{\cdots}}{\cdots}$$

input
```
Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x]
```

output
```
(x*(a - b*x^2)^(2/3))/(18*a*(3*a + b*x^2)^3) + ((x*(a - b*x^2)^(2/3))/(a*(
3*a + b*x^2)^2) + ((9*x*(a - b*x^2)^(2/3))/(8*a*(3*a + b*x^2)) + (14*a*(Ar
cTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) +
ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)
]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^
(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(
a - b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])) - (9*Sqrt[-(b*x^2)]*((-2*
Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqr
t[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/
3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*
x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b
*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*
(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
(a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
F[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((
a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))^2)])))/(2*b*x))/(8*a))/(3*a))/(54*a)
```

3.115. $\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^4}\,dx$

3.115.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

3.115. $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.115.4 Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{2}{3}}}{(b\,x^2+3a)^4}dx$$

input
```
int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x)
```

output
```
int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x)
```

3.115.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x, algorithm="fricas")
```

output
```
Timed out
```

3.115.6 Sympy [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \int \frac{(a-bx^2)^{\frac{2}{3}}}{(3a+bx^2)^4}\,dx$$

input
```
integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**4,x)
```

output
```
Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**4, x)
```

3.115.7 Maxima [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^4}\,dx$$

input
```
integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4, x)
```

3.115.8 Giac [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^4}\,dx$$

input
```
integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4, x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \int \frac{(a-b\,x^2)^{2/3}}{(b\,x^2+3\,a)^4}\,dx$$

input
```
int((a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x)
```

output
```
int((a - b*x^2)^(2/3)/(3*a + b*x^2)^4, x)
```

3.116 $\int (a-bx^2)^{5/3} (3a+bx^2)^3 \, dx$

3.116.1 Optimal result

Integrand size = 24, antiderivative size = 668

$$\int (a-bx^2)^{5/3} (3a+bx^2)^3 \, dx = \frac{2809728a^4x(a-bx^2)^{2/3}}{267995}$$
$$+\frac{1404864a^3x(a-bx^2)^{5/3}}{191425}-\frac{33264a^2x(a-bx^2)^{8/3}}{14725}$$

$$-\frac{432}{775}ax(a-bx^2)^{8/3}(3a+bx^2)-\frac{3}{31}x(a-bx^2)^{8/3}(3a+bx^2)^2-\frac{11238912a^5x}{267995\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{561945}{}$$

output
```
2809728/267995*a^4*x*(-b*x^2+a)^(2/3)+1404864/191425*a^3*x*(-b*x^2+a)^(5/3
)-33264/14725*a^2*x*(-b*x^2+a)^(8/3)-432/775*a*x*(-b*x^2+a)^(8/3)*(b*x^2+3
*a)-3/31*x*(-b*x^2+a)^(8/3)*(b*x^2+3*a)^2-11238912/267995*a^5*x/(-(-b*x^2+
a)^(1/3)+a^(1/3)*(1-3^(1/2)))+3746304/267995*3^(3/4)*a^(16/3)*(a^(1/3)-(-b
*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-
b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2
)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3
)*(1-3^(1/2)))^2)^(1/2)-5619456/267995*3^(1/4)*a^(16/3)*(a^(1/3)-(-b*x^2+a
)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1
/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)
+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6
^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.116.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.77 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.16

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^3 \, dx = \frac{3\left(5815935a^5x-5312355a^4bx^3-1675114a^3b^2x^5+749658a^2b^3x^7+378651ab^4x^9+43225b^5x^{11}\right.}{1339975\sqrt[3]{a-bx^2}}$$

input
```
Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x]
```

output
```
(3*(5815935*a^5*x - 5312355*a^4*b*x^3 - 1675114*a^3*b^2*x^5 + 749658*a^2*b
^3*x^7 + 378651*a*b^4*x^9 + 43225*b^5*x^11 + 6243840*a^5*x*(1 - (b*x^2)/a)
^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(1339975*(a - b*x^2)^
(1/3))
```

3.116.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {318, 27, 403, 27, 299, 211, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3\,dx$$

$$\Big\downarrow \text{318}$$

$$-\frac{3\int -48ab\left(a-bx^2\right)^{5/3}\left(bx^2+2a\right)\left(bx^2+3a\right)dx}{31b}-\frac{3}{31}x\left(3a+bx^2\right)^2\left(a-bx^2\right)^{8/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{144}{31}a\int \left(a-bx^2\right)^{5/3}\left(bx^2+2a\right)\left(bx^2+3a\right)dx-\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{403}$$

$$\frac{144}{31}a\left(-\frac{3\int -4ab\left(a-bx^2\right)^{5/3}\left(8bx^2+13a\right)dx}{25b}-\frac{3}{25}x\left(2a+bx^2\right)\left(a-bx^2\right)^{8/3}\right)-\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{144}{31}a\left(\frac{12}{25}a\int \left(a-bx^2\right)^{5/3}\left(8bx^2+13a\right)dx-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(2a+bx^2\right)\right)-\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{299}$$

$$\frac{144}{31}a\left(\frac{12}{25}a\left(\frac{271}{19}a\int \left(a-bx^2\right)^{5/3}dx-\frac{24}{19}x\left(a-bx^2\right)^{8/3}\right)-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(2a+bx^2\right)\right)-\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{211}$$

$$\frac{144}{31}a\left(\frac{12}{25}a\left(\frac{271}{19}a\left(\frac{10}{13}a\int \left(a-bx^2\right)^{2/3}dx+\frac{3}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{24}{19}x\left(a-bx^2\right)^{8/3}\right)-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(2a+bx\right.\right.$$
$$\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{211}$$

$$\frac{144}{31}a\left(\frac{12}{25}a\left(\frac{271}{19}a\left(\frac{10}{13}a\left(\frac{4}{7}a\int\frac{1}{\sqrt[3]{a-bx^2}}dx+\frac{3}{7}x\left(a-bx^2\right)^{2/3}\right)+\frac{3}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{24}{19}x\left(a-bx^2\right)^{8/3}\right)-\frac{3}{25}\right.$$
$$\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

$\big\downarrow$ 233

$$\frac{144}{31}a\left(\frac{12}{25}a\left(\frac{271}{19}a\left(\frac{10}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\int\frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}\right)+\frac{3}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{24}{19}x\left(a\right.\right.\right.$$
$$\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

$\big\downarrow$ 833

$$\frac{144}{31}a\left(\frac{12}{25}a\left(\frac{271}{19}a\left(\frac{10}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}{\sqrt{-bx^2}}\right.}{7bx}\right.\right.\right.\right.$$
$$\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

$\big\downarrow$ 760

$$\frac{144}{31}a\left(\frac{12}{25}a\left(\frac{271}{19}a\left(\frac{10}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{}}{}\right.}{}\right.\right.\right.\right.$$
$$\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

$\big\downarrow$ 2418

3.116. $\int\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3\,dx$

$$\frac{144}{31}a\left(\frac{12}{25}a\left(\frac{271}{19}a\left(\frac{10}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-b\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{\cdots}}{\left(\left(1\cdots\right.\right.}}}\cdots\right.}{\cdots}\right.\right.\right.\right.$$

$$\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

input
```
Int[(a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x]
```

output
```
(-3*x*(a - b*x^2)^(8/3)*(3*a + b*x^2)^2)/31 + (144*a*((-3*x*(a - b*x^2)^(8
/3)*(2*a + b*x^2))/25 + (12*a*((-24*x*(a - b*x^2)^(8/3))/19 + (271*a*((3*x
*(a - b*x^2)^(5/3))/13 + (10*a*((3*x*(a - b*x^2)^(2/3))/7 - (6*a*Sqrt[-(b*
x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (
3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2
/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3
])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^
2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2
)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2)])))/(7*b*x)))/13))/19))/25))/31
```

3.116. $\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3\,dx$

3.116.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.116. $\int (a-bx^2)^{5/3} (3a+bx^2)^3 \, dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.116.4 Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{\frac{5}{3}} \left(b\,x^{2}+3a\right)^{3}dx$$

input
```
int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x)
```

output
```
int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x)
```

3.116.5 Fricas [F]

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3\,dx=\int \left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x, algorithm="fricas")
```

output
```
integral(-(b^4*x^8 + 8*a*b^3*x^6 + 18*a^2*b^2*x^4 - 27*a^4)*(-b*x^2 + a)^(
2/3), x)
```

3.116.6 Sympy [A] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.21

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3\,dx=27a^{\frac{14}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)-\frac{18a^{\frac{8}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}-\frac{8a^{\frac{5}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7}-\frac{a^{\frac{2}{3}}b^4x^9\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{9}{2}\\ \frac{11}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{9}$$

input
```
integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a)**3,x)
```

output
```
27*a**(14/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) - 18
*a**(8/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)
/5 - 8*a**(5/3)*b**3*x**7*hyper((-2/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*
pi)/a)/7 - a**(2/3)*b**4*x**9*hyper((-2/3, 9/2), (11/2,), b*x**2*exp_polar
(2*I*pi)/a)/9
```

3.116.7 Maxima [F]

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^3 \, dx = \int \left(bx^2+3\,a\right)^3 \left(-bx^2+a\right)^{\frac{5}{3}} dx$$

input
```
integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3), x)
```

3.116.8 Giac [F]

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^3 \, dx = \int \left(bx^2+3\,a\right)^3 \left(-bx^2+a\right)^{\frac{5}{3}} dx$$

input
```
integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^3 \, dx = \int \left(a-b\,x^2\right)^{5/3} \left(b\,x^2+3\,a\right)^3 dx$$

input
```
int((a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x)
```

output
```
int((a - b*x^2)^(5/3)*(3*a + b*x^2)^3, x)
```

3.117 $\int (a-bx^2)^{5/3} (3a+bx^2)^2 \, dx$

3.117.1 Optimal result

Integrand size = 24, antiderivative size = 637

$$\int (a-bx^2)^{5/3} (3a+bx^2)^2 \, dx = \frac{28512a^3x(a-bx^2)^{2/3}}{8645} + \frac{14256a^2x(a-bx^2)^{5/3}}{6175}$$

$$-\frac{306}{475}ax(a-bx^2)^{8/3} - \frac{3}{25}x(a-bx^2)^{8/3}(3a+bx^2) - \frac{114048a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{57024\sqrt[4]{3}\sqrt{2+\sqrt{3}}a}{}$$

output
```
28512/8645*a^3*x*(-b*x^2+a)^(2/3)+14256/6175*a^2*x*(-b*x^2+a)^(5/3)-306/47
5*a*x*(-b*x^2+a)^(8/3)-3/25*x*(-b*x^2+a)^(8/3)*(b*x^2+3*a)-114048/8645*a^4
*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+38016/8645*3^(3/4)*a^(13/3)*(a^
(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/
(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a
^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^
(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-57024/8645*3^(1/4)*a^(13/3)*(a^(1/3)-(-b*
x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+
a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^
(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(
1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^
2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.117.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.54 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.27

$$\int (a-bx^2)^{5/3} (3a+bx^2)^2 \, dx = \frac{x(a-bx^2)^{2/3} \left(21a(45a^2+10abx^2+b^2x^4) \text{Gamma}\left(-\frac{5}{3}\right) \text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},\frac{7}{2},\frac{bx^2}{a}\right) + \right.}{}$$

input
```
Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x]
```

output
```
(x*(a - b*x^2)^(2/3)*(21*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*Gamma[-5/3]*Hyp
ergeometric2F1[-5/3, 1/2, 7/2, (b*x^2)/a] + 8*b*x^2*(18*a^2 + 9*a*b*x^2 +
b^2*x^4)*Gamma[-2/3]*Hypergeometric2F1[-2/3, 3/2, 9/2, (b*x^2)/a] + 4*b*(3
*a*x + b*x^3)^2*Gamma[-2/3]*HypergeometricPFQ[{-2/3, 3/2, 2}, {1, 9/2}, (b
*x^2)/a]))/(105*(1 - (b*x^2)/a)^(2/3)*Gamma[-5/3])
```

3.117.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {318, 27, 299, 211, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^2)^{5/3} (3a+bx^2)^2 \, dx$$

$$\Big\downarrow \text{318}$$

$$-\frac{3\int -2ab(a-bx^2)^{5/3}(17bx^2+39a)\,dx}{25b} - \frac{3}{25}x(3a+bx^2)(a-bx^2)^{8/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{25}a\int (a-bx^2)^{5/3}(17bx^2+39a)\,dx - \frac{3}{25}x(a-bx^2)^{8/3}(3a+bx^2)$$

$$\Big\downarrow \text{299}$$

$$\frac{6}{25}a\left(\frac{792}{19}a\int\left(a-bx^2\right)^{5/3}dx-\frac{51}{19}x\left(a-bx^2\right)^{8/3}\right)-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{6}{25}a\left(\frac{792}{19}a\left(\frac{10}{13}a\int\left(a-bx^2\right)^{2/3}dx+\frac{3}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{51}{19}x\left(a-bx^2\right)^{8/3}\right)-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{6}{25}a\left(\frac{792}{19}a\left(\frac{10}{13}a\left(\frac{4}{7}a\int\frac{1}{\sqrt[3]{a-bx^2}}dx+\frac{3}{7}x\left(a-bx^2\right)^{2/3}\right)+\frac{3}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{51}{19}x\left(a-bx^2\right)^{8/3}\right)-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)$$

$$\Big\downarrow \text{233}$$

$$\frac{6}{25}a\left(\frac{792}{19}a\left(\frac{10}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\int\frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}\right)+\frac{3}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{51}{19}x\left(a-bx^2\right)^{8}\right.$$
$$\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)$$

$$\Big\downarrow \text{833}$$

$$\frac{6}{25}a\left(\frac{792}{19}a\left(\frac{10}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{\right.}{7bx}\right.\right.\right.$$
$$\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)$$

$$\Big\downarrow \text{760}$$

3.117. $\int\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2\,dx$

$$\frac{6}{25}a\left(\frac{792}{19}a\left(\frac{10}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.\right.$$

$$\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)$$

$\downarrow$ 2418

$$\frac{6}{25}a\left(\frac{792}{19}a\left(\frac{10}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\cdots\right.}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}\right.}{\cdots}\right.\right.\right.$$

$$\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)$$

input
```
Int[(a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x]
```

3.117. $\int\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2\,dx$

output
```
(-3*x*(a - b*x^2)^(8/3)*(3*a + b*x^2))/25 + (6*a*((-51*x*(a - b*x^2)^(8/3)
)/19 + (792*a*((3*x*(a - b*x^2)^(5/3))/13 + (10*a*((3*x*(a - b*x^2)^(2/3))
/7 - (6*a*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2
)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2
)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - S
qrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3
^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(7*b*x)))/13))/19))/25
```

3.117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.117.4 Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{\frac{5}{3}} \left(b\,x^{2}+3a\right)^{2} dx$$

input
```
int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x)
```

output
```
int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x)
```

3.117.5 Fricas [F]

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^2 \, dx=\int \left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x, algorithm="fricas")
```

output
```
integral(-(b^3*x^6 + 5*a*b^2*x^4 + 3*a^2*b*x^2 - 9*a^3)*(-b*x^2 + a)^(2/3)
, x)
```

3.117.6 Sympy [A] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.21

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^2 \, dx = 9a^{\frac{11}{3}}x\,{}_2F_1\left(\left.\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\right| \frac{bx^2e^{2i\pi}}{a}\right) - a^{\frac{8}{3}}bx^3\,{}_2F_1\left(\left.\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\right| \frac{bx^2e^{2i\pi}}{a}\right) - a^{\frac{5}{3}}b^2x^5\,{}_2F_1\left(\left.\begin{matrix} -\frac{2}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\right| \frac{bx^2e^{2i\pi}}{a}\right) - \frac{a^{\frac{2}{3}}b^3x^7\,{}_2F_1\left(\left.\begin{matrix} -\frac{2}{3}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\right| \frac{bx^2e^{2i\pi}}{a}\right)}{7}$$

input
```
integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a)**2,x)
```

output
```
9*a**(11/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) - a**
(8/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) - a**(
5/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a) - a*
*(2/3)*b**3*x**7*hyper((-2/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/7
```

3.117.7 Maxima [F]

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^2 \, dx = \int \left(bx^2+3\,a\right)^2 \left(-bx^2+a\right)^{\frac{5}{3}} \, dx$$

input
```
integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3), x)
```

3.117.8 Giac [F]

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^2 \, dx = \int \left(bx^2+3\,a\right)^2 \left(-bx^2+a\right)^{\frac{5}{3}} \, dx$$

input
```
integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^2 \, dx = \int \left(a-b\,x^2\right)^{5/3} \left(b\,x^2+3\,a\right)^2 \, dx$$

input
```
int((a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x)
```

output
```
int((a - b*x^2)^(5/3)*(3*a + b*x^2)^2, x)
```

3.118 $\int (a-bx^2)^{5/3} (3a+bx^2)\, dx$

3.118.1 Optimal result

Integrand size = 22, antiderivative size = 608

$$\int (a-bx^2)^{5/3} (3a+bx^2)\, dx = \frac{1800a^2x(a-bx^2)^{2/3}}{1729} + \frac{180}{247}ax(a-bx^2)^{5/3}$$

$$-\frac{3}{19}x(a-bx^2)^{8/3} - \frac{7200a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{3600\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{\cdots}}{\left(\left(1-\cdots\right.\right.}}}{1729bx\sqrt{\cdots}}$$

output
```
1800/1729*a^2*x*(-b*x^2+a)^(2/3)+180/247*a*x*(-b*x^2+a)^(5/3)-3/19*x*(-b*x
^2+a)^(8/3)-7200/1729*a^3*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+2400/1
729*3^(3/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/
3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1
/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^
2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)
^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-3600/1729*3^(1/4)
*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*
(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*
(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b
*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.118.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.87 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.11

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\,dx=\frac{3}{19}x\big(a-bx^2\big)^{2/3}\left(-\left(a-bx^2\right)^2+\frac{20a^2\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{2/3}}\right)$$

input
```
Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2),x]
```

output
```
(3*x*(a - b*x^2)^(2/3)*(-(a - b*x^2)^2 + (20*a^2*Hypergeometric2F1[-5/3, 1
/2, 3/2, (b*x^2)/a])/(1 - (b*x^2)/a)^(2/3)))/19
```

3.118.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {299, 211, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\,dx$$

$$\Big\downarrow \text{ 299}$$

$$\frac{60}{19}a\int \left(a-bx^2\right)^{5/3}dx-\frac{3}{19}x\left(a-bx^2\right)^{8/3}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{60}{19}a\left(\frac{10}{13}a\int \left(a-bx^2\right)^{2/3}dx+\frac{3}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{3}{19}x\left(a-bx^2\right)^{8/3}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{60}{19}a\left(\frac{10}{13}a\left(\frac{4}{7}a\int \frac{1}{\sqrt[3]{a-bx^2}}dx+\frac{3}{7}x\left(a-bx^2\right)^{2/3}\right)+\frac{3}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{3}{19}x\left(a-bx^2\right)^{8/3}$$

$$\Big\downarrow \text{ 233}$$

$$\frac{60}{19}a\left(\frac{10}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\int\frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}\right)+\frac{3}{13}x\left(a-bx^2\right)^{5/3}\right)-\frac{3}{19}x\left(a-bx^2\right)^{8/3}$$

$\downarrow$ 833

$$\frac{60}{19}a\left(\frac{10}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right.}{7bx}\right.\right.$$

$$\frac{3}{19}x\left(a-bx^2\right)^{8/3}$$

$\downarrow$ 760

$$\frac{60}{19}a\left(\frac{10}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-\right.}{}\right.}{}\right.\right.$$

$$\frac{3}{19}x\left(a-bx^2\right)^{8/3}$$

$\downarrow$ 2418

3.118. $\int\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\,dx$

$$\frac{60}{19}a\left(\frac{10}{13}a\left(\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{6a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\mathrm{Ell}\cdots}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-\cdots}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-\cdots}\right.}}}\cdots\right)}{\cdots}\cdots\right.\right.$$

$$\frac{3}{19}x\left(a-bx^2\right)^{8/3}$$

input
```
Int[(a - b*x^2)^(5/3)*(3*a + b*x^2),x]
```

output
```
(-3*x*(a - b*x^2)^(8/3))/19 + (60*a*((3*x*(a - b*x^2)^(5/3))/13 + (10*a*((
3*x*(a - b*x^2)^(2/3))/7 - (6*a*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)
*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellipti
cE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)
*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)
^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(7*b*x
)))/13))/19
```

3.118. $\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\,dx$

3.118.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.118.4 Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{\frac{5}{3}} \left(b\,x^{2}+3a\right)dx$$

input
```
int((-b*x^2+a)^(5/3)*(b*x^2+3*a),x)
```

output
```
int((-b*x^2+a)^(5/3)*(b*x^2+3*a),x)
```

3.118.5 Fricas [F]

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)\,dx = \int \left(bx^{2}+3\,a\right)\left(-bx^{2}+a\right)^{\frac{5}{3}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a),x, algorithm="fricas")
```

output
```
integral(-(b^2*x^4 + 2*a*b*x^2 - 3*a^2)*(-b*x^2 + a)^(2/3), x)
```

3.118.6 Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.16

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)\,dx = 3a^{\frac{8}{3}}x\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\right|\frac{bx^{2}e^{2i\pi}}{a}\right) - \frac{2a^{\frac{5}{3}}bx^{3}\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\right|\frac{bx^{2}e^{2i\pi}}{a}\right)}{3} - \frac{a^{\frac{2}{3}}b^{2}x^{5}\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{5}{2}\\\frac{7}{2}\end{matrix}\right|\frac{bx^{2}e^{2i\pi}}{a}\right)}{5}$$

input
```
integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a),x)
```

output
```
3*a**(8/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) - 2*a*
*(5/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3 - a
**(2/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

3.118.7 Maxima [F]

$$\int (a-bx^2)^{5/3}(3a+bx^2)\,dx=\int (bx^2+3a)(-bx^2+a)^{\frac{5}{3}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3), x)
```

3.118.8 Giac [F]

$$\int (a-bx^2)^{5/3}(3a+bx^2)\,dx=\int (bx^2+3a)(-bx^2+a)^{\frac{5}{3}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3), x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int (a-bx^2)^{5/3}(3a+bx^2)\,dx=\int (a-b\,x^2)^{5/3}\,(b\,x^2+3\,a)\,dx$$

input
```
int((a - b*x^2)^(5/3)*(3*a + b*x^2),x)
```

output
```
int((a - b*x^2)^(5/3)*(3*a + b*x^2), x)
```

3.119 $\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx$

3.119.1 Optimal result

Integrand size = 24, antiderivative size = 765

$$
\begin{aligned}
\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = &-\frac{3}{7}x\left(a-bx^2\right)^{2/3}+\frac{96ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} \\
&+\frac{4\sqrt[3]{2}a^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}+\frac{4\sqrt[3]{2}a^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} \\
&-\frac{4\sqrt[3]{2}a^{7/6}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}+\frac{4\sqrt[3]{2}a^{7/6}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}} \\
&+\frac{48\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right|}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} \\
&-\frac{32\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}
\end{aligned}
$$

output
```
-3/7*x*(-b*x^2+a)^(2/3)+96/7*a*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+4
*2^(1/3)*a^(7/6)*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/
3)))/b^(1/2)-4/3*2^(1/3)*a^(7/6)*arctanh(x*b^(1/2)/a^(1/2))/b^(1/2)+4/3*2^
(1/3)*a^(7/6)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/
b^(1/2))*3^(1/2)/b^(1/2)+4/3*2^(1/3)*a^(7/6)*arctan(3^(1/2)*a^(1/2)/x/b^(1
/2))*3^(1/2)/b^(1/2)-32/7*3^(3/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*Ellip
ticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1
-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x
^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3
)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)+48/7*3^(1/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)
^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*
3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)
^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/
3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(
1/2)
```

3.119.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.21 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.30

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = \frac{x\left(-32bx^2\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+27\left(-a+bx^2+\frac{\cdots}{(3a+bx^2)\left(9a\operatorname{Appe}\cdots\right.}\right.\right.}{63\sqrt[3]{a-b\cdots}}$$

input
```
Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2),x]
```

output
```
(x*(-32*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a,
-1/3*(b*x^2)/a] + 27*(-a + b*x^2 + (48*a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x
^2)/a, -1/3*(b*x^2)/a])/((3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*
x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a,
-1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))
)))/(63*(a - b*x^2)^(1/3))
```

3.119.3 Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {318, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{5/3}}{3a+bx^2}dx$$

$$\Big\downarrow \text{318}$$

$$\frac{3\int \frac{16ab(a-2bx^2)}{3\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{7b}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{16}{7}a\int \frac{a-2bx^2}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

$$\Big\downarrow \text{405}$$

$$\frac{16}{7}a\left(7a\int \frac{1}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx-2\int \frac{1}{\sqrt[3]{a-bx^2}}dx\right)-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

$$\Big\downarrow \text{233}$$

$$\frac{16}{7}a\left(\frac{3\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{bx}+7a\int \frac{1}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx\right)-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

$$\Big\downarrow \text{305}$$

$$\frac{16}{7}a\left(\frac{3\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{bx}+7a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arcta}}{}\right.\right.$$

$$\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

$$\Big\downarrow \text{833}$$

$$\frac{16}{7}a\left(\frac{3\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{bx}+7a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}(\cdots}{\cdots}\right.}{2\ 2^{2/\cdots}}\right.\right.$$

$$\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

$\downarrow$ 760

$$\frac{16}{7}a\left(\frac{3\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a\cdots}\right.}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\cdots}{\left(\left(1-\cdots\right.\right.}}}\right.}{bx}\right.$$

$$\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

$\downarrow$ 2418

3.119. $\int\frac{\left(a-bx^2\right)^{5/3}}{3a+bx^2}\,dx$

$$\frac{16}{7}a\left(7a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\right.}\right.}{2\ 2^{2/3}a^{5/6}}\right.\right.$$

$$\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

input
```
Int[(a - b*x^2)^(5/3)/(3*a + b*x^2),x]
```

output
```
(-3*x*(a - b*x^2)^(2/3))/7 + (16*a*(7*a*(ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]
*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3
) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sq
rt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTan
h[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(2*2^(2/3)*
a^(5/6)*Sqrt[b])) + (3*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3)
 - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x
^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) -
(2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4
*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(b*x)))/7
```

3.119. $\int \frac{\left(a-bx^2\right)^{5/3}}{3a+bx^2}\,dx$

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

3.119. $\int \frac{(a-bx^2)^{5/3}}{3a+bx^2} \, dx$

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.119.4 Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{5}{3}}}{b\,x^2+3a}dx$$

input
```
int((-b*x^2+a)^(5/3)/(b*x^2+3*a),x)
```

output
```
int((-b*x^2+a)^(5/3)/(b*x^2+3*a),x)
```

3.119.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a),x, algorithm="fricas")
```

output
```
Timed out
```

3.119.6 Sympy [F]

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = \int \frac{(a-bx^2)^{\frac{5}{3}}}{3a+bx^2}\,dx$$

input
```
integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a),x)
```

output
```
Integral((a - b*x**2)**(5/3)/(3*a + b*x**2), x)
```

3.119.7 Maxima [F]

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{3}}}{bx^2+3\,a}\,dx$$

input
```
integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a), x)
```

3.119.8 Giac [F]

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{3}}}{bx^2+3\,a}\,dx$$

input
```
integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = \int \frac{(a-b\,x^2)^{5/3}}{b\,x^2+3\,a}\,dx$$

input
```
int((a - b*x^2)^(5/3)/(3*a + b*x^2),x)
```

output
```
int((a - b*x^2)^(5/3)/(3*a + b*x^2), x)
```

3.120 $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2} \, dx$

3.120.1 Optimal result

Integrand size = 24, antiderivative size = 775

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2} \, dx = \frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)} - \frac{11x}{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{\sqrt[3]{2}\sqrt[6]{a}\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}} - \frac{11\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7}{2\,3^{3/4}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} + \frac{11\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-}{3\sqrt[4]{3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
2/3*x*(-b*x^2+a)^(2/3)/(b*x^2+3*a)-11/3*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^
(1/2)))-2^(1/3)*a^(1/6)*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2
+a)^(1/3)))/b^(1/2)+1/3*2^(1/3)*a^(1/6)*arctanh(x*b^(1/2)/a^(1/2))/b^(1/2)
-1/3*2^(1/3)*a^(1/6)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(
1/2)/x/b^(1/2))*3^(1/2)/b^(1/2)-1/3*2^(1/3)*a^(1/6)*arctan(3^(1/2)*a^(1/2)
/x/b^(1/2))*3^(1/2)/b^(1/2)+11/9*3^(3/4)*a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3)
)*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(
1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3
)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(
-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)-11/6*3^(1/4)*a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b
*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))
,2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b
*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/
(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)
))^2)^(1/2)
```

3.120.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.30

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx = \frac{x\left(\frac{11bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a}+\frac{27\left(2a-2bx^2-\frac{9a^2\,\text{Ap}\ldots}{9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-\ldots\right.}\ldots\right.}{\ldots}\ldots\right.}{81\sqrt[3]{a-bx^2}\ldots}$$

input
```
Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x]
```

output
```
(x*((11*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a,
-1/3*(b*x^2)/a])/a + (27*(2*a - 2*b*x^2 - (9*a^2*AppellF1[1/2, 1/3, 1, 3/2
, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -
1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x
^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))/(3*a +
b*x^2)))/(81*(a - b*x^2)^(1/3))
```

3.120.3 Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 836, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {315, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 315$$

$$\frac{\int -\frac{2ab(3a-11bx^2)}{3\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{6ab}+\frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)}-\frac{1}{9}\int \frac{3a-11bx^2}{\sqrt[3]{a-bx^2}\,(bx^2+3a)}dx$$

$$\Big\downarrow \; 405$$

$$\frac{1}{9}\left(11\int \frac{1}{\sqrt[3]{a-bx^2}}dx-36a\int \frac{1}{\sqrt[3]{a-bx^2}\,(bx^2+3a)}dx\right)+\frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)}$$

$$\Big\downarrow \; 233$$

$$\frac{1}{9}\left(-\frac{33\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}-36a\int \frac{1}{\sqrt[3]{a-bx^2}\,(bx^2+3a)}dx\right)+\frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)}$$

$$\Big\downarrow \; 305$$

$$\frac{1}{9}\left(-\frac{33\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}-36a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arct}}{}\right.\right.$$

$$\frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)}$$

$$\Big\downarrow \; 833$$

$$\frac{1}{9}\left(-\frac{33\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{2bx}-36a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{\cdots}}{\cdots}\right.}{2\,2\cdots}\right.\right.$$

$$\frac{2x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)}$$

$\downarrow$ 760

$$\frac{1}{9}\left(-\frac{33\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{\cdots}}{\left(\left(1-\cdots\right.\right.}}}\right.}{2bx}\right.$$

$$\frac{2x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)}$$

$\downarrow$ 2418

3.120. $\int \frac{\left(a-bx^2\right)^{5/3}}{\left(3a+bx^2\right)^2}\,dx$

$$\frac{2(a-bx^2)^{2/3}x}{3(bx^2+3a)}+$$

$$\frac{1}{9}\left(-36a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\right.}\right.}{2\,2^{2/3}a^{5/6}}\right.\right.$$

input
```
Int[(a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x]
```

output
```
(2*x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)) + (-36*a*(ArcTan[(Sqrt[3]*Sqrt[a
])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1
/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]
*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b
]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/
(2*2^(2/3)*a^(5/6)*Sqrt[b])) - (33*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1
/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3
) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elli
pticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1
/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x
^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/
((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(2*
b*x))/9
```

3.120. $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx$

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

3.120. $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx$

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.120.4 Maple [F]

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{5}{3}}}{\left(b\,x^{2}+3a\right)^{2}}dx$$

input
```
int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x)
```

output
```
int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x)
```

3.120.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.120.6 Sympy [F]

$$\int \frac{\left(a-bx^2\right)^{5/3}}{\left(3a+bx^2\right)^2}\,dx=\int \frac{\left(a-bx^2\right)^{\frac{5}{3}}}{\left(3a+bx^2\right)^2}\,dx$$

input
```
integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a)**2,x)
```

output
```
Integral((a - b*x**2)**(5/3)/(3*a + b*x**2)**2, x)
```

3.120.7 Maxima [F]

$$\int \frac{\left(a-bx^2\right)^{5/3}}{\left(3a+bx^2\right)^2}\,dx=\int \frac{\left(-bx^2+a\right)^{\frac{5}{3}}}{\left(bx^2+3\,a\right)^2}\,dx$$

input
```
integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2, x)
```

3.120.8 Giac [F]

$$\int \frac{\left(a-bx^2\right)^{5/3}}{\left(3a+bx^2\right)^2}\,dx=\int \frac{\left(-bx^2+a\right)^{\frac{5}{3}}}{\left(bx^2+3\,a\right)^2}\,dx$$

input
```
integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2, x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx = \int \frac{(a-b\,x^2)^{5/3}}{(b\,x^2+3\,a)^2}\,dx$$

input
```
int((a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x)
```

output
```
int((a - b*x^2)^(5/3)/(3*a + b*x^2)^2, x)
```

3.121 $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx$

3.121.1 Optimal result

Integrand size = 24, antiderivative size = 815

$$
\begin{aligned}
\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx &= \frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2} - \frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)} + \frac{x}{18a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} \\
&+ \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{18\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} \\
&- \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{54\ 2^{2/3}a^{5/6}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{18\ 2^{2/3}a^{5/6}\sqrt{b}} \\
&+ \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}}{12\ 3^{3/4}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} \\
&- \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}}{9\sqrt{2}\sqrt[4]{3}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}
\end{aligned}
$$

output
```
1/3*x*(-b*x^2+a)^(2/3)/(b*x^2+3*a)^2-1/18*x*(-b*x^2+a)^(2/3)/a/(b*x^2+3*a)
+1/18*x/a/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/36*arctanh(x*b^(1/2)/a
^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(5/6)/b^(1/2)-1/108*a
rctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(5/6)/b^(1/2)+1/108*arctan(a^(1/6)*(a^
(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(5/6)*3^(1/2)
/b^(1/2)+1/108*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(5/6)*3^(1/2)/b
^(1/2)-1/54*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3
)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^
(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3
)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b
*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+1/36*(a^(1
/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-
(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b
*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(-
b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.121.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.31

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \frac{-\frac{bx^3\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^2}+\frac{27x\left(3a-4bx^2+\frac{b^2x^4}{a}+\frac{9a(3a+}{9a\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2}\right.}{(3}}{486\sqrt[3]{a-bx^2}}$$

input
```
Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x]
```

output
```
(-((b*x^3*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3
*(b*x^2)/a])/a^2) + (27*x*(3*a - 4*b*x^2 + (b^2*x^4)/a + (9*a*(3*a + b*x^2
)*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2
, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2
, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a,
-1/3*(b*x^2)/a]))))/(3*a + b*x^2)^2)/(486*(a - b*x^2)^(1/3))
```

3.121.3 Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {315, 27, 373, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{315}$$

$$\frac{\int \frac{16ab^2x^2}{3\sqrt[3]{a-bx^2}(bx^2+3a)^2}dx}{12ab}+\frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{9}b\int \frac{x^2}{\sqrt[3]{a-bx^2}\,(bx^2+3a)^2}dx+\frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2}$$

$$\Big\downarrow \text{373}$$

$$\frac{4}{9}b\left(\frac{\int \frac{3a-bx^2}{3\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{8ab}-\frac{x(a-bx^2)^{2/3}}{8ab(3a+bx^2)}\right)+\frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{9}b\left(\frac{\int \frac{3a-bx^2}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{24ab}-\frac{x(a-bx^2)^{2/3}}{8ab(3a+bx^2)}\right)+\frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2}$$

$$\Big\downarrow \text{405}$$

$$\frac{4}{9}b\left(\frac{6a\int \frac{1}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx-\int \frac{1}{\sqrt[3]{a-bx^2}}dx}{24ab}-\frac{x(a-bx^2)^{2/3}}{8ab(3a+bx^2)}\right)+\frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2}$$

$$\Big\downarrow \text{233}$$

$$\frac{4}{9}b\left(\frac{\frac{3\sqrt{-bx^2}\int\frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}+6a\int\frac{1}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{24ab}-\frac{x\left(a-bx^2\right)^{2/3}}{8ab\left(3a+bx^2\right)}\right)+$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)^2}$$

$$\Big\downarrow \text{305}$$

$$\frac{4}{9}b\left(\frac{\frac{3\sqrt{-bx^2}\int\frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}+6a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{}{\sqrt[6]{a}\left(\right.}\right.}{2\ 2}\right.}{24ab}\right.$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)^2}$$

$$\Big\downarrow \text{833}$$

$$\frac{4}{9}b\left(\frac{\frac{3\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{2bx}+6a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a\right.}{\sqrt{b}x}\right.}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}\right.}{24ab}\right.$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)^2}$$

3.121. $\int\frac{\left(a-bx^2\right)^{5/3}}{\left(3a+bx^2\right)^3}dx$

$\downarrow$ 760

$$\frac{4}{9}b\left(\frac{3\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}\right.}}}\right.}{2bx}\right.$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)^2}$$

$\downarrow$ 2418

3.121. $\int\frac{\left(a-bx^2\right)^{5/3}}{\left(3a+bx^2\right)^3}\,dx$

$$\frac{\left(a-bx^2\right)^{2/3}x}{3\left(bx^2+3a\right)^2}+$$

$$\frac{4}{9}b\left(\frac{6a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx\right.}}\right.}{2\ 2^{2/3}a^{5/6}\sqrt{b}}\right.}{}\right.$$

input
```
Int[(a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x]
```

3.121. $\int \frac{\left(a-bx^2\right)^{5/3}}{\left(3a+bx^2\right)^3}\,dx$

output
```
(x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)^2) + (4*b*(-1/8*(x*(a - b*x^2)^(2/3
))/(a*b*(3*a + b*x^2)) + (6*a*(ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^
(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3
)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - A
rcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]
*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sq
rt[b])) + (3*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*
x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*
x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2
- Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*
x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])
/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(2*b*x))/(24*a*b)))/9
```

3.121.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

3.121. $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\, dx$

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

3.121. $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx$

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.121.4 Maple [F]

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{5}{3}}}{\left(b\,x^{2}+3a\right)^{3}}dx$$

input
```
int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x)
```

output
```
int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x)
```

3.121.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.121.6 Sympy [F]

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \int \frac{(a-bx^2)^{\frac{5}{3}}}{(3a+bx^2)^3}\,dx$$

input
```
integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a)**3,x)
```

output
```
Integral((a - b*x**2)**(5/3)/(3*a + b*x**2)**3, x)
```

3.121.7 Maxima [F]

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{3}}}{(bx^2+3\,a)^3}\,dx$$

input
```
integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3, x)
```

3.121.8 Giac [F]

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{3}}}{(bx^2+3\,a)^3}\,dx$$

input
```
integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3, x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \int \frac{(a-b\,x^2)^{5/3}}{(b\,x^2+3\,a)^3}\,dx$$

input
```
int((a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x)
```

output
```
int((a - b*x^2)^(5/3)/(3*a + b*x^2)^3, x)
```

3.122 $\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx$

3.122.1 Optimal result

Integrand size = 24, antiderivative size = 659

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx = -\frac{1552608a^3x(a-bx^2)^{2/3}}{43225} - \frac{36288a^2x(a-bx^2)^{2/3}(3a+bx^2)}{6175}$$

$$-\frac{18}{19}ax(a-bx^2)^{2/3}(3a+bx^2)^2 - \frac{3}{25}x(a-bx^2)^{2/3}(3a+bx^2)^3 - \frac{3794688a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{189734\cdots}{\cdots}$$

output
```
-1552608/43225*a^3*x*(-b*x^2+a)^(2/3)-36288/6175*a^2*x*(-b*x^2+a)^(2/3)*(b
*x^2+3*a)-18/19*a*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)^2-3/25*x*(-b*x^2+a)^(2/3)
*(b*x^2+3*a)^3-3794688/8645*a^4*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+
1264896/8645*3^(3/4)*a^(13/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x
^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2
*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))
/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(
-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1897344/
8645*3^(1/4)*a^(13/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1
/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(
1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1
/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*
(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)
```

3.122.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.15

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx = \frac{3x\left(-941085a^4+727830a^3bx^2+184044a^2b^2x^4+27482ab^3x^6+1729b^4x^8+2108160a^4\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hyperg}\right.}{43225\sqrt[3]{a-bx^2}}$$

input
```
Integrate[(3*a + b*x^2)^4/(a - b*x^2)^(1/3),x]
```

output
```
(3*x*(-941085*a^4 + 727830*a^3*b*x^2 + 184044*a^2*b^2*x^4 + 27482*a*b^3*x^
6 + 1729*b^4*x^8 + 2108160*a^4*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3
, 1/2, 3/2, (b*x^2)/a]))/(43225*(a - b*x^2)^(1/3))
```

3.122.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {318, 27, 403, 27, 403, 27, 299, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx$$

$$\Big\downarrow \text{ 318}$$

$$-\frac{3\int -\frac{2ab(bx^2+3a)^2(25bx^2+39a)}{\sqrt[3]{a-bx^2}}dx}{25b}-\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

$$\Big\downarrow \text{ 27}$$

$$\frac{6}{25}a\int \frac{(bx^2+3a)^2(25bx^2+39a)}{\sqrt[3]{a-bx^2}}dx-\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

$$\Big\downarrow \text{ 403}$$

$$\frac{6}{25}a\left(-\frac{3\int -\frac{48ab(bx^2+3a)(14bx^2+17a)}{\sqrt[3]{a-bx^2}}dx}{19b}-\frac{75}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\right)-\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

$\downarrow$ 27

$$\frac{6}{25}a\left(\frac{144}{19}a\int\frac{\left(bx^2+3a\right)\left(14bx^2+17a\right)}{\sqrt[3]{a-bx^2}}dx-\frac{75}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\right)-\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

$\downarrow$ 403

$$\frac{6}{25}a\left(\frac{144}{19}a\left(-\frac{3\int -\frac{ab(599bx^2+789a)}{3\sqrt[3]{a-bx^2}}dx}{13b}-\frac{42}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\right)-\frac{75}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\right)-\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

$\downarrow$ 27

$$\frac{6}{25}a\left(\frac{144}{19}a\left(\frac{1}{13}a\int\frac{599bx^2+789a}{\sqrt[3]{a-bx^2}}dx-\frac{42}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\right)-\frac{75}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\right)-\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

$\downarrow$ 299

$$\frac{6}{25}a\left(\frac{144}{19}a\left(\frac{1}{13}a\left(\frac{7320}{7}a\int\frac{1}{\sqrt[3]{a-bx^2}}dx-\frac{1797}{7}x\left(a-bx^2\right)^{2/3}\right)-\frac{42}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\right)-\frac{75}{19}x\left(a-bx^2\right.\right.$$
$$\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

$\downarrow$ 233

$$\frac{6}{25}a\left(\frac{144}{19}a\left(\frac{1}{13}a\left(-\frac{10980a\sqrt{-bx^2}\int\frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}-\frac{1797}{7}x\left(a-bx^2\right)^{2/3}\right)-\frac{42}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx\right.\right.\right.$$
$$\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

$\downarrow$ 833

3.122. $\int\frac{\left(3a+bx^2\right)^4}{\sqrt[3]{a-bx^2}}dx$

$$\frac{6}{25}a\left(\frac{144}{19}a\left(\frac{1}{13}a\left(-\frac{10980a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{7bx}-\right.\right.\right.$$

$$\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

$\downarrow$ 760

$$\frac{6}{25}a\left(\frac{144}{19}a\left(\frac{1}{13}a\left(-\frac{10980a\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right.}{\quad}\right.}{7bx}\right.\right.\right.$$

$$\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

$\downarrow$ 2418

$$\frac{6}{25}a\left(\frac{144}{19}a\left(\frac{1}{13}a\left(-\frac{10980a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right.}}}\right.}{\quad}\right.\right.\right.$$

$$\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

3.122. $\int\frac{\left(3a+bx^2\right)^4}{\sqrt[3]{a-bx^2}}\,dx$

input
```
Int[(3*a + b*x^2)^4/(a - b*x^2)^(1/3),x]
```

output
```
(-3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^3)/25 + (6*a*((-75*x*(a - b*x^2)^(2/
3)*(3*a + b*x^2)^2)/19 + (144*a*((-42*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/1
3 + (a*((-1797*x*(a - b*x^2)^(2/3))/7 - (10980*a*Sqrt[-(b*x^2)]*((-2*Sqrt[
-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 +
 Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^
(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)
]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1
/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a -
b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/
3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2)])))/(7*b*x)))/13))/19))/25
```

3.122.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.122. $\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx$

3.122.4 Maple [F]

$$\int \frac{\left(b\,x^{2}+3a\right)^{4}}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}}}dx$$

input
```
int((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x)
```

output
```
int((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x)
```

3.122.5 Fricas [F]

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{\left(bx^2+3\,a\right)^4}{\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(b^4*x^8 + 12*a*b^3*x^6 + 54*a^2*b^2*x^4 + 108*a^3*b*x^2 + 81*a^
4)*(-b*x^2 + a)^(2/3)/(b*x^2 - a), x)
```

3.122.6 Sympy [A] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.25

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx = 81a^{\frac{11}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+36a^{\frac{8}{3}}bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + \frac{54a^{\frac{5}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5} + \frac{12a^{\frac{2}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7} + \frac{b^4x^9\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{9}{2}\\ \frac{11}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{9\sqrt[3]{a}}$$

input
```
integrate((b*x**2+3*a)**4/(-b*x**2+a)**(1/3),x)
```

output
```
81*a**(11/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 36*
a**(8/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 54
*a**(5/3)*b**2*x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/
5 + 12*a**(2/3)*b**3*x**7*hyper((1/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*p
i)/a)/7 + b**4*x**9*hyper((1/3, 9/2), (11/2,), b*x**2*exp_polar(2*I*pi)/a)
/(9*a**(1/3))
```

3.122.7 Maxima [F]

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^4}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3), x)
```

3.122.8 Giac [F]

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^4}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3), x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(b\,x^2+3\,a)^4}{(a-b\,x^2)^{1/3}}\,dx$$

input
```
int((3*a + b*x^2)^4/(a - b*x^2)^(1/3),x)
```

output
```
int((3*a + b*x^2)^4/(a - b*x^2)^(1/3), x)
```

3.123 $\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx$

3.123.1 Optimal result

Integrand size = 24, antiderivative size = 628

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = -\frac{15768a^2x(a-bx^2)^{2/3}}{1729} - \frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)$$
$$-\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 - \frac{215136a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{107568\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-\right.}{}$$

output
```
-15768/1729*a^2*x*(-b*x^2+a)^(2/3)-324/247*a*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a
)-3/19*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)^2-215136/1729*a^3*x/(-(-b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2)))+71712/1729*3^(3/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1
/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+
a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(
1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/
x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/
2)))^2)^(1/2)-107568/1729*3^(1/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*Elli
pticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(
1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(
2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^
(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)
```

3.123.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.14

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = \frac{3\left(-8343a^3x+7041a^2bx^3+1211ab^2x^5+91b^3x^7+23904a^3x\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right.}{1729\sqrt[3]{a-bx^2}}$$

input
```
Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(1/3),x]
```

output
```
(3*(-8343*a^3*x + 7041*a^2*b*x^3 + 1211*a*b^2*x^5 + 91*b^3*x^7 + 23904*a^3
*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(17
29*(a - b*x^2)^(1/3))
```

3.123.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {318, 27, 403, 27, 299, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx$$

$$\Big\downarrow \text{318}$$

$$-\frac{3\int -\frac{12ab(bx^2+3a)(3bx^2+5a)}{\sqrt[3]{a-bx^2}}dx}{19b}-\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{36}{19}a\int \frac{(bx^2+3a)(3bx^2+5a)}{\sqrt[3]{a-bx^2}}dx-\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{403}$$

$$\frac{36}{19}a\left(-\frac{3\int -\frac{2ab(73bx^2+111a)}{3\sqrt[3]{a-bx^2}}dx}{13b}-\frac{9}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\right)-\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{36}{19}a\left(\frac{2}{13}a\int \frac{73bx^2+111a}{\sqrt[3]{a-bx^2}}dx-\frac{9}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\right)-\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{299}$$

$$\frac{36}{19}a\left(\frac{2}{13}a\left(\frac{996}{7}a\int \frac{1}{\sqrt[3]{a-bx^2}}dx-\frac{219}{7}x\left(a-bx^2\right)^{2/3}\right)-\frac{9}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\right)-\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{233}$$

$$\frac{36}{19}a\left(\frac{2}{13}a\left(-\frac{1494a\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}-\frac{219}{7}x\left(a-bx^2\right)^{2/3}\right)-\frac{9}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\right)-\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{833}$$

$$\frac{36}{19}a\left(\frac{2}{13}a\left(-\frac{1494a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{7bx}-\frac{219}{7}x(a\right.\right.$$

$$\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2$$

$$\Big\downarrow \text{760}$$

3.123. $\int \frac{\left(3a+bx^2\right)^3}{\sqrt[3]{a-bx^2}}\,dx$

$$\frac{36}{19}a\left(\frac{2}{13}a\left(-\frac{1494a\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\cdots}{\left(\left(1\cdots\right.\right.}}}{\sqrt[4]{3}\sqrt{\cdots}}\right.}{7bx}\right.\right.$$

$$\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2$$

$\downarrow$ 2418

$$\frac{36}{19}a\left(\frac{2}{13}a\left(-\frac{1494a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\right.}{\cdots}\right.\right.$$

$$\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2$$

input
```
Int[(3*a + b*x^2)^3/(a - b*x^2)^(1/3),x]
```

3.123. $\int\frac{\left(3a+bx^2\right)^3}{\sqrt[3]{a-bx^2}}\,dx$

output
```
(-3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^2)/19 + (36*a*((-9*x*(a - b*x^2)^(2/
3)*(3*a + b*x^2))/13 + (2*a*((-219*x*(a - b*x^2)^(2/3))/7 - (1494*a*Sqrt[-
(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
+ (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[
3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqr
t[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b
*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*
x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))^2)])))/(7*b*x))/13))/19
```

3.123.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.123. $\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx$

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.123.4 Maple [F]

$$\int \frac{(b\,x^2+3a)^3}{(-b\,x^2+a)^{\frac{1}{3}}}dx$$

input
```
int((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x)
```

output
```
int((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x)
```

3.123.5 Fricas [F]

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/
3)/(b*x^2 - a), x)
```

3.123.6 Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.21

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = 27a^{\frac{8}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + 9a^{\frac{5}{3}}bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + \frac{9a^{\frac{2}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5} + \frac{b^3x^7\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7\sqrt[3]{a}}$$

input
```
integrate((b*x**2+3*a)**3/(-b*x**2+a)**(1/3),x)
```

output
```
27*a**(8/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a*
*(5/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a*
*(2/3)*b**2*x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5 +
 b**3*x**7*hyper((1/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(1/
3))
```

3.123.7 Maxima [F]

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3), x)
```

3.123.8 Giac [F]

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3), x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(b\,x^2+3\,a)^3}{(a-b\,x^2)^{1/3}}\,dx$$

input
```
int((3*a + b*x^2)^3/(a - b*x^2)^(1/3),x)
```

output
```
int((3*a + b*x^2)^3/(a - b*x^2)^(1/3), x)
```

3.124 $\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx$

3.124.1 Optimal result

Integrand size = 24, antiderivative size = 597

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = -\frac{198}{91}ax\left(a-bx^2\right)^{2/3} - \frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{3240a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$-\frac{1620\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b\cdots}}\right.\right.\cdots}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{1080\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-\cdots}}\right.\right.\cdots}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
-198/91*a*x*(-b*x^2+a)^(2/3)-3/13*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)-3240/91*a
^2*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1080/91*3^(3/4)*a^(7/3)*(a^(1
/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-
(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(
1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1
/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1620/91*3^(1/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)
^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+
(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^
(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(
1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.124.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.96 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.26

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx$$
$$= \frac{x\sqrt[3]{1-\frac{bx^2}{a}}\left(63a(45a^2+10abx^2+b^2x^4)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{7}{2},\frac{bx^2}{a}\right)+8bx^2(18a^2+9abx^2+b^2x^4)\,\text{H}\right.}{315a\sqrt[3]{a-bx^2}}$$

input
```
Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(1/3),x]
```

output
```
(x*(1 - (b*x^2)/a)^(1/3)*(63*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*Hypergeomet
ric2F1[1/3, 1/2, 7/2, (b*x^2)/a] + 8*b*x^2*(18*a^2 + 9*a*b*x^2 + b^2*x^4)*
Hypergeometric2F1[4/3, 3/2, 9/2, (b*x^2)/a] + 4*b*(3*a*x + b*x^3)^2*Hyperg
eometricPFQ[{4/3, 3/2, 2}, {1, 9/2}, (b*x^2)/a]))/(315*a*(a - b*x^2)^(1/3)
)
```

3.124.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {318, 27, 299, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}dx$$

$$\Big\downarrow \text{318}$$

$$-\frac{3\int -\frac{2ab(11bx^2+21a)}{\sqrt[3]{a-bx^2}}dx}{13b}-\frac{3}{13}x(a-bx^2)^{2/3}(3a+bx^2)$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{13}a\int \frac{11bx^2+21a}{\sqrt[3]{a-bx^2}}dx-\frac{3}{13}x(a-bx^2)^{2/3}(3a+bx^2)$$

$$\Big\downarrow \text{299}$$

$$\frac{6}{13}a\left(\frac{180}{7}a\int \frac{1}{\sqrt[3]{a-bx^2}}dx-\frac{33}{7}x(a-bx^2)^{2/3}\right)-\frac{3}{13}x(a-bx^2)^{2/3}(3a+bx^2)$$

$$\Big\downarrow \text{233}$$

$$\frac{6}{13}a\left(-\frac{270a\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}-\frac{33}{7}x(a-bx^2)^{2/3}\right)-\frac{3}{13}x(a-bx^2)^{2/3}(3a+bx^2)$$

$$\Big\downarrow \text{833}$$

$$\frac{6}{13}a\left(-\frac{270a\sqrt{-bx^2}\left((1+\sqrt{3})\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{7bx}-\frac{33}{7}x(a-bx^2)^{2/}\right.$$

$$\frac{3}{13}x(a-bx^2)^{2/3}(3a+bx^2)$$

$$\Big\downarrow \text{760}$$

$$\frac{6}{13}a\left(-\frac{270a\sqrt{-bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a \cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\cdots}}\right.}{7bx}\cdots\right.$$

$$\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)$$

$\downarrow$ 2418

$$\frac{6}{13}a\left(-\frac{270a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\right.}{\cdots}\cdots\right.$$

$$\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)$$

input
```
Int[(3*a + b*x^2)^2/(a - b*x^2)^(1/3),x]
```

3.124. $\int \frac{\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}\,dx$

output
```
(-3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/13 + (6*a*((-33*x*(a - b*x^2)^(2/3)
)/7 - (270*a*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*
x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*
x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2
- Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*
x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])
/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(7*b*x)))/13
```

3.124.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.124. $\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.124.4 Maple [F]

$$\int \frac{\left(b\,x^{2}+3a\right)^{2}}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}}}dx$$

input
```
int((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x)
```

output
```
int((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x)
```

3.124.5 Fricas [F]

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3)/(b*x^2 - a), x)
```

3.124.6 Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.16

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = 9a^{\frac{5}{3}}x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right) + 2a^{\frac{2}{3}}bx^3\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right) + \frac{b^2x^5\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{5\sqrt[3]{a}}$$

input
```
integrate((b*x**2+3*a)**2/(-b*x**2+a)**(1/3),x)
```

output
```
9*a**(5/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 2*a**
(2/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + b**2*
x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/3))
```

3.124.7 Maxima [F]

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3), x)
```

3.124.8 Giac [F]

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3), x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(b\,x^2+3\,a)^2}{(a-b\,x^2)^{1/3}}\,dx$$

input
```
int((3*a + b*x^2)^2/(a - b*x^2)^(1/3),x)
```

output
```
int((3*a + b*x^2)^2/(a - b*x^2)^(1/3), x)
```

3.125 $\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx$

3.125.1 Optimal result

Integrand size = 22, antiderivative size = 568

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx = -\frac{3}{7}x\left(a-bx^2\right)^{2/3} - \frac{72ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{24\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
-3/7*x*(-b*x^2+a)^(2/3)-72/7*a*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+2
4/7*3^(3/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3
)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/
2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^
(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-36/7*3^(1/4)*a^(4/
3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1
/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(
1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1
/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)
^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.125.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.11

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx=\frac{3x\left(-a+bx^2+8a\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{7\sqrt[3]{a-bx^2}}$$

input
```
Integrate[(3*a + b*x^2)/(a - b*x^2)^(1/3),x]
```

output
```
(3*x*(-a + b*x^2 + 8*a*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3
/2, (b*x^2)/a]))/(7*(a - b*x^2)^(1/3))
```

3.125.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {299, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx$$

$$\Big\downarrow \; 299$$

$$\frac{24}{7}a\int \frac{1}{\sqrt[3]{a-bx^2}}dx-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

$$\Big\downarrow \; 233$$

$$-\frac{36a\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

$$\Big\downarrow \; 833$$

$$-\frac{36a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{7bx}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

$$\Big\downarrow \; 760$$

$$-\frac{36a\sqrt{-bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}\right.}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}(}{\left(\left(1-\sqrt{3}\right)\right.}}}\right.}{7bx}$$

$$\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

$$\Big\downarrow \; 2418$$

$$-\frac{36a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\right.}{}$$

$$\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

input
```
Int[(3*a + b*x^2)/(a - b*x^2)^(1/3),x]
```

3.125. $\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx$

output
```
(-3*x*(a - b*x^2)^(2/3))/7 - (36*a*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1
/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3
) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elli
pticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1
/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x
^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/
((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(7*
b*x)
```

3.125.3.1 Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.125.4 Maple [F]

$$\int \frac{b\,x^{2}+3a}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}}}d x$$

input
```
int((b*x^2+3*a)/(-b*x^2+a)^(1/3),x)
```

output
```
int((b*x^2+3*a)/(-b*x^2+a)^(1/3),x)
```

3.125.5 Fricas [F]

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx=\int \frac{bx^{2}+3\,a}{{\left(-bx^{2}+a\right)}^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)/(-b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(b*x^2 + 3*a)*(-b*x^2 + a)^(2/3)/(b*x^2 - a), x)
```

3.125.6 Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.11

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx=3a^{\frac{2}{3}}x\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\right|\frac{bx^{2}e^{2i\pi}}{a}\right)+\frac{bx^{3}\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\right|\frac{bx^{2}e^{2i\pi}}{a}\right)}{3\sqrt[3]{a}}$$

input
```
integrate((b*x**2+3*a)/(-b*x**2+a)**(1/3),x)
```

output
```
3*a**(2/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + b*x**
3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/3))
```

3.125.7 Maxima [F]

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx=\int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)/(-b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3), x)
```

3.125.8 Giac [F]

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx=\int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)/(-b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3), x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx=\int \frac{b\,x^2+3\,a}{(a-b\,x^2)^{1/3}}\,dx$$

input
```
int((3*a + b*x^2)/(a - b*x^2)^(1/3),x)
```

output
```
int((3*a + b*x^2)/(a - b*x^2)^(1/3), x)
```

3.126 $\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx$

3.126.1 Optimal result

Integrand size = 24, antiderivative size = 204

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}$$

output
```
1/4*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/
a^(5/6)/b^(1/2)-1/12*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(5/6)/b^(1/2)+1/
12*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^
(1/3)/a^(5/6)*3^(1/2)/b^(1/2)+1/12*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/
3)/a^(5/6)*3^(1/2)/b^(1/2)
```

3.126.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$$
$$= \frac{9ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{a-bx^2}\,(3a+bx^2)\left(9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\text{Appe}\right.\right.}$$

input
```
Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

output
```
(9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((a - b*x^2)
^(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^
2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] +
AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

3.126.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {305}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$$
$$\Big\downarrow \ 305$$
$$\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}$$

input
```
Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

output
```
ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b])
+ ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*
x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*
2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)
*(a - b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])
```

3.126.3.1 Defintions of rubi rules used

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

3.126.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}} \left(b\,x^{2}+3 a\right)}d x$$

input
```
int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

output
```
int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

3.126.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="fricas")
```

output
```
Timed out
```

3.126.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{\sqrt[3]{a-bx^2}\cdot(3a+bx^2)}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
```

output
```
Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)), x)
```

3.126.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

3.126.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{\left(a-b\,x^2\right)^{1/3}\,\left(b\,x^2+3\,a\right)}\,dx$$

input
```
int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x)
```

output
```
int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)), x)
```

3.127 $\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx$

3.127.1 Optimal result

Integrand size = 24, antiderivative size = 787

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \frac{x(a-bx^2)^{2/3}}{24a^2(3a+bx^2)} - \frac{x}{24a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+ \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$
$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}}$$
$$- \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4}{16\ 3^{3/4}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+ \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4}{12\sqrt{2}\sqrt[4]{3}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
1/24*x*(-b*x^2+a)^(2/3)/a^2/(b*x^2+3*a)-1/24*x/a^2/(-(-b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2)))+1/16*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a
)^(1/3)))*2^(1/3)/a^(11/6)/b^(1/2)-1/48*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)
/a^(11/6)/b^(1/2)+1/48*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3
^(1/2)/x/b^(1/2))*2^(1/3)/a^(11/6)*3^(1/2)/b^(1/2)+1/48*arctan(3^(1/2)*a^(
1/2)/x/b^(1/2))*2^(1/3)/a^(11/6)*3^(1/2)/b^(1/2)+1/72*(a^(1/3)-(-b*x^2+a)^
(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(
-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/
a^(5/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1/48*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE(
(-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1
/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/
(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))
*3^(1/4)/a^(5/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.127.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \frac{x\left(\frac{bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^3}+\frac{27\left(\frac{a-bx^2}{a^2}+\frac{63\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\cdots\right.}\right.}{3a+bx^2}\cdots\right.}{648\sqrt[3]{a-bx^2}}$$

input
```
Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x]
```

output
```
(x*((b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/
3*(b*x^2)/a])/a^3 + (27*((a - b*x^2)/a^2 + (63*AppellF1[1/2, 1/3, 1, 3/2,
(b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/
3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2
)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))/(3*a + b*
x^2)))/(648*(a - b*x^2)^(1/3))
```

3.127.3 Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 842, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {316, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}dx$$

$$\Big\downarrow \ 316$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{24a^2\,(3a+bx^2)}-\frac{\int -\frac{b(bx^2+21a)}{3\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{24a^2b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{bx^2+21a}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{72a^2}+\frac{x\left(a-bx^2\right)^{2/3}}{24a^2\,(3a+bx^2)}$$

$$\Big\downarrow \ 405$$

$$\frac{\int \frac{1}{\sqrt[3]{a-bx^2}}dx+18a\int \frac{1}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{72a^2}+\frac{x\left(a-bx^2\right)^{2/3}}{24a^2\,(3a+bx^2)}$$

$$\Big\downarrow \ 233$$

$$\frac{18a\int \frac{1}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx-\frac{3\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}}{72a^2}+\frac{x\left(a-bx^2\right)^{2/3}}{24a^2\,(3a+bx^2)}$$

$$\Big\downarrow \ 305$$

$$\frac{18a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}\right)}{72a^2}$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{24a^2\,(3a+bx^2)}$$

$$\Big\downarrow \ 833$$

$$18a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}\right.$$

$$72a^2$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)}$$

$\downarrow$ 760

$$18a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}\right.$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)}$$

$\downarrow$ 2418

$$\frac{\left(a-bx^2\right)^{2/3}x}{24a^2\left(bx^2+3a\right)}+$$

$$18a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}\right.$$

input
```
Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x]
```

3.127. $\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)^2}\,dx$

output
```
(x*(a - b*x^2)^(2/3))/(24*a^2*(3*a + b*x^2)) + (18*a*(ArcTan[(Sqrt[3]*Sqrt
[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^
(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[
3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt
[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))
]/(2*2^(2/3)*a^(5/6)*Sqrt[b])) - (3*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/
3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ell
ipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*
x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) -
 (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(2
*b*x))/(72*a^2)
```

3.127.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.127. $\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)^2} \, dx$

3.127.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}} \left(b\,x^{2}+3a\right)^{2}}dx$$

input
```
int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x)
```

output
```
int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x)
```

3.127.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.127.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a)**2,x)
```

output
```
Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)**2), x)
```

3.127.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \int \frac{1}{(bx^2+3\,a)^2(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)), x)
```

3.127.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \int \frac{1}{(bx^2+3\,a)^2(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)), x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \int \frac{1}{(a-b\,x^2)^{1/3}\,(b\,x^2+3\,a)^2}\,dx$$

input
```
int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x)
```

output
```
int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2), x)
```

3.128 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^3} \, dx$

3.128.1 Optimal result

Integrand size = 24, antiderivative size = 818

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx$$
$$= \frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2} + \frac{5x(a-bx^2)^{2/3}}{288a^3(3a+bx^2)} - \frac{5x}{288a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+ \frac{5\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{5\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$- \frac{5\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{432\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{5\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{144\ 2^{2/3}a^{17/6}\sqrt{b}}$$
$$- \frac{5\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+}{192\ 3^{3/4}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+ \frac{5\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4}{144\sqrt{2}\sqrt[4]{3}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
1/48*x*(-b*x^2+a)^(2/3)/a^2/(b*x^2+3*a)^2+5/288*x*(-b*x^2+a)^(2/3)/a^3/(b*
x^2+3*a)-5/288*x/a^3/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+5/288*arctanh
(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(17/6)/b^
(1/2)-5/864*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(17/6)/b^(1/2)+5/864*arct
an(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a
^(17/6)*3^(1/2)/b^(1/2)+5/864*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^
(17/6)*3^(1/2)/b^(1/2)+5/864*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^
2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*
I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^
2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)/b/x*2^(1/2)/(-a^(
1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)-5/576*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/
3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a
^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(8/3)/b/x/(-a
^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^
2)^(1/2)
```

3.128.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx$$
$$= \frac{x\left(\frac{27a(a-bx^2)(21a+5bx^2)}{(3a+bx^2)^2}+5bx^2\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\frac{6\cdots}{(3a+bx^2)\left(9a\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\cdots\right.\right.}\right.}{7776a^4\sqrt[3]{a-bx^2}}$$

input
```
Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x]
```

output
```
(x*((27*a*(a - b*x^2)*(21*a + 5*b*x^2))/(3*a + b*x^2)^2 + 5*b*x^2*(1 - (b*
x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + (607
5*a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((3*a + b*x^2
)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-A
ppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3,
1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))))/(7776*a^4*(a - b*x^2)^(1/3))
```

3.128.3 Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {316, 27, 402, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2}-\frac{\int -\frac{5b(9a-bx^2)}{3\sqrt[3]{a-bx^2}(bx^2+3a)^2}dx}{48a^2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{9a-bx^2}{\sqrt[3]{a-bx^2}(bx^2+3a)^2}dx}{144a^2}+\frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{5\left(\frac{x(a-bx^2)^{2/3}}{2a(3a+bx^2)}-\frac{\int -\frac{4ab(bx^2+15a)}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{24a^2b}\right)}{144a^2}+\frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(\frac{\int \frac{bx^2+15a}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{6a}+\frac{x(a-bx^2)^{2/3}}{2a(3a+bx^2)}\right)}{144a^2}+\frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2}$$

$$\Big\downarrow \text{405}$$

$$\frac{5\left(\frac{\int \frac{1}{\sqrt[3]{a-bx^2}}dx+12a\int \frac{1}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{6a}+\frac{x(a-bx^2)^{2/3}}{2a(3a+bx^2)}\right)}{144a^2}+\frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2}$$

$$\Big\downarrow \text{233}$$

$$\frac{5\left(\frac{12a\int \frac{1}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx-\frac{3\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}}{6a}+\frac{x\left(a-bx^2\right)^{2/3}}{2a\left(3a+bx^2\right)}\right)}{144a^2}+\frac{x\left(a-bx^2\right)^{2/3}}{48a^2\left(3a+bx^2\right)^2}$$

$\downarrow$ 305

$$\frac{5\left(\frac{12a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}\right)-\cdots}{6a}\cdots\right)}{144a^2}$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{48a^2\left(3a+bx^2\right)^2}$$

$\downarrow$ 833

$$\frac{5\left(\frac{12a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}\right)-\cdots}{6a}\cdots\right)}{144a^2}$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{48a^2\left(3a+bx^2\right)^2}$$

3.128. $\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^3}\,dx$

$$\downarrow \text{760}$$

$$5\left(\frac{12a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}\right)--}{}\right.$$

$$\frac{x\left(a-bx^2\right)^{2/3}}{48a^2\left(3a+bx^2\right)^2}$$

$$\downarrow \text{2418}$$

3.128. $\displaystyle\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^3}\,dx$

$$\frac{\left(a-bx^2\right)^{2/3}x}{48a^2\left(bx^2+3a\right)^2}+$$

$$5\left(\frac{\left(a-bx^2\right)^{2/3}x}{2a(bx^2+3a)}+\frac{12a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a\right.}\right.}\right.}{2\ 2^{2/3}a^{5/6}\sqrt{b}}\right.}{}\right.$$

input
```
Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x]
```

3.128. $\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)^3}\,dx$

output
```
(x*(a - b*x^2)^(2/3))/(48*a^2*(3*a + b*x^2)^2) + (5*((x*(a - b*x^2)^(2/3))
/(2*a*(3*a + b*x^2)) + (12*a*(ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(
2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)
*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - Ar
cTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*
x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqr
t[b])) - (3*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x
^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/
((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x
^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 -
 Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/
(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(2*b*x))/(6*a)))/(144*a^2)
```

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

3.128. $\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)^3}\,dx$

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.128.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}}\left(b\,x^{2}+3a\right)^{3}}dx$$

input
```
int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x)
```

output
```
int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x)
```

3.128.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.128.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx = \int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a)**3,x)
```

output
```
Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)**3), x)
```

3.128.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx = \int \frac{1}{(bx^2+3\,a)^3(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)), x)
```

3.128.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx = \int \frac{1}{(bx^2+3\,a)^3(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)), x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx = \int \frac{1}{(a-b\,x^2)^{1/3}\,(b\,x^2+3\,a)^3}\,dx$$

input
```
int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x)
```

output
```
int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3), x)
```

3.129 $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx$

3.129.1 Optimal result

Integrand size = 24, antiderivative size = 623

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \frac{2538}{91}ax\left(a-bx^2\right)^{2/3}$$

$$+\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)+\frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}+\frac{20088a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{10044\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}\right.}{}$$

output
```
2538/91*a*x*(-b*x^2+a)^(2/3)+81/13*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)+6*x*(b*x
^2+3*a)^2/(-b*x^2+a)^(1/3)+20088/91*a^2*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^
(1/2)))-6696/91*3^(3/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b
*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))
,2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3
))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)
-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+10044/
91*3^(1/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)
+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2
))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^
(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.129.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.12

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = -\frac{3x\left(-3051a^2+132abx^2+7b^2x^4+2232a^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{91\sqrt[3]{a-bx^2}}$$

input
```
Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(4/3),x]
```

output
```
(-3*x*(-3051*a^2 + 132*a*b*x^2 + 7*b^2*x^4 + 2232*a^2*(1 - (b*x^2)/a)^(1/3
)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(91*(a - b*x^2)^(1/3))
```

3.129.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {315, 27, 403, 27, 299, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx$$

$$\Big\downarrow \; 315$$

$$\frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}-\frac{3\int \frac{6ab(bx^2+3a)(3bx^2+a)}{\sqrt[3]{a-bx^2}}dx}{2ab}$$

$$\Big\downarrow \; 27$$

$$\frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}-9\int \frac{(bx^2+3a)(3bx^2+a)}{\sqrt[3]{a-bx^2}}dx$$

$$\Big\downarrow \; 403$$

$$\frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}-9\left(-\frac{3\int -\frac{2ab\left(47bx^2+33a\right)}{3\sqrt[3]{a-bx^2}}dx}{13b}-\frac{9}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\right)$$

$$\Big\downarrow \; 27$$

$$\frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}-9\left(\frac{2}{13}a\int \frac{47bx^2+33a}{\sqrt[3]{a-bx^2}}dx-\frac{9}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\right)$$

$$\Big\downarrow \; 299$$

$$\frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}-9\left(\frac{2}{13}a\left(\frac{372}{7}a\int \frac{1}{\sqrt[3]{a-bx^2}}dx-\frac{141}{7}x\left(a-bx^2\right)^{2/3}\right)-\frac{9}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\right)$$

$$\Big\downarrow \; 233$$

$$\frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}-9\left(\frac{2}{13}a\left(-\frac{558a\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}-\frac{141}{7}x\left(a-bx^2\right)^{2/3}\right)-\frac{9}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\right)$$

$$\Big\downarrow \; 833$$

$$\frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}-9\left(\frac{2}{13}a\left(-\frac{558a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{7bx}-\frac{141}{7}x\left(a-b\right.\right.\right.$$

$$\Big\downarrow \; 760$$

3.129. $\int \frac{\left(3a+bx^2\right)^3}{\left(a-bx^2\right)^{4/3}}\,dx$

$$9\left(\frac{2}{13}a\left(\frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}-\frac{558a\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{\cdots}}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{\sqrt[4]{3}\sqrt{-bx^2}\cdots}\right.}{7bx}\right.\right.$$

$$\downarrow \text{2418}$$

$$9\left(\frac{2}{13}a\left(\frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}-\frac{558a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{\cdots}\right.}{\left(1-\sqrt{\cdots}\right.}\right.\right.}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\right.}{\cdots}\right.\right.$$

input
```
Int[(3*a + b*x^2)^3/(a - b*x^2)^(4/3),x]
```

3.129. $\int \frac{\left(3a+bx^2\right)^3}{\left(a-bx^2\right)^{4/3}}\,dx$

output
```
(6*x*(3*a + b*x^2)^2)/(a - b*x^2)^(1/3) - 9*((-9*x*(a - b*x^2)^(2/3)*(3*a
+ b*x^2))/13 + (2*a*((-141*x*(a - b*x^2)^(2/3))/7 - (558*a*Sqrt[-(b*x^2)]*
((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4
)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
 - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^
(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqr
t[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/
3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ell
ipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))^2)])))/(7*b*x)))/13)
```

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.129. $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}} dx$

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.129.4 Maple [F]

$$\int \frac{\left(b\,x^{2}+3a\right)^{3}}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}}}d x$$

input
```
int((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x)
```

output
```
int((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x)
```

3.129.5 Fricas [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3
)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

3.129.6 Sympy [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \int \frac{(3a+bx^2)^3}{(a-bx^2)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x**2+3*a)**3/(-b*x**2+a)**(4/3),x)
```

output
```
Integral((3*a + b*x**2)**3/(a - b*x**2)**(4/3), x)
```

3.129.7 Maxima [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3), x)
```

3.129.8 Giac [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3), x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \int \frac{(b\,x^2+3\,a)^3}{(a-b\,x^2)^{4/3}}\,dx$$

input
```
int((3*a + b*x^2)^3/(a - b*x^2)^(4/3),x)
```

output
```
int((3*a + b*x^2)^3/(a - b*x^2)^(4/3), x)
```

3.130 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx$

3.130.1 Optimal result

Integrand size = 24, antiderivative size = 592

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = \frac{45}{7}x\left(a-bx^2\right)^{2/3} + \frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}} + \frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\ldots\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{108\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\ldots\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
45/7*x*(-b*x^2+a)^(2/3)+6*x*(b*x^2+3*a)/(-b*x^2+a)^(1/3)+324/7*a*x/(-(-b*x
^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-108/7*3^(3/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)
^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a
)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^
(1/2)))^2)^(1/2)+162/7*3^(1/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*Elliptic
E((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^
(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3)
)/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2
))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-
3^(1/2)))^2)^(1/2)
```

3.130.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.28

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = \frac{x\sqrt[3]{1-\frac{bx^2}{a}}\,\mathrm{Gamma}\left(\frac{1}{3}\right)\left(63a(45a^2+10abx^2+b^2x^4)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{2},\frac{bx^2}{a}\right)\right.}{945}$$

input
```
Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(4/3),x]
```

output
```
(x*(1 - (b*x^2)/a)^(1/3)*Gamma[1/3]*(63*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*
Hypergeometric2F1[1/2, 4/3, 7/2, (b*x^2)/a] + 32*b*x^2*(18*a^2 + 9*a*b*x^2
 + b^2*x^4)*Hypergeometric2F1[3/2, 7/3, 9/2, (b*x^2)/a] + 16*b*(3*a*x + b*
x^3)^2*HypergeometricPFQ[{3/2, 2, 7/3}, {1, 9/2}, (b*x^2)/a]))/(945*a^2*(a
 - b*x^2)^(1/3)*Gamma[4/3])
```

3.130.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {315, 27, 299, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.130. $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx$

$$\int \frac{\left(3a+bx^2\right)^2}{\left(a-bx^2\right)^{4/3}}dx$$

$$\Big\downarrow \text{315}$$

$$\frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}}-\frac{3\int \frac{2ab\left(5bx^2+3a\right)}{\sqrt[3]{a-bx^2}}dx}{2ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}}-3\int \frac{5bx^2+3a}{\sqrt[3]{a-bx^2}}dx$$

$$\Big\downarrow \text{299}$$

$$\frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}}-3\left(\frac{36}{7}a\int \frac{1}{\sqrt[3]{a-bx^2}}dx-\frac{15}{7}x\left(a-bx^2\right)^{2/3}\right)$$

$$\Big\downarrow \text{233}$$

$$\frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}}-3\left(-\frac{54a\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}-\frac{15}{7}x\left(a-bx^2\right)^{2/3}\right)$$

$$\Big\downarrow \text{833}$$

$$\frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}}-3\left(-\frac{54a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{7bx}-\frac{15}{7}x\left(a-bx^2\right)^{2/3}\right)$$

$$\Big\downarrow \text{760}$$

3.130. $\int \frac{\left(3a+bx^2\right)^2}{\left(a-bx^2\right)^{4/3}}dx$

$$3\left(\frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}}-\frac{54a\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}\right.}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{}}{\left(\left(1-\right.\right.}}}\right.}{7bx}\right.$$

$$\Big\downarrow \text{ 2418}$$

$$3\left(\frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}}-\frac{54a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\right.}{}\right.$$

input
```
Int[(3*a + b*x^2)^2/(a - b*x^2)^(4/3),x]
```

3.130. $\int \frac{\left(3a+bx^2\right)^2}{\left(a-bx^2\right)^{4/3}}\,dx$

output
```
(6*x*(3*a + b*x^2))/(a - b*x^2)^(1/3) - 3*((-15*x*(a - b*x^2)^(2/3))/7 - (
54*a*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -
7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/
3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3
]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4
)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(7*b*x))
```

3.130.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.130.4 Maple [F]

$$\int \frac{(b\,x^2+3a)^2}{(-b\,x^2+a)^{\frac{4}{3}}}dx$$

input
```
int((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x)
```

output
```
int((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x)
```

3.130.5 Fricas [F]

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx=\int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b
*x^2 + a^2), x)
```

3.130.6 Sympy [F]

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx=\int \frac{(3a+bx^2)^2}{(a-bx^2)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x**2+3*a)**2/(-b*x**2+a)**(4/3),x)
```

output
```
Integral((3*a + b*x**2)**2/(a - b*x**2)**(4/3), x)
```

3.130.7 Maxima [F]

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx=\int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3), x)
```

3.130.8 Giac [F]

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3), x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{(b\,x^2+3\,a)^2}{(a-b\,x^2)^{4/3}}\,dx$$

input
```
int((3*a + b*x^2)^2/(a - b*x^2)^(4/3),x)
```

output
```
int((3*a + b*x^2)^2/(a - b*x^2)^(4/3), x)
```

3.131 $\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx$

3.131.1 Optimal result

Integrand size = 22, antiderivative size = 561

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx = \frac{6x}{\sqrt[3]{a-bx^2}} + \frac{9x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}$$

$$+ \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$- \frac{3\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
6*x/(-b*x^2+a)^(1/3)+9*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-3*3^(3/4)
*a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(
1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)
*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a
^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+9/2*3^(1/4)*a^(1/3)*(a^(1/3)-
(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2
+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.131.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.09

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx=\frac{6x-3x\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[3]{a-bx^2}}$$

input
```
Integrate[(3*a + b*x^2)/(a - b*x^2)^(4/3),x]
```

output
```
(6*x - 3*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/
a])/(a - b*x^2)^(1/3)
```

3.131.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {298, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx$$

$$\Big\downarrow \text{298}$$

$$\frac{6x}{\sqrt[3]{a-bx^2}}-3\int \frac{1}{\sqrt[3]{a-bx^2}}dx$$

$$\Big\downarrow \; 233$$

$$\frac{9\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}+\frac{6x}{\sqrt[3]{a-bx^2}}$$

$$\Big\downarrow \; 833$$

$$\frac{9\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{2bx}+\frac{6x}{\sqrt[3]{a-bx^2}}$$

$$\Big\downarrow \; 760$$

$$\frac{9\sqrt{-bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\cdots\right.}}\cdots}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}\cdots\right.}{2bx}\cdots$$

$$\frac{6x}{\sqrt[3]{a-bx^2}}$$

$$\Big\downarrow \; 2418$$

$$\frac{9\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b\cdots}}\cdots\right.\right.}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\cdots\right.}{\cdots}$$

$$\frac{6x}{\sqrt[3]{a-bx^2}}$$

input
```
Int[(3*a + b*x^2)/(a - b*x^2)^(4/3),x]
```

3.131. $\int \frac{3a+bx^2}{\left(a-bx^2\right)^{4/3}}\,dx$

output
```
(6*x)/(a - b*x^2)^(1/3) + (9*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
 - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[
ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a
^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1
/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b
*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(2*b*x)
```

3.131.3.1 Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

3.131. $\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx$

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.131.4 Maple [F]

$$\int \frac{b\,x^{2}+3a}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}}}d x$$

input
```
int((b*x^2+3*a)/(-b*x^2+a)^(4/3),x)
```

output
```
int((b*x^2+3*a)/(-b*x^2+a)^(4/3),x)
```

3.131.5 Fricas [F]

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)/(-b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

3.131.6 Sympy [A] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.11

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx = \frac{3x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{\sqrt[3]{a}} + \frac{bx^3\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{4}{3}}}$$

input
```
integrate((b*x**2+3*a)/(-b*x**2+a)**(4/3),x)
```

output
```
3*x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(1/3) + b*x**
3*hyper((4/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(4/3))
```

3.131.7 Maxima [F]

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)/(-b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3), x)
```

3.131.8 Giac [F]

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)/(-b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3), x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{b\,x^2+3\,a}{(a-b\,x^2)^{4/3}}\,dx$$

input
```
int((3*a + b*x^2)/(a - b*x^2)^(4/3),x)
```

output
```
int((3*a + b*x^2)/(a - b*x^2)^(4/3), x)
```

3.132 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx$

3.132.1 Optimal result

Integrand size = 24, antiderivative size = 776

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx = \frac{3x}{8a^2\sqrt[3]{a-bx^2}} + \frac{3x}{8a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+ \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$

$$- \frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}} + \frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}}$$

$$+ \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+\right.}{16a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$- \frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\right.}{4\sqrt{2}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
3/8*x/a^2/(-b*x^2+a)^(1/3)+3/8*x/a^2/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)
))+1/16*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1
/3)/a^(11/6)/b^(1/2)-1/48*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(11/6)/b^(1
/2)+1/48*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/
2))*2^(1/3)/a^(11/6)*3^(1/2)/b^(1/2)+1/48*arctan(3^(1/2)*a^(1/2)/x/b^(1/2)
)*2^(1/3)/a^(11/6)*3^(1/2)/b^(1/2)-1/8*(a^(1/3)-(-b*x^2+a)^(1/3))*Elliptic
F((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^
(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3)
)/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b/x*2^(
1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^
(1/2)))^2)^(1/2)+3/16*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1
/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(
1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1
/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(5/3
)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)
```

3.132.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx = \frac{x\left(-\frac{bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^3}+27\left(\frac{1}{a^2}-\frac{}{(3a+bx^2)\left(9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},\ldots\right.\right.}\right.\right.}{72\sqrt[3]{a-\ldots}}$$

input
```
Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x]
```

output
```
(x*(-((b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
1/3*(b*x^2)/a])/a^3) + 27*(a^(-2) - (3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/
a, -1/3*(b*x^2)/a])/((3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)
/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3
*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))))/
(72*(a - b*x^2)^(1/3))
```

3.132.3 Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {316, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{3\int -\frac{b(bx^2+a)}{3\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{8a^2b}+\frac{3x}{8a^2\sqrt[3]{a-bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3x}{8a^2\sqrt[3]{a-bx^2}}-\frac{\int \frac{bx^2+a}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{8a^2}$$

$$\Big\downarrow \text{405}$$

$$\frac{3x}{8a^2\sqrt[3]{a-bx^2}}-\frac{\int \frac{1}{\sqrt[3]{a-bx^2}}dx-2a\int \frac{1}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{8a^2}$$

$$\Big\downarrow \text{233}$$

$$\frac{3x}{8a^2\sqrt[3]{a-bx^2}}-\frac{-\frac{3\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}-2a\int \frac{1}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{8a^2}$$

$$\Big\downarrow \text{305}$$

$$\frac{3x}{8a^2\sqrt[3]{a-bx^2}}-\frac{-\frac{3\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}-2a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\cdots}{\sqrt[6]{a}\left(\sqrt[3]{2}\cdots\right.}\right.}{2\ 2^{2/3}\cdots}\right.}{8a^2}$$

$$\Big\downarrow \text{833}$$

$$\frac{3x}{8a^2\sqrt[3]{a-bx^2}}-\frac{-\frac{3\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{2bx}-2a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-b\right.}}{\sqrt{b}x}\right.}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}\right.}{8a^2}$$

$\downarrow$ 760

$$\frac{3x}{8a^2\sqrt[3]{a-bx^2}}-\frac{-\frac{3\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{Ell}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-b\right.}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-\right.}}}}\right)}{2bx}}{}$$

$\downarrow$ 2418

$$\frac{3x}{8a^2\sqrt[3]{a-bx^2}}-\frac{-2a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right.}{2\,2^{2/3}a^{5/6}\sqrt{b}}\right.}{}$$

input
```
Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x]
```

3.132. $\int\frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}\,dx$

output
```
(3*x)/(8*a^2*(a - b*x^2)^(1/3)) - (-2*a*(ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]
*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3
) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sq
rt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTan
h[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(2*2^(2/3)*
a^(5/6)*Sqrt[b])) - (3*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3)
 - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x
^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) -
(2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4
*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(2*b*x))/(8*a^2
)
```

3.132.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.132. $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx$

3.132.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}} \left(b\,x^{2}+3a\right)}dx$$

input
```
int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x)
```

output
```
int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x)
```

3.132.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x, algorithm="fricas")
```

output
```
Timed out
```

3.132.6 Sympy [F]

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)}\,dx = \int \frac{1}{(a-bx^2)^{\frac{4}{3}} \cdot (3a+bx^2)}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a),x)
```

output
```
Integral(1/((a - b*x**2)**(4/3)*(3*a + b*x**2)), x)
```

3.132.7 Maxima [F]

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx = \int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)), x)
```

3.132.8 Giac [F]

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx = \int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)), x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx = \int \frac{1}{(a-b\,x^2)^{4/3}\,(b\,x^2+3\,a)}\,dx$$

input
```
int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x)
```

output
```
int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)), x)
```

3.133 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx$

3.133.1 Optimal result

Integrand size = 24, antiderivative size = 807

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx = \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}(3a+bx^2)} + \frac{x}{12a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{16\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{16\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48\,2^{2/3}a^{17/6}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{16\,2^{2/3}a^{17/6}\sqrt{b}} + \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8\,3^{3/4}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} - \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right)}{6\sqrt{2}\sqrt[4]{3}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
1/12*x/a^3/(-b*x^2+a)^(1/3)+1/24*x/a^2/(-b*x^2+a)^(1/3)/(b*x^2+3*a)+1/12*x
/a^3/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/32*arctanh(x*b^(1/2)/a^(1/6
)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(17/6)/b^(1/2)-1/96*arctan
h(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(17/6)/b^(1/2)+1/96*arctan(a^(1/6)*(a^(1/3)
-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(
1/2)+1/96*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/
2)-1/36*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1
+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3
)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1
-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2
+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+1/24*(a^(1/3)-
(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2
+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(8/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^
2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.133.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx = \frac{x\left(-2bx^2\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\frac{27a\left(7a+2bx^2+\frac{\cdots}{9a\operatorname{AppellF1}\cdots}\right.}{\cdots}\right.}{648a^4\sqrt[3]{a\cdots}}$$

input
```
Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x]
```

output
```
(x*(-2*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
1/3*(b*x^2)/a] + (27*a*(7*a + 2*b*x^2 + (9*a^2*AppellF1[1/2, 1/3, 1, 3/2,
(b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/
3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2
)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))/(3*a + b*
x^2)))/(648*a^4*(a - b*x^2)^(1/3))
```

3.133.3 Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {316, 27, 402, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}dx$$

$$\Big\downarrow \text{ 316}$$

$$\frac{x}{24a^2\sqrt[3]{a-bx^2}(3a+bx^2)}-\frac{\int -\frac{b(21a-5bx^2)}{3(a-bx^2)^{4/3}(bx^2+3a)}dx}{24a^2b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{21a-5bx^2}{(a-bx^2)^{4/3}(bx^2+3a)}dx}{72a^2}+\frac{x}{24a^2\sqrt[3]{a-bx^2}(3a+bx^2)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{3\int \frac{8ab(3a-2bx^2)}{3\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{8a^2b}+\frac{6x}{a\sqrt[3]{a-bx^2}}}{72a^2}+\frac{x}{24a^2\sqrt[3]{a-bx^2}(3a+bx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{3a-2bx^2}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{a}+\frac{6x}{a\sqrt[3]{a-bx^2}}}{72a^2}+\frac{x}{24a^2\sqrt[3]{a-bx^2}(3a+bx^2)}$$

$$\Big\downarrow \text{ 405}$$

$$\frac{\frac{9a\int \frac{1}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx-2\int \frac{1}{\sqrt[3]{a-bx^2}}dx}{a}+\frac{6x}{a\sqrt[3]{a-bx^2}}}{72a^2}+\frac{x}{24a^2\sqrt[3]{a-bx^2}(3a+bx^2)}$$

$$\Big\downarrow \text{ 233}$$

$$\frac{\frac{\frac{3\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{bx}+9a\int \frac{1}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{a}+\frac{6x}{a\sqrt[3]{a-bx^2}}}{72a^2}+\frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)}$$

$\Big\downarrow$ 305

$$\frac{\frac{\frac{3\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{bx}+9a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}\right.}\right.}{2\ 2^{2/3}a^{5/6}\sqrt{b}}\right.}{a}}{72a^2}$$

$$\frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)}$$

$\Big\downarrow$ 833

$$\frac{\frac{\frac{3\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{bx}+9a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\dots\right.}{a}}{72a^2}$$

$$\frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)}$$

$\Big\downarrow$ 760

3.133. $\displaystyle\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx$

$$\frac{3\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{Elliptic}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right.}}}\right.}{bx}$$

$$\frac{x}{24a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}$$

$$\Big\downarrow \text{ 2418}$$

$$\frac{x}{24a^2\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}+$$

$$\frac{6x}{a\sqrt[3]{a-bx^2}}+\frac{9a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-\right.}}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}\right.}{}$$

input
```
Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x]
```

3.133. $\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)^2}\,dx$

output
```
x/(24*a^2*(a - b*x^2)^(1/3)*(3*a + b*x^2)) + ((6*x)/(a*(a - b*x^2)^(1/3))
+ (9*a*(ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*S
qrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(
Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt
[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) +
 2^(1/3)*(a - b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])) + (3*Sqrt[-(b*x
^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3
^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*
x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])
/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3]
)*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2
)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)
]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2)])))/(b*x))/a)/(72*a^2)
```

3.133.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

3.133. $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx$

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

3.133. $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx$

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.133.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}}\left(b\,x^{2}+3a\right)^{2}}dx$$

input
```
int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x)
```

output
```
int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x)
```

3.133.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.133.6 Sympy [F]

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)^2}\,dx = \int \frac{1}{(a-bx^2)^{\frac{4}{3}}\,(3a+bx^2)^2}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a)**2,x)
```

output
```
Integral(1/((a - b*x**2)**(4/3)*(3*a + b*x**2)**2), x)
```

3.133.7 Maxima [F]

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx = \int \frac{1}{(bx^2+3\,a)^2(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)), x)
```

3.133.8 Giac [F]

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx = \int \frac{1}{(bx^2+3\,a)^2(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)), x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx = \int \frac{1}{(a-b\,x^2)^{4/3}\,(b\,x^2+3\,a)^2}\,dx$$

input
```
int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x)
```

output
```
int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2), x)
```

3.134 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx$

3.134.1 Optimal result

Integrand size = 24, antiderivative size = 849

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx = \frac{x}{48a^2\sqrt[3]{a-bx^2}(3a+bx^2)^2}+\frac{17x}{192a^3\sqrt[3]{a-bx^2}(3a+bx^2)}$$
$$-\frac{19x(a-bx^2)^{2/3}}{1152a^4(3a+bx^2)}+\frac{19x}{1152a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+\frac{7\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{288\ 2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}}+\frac{7\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{288\ 2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}}$$
$$-\frac{7\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{864\ 2^{2/3}a^{23/6}\sqrt{b}}+\frac{7\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{288\ 2^{2/3}a^{23/6}\sqrt{b}}$$
$$+\frac{19\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4}{768\ 3^{3/4}a^{11/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{19\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4}{576\sqrt{2}\sqrt[4]{3}a^{11/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
1/48*x/a^2/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2+17/192*x/a^3/(-b*x^2+a)^(1/3)/(b
*x^2+3*a)-19/1152*x*(-b*x^2+a)^(2/3)/a^4/(b*x^2+3*a)+19/1152*x/a^4/(-(-b*x
^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+7/576*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+
2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(23/6)/b^(1/2)-7/1728*arctanh(x*b^(1/
2)/a^(1/2))*2^(1/3)/a^(23/6)/b^(1/2)+7/1728*arctan(a^(1/6)*(a^(1/3)-2^(1/3
)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(23/6)*3^(1/2)/b^(1/2)+7/
1728*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(23/6)*3^(1/2)/b^(1/2)-19
/3456*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3
^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+
a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)*3^(3/4)/a^(11/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+
a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+19/2304*(a^(1/3
)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x
^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(
1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(11/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b
*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.134.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.30

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx = \frac{x\left(-19bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\frac{27a\left(273a^2+140abx^2+19\cdots\right.}{\cdots}\right)}{3110\cdots}$$

input
```
Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x]
```

output
```
(x*(-19*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a,
-1/3*(b*x^2)/a] + (27*a*(273*a^2 + 140*a*b*x^2 + 19*b^2*x^4 + (333*a^2*(3*
a + b*x^2)*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*App
ellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/
2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b
*x^2)/a, -1/3*(b*x^2)/a]))))/(3*a + b*x^2)^2))/(31104*a^5*(a - b*x^2)^(1/3
))
```

3.134.3 Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {316, 27, 402, 27, 402, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{x}{48a^2\sqrt[3]{a-bx^2}(3a+bx^2)^2}-\frac{\int -\frac{b(45a-11bx^2)}{3(a-bx^2)^{4/3}(bx^2+3a)^2}dx}{48a^2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{45a-11bx^2}{(a-bx^2)^{4/3}(bx^2+3a)^2}dx}{144a^2}+\frac{x}{48a^2\sqrt[3]{a-bx^2}(3a+bx^2)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{3\int \frac{2ab(85bx^2+27a)}{3\sqrt[3]{a-bx^2}(bx^2+3a)^2}dx}{8a^2b}+\frac{51x}{4a\sqrt[3]{a-bx^2}(3a+bx^2)}}{144a^2}+\frac{x}{48a^2\sqrt[3]{a-bx^2}(3a+bx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{85bx^2+27a}{\sqrt[3]{a-bx^2}(bx^2+3a)^2}dx}{4a}+\frac{51x}{4a\sqrt[3]{a-bx^2}(3a+bx^2)}}{144a^2}+\frac{x}{48a^2\sqrt[3]{a-bx^2}(3a+bx^2)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{-\frac{\int -\frac{4ab(111a-19bx^2)}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{24a^2b}-\frac{19x(a-bx^2)^{2/3}}{2a(3a+bx^2)}}{4a}+\frac{51x}{4a\sqrt[3]{a-bx^2}(3a+bx^2)}}{144a^2}+\frac{x}{48a^2\sqrt[3]{a-bx^2}(3a+bx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{111a-19bx^2}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx}{6a}-\frac{19x\left(a-bx^2\right)^{2/3}}{2a\left(3a+bx^2\right)}}{4a}+\frac{51x}{4a\sqrt[3]{a-bx^2}(3a+bx^2)}}{144a^2}+\frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}$$

$\big\downarrow$ 405

$$\frac{\frac{\frac{168a\int \frac{1}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx-19\int \frac{1}{\sqrt[3]{a-bx^2}}dx}{6a}-\frac{19x\left(a-bx^2\right)^{2/3}}{2a\left(3a+bx^2\right)}}{4a}+\frac{51x}{4a\sqrt[3]{a-bx^2}(3a+bx^2)}}{144a^2}+\frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}$$

$\big\downarrow$ 233

$$\frac{\frac{\frac{\frac{57\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}+168a\int \frac{1}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx}{6a}-\frac{19x\left(a-bx^2\right)^{2/3}}{2a\left(3a+bx^2\right)}}{4a}+\frac{51x}{4a\sqrt[3]{a-bx^2}(3a+bx^2)}}{144a^2}+\frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}$$

$\big\downarrow$ 305

$$\frac{\frac{\frac{\frac{57\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}+168a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}\cdots\right.}\right.}{2\ 2^{2/3}a^{5/6}\sqrt{b}}\cdots}{6a}\cdots}{4a}\cdots}{144a^2}\cdots$$

$$\frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}$$

$\big\downarrow$ 833

3.134. $\displaystyle\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)^3}dx$

$$\frac{57\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{2bx}+168a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}\right.$$

$$6a$$

$$4a$$

$$144a^2$$

$$\frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}$$

$$\Big\downarrow \; 760$$

$$\frac{57\sqrt{-bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\ \text{Ellipt}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx}\right.}}}\right.}{2bx}$$

$$\frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}$$

$$\Big\downarrow \; 2418$$

3.134. $\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)^3}\,dx$

$$\frac{x}{48a^2\sqrt[3]{a-bx^2}\left(bx^2+3a\right)^2}+$$

$$\frac{\frac{51x}{4a\sqrt[3]{a-bx^2}(bx^2+3a)}+\frac{168a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\cdots}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right)}\right)}{2\ 2^{2/3}a^{5/\cdots}}\cdots\right.}{\cdots}}{\cdots}$$

input

```
Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x]
```

output

```
x/(48*a^2*(a - b*x^2)^(1/3)*(3*a + b*x^2)^2) + ((51*x)/(4*a*(a - b*x^2)^(1
/3)*(3*a + b*x^2)) + ((-19*x*(a - b*x^2)^(2/3))/(2*a*(3*a + b*x^2)) + (168
*a*(ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[
b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt
[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]
/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(
1/3)*(a - b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])) + (57*Sqrt[-(b*x^2)
]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(S
qrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a
^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(
1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*S
qrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))^2)])))/(2*b*x))/(6*a))/(4*a))/(144*a^2)
```

3.134. $\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)^3}\,dx$

3.134.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.134.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}} \left(b\,x^{2}+3 a\right)^{3}}dx$$

input
```
int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x)
```

output
```
int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x)
```

3.134.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.134.6 Sympy [F]

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx = \int \frac{1}{(a-bx^2)^{\frac{4}{3}}(3a+bx^2)^3}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a)**3,x)
```

output
```
Integral(1/((a - b*x**2)**(4/3)*(3*a + b*x**2)**3), x)
```

3.134.7 Maxima [F]

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx = \int \frac{1}{(bx^2+3\,a)^3(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)), x)
```

3.134.8 Giac [F]

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx = \int \frac{1}{(bx^2+3\,a)^3(-bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)), x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx = \int \frac{1}{(a-b\,x^2)^{4/3}\,(b\,x^2+3\,a)^3}\,dx$$

input
```
int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x)
```

output
```
int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3), x)
```

3.135 $\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}} \, dx$

3.135.1 Optimal result

Integrand size = 24, antiderivative size = 653

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}} \, dx = -\frac{3240}{91}ax\left(a-bx^2\right)^{2/3}$$

$$-\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}}+\frac{3x(3a+bx^2)^3}{2\left(a-bx^2\right)^{4/3}}-\frac{36936a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{18468\sqrt[4]{3}\cdots}{\cdots}$$

output
```
-3240/91*a*x*(-b*x^2+a)^(2/3)-81/13*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)-9/2*x*(
b*x^2+3*a)^2/(-b*x^2+a)^(1/3)+3/2*x*(b*x^2+3*a)^3/(-b*x^2+a)^(4/3)-36936/9
1*a^2*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+12312/91*3^(3/4)*a^(7/3)*(
a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2))
)/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)
+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-
3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(
1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-18468/91*3^(1/4)*a^(7/3)*(a^(1/3)-(-b*x
^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a
)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(
1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1
/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.135.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.15

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx =$$
$$-\frac{3\left(1647a^3x-4743a^2bx^3+177ab^2x^5+7b^3x^7-4104a^2x(a-bx^2)\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\right.\right.}{91\,(a-bx^2)^{4/3}}$$

input
```
Integrate[(3*a + b*x^2)^4/(a - b*x^2)^(7/3),x]
```

output
```
(-3*(1647*a^3*x - 4743*a^2*b*x^3 + 177*a*b^2*x^5 + 7*b^3*x^7 - 4104*a^2*x*
(a - b*x^2)*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)
/a]))/(91*(a - b*x^2)^(4/3))
```

3.135.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {315, 27, 401, 27, 403, 27, 299, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}dx$$

$$\Big\downarrow \text{315}$$

$$\frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}-\frac{3\int -\frac{4ab(3a-5bx^2)(bx^2+3a)^2}{(a-bx^2)^{4/3}}dx}{8ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\int \frac{(3a-5bx^2)(bx^2+3a)^2}{(a-bx^2)^{4/3}}dx+\frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \text{401}$$

$$\frac{3}{2}\left(\frac{3\int \frac{12ab(bx^2+a)(bx^2+3a)}{\sqrt[3]{a-bx^2}}dx}{2ab}-\frac{3x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\right)+\frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{2}\left(18\int \frac{(bx^2+a)(bx^2+3a)}{\sqrt[3]{a-bx^2}}dx-\frac{3x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\right)+\frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \; 403$$

$$\frac{3}{2}\left(18\left(-\frac{3\int -\frac{2ab(9bx^2+7a)}{\sqrt[3]{a-bx^2}}dx}{13b}-\frac{3}{13}x(a-bx^2)^{2/3}(a+bx^2)\right)-\frac{3x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\right)+\frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{2}\left(18\left(\frac{6}{13}a\int \frac{9bx^2+7a}{\sqrt[3]{a-bx^2}}dx-\frac{3}{13}x(a-bx^2)^{2/3}(a+bx^2)\right)-\frac{3x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\right)+\frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \; 299$$

$$\frac{3}{2}\left(18\left(\frac{6}{13}a\left(\frac{76}{7}a\int \frac{1}{\sqrt[3]{a-bx^2}}dx-\frac{27}{7}x(a-bx^2)^{2/3}\right)-\frac{3}{13}x(a-bx^2)^{2/3}(a+bx^2)\right)-\frac{3x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\right)+\frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \; 233$$

$$\frac{3}{2}\left(18\left(\frac{6}{13}a\left(-\frac{114a\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}-\frac{27}{7}x(a-bx^2)^{2/3}\right)-\frac{3}{13}x(a-bx^2)^{2/3}(a+bx^2)\right)-\frac{3x(3\cdots}{\sqrt[3]{\cdots}}\right.$$

$$\frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \; 833$$

3.135. $\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx$

$$\frac{3}{2}\left(18\left(\frac{6}{13}a\left(-\frac{114a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{7bx}-\frac{27}{7}x(a\right.\right.\right.$$

$$\frac{3x\left(3a+bx^2\right)^3}{2\left(a-bx^2\right)^{4/3}}$$

$\downarrow$ 760

$$\frac{3}{2}\left(18\left(\frac{6}{13}a\left(-\frac{114a\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+}{\left(\left(\right.\right.}}}{\sqrt[4]{3}}\right.}{7bx}\right.\right.\right.$$

$$\frac{3x\left(3a+bx^2\right)^3}{2\left(a-bx^2\right)^{4/3}}$$

$\downarrow$ 2418

3.135. $\int\frac{\left(3a+bx^2\right)^4}{\left(a-bx^2\right)^{7/3}}\,dx$

$$\frac{3}{2}\left(18\left(\frac{6}{13}a\left(-\frac{114a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\right.}{}\right.\right.\right.$$

$$\frac{3x\left(3a+bx^2\right)^3}{2\left(a-bx^2\right)^{4/3}}$$

input
```
Int[(3*a + b*x^2)^4/(a - b*x^2)^(7/3),x]
```

output
```
(3*x*(3*a + b*x^2)^3)/(2*(a - b*x^2)^(4/3)) + (3*((-3*x*(3*a + b*x^2)^2)/(
a - b*x^2)^(1/3) + 18*((-3*x*(a - b*x^2)^(2/3)*(a + b*x^2))/13 + (6*a*((-2
7*x*(a - b*x^2)^(2/3))/7 - (114*a*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/
3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3)
 + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellip
ticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/
3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^
2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/(
(1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (
a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(7*b
*x)))/13)))/2
```

3.135. $\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx$

3.135.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.135. $\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.135.4 Maple [F]

$$\int \frac{\left(b\,x^{2}+3a\right)^{4}}{\left(-b\,x^{2}+a\right)^{\frac{7}{3}}}d x$$

input
```
int((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x)
```

output
```
int((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x)
```

3.135.5 Fricas [F]

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^4}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

output
```
integral(-(b^4*x^8 + 12*a*b^3*x^6 + 54*a^2*b^2*x^4 + 108*a^3*b*x^2 + 81*a^
4)*(-b*x^2 + a)^(2/3)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x)
```

3.135.6 Sympy [F]

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = \int \frac{(3a+bx^2)^4}{(a-bx^2)^{\frac{7}{3}}}\,dx$$

input
```
integrate((b*x**2+3*a)**4/(-b*x**2+a)**(7/3),x)
```

output
```
Integral((3*a + b*x**2)**4/(a - b*x**2)**(7/3), x)
```

3.135.7 Maxima [F]

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^4}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3), x)
```

3.135.8 Giac [F]

$$\int \frac{(3a + bx^2)^4}{(a - bx^2)^{7/3}}\, dx = \int \frac{(bx^2 + 3\,a)^4}{(-bx^2 + a)^{\frac{7}{3}}}\, dx$$

input
```
integrate((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3), x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3a + bx^2)^4}{(a - bx^2)^{7/3}}\, dx = \int \frac{(b\,x^2 + 3\,a)^4}{(a - b\,x^2)^{7/3}}\, dx$$

input
```
int((3*a + b*x^2)^4/(a - b*x^2)^(7/3),x)
```

output
```
int((3*a + b*x^2)^4/(a - b*x^2)^(7/3), x)
```

3.136 $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx$

3.136.1 Optimal result

Integrand size = 24, antiderivative size = 596

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = -\frac{27}{14}x\left(a-bx^2\right)^{2/3}+\frac{3x(3a+bx^2)^2}{2\left(a-bx^2\right)^{4/3}}-\frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$-\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
-27/14*x*(-b*x^2+a)^(2/3)+3/2*x*(b*x^2+3*a)^2/(-b*x^2+a)^(4/3)-324/7*a*x/(
-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+108/7*3^(3/4)*a^(4/3)*(a^(1/3)-(-b*
x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+
a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b
*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)-162/7*3^(1/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*El
lipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)
*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)
^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*
2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2)))^2)^(1/2)
```

3.136.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \frac{81a^2x+90abx^3-3b^2x^5+108ax(a-bx^2)\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{7(a-bx^2)^{4/3}}$$

input
```
Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(7/3),x]
```

output
```
(81*a^2*x + 90*a*b*x^3 - 3*b^2*x^5 + 108*a*x*(a - b*x^2)*(1 - (b*x^2)/a)^(
1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a])/(7*(a - b*x^2)^(4/3))
```

3.136.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {315, 27, 299, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx$$

$$\Big\downarrow \; 315$$

$$\frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}}-\frac{3\int -\frac{12ab\left(bx^2+3a\right)}{\sqrt[3]{a-bx^2}}dx}{8ab}$$

$$\Big\downarrow \; 27$$

$$\frac{9}{2}\int \frac{bx^2+3a}{\sqrt[3]{a-bx^2}}dx+\frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}}$$

$$\Big\downarrow \; 299$$

$$\frac{9}{2}\left(\frac{24}{7}a\int \frac{1}{\sqrt[3]{a-bx^2}}dx-\frac{3}{7}x\left(a-bx^2\right)^{2/3}\right)+\frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}}$$

$$\Big\downarrow \; 233$$

$$\frac{9}{2}\left(-\frac{36a\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{7bx}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}\right)+\frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}}$$

$$\Big\downarrow \; 833$$

$$\frac{9}{2}\left(-\frac{36a\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{7bx}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}\right)+$$

$$\frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}}$$

$$\Big\downarrow \; 760$$

3.136. $\int \frac{\left(3a+bx^2\right)^3}{\left(a-bx^2\right)^{7/3}}\,dx$

$$\frac{9}{2}\left(-\frac{36a\sqrt{-bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-b\ldots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\ldots}\right.}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{\ldots}}{\left(\left(1\ldots\right.\right.}}}\right.}{7bx}\right.$$

$$\frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}}$$

$$\Big\downarrow \text{2418}$$

$$\frac{9}{2}\left(-\frac{36a\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\ldots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\ldots}\right.\right.}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\right.}{\ldots}\right.$$

$$\frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}}$$

input
```
Int[(3*a + b*x^2)^3/(a - b*x^2)^(7/3),x]
```

3.136. $\int \frac{\left(3a+bx^2\right)^3}{\left(a-bx^2\right)^{7/3}}\,dx$

output
```
(3*x*(3*a + b*x^2)^2)/(2*(a - b*x^2)^(4/3)) + (9*((-3*x*(a - b*x^2)^(2/3))
/7 - (36*a*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a
 - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^
2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/(
(1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^
2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 -
Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
 + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(
3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(7*b*x)))/2
```

3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.136. $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}} \, dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.136.4 Maple [F]

$$\int \frac{\left(b\,x^{2}+3a\right)^{3}}{\left(-b\,x^{2}+a\right)^{\frac{7}{3}}}dx$$

input
```
int((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x)
```

output
```
int((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x)
```

3.136.5 Fricas [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

output
```
integral(-(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/
3)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x)
```

3.136.6 Sympy [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \int \frac{(3a+bx^2)^3}{(a-bx^2)^{\frac{7}{3}}}\,dx$$

input
```
integrate((b*x**2+3*a)**3/(-b*x**2+a)**(7/3),x)
```

output
```
Integral((3*a + b*x**2)**3/(a - b*x**2)**(7/3), x)
```

3.136.7 Maxima [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3), x)
```

3.136.8 Giac [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3), x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \int \frac{(b\,x^2+3\,a)^3}{(a-b\,x^2)^{7/3}}\,dx$$

input
```
int((3*a + b*x^2)^3/(a - b*x^2)^(7/3),x)
```

output
```
int((3*a + b*x^2)^3/(a - b*x^2)^(7/3), x)
```

3.137 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx$

3.137.1 Optimal result

Integrand size = 24, antiderivative size = 44

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx = \frac{9x}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)}{2\,(a-bx^2)^{4/3}}$$

output
```
9/2*x/(-b*x^2+a)^(1/3)+3/2*x*(b*x^2+3*a)/(-b*x^2+a)^(4/3)
```

3.137.2 Mathematica [A] (verified)

Time = 15.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx = \frac{9ax-3bx^3}{(a-bx^2)^{4/3}}$$

input
```
Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
```

output
```
(9*a*x - 3*b*x^3)/(a - b*x^2)^(4/3)
```

3.137.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {315, 27, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx$$

$$\Big\downarrow \; 315$$

$$\frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}-\frac{3\int -\frac{4ab(3a-bx^2)}{(a-bx^2)^{4/3}}dx}{8ab}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{2}\int \frac{3a-bx^2}{(a-bx^2)^{4/3}}dx+\frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \; 297$$

$$\frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}+\frac{9x}{2\sqrt[3]{a-bx^2}}$$

input
```
Int[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
```

output
```
(9*x)/(2*(a - b*x^2)^(1/3)) + (3*x*(3*a + b*x^2))/(2*(a - b*x^2)^(4/3))
```

3.137.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.137.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{3x\left(-b\,x^{2}+3a\right)}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}}}$	24
trager	$\frac{3x\left(-b\,x^{2}+3a\right)}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}}}$	24

input
```
int((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
3/(-b*x^2+a)^(4/3)*x*(-b*x^2+3*a)
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\left(3a+bx^2\right)^2}{\left(a-bx^2\right)^{7/3}}\,dx=-\frac{3\left(bx^3-3\,ax\right)\left(-bx^2+a\right)^{\frac{2}{3}}}{b^2x^4-2\,abx^2+a^2}$$

input
```
integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

output
```
-3*(b*x^3 - 3*a*x)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*x^2 + a^2)
```

3.137.6 Sympy [F]

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx = \int \frac{(3a+bx^2)^2}{(a-bx^2)^{\frac{7}{3}}}\,dx$$

input
```
integrate((b*x**2+3*a)**2/(-b*x**2+a)**(7/3),x)
```

output
```
Integral((3*a + b*x**2)**2/(a - b*x**2)**(7/3), x)
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx = \frac{3\,(bx^3-3\,ax)}{(bx^2-a)(-bx^2+a)^{\frac{1}{3}}}$$

input
```
integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="maxima")
```

output
```
3*(b*x^3 - 3*a*x)/((b*x^2 - a)*(-b*x^2 + a)^(1/3))
```

3.137.8 Giac [F]

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(7/3), x)
```

3.137.9 Mupad [B] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx = \frac{3\,x\,(a-b\,x^2)+6\,a\,x}{(a-b\,x^2)^{4/3}}$$

input
```
int((3*a + b*x^2)^2/(a - b*x^2)^(7/3),x)
```

output
```
(3*x*(a - b*x^2) + 6*a*x)/(a - b*x^2)^(4/3)
```

3.138 $\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx$

3.138.1 Optimal result

Integrand size = 22, antiderivative size = 590

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx = \frac{3x}{2\left(a-bx^2\right)^{4/3}}+\frac{9x}{4a\sqrt[3]{a-bx^2}}+\frac{9x}{4a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+\right.}{8a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{3\,3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+\right.}{2\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
3/2*x/(-b*x^2+a)^(4/3)+9/4*x/a/(-b*x^2+a)^(1/3)+9/4*x/a/(-(-b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2)))-3/4*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a
)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I
*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a
)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b/x*2^(1/2)/(-a^(1/3
)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)+9/8*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1
+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3
)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1
-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(2/3)/b/x/(-a^(1/3
)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)
```

3.138.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.13

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx = \frac{15ax-9bx^3-3x(a-bx^2)\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{4a(a-bx^2)^{4/3}}$$

input
```
Integrate[(3*a + b*x^2)/(a - b*x^2)^(7/3),x]
```

output
```
(15*a*x - 9*b*x^3 - 3*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F
1[1/3, 1/2, 3/2, (b*x^2)/a])/(4*a*(a - b*x^2)^(4/3))
```

3.138.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {298, 215, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx$$

$$\Big\downarrow \text{298}$$

$$\frac{3}{2}\int \frac{1}{\left(a-bx^2\right)^{4/3}}dx+\frac{3x}{2\left(a-bx^2\right)^{4/3}}$$

$$\Big\downarrow \text{215}$$

$$\frac{3}{2}\left(\frac{3x}{2a\sqrt[3]{a-bx^2}}-\frac{\int \frac{1}{\sqrt[3]{a-bx^2}}dx}{2a}\right)+\frac{3x}{2\left(a-bx^2\right)^{4/3}}$$

$$\Big\downarrow \text{233}$$

$$\frac{3}{2}\left(\frac{3\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{4abx}+\frac{3x}{2a\sqrt[3]{a-bx^2}}\right)+\frac{3x}{2\left(a-bx^2\right)^{4/3}}$$

$$\Big\downarrow \text{833}$$

$$\frac{3}{2}\left(\frac{3\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{4abx}+\frac{3x}{2a\sqrt[3]{a-bx^2}}\right)+$$

$$\frac{3x}{2\left(a-bx^2\right)^{4/3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{3}{2}\left(\frac{3\sqrt{-bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-\cdots}}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\cdots\right.}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}\right.}{4abx}\cdots\right.$$

$$\frac{3x}{2\left(a-bx^2\right)^{4/3}}$$

3.138. $\int \frac{3a+bx^2}{\left(a-bx^2\right)^{7/3}}dx$

$$\Big\downarrow 2418$$

$$\frac{3}{2}\left(\frac{3\sqrt{-bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\right.}{}\right.$$

$$\frac{3x}{2\left(a-bx^2\right)^{4/3}}$$

input
```
Int[(3*a + b*x^2)/(a - b*x^2)^(7/3),x]
```

output
```
(3*x)/(2*(a - b*x^2)^(4/3)) + (3*((3*x)/(2*a*(a - b*x^2)^(1/3)) + (3*Sqrt[
-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
 + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(
a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
 - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt
[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sq
rt[3])*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a -
b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b
*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2)])))/(4*a*b*x)))/2
```

3.138. $\int \frac{3a+bx^2}{\left(a-bx^2\right)^{7/3}}\,dx$

3.138.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.138. $\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx$

3.138.4 Maple [F]

$$\int \frac{b\,x^{2}+3a}{\left(-b\,x^{2}+a\right)^{\frac{7}{3}}}d x$$

input
```
int((b*x^2+3*a)/(-b*x^2+a)^(7/3),x)
```

output
```
int((b*x^2+3*a)/(-b*x^2+a)^(7/3),x)
```

3.138.5 Fricas [F]

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx = \int \frac{bx^{2}+3\,a}{\left(-bx^{2}+a\right)^{\frac{7}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

output
```
integral(-(b*x^2 + 3*a)*(-b*x^2 + a)^(2/3)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*
b*x^2 - a^3), x)
```

3.138.6 Sympy [A] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.10

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx = \frac{3x\,{}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{7}{3} \\ \frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{2i\pi}}{a}} \right)}{a^{\frac{4}{3}}} + \frac{bx^{3}\,{}_{2}F_{1}\left(\begin{matrix} \frac{3}{2}, \frac{7}{3} \\ \frac{5}{2} \end{matrix}\middle| {\frac{bx^{2}e^{2i\pi}}{a}} \right)}{3a^{\frac{7}{3}}}$$

input
```
integrate((b*x**2+3*a)/(-b*x**2+a)**(7/3),x)
```

output
```
3*x*hyper((1/2, 7/3), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(4/3) + b*x**
3*hyper((3/2, 7/3), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(7/3))
```

3.138.7 Maxima [F]

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx = \int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)/(-b*x^2+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3), x)
```

3.138.8 Giac [F]

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx = \int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((b*x^2+3*a)/(-b*x^2+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3), x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx = \int \frac{b\,x^2+3\,a}{(a-b\,x^2)^{7/3}}\,dx$$

input
```
int((3*a + b*x^2)/(a - b*x^2)^(7/3),x)
```

output
```
int((3*a + b*x^2)/(a - b*x^2)^(7/3), x)
```

3.139 $\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx$

3.139.1 Optimal result

Integrand size = 24, antiderivative size = 796

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx = \frac{3x}{32a^2(a-bx^2)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+ \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{96\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{32\ 2^{2/3}a^{17/6}\sqrt{b}}$$

$$+ \frac{21\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7 \cdots\right)}{128a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$- \frac{7\ 3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7 \cdots\right)}{32\sqrt{2}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
3/32*x/a^2/(-b*x^2+a)^(4/3)+21/64*x/a^3/(-b*x^2+a)^(1/3)+21/64*x/a^3/(-(-b
*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/64*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)
+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(17/6)/b^(1/2)-1/192*arctanh(x*b^(1/
2)/a^(1/2))*2^(1/3)/a^(17/6)/b^(1/2)+1/192*arctan(a^(1/6)*(a^(1/3)-2^(1/3)
*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)+1/1
92*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)-7/64
*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2
)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/
3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2
)))^2)^(1/2)*3^(3/4)/a^(8/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/
3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+21/128*(a^(1/3)-(-b*x
^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a
)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(
1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1
/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(8/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^
(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.139.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.32 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.31

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx = \frac{x\left(-7bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+27a\left(\frac{9a-7bx^2}{a-bx^2}-\frac{}{(3a+bx}\right.\right.}{57}$$

input
```
Integrate[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x]
```

output
```
(x*(-7*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
1/3*(b*x^2)/a] + 27*a*((9*a - 7*b*x^2)/(a - b*x^2) - (51*a^2*AppellF1[1/2,
 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((3*a + b*x^2)*(9*a*AppellF1[1/2
, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2
, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a,
-1/3*(b*x^2)/a])))))/(576*a^4*(a - b*x^2)^(1/3))
```

3.139.3 Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 859, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {316, 27, 402, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{3\int \frac{b(5bx^2+23a)}{3(a-bx^2)^{4/3}(bx^2+3a)}dx}{32a^2b}+\frac{3x}{32a^2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5bx^2+23a}{(a-bx^2)^{4/3}(bx^2+3a)}dx}{32a^2}+\frac{3x}{32a^2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{3\int -\frac{4ab(7bx^2+17a)}{3\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{8a^2b}+\frac{21x}{2a\sqrt[3]{a-bx^2}}}{32a^2}+\frac{3x}{32a^2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{21x}{2a\sqrt[3]{a-bx^2}}-\frac{\int \frac{7bx^2+17a}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{2a}}{32a^2}+\frac{3x}{32a^2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \text{405}$$

$$\frac{\frac{21x}{2a\sqrt[3]{a-bx^2}}-\frac{7\int \frac{1}{\sqrt[3]{a-bx^2}}dx-4a\int \frac{1}{\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{2a}}{32a^2}+\frac{3x}{32a^2(a-bx^2)^{4/3}}$$

$$\Big\downarrow \text{233}$$

$$\frac{\frac{21x}{2a\sqrt[3]{a-bx^2}}-\frac{-\frac{21\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}-4a\int \frac{1}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx}{2a}}{32a^2}+\frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}$$

$\big\downarrow$ 305

$$\frac{\frac{21x}{2a\sqrt[3]{a-bx^2}}-\frac{-\frac{21\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}-4a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\cdots\right.}{\cdots}\right.}{2a}}{32a^2}$$

$$\frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}$$

$\big\downarrow$ 833

$$\frac{\frac{21x}{2a\sqrt[3]{a-bx^2}}-\frac{-\frac{21\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{2bx}-4a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}{\cdots}\right.}{2\ 2^{2/3}\sqrt{3}a\cdots}\right.}{2a}}{32a^2}$$

$$\frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}$$

$\big\downarrow$ 760

3.139. $\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}\,dx$

$$-\frac{21\sqrt{-bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-b}\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{\cdots}}{\left(\left(1-\cdots\right.\right.}}}\right.}{2bx}$$

$$\frac{\frac{21x}{2a\sqrt[3]{a-bx^2}}-\cdots}{\cdots}$$

$$\frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}$$

$$\Big\downarrow \ 2418$$

$$\frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}+$$

$$\frac{\frac{21x}{2a\sqrt[3]{a-bx^2}}-\frac{-4a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a}\cdots\right.}\right.}{2\ 2^{2/3}a^{5/6}\sqrt{b}}\cdots\right.}{\cdots}}{\cdots}$$

input

```
Int[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x]
```

3.139. $\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}\,dx$

output
```
(3*x)/(32*a^2*(a - b*x^2)^(4/3)) + ((21*x)/(2*a*(a - b*x^2)^(1/3)) - (-4*a
*(ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]
) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b
]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(
6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/
3)*(a - b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])) - (21*Sqrt[-(b*x^2)]*
((-2*Sqrt[-(b*x^2)])/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4
)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
 - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^
(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqr
t[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/
3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ell
ipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))^2)])))/(2*b*x))/(2*a))/(32*a^2)
```

3.139.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

3.139. $\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx$

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.139.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{7}{3}} \left(b\,x^{2}+3 a\right)}d x$$

input
```
int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x)
```

output
```
int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x)
```

3.139.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x, algorithm="fricas")
```

output
```
Timed out
```

3.139.6 Sympy [F]

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx=\int \frac{1}{(a-bx^2)^{\frac{7}{3}}\cdot(3a+bx^2)}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(7/3)/(b*x**2+3*a),x)
```

output
```
Integral(1/((a - b*x**2)**(7/3)*(3*a + b*x**2)), x)
```

3.139.7 Maxima [F]

$$\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}\,dx=\int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)), x)
```

3.139.8 Giac [F]

$$\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}\,dx=\int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)), x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}\,dx=\int \frac{1}{\left(a-b\,x^2\right)^{7/3}\,\left(b\,x^2+3\,a\right)}\,dx$$

input
```
int(1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x)
```

output
```
int(1/((a - b*x^2)^(7/3)*(3*a + b*x^2)), x)
```

3.140 $\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx$

3.140.1 Optimal result

Integrand size = 24, antiderivative size = 827

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx = \frac{5x}{384a^3(a-bx^2)^{4/3}}+\frac{79x}{768a^4\sqrt[3]{a-bx^2}}+\frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)}+\frac{79x}{768a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}}+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}}+\frac{79\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4}{512\ 3^{3/4}a^{11/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{79\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{}}{384\sqrt{2}\sqrt[4]{3}a^{11/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

output
```
5/384*x/a^3/(-b*x^2+a)^(4/3)+79/768*x/a^4/(-b*x^2+a)^(1/3)+1/24*x/a^2/(-b*
x^2+a)^(4/3)/(b*x^2+3*a)+79/768*x/a^4/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2
)))+3/256*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^
(1/3)/a^(23/6)/b^(1/2)-1/256*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(23/6)/b
^(1/2)+1/256*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b
^(1/2))*2^(1/3)/a^(23/6)*3^(1/2)/b^(1/2)+1/256*arctan(3^(1/2)*a^(1/2)/x/b^
(1/2))*2^(1/3)/a^(23/6)*3^(1/2)/b^(1/2)-79/2304*(a^(1/3)-(-b*x^2+a)^(1/3))
*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2
+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(11/
3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(
1/3)*(1-3^(1/2)))^2)^(1/2)+79/1536*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-
(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2
))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-
(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3
^(1/4)/a^(11/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.140.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.31

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx = \frac{x\left(-79bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\frac{27a\left(\frac{299a^2-148abx^2-79b^2\ldots}{a-bx^2}\ldots\right.}{\ldots}\right.}{2073\ldots}$$

input
```
Integrate[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x]
```

output
```
(x*(-79*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a,
-1/3*(b*x^2)/a] + (27*a*((299*a^2 - 148*a*b*x^2 - 79*b^2*x^4)/(a - b*x^2)
- (387*a^2*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*App
ellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/
2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b
*x^2)/a, -1/3*(b*x^2)/a]))))/(3*a + b*x^2)))/(20736*a^5*(a - b*x^2)^(1/3))
```

3.140.3 Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {316, 27, 402, 27, 402, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)}-\frac{\int -\frac{b(21a-11bx^2)}{3(a-bx^2)^{7/3}(bx^2+3a)}dx}{24a^2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{21a-11bx^2}{(a-bx^2)^{7/3}(bx^2+3a)}dx}{72a^2}+\frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{3\int \frac{2ab(25bx^2+291a)}{3(a-bx^2)^{4/3}(bx^2+3a)}dx}{32a^2b}+\frac{15x}{16a(a-bx^2)^{4/3}}}{72a^2}+\frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{25bx^2+291a}{(a-bx^2)^{4/3}(bx^2+3a)}dx}{16a}+\frac{15x}{16a(a-bx^2)^{4/3}}}{72a^2}+\frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{3\int -\frac{4ab(79bx^2+129a)}{3\sqrt[3]{a-bx^2}(bx^2+3a)}dx}{8a^2b}+\frac{237x}{2a\sqrt[3]{a-bx^2}}}{16a}+\frac{15x}{16a(a-bx^2)^{4/3}}}{72a^2}+\frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{237x}{2a\sqrt[3]{a-bx^2}}-\frac{\int \frac{79bx^2+129a}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx}{2a}}{16a}+\frac{15x}{16a\left(a-bx^2\right)^{4/3}}}{72a^2}+\frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}$$

$$\Big\downarrow \; 405$$

$$\frac{\frac{\frac{237x}{2a\sqrt[3]{a-bx^2}}-\frac{79\int \frac{1}{\sqrt[3]{a-bx^2}}dx-108a\int \frac{1}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx}{2a}}{16a}+\frac{15x}{16a\left(a-bx^2\right)^{4/3}}}{72a^2}+\frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}$$

$$\Big\downarrow \; 233$$

$$\frac{\frac{\frac{237x}{2a\sqrt[3]{a-bx^2}}-\frac{-\frac{237\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}-108a\int \frac{1}{\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}dx}{2a}}{16a}+\frac{15x}{16a\left(a-bx^2\right)^{4/3}}}{72a^2}+\frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}$$

$$\Big\downarrow \; 305$$

$$\frac{\frac{\frac{237x}{2a\sqrt[3]{a-bx^2}}-\frac{-\frac{237\sqrt{-bx^2}\int \frac{\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}}{2bx}-108a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\cdots\right)}{\cdots}\right)}{2a}}{16a}\cdots}{72a^2}+\frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}$$

$$\Big\downarrow \; 833$$

3.140. $\displaystyle\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)^2}\,dx$

$$\frac{\frac{237x}{2a\sqrt[3]{a-bx^2}}-\frac{-\frac{237\sqrt{-bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}\right)}{2bx}-108a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{\cdots}\right)}{\cdots}\right)}{2\ 2^{2/3}\sqrt{3}a\cdots}\cdots\right)}{2a}}{16a}}{72a^2}$$

$$\frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}$$

$\downarrow$ 760

$$\frac{\frac{237x}{2a\sqrt[3]{a-bx^2}}-\frac{-\frac{237\sqrt{-bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\sqrt{-bx^2}}d\sqrt[3]{a-bx^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}\right)\cdots}}}{\sqrt[4]{3}\sqrt{-bx^2}\sqrt{-\frac{\sqrt[3]{a}\cdots}{\left(\left(1-\sqrt{\cdots}\right)\cdots\right)\cdots}}}\cdots\right)}{2bx}\cdots}{\cdots}}{\cdots}$$

$$\frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}$$

$\downarrow$ 2418

$$\frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(bx^2+3a\right)}+$$

$$\frac{\frac{15x}{16a\left(a-bx^2\right)^{4/3}}+\frac{\frac{237x}{2a\sqrt[3]{a-bx^2}}-\frac{-108a\left(\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\cdots}{\sqrt[6]{a}\cdots}\right)}{\cdots}\cdots\right)}{\cdots}}{\cdots}}{\cdots}$$

3.140. $\int\frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)^2}\,dx$

input
```
Int[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x]
```

output
```
x/(24*a^2*(a - b*x^2)^(4/3)*(3*a + b*x^2)) + ((15*x)/(16*a*(a - b*x^2)^(4/
3)) + ((237*x)/(2*a*(a - b*x^2)^(1/3)) - (-108*a*(ArcTan[(Sqrt[3]*Sqrt[a])
/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6
)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a
^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b])
 + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(2
*2^(2/3)*a^(5/6)*Sqrt[b])) - (237*Sqrt[-(b*x^2)]*((-2*Sqrt[-(b*x^2)])/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/
3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3)
 + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellip
ticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b*x^2)]*Sqrt[-((a^(1/
3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a - b*x^
2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/(
(1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b*x^2)]*Sqrt[-((a^(1/3)*(a^(1/3) - (
a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])))/(2*b
*x))/(2*a))/(16*a))/(72*a^2)
```

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

3.140. $\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx$

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.140.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{7}{3}} \left(b\,x^{2}+3a\right)^{2}}dx$$

input
```
int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x)
```

output
```
int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x)
```

3.140.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.140.6 Sympy [F]

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx = \int \frac{1}{(a-bx^2)^{\frac{7}{3}}(3a+bx^2)^2}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(7/3)/(b*x**2+3*a)**2,x)
```

output
```
Integral(1/((a - b*x**2)**(7/3)*(3*a + b*x**2)**2), x)
```

3.140.7 Maxima [F]

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx = \int \frac{1}{(bx^2+3\,a)^2(-bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)), x)
```

3.140.8 Giac [F]

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx = \int \frac{1}{(bx^2+3\,a)^2(-bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)), x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{7/3}\,(3a+bx^2)^2}\,dx = \int \frac{1}{\left(a-b\,x^2\right)^{7/3}\,\left(b\,x^2+3\,a\right)^2}\,dx$$

input
```
int(1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x)
```

output
```
int(1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2), x)
```

3.141 $\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx$

3.141.1 Optimal result

Integrand size = 26, antiderivative size = 252

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} - \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{-a+bx^2}\right)}{\sqrt[3]{-a}\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} + \frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} - \frac{\text{arctanh}\left(\frac{\sqrt[3]{-a}\sqrt{b}x}{\sqrt{a}\left(\sqrt[3]{-a}+\sqrt[3]{2}\sqrt[3]{-a+bx^2}\right)}\right)}{2\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}$$

output
```
1/12*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/(-a)^(1/3)/a^(1/2)/b^(1/2)-1/4*arc
tanh((-a)^(1/3)*x*b^(1/2)/((-a)^(1/3)+2^(1/3)*(b*x^2-a)^(1/3))/a^(1/2))*2^
(1/3)/(-a)^(1/3)/a^(1/2)/b^(1/2)-1/12*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^
(1/3)/(-a)^(1/3)*3^(1/2)/a^(1/2)/b^(1/2)-1/12*arctan(((-a)^(1/3)-2^(1/3)*(
b*x^2-a)^(1/3))*3^(1/2)*a^(1/2)/(-a)^(1/3)/x/b^(1/2))*2^(1/3)/(-a)^(1/3)*3
^(1/2)/a^(1/2)/b^(1/2)
```

3.141.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.41 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.65

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = -\frac{9ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{-a+bx^2}\,(3a+bx^2)\left(9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+A\right.\right.}$$

input
```
Integrate[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]
```

output
```
(-9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((-a + b*x^
2)^(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*
x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]
+ AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

3.141.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {305}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{bx^2-a}}\,dx$$

$$\Big\downarrow \text{305}$$

$$-\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{bx^2-a}\right)}{\sqrt[3]{-a}\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt[3]{-a}\sqrt{b}x}{\sqrt{a}\left(\sqrt[3]{2}\sqrt[3]{bx^2-a}+\sqrt[3]{-a}\right)}\right)}{2\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}$$

input
```
Int[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]
```

output
```
-1/2*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2^(2/3)*Sqrt[3]*(-a)^(1/3)*Sqr
t[a]*Sqrt[b]) - ArcTan[(Sqrt[3]*Sqrt[a]*((-a)^(1/3) - 2^(1/3)*(-a + b*x^2)
^(1/3)))/((-a)^(1/3)*Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*(-a)^(1/3)*Sqrt[a]*Sqr
t[b]) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b]
) - ArcTanh[((-a)^(1/3)*Sqrt[b]*x)/(Sqrt[a]*((-a)^(1/3) + 2^(1/3)*(-a + b*
x^2)^(1/3)))]/(2*2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b])
```

3.141.3.1 Defintions of rubi rules used

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

3.141.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}-3a\right)\left(b\,x^{2}-a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x)
```

output
```
int(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x)
```

3.141.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.141.6 Sympy [F]

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = -\int \frac{1}{3a\sqrt[3]{-a+bx^2}+bx^2\sqrt[3]{-a+bx^2}}\,dx$$

input
```
integrate(1/(-b*x**2-3*a)/(b*x**2-a)**(1/3),x)
```

output
```
-Integral(1/(3*a*(-a + b*x**2)**(1/3) + b*x**2*(-a + b*x**2)**(1/3)), x)
```

3.141.7 Maxima [F]

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = \int -\frac{1}{(bx^2+3\,a)(bx^2-a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)), x)
```

3.141.8 Giac [F]

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = \int -\frac{1}{(bx^2+3\,a)(bx^2-a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)), x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx=-\int \frac{1}{{\left(b\,x^2-a\right)}^{1/3}\,\left(b\,x^2+3\,a\right)} \,d x$$

input
```
int(-1/((b*x^2 - a)^(1/3)*(3*a + b*x^2)),x)
```

output
```
-int(1/((b*x^2 - a)^(1/3)*(3*a + b*x^2)), x)
```

3.142 $\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx$

3.142.1 Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$

output
```
1/4*arctan(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(b*x^2+a)^(1/3)))*2^(1/3)/a^
(5/6)/b^(1/2)-1/12*arctan(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(5/6)/b^(1/2)-1/12*
arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*(b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/
3)/a^(5/6)*3^(1/2)/b^(1/2)-1/12*arctanh(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)
/a^(5/6)*3^(1/2)/b^(1/2)
```

3.142.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.82

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx$$
$$=\frac{9ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},\frac{bx^2}{3a}\right)}{(3a-bx^2)\sqrt[3]{a+bx^2}\left(9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},\frac{bx^2}{3a}\right)+2bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{bx^2}{a},\frac{bx^2}{3a}\right)-\text{AppellF1}\cdots\right.\right.}$$

input
```
Integrate[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
```

output
```
(9*a*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), (b*x^2)/(3*a)])/((3*a - b*
x^2)*(a + b*x^2)^(1/3)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), (b*x^
2)/(3*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), (b*x^2)/(3*a
)] - AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), (b*x^2)/(3*a)])))
```

3.142.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx$$
$$\Big\downarrow \text{304}$$
$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a+bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$

input
```
Int[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
```

output
```
-1/6*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[
b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a + b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*
Sqrt[b]) - ArcTanh[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/
6)*Sqrt[b]) - ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3
)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b])
```

3.142.3.1 Defintions of rubi rules used

rule 304
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

3.142.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+3a\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x)
```

output
```
int(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x)
```

3.142.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.142.6 Sympy [F]

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx = -\int \frac{1}{-3a\sqrt[3]{a+bx^2}+bx^2\sqrt[3]{a+bx^2}}\,dx$$

input
```
integrate(1/(-b*x**2+3*a)/(b*x**2+a)**(1/3),x)
```

output
```
-Integral(1/(-3*a*(a + b*x**2)**(1/3) + b*x**2*(a + b*x**2)**(1/3)), x)
```

3.142.7 Maxima [F]

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx = \int -\frac{1}{(bx^2+a)^{\frac{1}{3}}(bx^2-3\,a)}\,dx$$

input
```
integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)), x)
```

3.142.8 Giac [F]

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx = \int -\frac{1}{(bx^2+a)^{\frac{1}{3}}(bx^2-3\,a)}\,dx$$

input
```
integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)), x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{1/3}\,(3\,a-b\,x^2)}\,dx$$

input
```
int(1/((a + b*x^2)^(1/3)*(3*a - b*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(1/3)*(3*a - b*x^2)), x)
```

3.143 $\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx$

3.143.1 Optimal result

Integrand size = 23, antiderivative size = 204

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}$$

output
```
-1/4*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*(3*d*x^2+c)^(1/3))/x/d^(1/2))*2^(1/3
)/c^(5/6)/d^(1/2)-1/4*arctanh(1/x/d^(1/2)*c^(1/2))*2^(1/3)/c^(5/6)/d^(1/2)
-1/12*arctan(x*3^(1/2)*d^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)/d^(1/2)+1/
4*arctan(x*3^(1/2)*d^(1/2)/c^(1/6)/(c^(1/3)+2^(1/3)*(3*d*x^2+c)^(1/3)))*3^
(1/2)*2^(1/3)/c^(5/6)/d^(1/2)
```

3.143.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.94 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.75

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx$$
$$= \frac{3cx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3dx^2}{c},\frac{dx^2}{c}\right)}{(c-dx^2)\sqrt[3]{c+3dx^2}\left(3c\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3dx^2}{c},\frac{dx^2}{c}\right)+2dx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{3dx^2}{c},\frac{dx^2}{c}\right)-\text{Appe}\right.\right.}$$

input
```
Integrate[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
```

output
```
(3*c*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*d*x^2)/c, (d*x^2)/c])/((c - d*x^2)*(
c + 3*d*x^2)^(1/3)*(3*c*AppellF1[1/2, 1/3, 1, 3/2, (-3*d*x^2)/c, (d*x^2)/c
] + 2*d*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (-3*d*x^2)/c, (d*x^2)/c] - AppellF
1[3/2, 4/3, 1, 5/2, (-3*d*x^2)/c, (d*x^2)/c])))
```

3.143.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx$$
$$\Big\downarrow \; 304$$
$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c+3dx^2}+\sqrt[3]{c}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}-\frac{\text{arctanh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}$$

input
```
Int[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
```

output
```
-1/2*ArcTan[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2^(2/3)*Sqrt[3]*c^(5/6)*Sqrt[d])
 + (Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c + 3*
d*x^2)^(1/3)))]])/(2*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[Sqrt[c]/(Sqrt[d]*x)
]/(2*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*(c + 3
*d*x^2)^(1/3)))/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d])
```

3.143.3.1 Defintions of rubi rules used

rule 304
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

3.143.4 Maple [F]

$$\int \frac{1}{\left(-d\,x^{2}+c \right) \left(3 d\,x^{2}+c \right)^{\frac{1}{3}}}d x$$

input
```
int(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x)
```

output
```
int(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x)
```

3.143.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c-dx^2)\,\sqrt[3]{c+3dx^2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.143.6 Sympy [F]

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = -\int \frac{1}{-c\sqrt[3]{c+3dx^2}+dx^2\sqrt[3]{c+3dx^2}}\,dx$$

input
```
integrate(1/(-d*x**2+c)/(3*d*x**2+c)**(1/3),x)
```

output
```
-Integral(1/(-c*(c + 3*d*x**2)**(1/3) + d*x**2*(c + 3*d*x**2)**(1/3)), x)
```

3.143.7 Maxima [F]

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = \int -\frac{1}{(3\,dx^2+c)^{\frac{1}{3}}(dx^2-c)}\,dx$$

input
```
integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="maxima")
```

output
```
-integrate(1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)), x)
```

3.143.8 Giac [F]

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = \int -\frac{1}{(3\,dx^2+c)^{\frac{1}{3}}(dx^2-c)}\,dx$$

input
```
integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="giac")
```

output
```
integrate(-1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)), x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = \int \frac{1}{(c-d\,x^2)\,(3\,d\,x^2+c)^{1/3}}\,dx$$

input
```
int(1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x)
```

output
```
int(1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)), x)
```

3.144 $\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx$

3.144.1 Optimal result

Integrand size = 24, antiderivative size = 204

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}$$

output
```
1/4*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/
a^(5/6)/b^(1/2)-1/12*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(5/6)/b^(1/2)+1/
12*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^
(1/3)/a^(5/6)*3^(1/2)/b^(1/2)+1/12*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/
3)/a^(5/6)*3^(1/2)/b^(1/2)
```

3.144.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$$
$$= \frac{9ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{a-bx^2}\,(3a+bx^2)\left(9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\text{Appe}\right.\right.}$$

input
```
Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

output
```
(9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((a - b*x^2)
^(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^
2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] +
AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

3.144.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {305}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$$
$$\Big\downarrow \ 305$$
$$\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}$$

input
```
Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

output
```
ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b])
+ ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*
x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*
2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)
*(a - b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])
```

3.144.3.1 Defintions of rubi rules used

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

3.144.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}} \left(b\,x^{2}+3 a\right)}d x$$

input
```
int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

output
```
int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

3.144.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="fricas")
```

output
```
Timed out
```

3.144.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx=\int \frac{1}{\sqrt[3]{a-bx^2}\cdot(3a+bx^2)}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
```

output
```
Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)), x)
```

3.144.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx=\int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

3.144.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx=\int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{\left(a-b\,x^2\right)^{1/3}\,\left(b\,x^2+3\,a\right)}\,dx$$

input
```
int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x)
```

output
```
int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)), x)
```

3.145 $\int \frac{1}{\sqrt[3]{c-3dx^2}(c+dx^2)}\,dx$

3.145.1 Optimal result

Integrand size = 22, antiderivative size = 204

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}+\frac{\arctan\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}$$

output
```
1/4*arctan(c^(1/6)*(c^(1/3)-2^(1/3)*(-3*d*x^2+c)^(1/3))/x/d^(1/2))*2^(1/3)
/c^(5/6)/d^(1/2)+1/4*arctan(1/x/d^(1/2)*c^(1/2))*2^(1/3)/c^(5/6)/d^(1/2)-1
/12*arctanh(x*3^(1/2)*d^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)/d^(1/2)+1/4
*arctanh(x*3^(1/2)*d^(1/2)/c^(1/6)/(c^(1/3)+2^(1/3)*(-3*d*x^2+c)^(1/3)))*3
^(1/2)*2^(1/3)/c^(5/6)/d^(1/2)
```

3.145.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx$$
$$= \frac{3cx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3dx^2}{c},-\frac{dx^2}{c}\right)}{\sqrt[3]{c-3dx^2}\,(c+dx^2)\left(3c\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3dx^2}{c},-\frac{dx^2}{c}\right)+2dx^2\left(-\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{3dx^2}{c},-\frac{dx^2}{c}\right)+\text{Ap}\right.\right.}$$

input
```
Integrate[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
```

output
```
(3*c*x*AppellF1[1/2, 1/3, 1, 3/2, (3*d*x^2)/c, -((d*x^2)/c)])/((c - 3*d*x^
2)^(1/3)*(c + d*x^2)*(3*c*AppellF1[1/2, 1/3, 1, 3/2, (3*d*x^2)/c, -((d*x^2
)/c)] + 2*d*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (3*d*x^2)/c, -((d*x^2)/c)] +
AppellF1[3/2, 4/3, 1, 5/2, (3*d*x^2)/c, -((d*x^2)/c)])))
```

3.145.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {305}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx$$
$$\Big\downarrow \; 305$$
$$\frac{\arctan\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}+\frac{\arctan\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c-3dx^2}+\sqrt[3]{c}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}$$

input
```
Int[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
```

output
```
ArcTan[Sqrt[c]/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d]) + ArcTan[(c^(1/6)*
(c^(1/3) - 2^(1/3)*(c - 3*d*x^2)^(1/3)))/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*S
qrt[d]) - ArcTanh[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2*2^(2/3)*Sqrt[3]*c^(5/6)*
Sqrt[d]) + (Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3
)*(c - 3*d*x^2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d])
```

3.145.3.1 Defintions of rubi rules used

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

3.145.4 Maple [F]

$$\int \frac{1}{\left(-3d\,x^{2}+c\right)^{\frac{1}{3}}\left(d\,x^{2}+c\right)}dx$$

input
```
int(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x)
```

output
```
int(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x)
```

3.145.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.145.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx = \int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx$$

input
```
integrate(1/(-3*d*x**2+c)**(1/3)/(d*x**2+c),x)
```

output
```
Integral(1/((c - 3*d*x**2)**(1/3)*(c + d*x**2)), x)
```

3.145.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx = \int \frac{1}{(dx^2+c)(-3\,dx^2+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)), x)
```

3.145.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx = \int \frac{1}{(dx^2+c)(-3\,dx^2+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx = \int \frac{1}{(d\,x^2+c)\,\left(c-3\,d\,x^2\right)^{1/3}}\,dx$$

input
```
int(1/((c + d*x^2)*(c - 3*d*x^2)^(1/3)),x)
```

output
```
int(1/((c + d*x^2)*(c - 3*d*x^2)^(1/3)), x)
```

3.146 $\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.146.1 Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

output
```
-1/12*arctanh(x)*2^(1/3)+1/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)
+1/12*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*arctan((1-2^(1/3)*(-x^2+1)^(1
/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

3.146.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx =$$

$$-\frac{9x\,\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)-\operatorname{AppellF1}\left(\right.\right.\right.}$$

input
```
Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2
)*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3
, 2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

3.146.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {305}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx$$

$$\Big\downarrow \text{305}$$

$$\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\text{arctanh}(x)}{6\ 2^{2/3}}$$

input
```
Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 -
x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(
1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

3.146.3.1 Defintions of rubi rules used

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

3.146.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 64.53 (sec) , antiderivative size = 704, normalized size of antiderivative = 6.23

method	result	size
trager	Expression too large to display	704

input
```
int(1/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-1/144*ln((2*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x^2-RootOf(_Z^6+108)^4*x^3-
3*RootOf(_Z^6+108)^4*x-36*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x+216*(-x^2+1)
^(2/3)*x+126*RootOf(_Z^6+108)*x^2-54*RootOf(_Z^6+108))/(RootOf(_Z^6+108)^3
*x+18)^2/(RootOf(_Z^6+108)^3*x-18))*RootOf(_Z^6+108)^4-1/24*ln((2*(-x^2+1)
^(1/3)*RootOf(_Z^6+108)^5*x^2-RootOf(_Z^6+108)^4*x^3-3*RootOf(_Z^6+108)^4*
x-36*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x+216*(-x^2+1)^(2/3)*x+126*RootOf(_
Z^6+108)*x^2-54*RootOf(_Z^6+108))/(RootOf(_Z^6+108)^3*x+18)^2/(RootOf(_Z^6
+108)^3*x-18))*RootOf(_Z^6+108)+1/216*RootOf(_Z^6+108)^4*ln((72*RootOf(_Z^
6+108)^4*x^3-1296*RootOf(_Z^6+108)*x^3-RootOf(_Z^6+108)^4*x^6-225*RootOf(_
Z^6+108)^4*x^4+4050*RootOf(_Z^6+108)*x^4-72*x^5*RootOf(_Z^6+108)^4+1296*x^
5*RootOf(_Z^6+108)+486*RootOf(_Z^6+108)-27*RootOf(_Z^6+108)^4+189*RootOf(_
Z^6+108)^4*x^2-3402*RootOf(_Z^6+108)*x^2+18*RootOf(_Z^6+108)*x^6-108*(-x^2
+1)^(1/3)*RootOf(_Z^6+108)^5*x^2+324*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x+6
*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x^5+108*(-x^2+1)^(1/3)*RootOf(_Z^6+108)
^5*x^4-3888*(-x^2+1)^(2/3)*x+1296*(-x^2+1)^(2/3)*x^4+9072*(-x^2+1)^(2/3)*x
^3+3888*(-x^2+1)^(2/3)*x^2+144*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x^3-36*(-
x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x^5-54*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x
-648*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x^4-864*(-x^2+1)^(1/3)*RootOf(_Z^6+
108)^2*x^3+648*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x^2)/(x^2+3)^3)
```

3.146.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 1232, normalized size of antiderivative = 10.90

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log((432^(5/6)*(-1)^(1/6)*(x^6
 - 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 - 69*x^4 + 63*x^2 - 27) - 27) + 432*2^(
1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 + sqrt(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) +
 1728*(9*x^3 - sqrt(3)*(I*x^4 - 9*I*x^2) - 9*x)*(-x^2 + 1)^(2/3) - 432*(2^
(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 - sqrt(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) + 4*
432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 - sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1
/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3)
 + 1)*log(-(432^(5/6)*(-1)^(1/6)*(x^6 - 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 -
69*x^4 + 63*x^2 - 27) - 27) - 432*2^(1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 + sqr
t(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) - 1728*(9*x^3 - sqrt(3)*(-I*x^4 + 9*I*
x^2) - 9*x)*(-x^2 + 1)^(2/3) + 432*(2^(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 - sqr
t(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) - 4*432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 -
sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log((432^(5/6)*(-1)^(1/6)*(x^6
 - 69*x^4 + 63*x^2 - sqrt(-3)*(x^6 - 69*x^4 + 63*x^2 - 27) - 27) + 432*2^(
1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 - sqrt(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) +
 1728*(9*x^3 - sqrt(3)*(I*x^4 - 9*I*x^2) - 9*x)*(-x^2 + 1)^(2/3) - 432*(2^
(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 + sqrt(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) + 4*
432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 + sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1
/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(...
```

3.146.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

input
```
integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.146.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.146.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

input
```
int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.147 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

3.147.1 Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\frac{\arctan(x)}{6\ 2^{2/3}}+\frac{\arctan\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

output
```
-1/12*arctan(x)*2^(1/3)+1/4*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)-1/
12*arctanh(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/12*arctanh((1-2^(1/3)*(x^2+1)^(1/3
))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

3.147.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right)}{(-3+x^2)\sqrt[3]{1+x^2}\left(9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,\frac{x^2}{3}\right)-\text{AppellF1}\left(\right.\right.\right.}$$

input
```
Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

output
```
(-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)
*(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2,
 5/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

3.147.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-x^2)\sqrt[3]{x^2+1}}dx$$

$$\Big\downarrow \ 304$$

$$\frac{\arctan\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\arctan(x)}{6\ 2^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

input
```
Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

output
```
-1/6*ArcTan[x]/2^(2/3) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3
)) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3
)*(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

3.147.3.1 Defintions of rubi rules used

rule 304
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

3.147.4 Maple [F]

$$\int \frac{1}{\left(-x^{2}+3\right) \left(x^{2}+1\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

output
```
int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

3.147.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. $2(77) = 154$.

Time = 0.70 (sec) , antiderivative size = 1103, normalized size of antiderivative = 10.12

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

output
```
-1/10368*432^(5/6)*(sqrt(-3) + 1)*log((432^(5/6)*(x^6 + 69*x^4 + 63*x^2 +
sqrt(-3)*(x^6 + 69*x^4 + 63*x^2 + 27) + 27) - 1728*(9*x^3 + sqrt(3)*(x^4 +
 9*x^2) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + sqrt(-3)*(5
*x^5 + 30*x^3 + 9*x) + 9*x) + 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 -
 sqrt(-3)*(x^5 + 18*x^3 + 9*x) + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2 - sqrt(-3
)*(x^4 + 3*x^2))))/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*(sqrt(
-3) + 1)*log(-(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 + 69*x^4 +
 63*x^2 + 27) + 27) + 1728*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)
^(2/3) - 432*2^(1/3)*(5*x^5 + 30*x^3 + sqrt(-3)*(5*x^5 + 30*x^3 + 9*x) + 9
*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 - sqrt(-3)*(x^5 + 18*x^3
+ 9*x) + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2 - sqrt(-3)*(x^4 + 3*x^2))))/(x^6
- 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*(sqrt(-3) - 1)*log((432^(5/6)*
(x^6 + 69*x^4 + 63*x^2 - sqrt(-3)*(x^6 + 69*x^4 + 63*x^2 + 27) + 27) - 172
8*(9*x^3 + sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x
^5 + 30*x^3 - sqrt(-3)*(5*x^5 + 30*x^3 + 9*x) + 9*x) + 432*(x^2 + 1)^(1/3)
*(2^(2/3)*(x^5 + 18*x^3 + sqrt(-3)*(x^5 + 18*x^3 + 9*x) + 9*x) + 4*432^(1/
6)*(x^4 + 3*x^2 + sqrt(-3)*(x^4 + 3*x^2))))/(x^6 - 9*x^4 + 27*x^2 - 27)) -
 1/10368*432^(5/6)*(sqrt(-3) - 1)*log(-(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 -
 sqrt(-3)*(x^6 + 69*x^4 + 63*x^2 + 27) + 27) + 1728*(9*x^3 - sqrt(3)*(x^4
+ 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 432*2^(1/3)*(5*x^5 + 30*x^3 - sqrt(-3...
```

3.147.6 Sympy [F]

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\int \frac{1}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

input
```
integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
```

output
```
-Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

3.147.7 Maxima [F]

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

input
```
integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
```

output
```
-integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.147.8 Giac [F]

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

input
```
integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\int \frac{1}{(x^2+1)^{1/3}\,(x^2-3)}\,dx$$

input
```
int(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
```

output
```
-int(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.148 $\int \frac{3-x}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.148.1 Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1+x)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}-\frac{\log(3+x^2)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x}+(1+x)^{2/3}\right)}{2\ 2^{2/3}}$$

output
```
-1/4*ln(x^2+3)*2^(1/3)+3/4*ln(2^(1/3)*(1-x)^(1/3)+(1+x)^(2/3))*2^(1/3)+1/2
*arctan(-1/3*3^(1/2)+1/3*2^(2/3)*(1+x)^(2/3)/(1-x)^(1/3)*3^(1/2))*3^(1/2)*
2^(1/3)
```

3.148.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.61

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$
$$= \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^2}}{-2^{2/3}-2^{2/3}x+\sqrt[3]{1-x^2}}\right)+2\log\left(2^{2/3}+2^{2/3}x+2\sqrt[3]{1-x^2}\right)-\log\left(-\sqrt[3]{2}-2\sqrt[3]{2}x-\sqrt[3]{2}x^2+2\right.}{2\ 2^{2/3}}$$

input
```
Integrate[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^2)^(1/3))/(-2^(2/3) - 2^(2/3)*x + (1 - x
^2)^(1/3))] + 2*Log[2^(2/3) + 2^(2/3)*x + 2*(1 - x^2)^(1/3)] - Log[-2^(1/3
) - 2*2^(1/3)*x - 2^(1/3)*x^2 + 2^(2/3)*(1 + x)*(1 - x^2)^(1/3) - 2*(1 - x
^2)^(2/3)])/(2*2^(2/3))
```

3.148.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1341}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(x^2+3)}dx$$

$$\Big\downarrow \text{1341}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left((x+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{1-x}\right)}{2\ 2^{2/3}}$$

input
```
Int[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
-((Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 + x)^(2/3))/(Sqrt[3]*(1 - x)^(1/
3))])/2^(2/3)) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(1/3)*(1 - x)^(1/3) +
 (1 + x)^(2/3)])/(2*2^(2/3))
```

3.148.3.1 Defintions of rubi rules used

rule 1341
```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[Sqrt[3]*h*(ArcTan[1/Sqrt[3] - 2^(2/3)*((1 - 3*h*(x/g))^
(2/3)/(Sqrt[3]*(1 + 3*h*(x/g))^(1/3)))]/(2^(2/3)*a^(1/3)*f)), x] + (-Simp[3
*h*(Log[(1 - 3*h*(x/g))^(2/3) + 2^(1/3)*(1 + 3*h*(x/g))^(1/3)]/(2^(5/3)*a^(
1/3)*f)), x] + Simp[h*(Log[d + f*x^2]/(2^(5/3)*a^(1/3)*f)), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

3.148.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.23 (sec) , antiderivative size = 1033, normalized size of antiderivative = 10.76

method	result	size
trager	Expression too large to display	1033

input
```
int((3-x)/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
1/2*RootOf(_Z^3-2)*ln((12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)^2*RootOf(_Z^3-2)^2*x^2+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4
*_Z^2)*RootOf(_Z^3-2)^3*x^2+36*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)
+4*_Z^2)^2*RootOf(_Z^3-2)^2*x+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2
)+4*_Z^2)*RootOf(_Z^3-2)^3*x-18*(-x^2+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_
Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-12*(-x^2+1)^(1/3)*RootOf(RootOf(
_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-9*(-x^2+1)^(1/3)*Ro
otOf(_Z^3-2)^2*x-12*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+4*_Z^2)*RootOf(_Z^3-2)+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+
4*_Z^2)*x^2-9*(-x^2+1)^(1/3)*RootOf(_Z^3-2)^2+RootOf(_Z^3-2)*x^2+36*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+6*RootOf(_Z^3-2)*x-6*(-x^2
+1)^(2/3)-18*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-3*RootOf(
_Z^3-2))/(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^
3-2)^2*x+x+3)/(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootO
f(_Z^3-2)^2*x+x-3))+RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln
(-(4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^
2*x^2+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)
^3*x^2+12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^
3-2)^2*x+18*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^
3-2)^3*x-18*(-x^2+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+...
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(70) = 140$.

Time = 3.21 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.97

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{1}{6}$$

$$\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\Big(12\cdot 4^{\frac{2}{3}}(x^4+3\,x^3+3\,x^2+9\,x)(-x^2+1)^{\frac{2}{3}}+4^{\frac{1}{3}}(x^6-18\,x^5-117\,x^4-36\,x^3+207}{6\,(x^6+54\,x^5+171\,x^4+108\,x^3-8}\right.$$

$$-\frac{1}{24}$$

$$\cdot 4^{\frac{2}{3}}\log\left(\frac{6\cdot 4^{\frac{2}{3}}(x^2+3\,x)(-x^2+1)^{\frac{2}{3}}+4^{\frac{1}{3}}(x^4+18\,x^3+24\,x^2-18\,x-9)-6\,(x^3+7\,x^2+3\,x-3)(-x^2+}{x^4+6\,x^2+9}\right.$$

$$+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}(x^2+3)+6\cdot 4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}(x+1)+12\,(-x^2+1)^{\frac{2}{3}}}{x^2+3}\right)$$

input
```
integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(x^4 + 3*x^3 +
 3*x^2 + 9*x)*(-x^2 + 1)^(2/3) + 4^(1/3)*(x^6 - 18*x^5 - 117*x^4 - 36*x^3
+ 207*x^2 + 54*x - 27) + 12*(x^5 + 19*x^4 + 42*x^3 + 6*x^2 - 27*x - 9)*(-x
^2 + 1)^(1/3))/(x^6 + 54*x^5 + 171*x^4 + 108*x^3 - 81*x^2 - 162*x - 27)) -
 1/24*4^(2/3)*log((6*4^(2/3)*(x^2 + 3*x)*(-x^2 + 1)^(2/3) + 4^(1/3)*(x^4 +
 18*x^3 + 24*x^2 - 18*x - 9) - 6*(x^3 + 7*x^2 + 3*x - 3)*(-x^2 + 1)^(1/3))
/(x^4 + 6*x^2 + 9)) + 1/12*4^(2/3)*log((4^(2/3)*(x^2 + 3) + 6*4^(1/3)*(-x^
2 + 1)^(1/3)*(x + 1) + 12*(-x^2 + 1)^(2/3))/(x^2 + 3))
```

3.148.6 Sympy [F]

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\int \frac{x}{x^2\sqrt[3]{1-x^2}+3\sqrt[3]{1-x^2}}\,dx - \int \left(-\frac{3}{x^2\sqrt[3]{1-x^2}+3\sqrt[3]{1-x^2}}\right)dx$$

input
```
integrate((3-x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
-Integral(x/(x**2*(1 - x**2)**(1/3) + 3*(1 - x**2)**(1/3)), x) - Integral(
-3/(x**2*(1 - x**2)**(1/3) + 3*(1 - x**2)**(1/3)), x)
```

3.148.7 Maxima [F]

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int -\frac{x-3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
-integrate((x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.148.8 Giac [F]

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int -\frac{x-3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
integrate(-(x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\int \frac{x-3}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

input
```
int(-(x - 3)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
-int((x - 3)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.149 $\int \frac{3+x}{\sqrt[3]{1-x^2}(3+x^2)} \, dx$

3.149.1 Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)} \, dx = \frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{2^{2/3}}+\frac{\log(3+x^2)}{2\ 2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\sqrt[3]{1+x}\right)}{2\ 2^{2/3}}$$

output
```
1/4*ln(x^2+3)*2^(1/3)-3/4*ln((1-x)^(2/3)+2^(1/3)*(1+x)^(1/3))*2^(1/3)-1/2*
arctan(-1/3*3^(1/2)+1/3*2^(2/3)*(1-x)^(2/3)/(1+x)^(1/3)*3^(1/2))*3^(1/2)*2
^(1/3)
```

3.149.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.59

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)} \, dx = \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^2}}{-2^{2/3}+2^{2/3}x+\sqrt[3]{1-x^2}}\right)-2\log\left(-2^{2/3}+2^{2/3}x-2\sqrt[3]{1-x^2}\right)+\log\left(\sqrt[3]{2}-2\sqrt[3]{2}x+\sqrt[3]{2}x^2+\right.}{2\ 2^{2/3}}$$

input
```
Integrate[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^2)^(1/3))/(-2^(2/3) + 2^(2/3)*x + (1 -
x^2)^(1/3))] - 2*Log[-2^(2/3) + 2^(2/3)*x - 2*(1 - x^2)^(1/3)] + Log[2^(1/
3) - 2*2^(1/3)*x + 2^(1/3)*x^2 + 2^(2/3)*(-1 + x)*(1 - x^2)^(1/3) + 2*(1 -
 x^2)^(2/3)])/(2*2^(2/3))
```

3.149.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{1341\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+3}{\sqrt[3]{1-x^2}\,(x^2+3)}dx$$

$$\Big\downarrow \text{1341}$$

$$\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{2^{2/3}}+\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{2\ 2^{2/3}}$$

input
```
Int[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - x)^(2/3))/(Sqrt[3]*(1 + x)^(1/3)
)])/2^(2/3) + Log[3 + x^2]/(2*2^(2/3)) - (3*Log[(1 - x)^(2/3) + 2^(1/3)*(1
 + x)^(1/3)])/(2*2^(2/3))
```

3.149.3.1 Defintions of rubi rules used

rule 1341
```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[Sqrt[3]*h*(ArcTan[1/Sqrt[3] - 2^(2/3)*((1 - 3*h*(x/g))^
(2/3)/(Sqrt[3]*(1 + 3*h*(x/g))^(1/3)))]/(2^(2/3)*a^(1/3)*f)), x] + (-Simp[3
*h*(Log[(1 - 3*h*(x/g))^(2/3) + 2^(1/3)*(1 + 3*h*(x/g))^(1/3)]/(2^(5/3)*a^(
1/3)*f)), x] + Simp[h*(Log[d + f*x^2]/(2^(5/3)*a^(1/3)*f)), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

3.149.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.11 (sec) , antiderivative size = 1552, normalized size of antiderivative = 16.34

method	result	size
trager	Expression too large to display	1552

input
```
int((3+x)/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*ln((-8*RootOf(RootOf(_
Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^2-6*RootOf(RootO
f(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^2+18*(-x^2+1)^(
2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2+
24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*
x+18*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*
x-18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*Ro
otOf(_Z^3+2)*x-3*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2*x+18*(-x^2+1)^(1/3)*RootO
f(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)+3*(-x^2+1)^(
1/3)*RootOf(_Z^3+2)^2+4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2
)*x^2+3*RootOf(_Z^3+2)*x^2-6*(-x^2+1)^(2/3)-24*x*RootOf(RootOf(_Z^3+2)^2+2
*_Z*RootOf(_Z^3+2)+4*_Z^2)-18*RootOf(_Z^3+2)*x-12*RootOf(RootOf(_Z^3+2)^2+
2*_Z*RootOf(_Z^3+2)+4*_Z^2)-9*RootOf(_Z^3+2))/(2*RootOf(RootOf(_Z^3+2)^2+2
*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x-3)/(2*RootOf(RootOf(_Z^3+2
)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x+3))-1/2*ln(-(8*RootOf
(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^2-2*Roo
tOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^2+18*(
-x^2+1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_
Z^3+2)^2-24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_
Z^3+2)^2*x+6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf...
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(70) = 140.

Time = 3.18 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.32

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{1}{6} \cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}} \arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}(-1)^{\frac{2}{3}}\,(x^4-3\,x^3+3\,x^2-9\,x)(-x^2+1)^{\frac{2}{3}}+12\,(-1)^{\frac{1}{3}}\,(x^5-19\,x^4+4\cdots\right.}{6\,(x^6-54\,x^5+171\,x^4\cdots}\right. - \frac{1}{24} \cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{6\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^2-3\,x)(-x^2+1)^{\frac{2}{3}}-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(x^4-18\,x^3+24\,x^2+18\,x-9)-6\,(x^3-\cdots}{x^4+6\,x^2+9}\right. + \frac{1}{12} \cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{6\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(-x^2+1)^{\frac{1}{3}}(x-1)+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^2+3)-12\,(-x^2+1)^{\frac{2}{3}}}{x^2+3}\right)$$

input
```
integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(-1
)^(2/3)*(x^4 - 3*x^3 + 3*x^2 - 9*x)*(-x^2 + 1)^(2/3) + 12*(-1)^(1/3)*(x^5
- 19*x^4 + 42*x^3 - 6*x^2 - 27*x + 9)*(-x^2 + 1)^(1/3) + 4^(1/3)*(x^6 + 18
*x^5 - 117*x^4 + 36*x^3 + 207*x^2 - 54*x - 27))/(x^6 - 54*x^5 + 171*x^4 -
108*x^3 - 81*x^2 + 162*x - 27)) - 1/24*4^(2/3)*(-1)^(1/3)*log(-(6*4^(2/3)*
(-1)^(1/3)*(x^2 - 3*x)*(-x^2 + 1)^(2/3) - 4^(1/3)*(-1)^(2/3)*(x^4 - 18*x^3
 + 24*x^2 + 18*x - 9) - 6*(x^3 - 7*x^2 + 3*x + 3)*(-x^2 + 1)^(1/3))/(x^4 +
 6*x^2 + 9)) + 1/12*4^(2/3)*(-1)^(1/3)*log(-(6*4^(1/3)*(-1)^(2/3)*(-x^2 +
1)^(1/3)*(x - 1) + 4^(2/3)*(-1)^(1/3)*(x^2 + 3) - 12*(-x^2 + 1)^(2/3))/(x^
2 + 3))
```

3.149.6 Sympy [F]

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x+3}{\sqrt[3]{-(x-1)\,(x+1)}\,(x^2+3)}\,dx$$

input
```
integrate((3+x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral((x + 3)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.149.7 Maxima [F]

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x+3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.149.8 Giac [F]

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x+3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x+3}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

input
```
int((x + 3)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
int((x + 3)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.150 $\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)} \, dx$

3.150.1 Optimal result

Integrand size = 27, antiderivative size = 151

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)} \, dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}$$

output
```
1/12*arctan(1/3*(a^(1/3)-(b*x^2+a)^(1/3))^2/a^(1/6)/x/b^(1/2))*b^(1/2)/a^(
5/6)/d+1/12*arctan(1/3*x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/6)/d-1/12*arctanh(a
^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)/a^(5/6)/d*3^(1
/2)
```

3.150.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.64 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$$
$$= \frac{27abx\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{d\sqrt[3]{a+bx^2}\,(9a+bx^2)\left(27a\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)-2bx^2\left(\mathrm{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)+3\right.\right.}$$

input
```
Integrate[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

output
```
(27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), -1/9*(b*x^2)/a])/(d*(a
+ b*x^2)^(1/3)*(9*a + b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a)
, -1/9*(b*x^2)/a] - 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), -1/9
*(b*x^2)/a] + 3*AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), -1/9*(b*x^2)/a]))
)
```

3.150.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {306}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$$
$$\Big\downarrow \; 306$$
$$\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}$$

input
```
Int[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

output
```
(Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) + (Sqrt[b]*ArcTan
[(a^(1/3) - (a + b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d) -
(Sqrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a + b*x^2)^(1/3)))/(Sqrt[b]*
x)])/(4*Sqrt[3]*a^(5/6)*d)
```

3.150.3.1 Defintions of rubi rules used

rule 306
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

3.150.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{3}} \left(\frac{9 a d}{b}+d\,x^{2}\right)}dx$$

input
```
int(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x)
```

output
```
int(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x)
```

3.150.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="fricas")
```

output
```
Timed out
```

3.150.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = \frac{b\int \frac{1}{9a\sqrt[3]{a+bx^2}+bx^2\sqrt[3]{a+bx^2}}\,dx}{d}$$

input
```
integrate(1/(b*x**2+a)**(1/3)/(9*a*d/b+d*x**2),x)
```

output
```
b*Integral(1/(9*a*(a + b*x**2)**(1/3) + b*x**2*(a + b*x**2)**(1/3)), x)/d
```

3.150.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

3.150.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\int \frac{1}{{\left(b\,x^2+a\right)}^{1/3}\,\left(d\,x^2+\frac{9\,a\,d}{b}\right)} \,d x$$

input
```
int(1/((a + b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)),x)
```

output
```
int(1/((a + b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)), x)
```

3.151 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$

3.151.1 Optimal result

Integrand size = 28, antiderivative size = 153

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d} - \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}$$

output
```
1/12*arctanh(1/3*(a^(1/3)-(-b*x^2+a)^(1/3))^2/a^(1/6)/x/b^(1/2))*b^(1/2)/a
^(5/6)/d-1/12*arctanh(1/3*x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/6)/d-1/12*arctan
(a^(1/6)*(a^(1/3)-(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)/a^(5/6)/d*3
^(1/2)
```

3.151.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.77 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = -\frac{27abx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{d\sqrt[3]{a-bx^2}\,(9a-bx^2)\left(27a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)+2bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)+3\,\text{Appe}\right.\right.}$$

input
```
Integrate[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

output
```
(-27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2)/(9*a)])/(d*(a - b
*x^2)^(1/3)*(9*a - b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x
^2)/(9*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, (b*x^2)/(9*a)]
 + 3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, (b*x^2)/(9*a)])))
```

3.151.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(dx^2-\frac{9ad}{b}\right)}dx$$

$$\Big\downarrow \ 307$$

$$-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

input
```
Int[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

output
```
-1/4*(Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a - b*x^2)^(1/3)))/(Sqrt
[b]*x)])/(Sqrt[3]*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[a])])/
(12*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(a^(1/3) - (a - b*x^2)^(1/3))^2/(3*a^(1/
6)*Sqrt[b]*x)])/(12*a^(5/6)*d)
```

3.151.3.1 Defintions of rubi rules used

rule 307
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

3.151.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}} \left(-\frac{9 a d}{b}+d\,x^{2}\right)}d x$$

input
```
int(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

output
```
int(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

3.151.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="fricas")
```

output
```
Timed out
```

3.151.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx=\frac{b\int \frac{1}{-9a\sqrt[3]{a-bx^2}+bx^2\sqrt[3]{a-bx^2}}\,dx}{d}$$

input
```
integrate(1/(-b*x**2+a)**(1/3)/(-9*a*d/b+d*x**2),x)
```

output
```
b*Integral(1/(-9*a*(a - b*x**2)**(1/3) + b*x**2*(a - b*x**2)**(1/3)), x)/d
```

3.151.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

3.151.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx=\int \frac{1}{{\left(a-b\,x^2\right)}^{1/3}\,\left(d\,x^2-\frac{9\,a\,d}{b}\right)}\,dx$$

input
```
int(1/((a - b*x^2)^(1/3)*(d*x^2 - (9*a*d)/b)),x)
```

output
```
int(1/((a - b*x^2)^(1/3)*(d*x^2 - (9*a*d)/b)), x)
```

3.152 $\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$

3.152.1 Optimal result

Integrand size = 29, antiderivative size = 151

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}$$

output

```
-1/12*arctanh(1/3*(a^(1/3)+(b*x^2-a)^(1/3))^2/a^(1/6)/x/b^(1/2))*b^(1/2)/a
^(5/6)/d+1/12*arctanh(1/3*x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/6)/d+1/12*arctan
(a^(1/6)*(a^(1/3)+(b*x^2-a)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)/a^(5/6)/d*3^
(1/2)
```

3.152.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.86 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx =$$

$$-\frac{27abx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{d\left(9a-bx^2\right)\sqrt[3]{-a+bx^2}\left(27a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)+2bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)+3\,\text{Ap}\right.\right.}$$

input
```
Integrate[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

output
```
(-27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2)/(9*a)])/(d*(9*a -
 b*x^2)*(-a + b*x^2)^(1/3)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*
x^2)/(9*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, (b*x^2)/(9*a)
] + 3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, (b*x^2)/(9*a)])))
```

3.152.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{bx^2-a}\left(dx^2-\frac{9ad}{b}\right)}dx$$

$$\Big\downarrow \; 307$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

input
```
Int[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

output
```
(Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a + b*x^2)^(1/3)))/(Sqrt[b]*
x)])/(4*Sqrt[3]*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[a])])/(1
2*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(a^(1/3) + (-a + b*x^2)^(1/3))^2/(3*a^(1/6
)*Sqrt[b]*x)])/(12*a^(5/6)*d)
```

3.152.3.1 Defintions of rubi rules used

rule 307
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

3.152.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}-a\right)^{\frac{1}{3}} \left(-\frac{9 a d}{b}+d\,x^{2}\right)}dx$$

input
```
int(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

output
```
int(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

3.152.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="fricas")
```

output
```
Timed out
```

3.152.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \frac{b\int \frac{1}{-9a\sqrt[3]{-a+bx^2}+bx^2\sqrt[3]{-a+bx^2}}\,dx}{d}$$

input
```
integrate(1/(b*x**2-a)**(1/3)/(-9*a*d/b+d*x**2),x)
```

output
```
b*Integral(1/(-9*a*(-a + b*x**2)**(1/3) + b*x**2*(-a + b*x**2)**(1/3)), x)
/d
```

3.152.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

input
```
integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

3.152.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

input
```
integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{\left(b\,x^2-a\right)^{1/3}\,\left(d\,x^2-\frac{9\,a\,d}{b}\right)}\,dx$$

input
```
int(1/((b*x^2 - a)^(1/3)*(d*x^2 - (9*a*d)/b)),x)
```

output
```
int(1/((b*x^2 - a)^(1/3)*(d*x^2 - (9*a*d)/b)), x)
```

3.153 $\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$

3.153.1 Optimal result

Integrand size = 30, antiderivative size = 153

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}$$

output
```
-1/12*arctan(1/3*(a^(1/3)+(-b*x^2-a)^(1/3))^2/a^(1/6)/x/b^(1/2))*b^(1/2)/a
^(5/6)/d-1/12*arctan(1/3*x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/6)/d+1/12*arctanh
(a^(1/6)*(a^(1/3)+(-b*x^2-a)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)/a^(5/6)/d*3
^(1/2)
```

3.153.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.80 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = \frac{27abx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{d\sqrt[3]{-a-bx^2}\,(9a+bx^2)\left(27a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)-2bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)+\right.\right.}$$

input
```
Integrate[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

output
```
(27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), -1/9*(b*x^2)/a])/(d*(-a
 - b*x^2)^(1/3)*(9*a + b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a
), -1/9*(b*x^2)/a] - 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), -1/
9*(b*x^2)/a] + 3*AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), -1/9*(b*x^2)/a])
))
```

3.153.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {306}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$$

$$\Big\downarrow \; 306$$

$$-\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}$$

input
```
Int[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

output
```
-1/12*(Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(a^(5/6)*d) - (Sqrt[b]*Arc
Tan[(a^(1/3) + (-a - b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d
) + (Sqrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a - b*x^2)^(1/3)))/(Sqr
t[b]*x)])/(4*Sqrt[3]*a^(5/6)*d)
```

3.153.3.1 Defintions of rubi rules used

rule 306
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

3.153.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}-a\right)^{\frac{1}{3}}\left(\frac{9ad}{b}+d\,x^{2}\right)}dx$$

input
```
int(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x)
```

output
```
int(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x)
```

3.153.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="fricas")
```

output
```
Timed out
```

3.153.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\frac{b\int \frac{1}{9a\sqrt[3]{-a-bx^2}+bx^2\sqrt[3]{-a-bx^2}}\,dx}{d}$$

input
```
integrate(1/(-b*x**2-a)**(1/3)/(9*a*d/b+d*x**2),x)
```

output
```
b*Integral(1/(9*a*(-a - b*x**2)**(1/3) + b*x**2*(-a - b*x**2)**(1/3)), x)/
d
```

3.153.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\int \frac{1}{(-bx^2-a)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

input
```
integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

3.153.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\int \frac{1}{(-bx^2-a)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

input
```
integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\int \frac{1}{{\left(-b\,x^2-a\right)}^{1/3}\,\left(d\,x^2+\frac{9\,a\,d}{b}\right)} \,dx$$

input
```
int(1/((- a - b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)),x)
```

output
```
int(1/((- a - b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)), x)
```

3.154 $\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx$

3.154.1 Optimal result

Integrand size = 26, antiderivative size = 151

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d} + \frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d} - \frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

output

```
1/24*arctan(1/6*(2^(1/3)-(b*x^2+2)^(1/3))^2*2^(5/6)/x/b^(1/2))*b^(1/2)*2^(
1/6)/d+1/24*arctan(1/6*x*b^(1/2)*2^(1/2))*b^(1/2)*2^(1/6)/d-1/24*arctanh(2
^(1/6)*(2^(1/3)-(b*x^2+2)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)*2^(1/6)/d*3^(1
/2)
```

3.154.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.40 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx =$$

$$-\frac{27bx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{2},-\frac{bx^2}{18}\right)}{d\sqrt[3]{2+bx^2}\,(18+bx^2)\left(-27\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{2},-\frac{bx^2}{18}\right)+bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{bx^2}{2},-\frac{bx^2}{18}\right)+\right.\right.}$$

input
```
Integrate[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
```

output
```
(-27*b*x*AppellF1[1/2, 1/3, 1, 3/2, -1/2*(b*x^2), -1/18*(b*x^2)])/(d*(2 +
b*x^2)^(1/3)*(18 + b*x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -1/2*(b*x^2), -1
/18*(b*x^2)] + b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -1/2*(b*x^2), -1/18*(b*x^
2)] + 3*AppellF1[3/2, 4/3, 1, 5/2, -1/2*(b*x^2), -1/18*(b*x^2)])))
```

3.154.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {306}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{bx^2+2}\left(\frac{18d}{b}+dx^2\right)}\,dx$$

$$\Big\downarrow \; 306$$

$$\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\;2^{5/6}d}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\;2^{5/6}d}-\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)}{\sqrt{b}x}\right)}{4\;2^{5/6}\sqrt{3}d}$$

input
```
Int[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
```

output
```
(Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*2^(5/6)*d) + (Sqrt[b]*ArcTan
[(2^(1/3) - (2 + b*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[b]*x)])/(12*2^(5/6)*d) -
(Sqrt[b]*ArcTanh[(2^(1/6)*Sqrt[3]*(2^(1/3) - (2 + b*x^2)^(1/3)))/(Sqrt[b]*
x)])/(4*2^(5/6)*Sqrt[3]*d)
```

3.154.3.1 Defintions of rubi rules used

rule 306
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

3.154.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+2\right)^{\frac{1}{3}} \left(\frac{18d}{b}+d\,x^{2}\right)}dx$$

input
```
int(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x)
```

output
```
int(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x)
```

3.154.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="fricas")
```

output
```
Timed out
```

3.154.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx = \frac{b\int \frac{1}{bx^2\sqrt[3]{bx^2+2}+18\sqrt[3]{bx^2+2}}\,dx}{d}$$

input
```
integrate(1/(b*x**2+2)**(1/3)/(18*d/b+d*x**2),x)
```

output
```
b*Integral(1/(b*x**2*(b*x**2 + 2)**(1/3) + 18*(b*x**2 + 2)**(1/3)), x)/d
```

3.154.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2+2)^{\frac{1}{3}}\left(dx^2+\frac{18\,d}{b}\right)}\,dx$$

input
```
integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)), x)
```

3.154.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2+2)^{\frac{1}{3}}\left(dx^2+\frac{18\,d}{b}\right)}\,dx$$

input
```
integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)), x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx = \int \frac{1}{\left(\frac{18\,d}{b}+d\,x^2\right)\,\left(b\,x^2+2\right)^{1/3}}\,dx$$

input
```
int(1/(((18*d)/b + d*x^2)*(b*x^2 + 2)^(1/3)),x)
```

output
```
int(1/(((18*d)/b + d*x^2)*(b*x^2 + 2)^(1/3)), x)
```

3.155 $\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx$

3.155.1 Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}+\sqrt[3]{-2+bx^2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2+bx^2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}$$

output
```
-1/24*arctanh(1/6*(2^(1/3)+(b*x^2-2)^(1/3))^2*2^(5/6)/x/b^(1/2))*b^(1/2)*2
^(1/6)/d+1/24*arctanh(1/6*x*b^(1/2)*2^(1/2))*b^(1/2)*2^(1/6)/d+1/24*arctan
(2^(1/6)*(2^(1/3)+(b*x^2-2)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)*2^(1/6)/d*3^
(1/2)
```

3.155.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.66 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx$$
$$= \frac{27bx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{2},\frac{bx^2}{18}\right)}{d\left(-18+bx^2\right)\sqrt[3]{-2+bx^2}\left(27\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{2},\frac{bx^2}{18}\right)+bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{2},\frac{bx^2}{18}\right)+3\,\text{Appe}\right.\right.}$$

input
```
Integrate[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
```

output
```
(27*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/2, (b*x^2)/18])/(d*(-18 + b*x^2
)*(-2 + b*x^2)^(1/3)*(27*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/2, (b*x^2)/18]
 + b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/2, (b*x^2)/18] + 3*AppellF1[3
/2, 4/3, 1, 5/2, (b*x^2)/2, (b*x^2)/18])))
```

3.155.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{bx^2-2}\left(dx^2-\frac{18d}{b}\right)}dx$$
$$\Big\downarrow \; 307$$
$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}$$

input
```
Int[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
```

output
```
(Sqrt[b]*ArcTan[(2^(1/6)*Sqrt[3]*(2^(1/3) + (-2 + b*x^2)^(1/3)))/(Sqrt[b]*
x)])/(4*2^(5/6)*Sqrt[3]*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[2])])/(1
2*2^(5/6)*d) - (Sqrt[b]*ArcTanh[(2^(1/3) + (-2 + b*x^2)^(1/3))^2/(3*2^(1/6
)*Sqrt[b]*x)])/(12*2^(5/6)*d)
```

3.155.3.1 Defintions of rubi rules used

rule 307
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

3.155.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}-2\right)^{\frac{1}{3}} \left(-\frac{18 d}{b}+d\,x^{2}\right)}dx$$

input
```
int(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x)
```

output
```
int(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x)
```

3.155.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="fricas")
```

output
```
Timed out
```

3.155.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \frac{b\int \frac{1}{bx^2\sqrt[3]{bx^2-2}-18\sqrt[3]{bx^2-2}}\,dx}{d}$$

input
```
integrate(1/(b*x**2-2)**(1/3)/(-18*d/b+d*x**2),x)
```

output
```
b*Integral(1/(b*x**2*(b*x**2 - 2)**(1/3) - 18*(b*x**2 - 2)**(1/3)), x)/d
```

3.155.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2-2)^{\frac{1}{3}}\left(dx^2-\frac{18\,d}{b}\right)}\,dx$$

input
```
integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)), x)
```

3.155.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2-2)^{\frac{1}{3}}\left(dx^2-\frac{18\,d}{b}\right)}\,dx$$

input
```
integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)), x)
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx=\int -\frac{1}{\left(\frac{18\,d}{b}-d\,x^2\right)\,\left(b\,x^2-2\right)^{1/3}}\,dx$$

input
```
int(-1/(((18*d)/b - d*x^2)*(b*x^2 - 2)^(1/3)),x)
```

output
```
int(-1/(((18*d)/b - d*x^2)*(b*x^2 - 2)^(1/3)), x)
```

3.156 $\int \frac{1}{\sqrt[3]{2+3x^2}(6d+dx^2)}\,dx$

3.156.1 Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx=\frac{\arctan\left(\frac{x}{\sqrt{6}}\right)}{4\;2^{5/6}\sqrt{3}d}+\frac{\arctan\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\;2^{5/6}\sqrt{3}d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)}{x}\right)}{4\;2^{5/6}d}$$

output
```
-1/8*arctanh(2^(1/6)*(2^(1/3)-(3*x^2+2)^(1/3))/x)*2^(1/6)/d+1/24*arctan(1/
18*(2^(1/3)-(3*x^2+2)^(1/3))^2*2^(5/6)/x*3^(1/2))*2^(1/6)/d*3^(1/2)+1/24*a
rctan(1/6*x*6^(1/2))*2^(1/6)/d*3^(1/2)
```

3.156.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx=$$

$$-\frac{9x\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{d\,(6+x^2)\sqrt[3]{2+3x^2}\left(-9\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)+x^2\left(\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)+3\operatorname{Ap}\right.\right.}$$

input
```
Integrate[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

output
```
(-9*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2])/(d*(6 + x^2)*(2 +
3*x^2)^(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2] + x^2*(A
ppellF1[3/2, 1/3, 2, 5/2, (-3*x^2)/2, -1/6*x^2] + 3*AppellF1[3/2, 4/3, 1,
5/2, (-3*x^2)/2, -1/6*x^2])))
```

3.156.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {306}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{3x^2+2}\,(dx^2+6d)}dx$$

$$\Big\downarrow \; 306$$

$$\frac{\arctan\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\arctan\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)}{x}\right)}{4\ 2^{5/6}d}$$

input
```
Int[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

output
```
ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTan[(2^(1/3) - (2 + 3*x^2)^(1
/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/6)*(2^
(1/3) - (2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

3.156.3.1 Defintions of rubi rules used

rule 306
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

3.156.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 76.62 (sec) , antiderivative size = 1066, normalized size of antiderivative = 8.67

method	result	size
trager	Expression too large to display	1066

input
```
int(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(ln(-(16*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*Ro
otOf(_Z^6+54)^6*x-768*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_
Z^2)^2*RootOf(_Z^6+54)^5*x-(3*x^2+2)^(1/3)*RootOf(_Z^6+54)^5*x+72*RootOf(_
Z^6+54)^4*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*(3*x^2+
2)^(1/3)*x-1152*RootOf(_Z^6+54)^3*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z
^6+54)+576*_Z^2)^2*(3*x^2+2)^(1/3)*x-36*RootOf(_Z^6+54)^3*RootOf(RootOf(_Z
^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x^2+72*RootOf(RootOf(_Z^6+54)^2-2
4*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3+18*RootOf(_Z^6+54)^2*(3*x
^2+2)^(1/3)-432*RootOf(_Z^6+54)*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6
+54)+576*_Z^2)*(3*x^2+2)^(1/3)+54*(3*x^2+2)^(2/3))/(x^2+6))*RootOf(_Z^6+54
)-24*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*ln(-(4*RootO
f(_Z^6+54)^7*x-288*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2
)*RootOf(_Z^6+54)^6*x+4608*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+
576*_Z^2)^2*RootOf(_Z^6+54)^5*x-144*RootOf(_Z^6+54)^4*RootOf(RootOf(_Z^6+5
4)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*(3*x^2+2)^(1/3)*x+6912*RootOf(_Z^6+54
)^3*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)^2*(3*x^2+2)^(
1/3)*x-9*x^2*RootOf(_Z^6+54)^4+216*RootOf(_Z^6+54)^3*RootOf(RootOf(_Z^6+54
)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x^2+18*RootOf(_Z^6+54)^4-432*RootOf(Ro
otOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3+2592*Roo
tOf(_Z^6+54)*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*(...
```

3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1843 vs. $2(90) = 180$.

Time = 192.78 (sec) , antiderivative size = 1843, normalized size of antiderivative = 14.98

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="fricas")
```

output
```
1/48*(1/864)^(1/6)*(sqrt(-3) + 1)*(-1/d^6)^(1/6)*log(-1/4*(4*(1/4)^(2/3)*(
7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x + sqrt(-3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*
d^4*x))*(-1/d^6)^(2/3) + 4*(10*x^3 + 3*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^2 + 1
2*d^3)*sqrt(-1/d^6) - 36*x)*(3*x^2 + 2)^(2/3) - 2*(432*(1/864)^(5/6)*(5*d^
5*x^4 - 36*d^5*x^2 - 12*d^5 - sqrt(-3)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5))*
(-1/d^6)^(5/6) - (1/4)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x - sqrt(-3)*(
d^2*x^5 - 52*d^2*x^3 + 36*d^2*x))*(-1/d^6)^(1/3))*(3*x^2 + 2)^(1/3) - (1/8
64)^(1/6)*(d*x^6 - 210*d*x^4 + 252*d*x^2 + sqrt(-3)*(d*x^6 - 210*d*x^4 + 2
52*d*x^2 + 72*d) + 72*d)*(-1/d^6)^(1/6))/(x^6 + 18*x^4 + 108*x^2 + 216)) -
 1/48*(1/864)^(1/6)*(sqrt(-3) + 1)*(-1/d^6)^(1/6)*log(-1/4*(4*(1/4)^(2/3)*
(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x + sqrt(-3)*(7*d^4*x^5 - 92*d^4*x^3 - 36
*d^4*x))*(-1/d^6)^(2/3) + 4*(10*x^3 - 3*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^2 +
12*d^3)*sqrt(-1/d^6) - 36*x)*(3*x^2 + 2)^(2/3) + 2*(432*(1/864)^(5/6)*(5*d
^5*x^4 - 36*d^5*x^2 - 12*d^5 - sqrt(-3)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5))
*(-1/d^6)^(5/6) + (1/4)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x - sqrt(-3)*
(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x))*(-1/d^6)^(1/3))*(3*x^2 + 2)^(1/3) + (1/
864)^(1/6)*(d*x^6 - 210*d*x^4 + 252*d*x^2 + sqrt(-3)*(d*x^6 - 210*d*x^4 +
252*d*x^2 + 72*d) + 72*d)*(-1/d^6)^(1/6))/(x^6 + 18*x^4 + 108*x^2 + 216))
- 1/48*(1/864)^(1/6)*(sqrt(-3) - 1)*(-1/d^6)^(1/6)*log(-1/4*(4*(1/4)^(2/3)
*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x - sqrt(-3)*(7*d^4*x^5 - 92*d^4*x^3 ...
```

3.156.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx = \frac{\int \frac{1}{x^2\sqrt[3]{3x^2+2}+6\sqrt[3]{3x^2+2}}\,dx}{d}$$

input
```
integrate(1/(3*x**2+2)**(1/3)/(d*x**2+6*d),x)
```

output
```
Integral(1/(x**2*(3*x**2 + 2)**(1/3) + 6*(3*x**2 + 2)**(1/3)), x)/d
```

3.156.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx = \int \frac{1}{(dx^2+6\,d)(3\,x^2+2)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)), x)
```

3.156.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx = \int \frac{1}{(dx^2+6\,d)(3\,x^2+2)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="giac")
```

output
```
integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)), x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx = \int \frac{1}{(3\,x^2+2)^{1/3}\,(d\,x^2+6\,d)}\,dx$$

input
```
int(1/((3*x^2 + 2)^(1/3)*(6*d + d*x^2)),x)
```

output
```
int(1/((3*x^2 + 2)^(1/3)*(6*d + d*x^2)), x)
```

3.157 $\int \frac{1}{\sqrt[3]{2-3x^2}(-6d+dx^2)}\,dx$

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = -\frac{\arctan\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

output
```
-1/8*arctan(2^(1/6)*(2^(1/3)-(-3*x^2+2)^(1/3))/x)*2^(1/6)/d+1/24*arctanh(1
/18*(2^(1/3)-(-3*x^2+2)^(1/3))^2*2^(5/6)/x*3^(1/2))*2^(1/6)/d*3^(1/2)-1/24
*arctanh(1/6*x*6^(1/2))*2^(1/6)/d*3^(1/2)
```

3.157.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx$$
$$= \frac{9x\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)}{d\sqrt[3]{2-3x^2}\,(-6+x^2)\left(9\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)+x^2\left(\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)+3\operatorname{AppellF1}\left(\frac{3}{2},\right.\right.\right.}$$

input
```
Integrate[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

output
```
(9*x*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6])/(d*(2 - 3*x^2)^(1/3)*(-
6 + x^2)*(9*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6] + x^2*(AppellF1[3
/2, 1/3, 2, 5/2, (3*x^2)/2, x^2/6] + 3*AppellF1[3/2, 4/3, 1, 5/2, (3*x^2)/
2, x^2/6])))
```

3.157.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(dx^2-6d)}\,dx$$

$$\Big\downarrow \; 307$$

$$-\frac{\arctan\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}+\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

input
```
Int[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

output
```
-1/4*ArcTan[(2^(1/6)*(2^(1/3) - (2 - 3*x^2)^(1/3)))/x]/(2^(5/6)*d) - ArcTa
nh[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/3) - (2 - 3*x^2)^(1/3)
)^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

3.157.3.1 Defintions of rubi rules used

rule 307
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

3.157.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 81.70 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.45

method	result	size
trager	Expression too large to display	547

input
```
int(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(24*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*ln((-19
2*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)
^6*x+4*RootOf(_Z^6-54)^7*x-288*RootOf(_Z^6-54)^4*RootOf(RootOf(_Z^6-54)^2-
24*_Z*RootOf(_Z^6-54)+576*_Z^2)*(-3*x^2+2)^(1/3)*x+6*RootOf(_Z^6-54)^5*(-3
*x^2+2)^(1/3)*x-9*x^2*RootOf(_Z^6-54)^4-18*RootOf(_Z^6-54)^4+108*(-3*x^2+2
)^(1/3)*RootOf(_Z^6-54)^2+324*(-3*x^2+2)^(2/3))/(x^2-6))+RootOf(_Z^6-54)*l
n((768*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_
Z^6-54)^5*x-16*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*Ro
otOf(_Z^6-54)^6*x-1152*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*
_Z^2)^2*RootOf(_Z^6-54)^3*(-3*x^2+2)^(1/3)*x+72*RootOf(_Z^6-54)^4*RootOf(R
ootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*(-3*x^2+2)^(1/3)*x-RootOf
(_Z^6-54)^5*(-3*x^2+2)^(1/3)*x+36*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z
^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3*x^2+72*RootOf(RootOf(_Z^6-54)^2-24*_Z*R
ootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3+432*RootOf(RootOf(_Z^6-54)^2-24
*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)*(-3*x^2+2)^(1/3)-18*(-3*x^2+
2)^(1/3)*RootOf(_Z^6-54)^2+54*(-3*x^2+2)^(2/3))/(x^2-6)))/d
```

3.157.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1867 vs. $2(90) = 180$.

Time = 193.66 (sec) , antiderivative size = 1867, normalized size of antiderivative = 15.18

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="fricas")
```

output
```
-1/48*(1/864)^(1/6)*(sqrt(-3) - 1)*(d^(-6))^(1/6)*log(-1/4*(864*(1/864)^(5
/6)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5 + sqrt(-3)*(5*d^5*x^4 + 36*d^5*x^2 -
12*d^5))*(-3*x^2 + 2)^(1/3)*(d^(-6))^(5/6) + 12*sqrt(1/6)*(d^3*x^4 + 24*d^
3*x^2 + 12*d^3)*(-3*x^2 + 2)^(2/3)*sqrt(d^(-6)) + 4*(1/4)^(2/3)*(7*d^4*x^5
 + 92*d^4*x^3 - 36*d^4*x - sqrt(-3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x))*(
d^(-6))^(2/3) - 2*(1/4)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x + sqrt(-3)*
(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x))*(-3*x^2 + 2)^(1/3)*(d^(-6))^(1/3) - 8*(
5*x^3 + 18*x)*(-3*x^2 + 2)^(2/3) - (1/864)^(1/6)*(d*x^6 + 210*d*x^4 + 252*
d*x^2 - sqrt(-3)*(d*x^6 + 210*d*x^4 + 252*d*x^2 - 72*d) - 72*d)*(d^(-6))^(
1/6))/(x^6 - 18*x^4 + 108*x^2 - 216)) + 1/48*(1/864)^(1/6)*(sqrt(-3) - 1)*
(d^(-6))^(1/6)*log(1/4*(864*(1/864)^(5/6)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5
 + sqrt(-3)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5))*(-3*x^2 + 2)^(1/3)*(d^(-6))
^(5/6) + 12*sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 + 12*d^3)*(-3*x^2 + 2)^(2/3)*s
qrt(d^(-6)) - 4*(1/4)^(2/3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x - sqrt(-3)*
(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x))*(d^(-6))^(2/3) + 2*(1/4)^(1/3)*(d^2*x
^5 + 52*d^2*x^3 + 36*d^2*x + sqrt(-3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x))*(
-3*x^2 + 2)^(1/3)*(d^(-6))^(1/3) + 8*(5*x^3 + 18*x)*(-3*x^2 + 2)^(2/3) - (
1/864)^(1/6)*(d*x^6 + 210*d*x^4 + 252*d*x^2 - sqrt(-3)*(d*x^6 + 210*d*x^4
+ 252*d*x^2 - 72*d) - 72*d)*(d^(-6))^(1/6))/(x^6 - 18*x^4 + 108*x^2 - 216)
) + 1/48*(1/864)^(1/6)*(sqrt(-3) + 1)*(d^(-6))^(1/6)*log(-1/4*(864*(1/8...
```

3.157.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = \frac{\int \frac{1}{x^2\sqrt[3]{2-3x^2}-6\sqrt[3]{2-3x^2}}\,dx}{d}$$

input
```
integrate(1/(-3*x**2+2)**(1/3)/(d*x**2-6*d),x)
```

output
```
Integral(1/(x**2*(2 - 3*x**2)**(1/3) - 6*(2 - 3*x**2)**(1/3)), x)/d
```

3.157.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = \int \frac{1}{(dx^2-6\,d)(-3\,x^2+2)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)), x)
```

3.157.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = \int \frac{1}{(dx^2-6\,d)(-3\,x^2+2)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="giac")
```

output
```
integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)), x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = -\int \frac{1}{(2-3\,x^2)^{1/3}\,(6\,d-d\,x^2)}\,dx$$

input
```
int(-1/((2 - 3*x^2)^(1/3)*(6*d - d*x^2)),x)
```

output
```
-int(1/((2 - 3*x^2)^(1/3)*(6*d - d*x^2)), x)
```

3.158 $\int \frac{1}{\sqrt[3]{-2+3x^2}(-6d+dx^2)}\,dx$

3.158.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)}{x}\right)}{4\,2^{5/6}d}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}$$

output
```
1/8*arctan(2^(1/6)*(2^(1/3)+(3*x^2-2)^(1/3))/x)*2^(1/6)/d-1/24*arctanh(1/1
8*(2^(1/3)+(3*x^2-2)^(1/3))^2*2^(5/6)/x*3^(1/2))*2^(1/6)/d*3^(1/2)+1/24*ar
ctanh(1/6*x*6^(1/2))*2^(1/6)/d*3^(1/2)
```

3.158.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx$$
$$= \frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)}{d\,(-6+x^2)\sqrt[3]{-2+3x^2}\left(9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)+x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)+3\,\text{AppellF1}\left(\right.\right.\right.}$$

input
```
Integrate[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

output
```
(9*x*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6])/(d*(-6 + x^2)*(-2 + 3*x
^2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6] + x^2*(AppellF1[
3/2, 1/3, 2, 5/2, (3*x^2)/2, x^2/6] + 3*AppellF1[3/2, 4/3, 1, 5/2, (3*x^2)
/2, x^2/6])))
```

3.158.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{3x^2-2}\,(dx^2-6d)}\,dx$$

$$\Big\downarrow \text{307}$$

$$\frac{\arctan\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\ 2^{5/6}d}-\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

input
```
Int[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

output
```
ArcTan[(2^(1/6)*(2^(1/3) + (-2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d) + ArcTanh
[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/3) + (-2 + 3*x^2)^(1/3))
^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

3.158.3.1 Defintions of rubi rules used

rule 307
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

3.158.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 74.92 (sec) , antiderivative size = 1063, normalized size of antiderivative = 8.93

method	result	size
trager	Expression too large to display	1063

input
```
int(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(24*ln(-(768*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2
)^2*RootOf(_Z^6-54)^5*x+16*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+
576*_Z^2)*RootOf(_Z^6-54)^6*x+1152*(3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6-54)^
2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)^3*x+72*(3*x^2-2)^(1/3)
*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^
4*x+(3*x^2-2)^(1/3)*RootOf(_Z^6-54)^5*x+36*RootOf(RootOf(_Z^6-54)^2+24*_Z*
RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3*x^2+72*RootOf(RootOf(_Z^6-54)^
2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3-432*(3*x^2-2)^(1/3)*Ro
otOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)-18*
RootOf(_Z^6-54)^2*(3*x^2-2)^(1/3)-54*(3*x^2-2)^(2/3))/(x^2-6))*RootOf(Root
Of(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)+24*RootOf(RootOf(_Z^6-54)^2+
24*_Z*RootOf(_Z^6-54)+576*_Z^2)*ln((4608*RootOf(RootOf(_Z^6-54)^2+24*_Z*Ro
otOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)^5*x+288*RootOf(RootOf(_Z^6-54)^2
+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^6*x+4*RootOf(_Z^6-54)^7*x
+6912*(3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_
Z^2)^2*RootOf(_Z^6-54)^3*x+144*(3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6-54)^2+24
*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^4*x+216*RootOf(RootOf(_Z^6-5
4)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3*x^2+9*x^2*RootOf(_Z
^6-54)^4+432*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*Root
Of(_Z^6-54)^3+18*RootOf(_Z^6-54)^4-2592*(3*x^2-2)^(1/3)*RootOf(RootOf(_...
```

3.158.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1771 vs. $2(86) = 172$.

Time = 190.80 (sec) , antiderivative size = 1771, normalized size of antiderivative = 14.88

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="fricas")
```

output
```
1/48*(1/864)^(1/6)*(sqrt(-3) + 1)*(d^(-6))^(1/6)*log(-1/4*(4*(1/4)^(2/3)*(
7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x + sqrt(-3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*
d^4*x))*(d^(-6))^(2/3) - 4*(10*x^3 + 3*sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 + 1
2*d^3)*sqrt(d^(-6)) + 36*x)*(3*x^2 - 2)^(2/3) + 2*(432*(1/864)^(5/6)*(5*d^
5*x^4 + 36*d^5*x^2 - 12*d^5 - sqrt(-3)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5))*
(d^(-6))^(5/6) + (1/4)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x - sqrt(-3)*(
d^2*x^5 + 52*d^2*x^3 + 36*d^2*x))*(d^(-6))^(1/3))*(3*x^2 - 2)^(1/3) + (1/8
64)^(1/6)*(d*x^6 + 210*d*x^4 + 252*d*x^2 + sqrt(-3)*(d*x^6 + 210*d*x^4 + 2
52*d*x^2 - 72*d) - 72*d)*(d^(-6))^(1/6))/(x^6 - 18*x^4 + 108*x^2 - 216)) -
 1/48*(1/864)^(1/6)*(sqrt(-3) + 1)*(d^(-6))^(1/6)*log(-1/4*(4*(1/4)^(2/3)*
(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x + sqrt(-3)*(7*d^4*x^5 + 92*d^4*x^3 - 36
*d^4*x))*(d^(-6))^(2/3) - 4*(10*x^3 - 3*sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 +
12*d^3)*sqrt(d^(-6)) + 36*x)*(3*x^2 - 2)^(2/3) - 2*(432*(1/864)^(5/6)*(5*d
^5*x^4 + 36*d^5*x^2 - 12*d^5 - sqrt(-3)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5))
*(d^(-6))^(5/6) - (1/4)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x - sqrt(-3)*
(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x))*(d^(-6))^(1/3))*(3*x^2 - 2)^(1/3) - (1/
864)^(1/6)*(d*x^6 + 210*d*x^4 + 252*d*x^2 + sqrt(-3)*(d*x^6 + 210*d*x^4 +
252*d*x^2 - 72*d) - 72*d)*(d^(-6))^(1/6))/(x^6 - 18*x^4 + 108*x^2 - 216))
- 1/48*(1/864)^(1/6)*(sqrt(-3) - 1)*(d^(-6))^(1/6)*log(-1/4*(4*(1/4)^(2/3)
*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x - sqrt(-3)*(7*d^4*x^5 + 92*d^4*x^3 ...
```

3.158.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = \frac{\int \frac{1}{x^2\sqrt[3]{3x^2-2}-6\sqrt[3]{3x^2-2}}\,dx}{d}$$

input
```
integrate(1/(3*x**2-2)**(1/3)/(d*x**2-6*d),x)
```

output
```
Integral(1/(x**2*(3*x**2 - 2)**(1/3) - 6*(3*x**2 - 2)**(1/3)), x)/d
```

3.158.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = \int \frac{1}{(dx^2-6\,d)(3\,x^2-2)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)), x)
```

3.158.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = \int \frac{1}{(dx^2-6\,d)(3\,x^2-2)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="giac")
```

output
```
integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = -\int \frac{1}{(3\,x^2-2)^{1/3}\,(6\,d-d\,x^2)}\,dx$$

input
```
int(-1/((3*x^2 - 2)^(1/3)*(6*d - d*x^2)),x)
```

output
```
-int(1/((3*x^2 - 2)^(1/3)*(6*d - d*x^2)), x)
```

3.159 $\int \frac{1}{\sqrt[3]{-2-3x^2}(6d+dx^2)}\,dx$

3.159.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx=-\frac{\arctan\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\arctan\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}$$

output
```
1/8*arctanh(2^(1/6)*(2^(1/3)+(-3*x^2-2)^(1/3))/x)*2^(1/6)/d-1/24*arctan(1/
18*(2^(1/3)+(-3*x^2-2)^(1/3))^2*2^(5/6)/x*3^(1/2))*2^(1/6)/d*3^(1/2)-1/24*
arctan(1/6*x*6^(1/2))*2^(1/6)/d*3^(1/2)
```

3.159.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.57 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx=$$

$$-\frac{9x\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{d\sqrt[3]{-2-3x^2}\,(6+x^2)\left(-9\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)+x^2\left(\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)+3\,\mathrm{A}\right.\right.}$$

input
```
Integrate[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

output
```
(-9*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2])/(d*(-2 - 3*x^2)^(1
/3)*(6 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2] + x^2*(
AppellF1[3/2, 1/3, 2, 5/2, (-3*x^2)/2, -1/6*x^2] + 3*AppellF1[3/2, 4/3, 1,
 5/2, (-3*x^2)/2, -1/6*x^2])))
```

3.159.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {306}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{-3x^2-2}\,(dx^2+6d)}dx$$

$$\Big\downarrow \text{306}$$

$$-\frac{\arctan\left(\frac{\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\arctan\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\text{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\ 2^{5/6}d}$$

input
```
Int[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

output
```
-1/4*ArcTan[x/Sqrt[6]]/(2^(5/6)*Sqrt[3]*d) - ArcTan[(2^(1/3) + (-2 - 3*x^2
)^(1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/6)
*(2^(1/3) + (-2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

3.159.3.1 Defintions of rubi rules used

rule 306
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

3.159.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 73.39 (sec) , antiderivative size = 1061, normalized size of antiderivative = 8.92

method	result	size
trager	Expression too large to display	1061

input
```
int(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(ln((16*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*Roo
tOf(_Z^6+54)^6*x-768*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z
^2)^2*RootOf(_Z^6+54)^5*x+(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)^5*x-72*(-3*x^2-
2)^(1/3)*RootOf(_Z^6+54)^4*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+
576*_Z^2)*x+1152*(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)^3*RootOf(RootOf(_Z^6+54)
^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)^2*x+36*RootOf(_Z^6+54)^3*RootOf(RootOf(
_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x^2-72*RootOf(RootOf(_Z^6+54)^2
-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3+18*(-3*x^2-2)^(1/3)*Roo
tOf(_Z^6+54)^2-432*(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)*RootOf(RootOf(_Z^6+54)
^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)-54*(-3*x^2-2)^(2/3))/(x^2+6))*RootOf(_Z
^6+54)-24*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*ln((4*R
ootOf(_Z^6+54)^7*x-288*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*
_Z^2)*RootOf(_Z^6+54)^6*x+4608*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+
54)+576*_Z^2)^2*RootOf(_Z^6+54)^5*x+144*(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)^4
*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x-6912*(-3*x^2-2
)^(1/3)*RootOf(_Z^6+54)^3*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+5
76*_Z^2)^2*x+9*x^2*RootOf(_Z^6+54)^4-216*RootOf(_Z^6+54)^3*RootOf(RootOf(_
Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x^2-18*RootOf(_Z^6+54)^4+432*Roo
tOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3+25
92*(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)*RootOf(RootOf(_Z^6+54)^2-24*_Z*Root...
```

3.159.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1939 vs. $2(86) = 172$.

Time = 191.30 (sec) , antiderivative size = 1939, normalized size of antiderivative = 16.29

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="fricas")
```

output
```
1/48*(1/864)^(1/6)*(sqrt(-3) - 1)*(-1/d^6)^(1/6)*log(-1/4*(864*(1/864)^(5/
6)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5 + sqrt(-3)*(5*d^5*x^4 - 36*d^5*x^2 - 1
2*d^5))*(-3*x^2 - 2)^(1/3)*(-1/d^6)^(5/6) + 12*sqrt(1/6)*(d^3*x^4 - 24*d^3
*x^2 + 12*d^3)*(-3*x^2 - 2)^(2/3)*sqrt(-1/d^6) + 4*(1/4)^(2/3)*(7*d^4*x^5
- 92*d^4*x^3 - 36*d^4*x - sqrt(-3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x))*(-
1/d^6)^(2/3) - 2*(1/4)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x + sqrt(-3)*(
d^2*x^5 - 52*d^2*x^3 + 36*d^2*x))*(-3*x^2 - 2)^(1/3)*(-1/d^6)^(1/3) + 8*(5
*x^3 - 18*x)*(-3*x^2 - 2)^(2/3) - (1/864)^(1/6)*(d*x^6 - 210*d*x^4 + 252*d
*x^2 - sqrt(-3)*(d*x^6 - 210*d*x^4 + 252*d*x^2 + 72*d) + 72*d)*(-1/d^6)^(1
/6))/(x^6 + 18*x^4 + 108*x^2 + 216)) - 1/48*(1/864)^(1/6)*(sqrt(-3) - 1)*(
-1/d^6)^(1/6)*log(1/4*(864*(1/864)^(5/6)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5
+ sqrt(-3)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5))*(-3*x^2 - 2)^(1/3)*(-1/d^6)^
(5/6) + 12*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^2 + 12*d^3)*(-3*x^2 - 2)^(2/3)*sq
rt(-1/d^6) - 4*(1/4)^(2/3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x - sqrt(-3)*(
7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x))*(-1/d^6)^(2/3) + 2*(1/4)^(1/3)*(d^2*x^
5 - 52*d^2*x^3 + 36*d^2*x + sqrt(-3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x))*(-
3*x^2 - 2)^(1/3)*(-1/d^6)^(1/3) - 8*(5*x^3 - 18*x)*(-3*x^2 - 2)^(2/3) - (1
/864)^(1/6)*(d*x^6 - 210*d*x^4 + 252*d*x^2 - sqrt(-3)*(d*x^6 - 210*d*x^4 +
 252*d*x^2 + 72*d) + 72*d)*(-1/d^6)^(1/6))/(x^6 + 18*x^4 + 108*x^2 + 216))
 - 1/48*(1/864)^(1/6)*(sqrt(-3) + 1)*(-1/d^6)^(1/6)*log(-1/4*(864*(1/86...
```

3.159.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = \frac{\int \frac{1}{x^2\sqrt[3]{-3x^2-2}+6\sqrt[3]{-3x^2-2}}\,dx}{d}$$

input
```
integrate(1/(-3*x**2-2)**(1/3)/(d*x**2+6*d),x)
```

output
```
Integral(1/(x**2*(-3*x**2 - 2)**(1/3) + 6*(-3*x**2 - 2)**(1/3)), x)/d
```

3.159.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = \int \frac{1}{(dx^2+6\,d)(-3\,x^2-2)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)), x)
```

3.159.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = \int \frac{1}{(dx^2+6\,d)(-3\,x^2-2)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="giac")
```

output
```
integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = \int \frac{1}{(-3\,x^2-2)^{1/3}\,(d\,x^2+6\,d)}\,dx$$

input
```
int(1/((- 3*x^2 - 2)^(1/3)*(6*d + d*x^2)),x)
```

output
```
int(1/((- 3*x^2 - 2)^(1/3)*(6*d + d*x^2)), x)
```

3.160 $\int \frac{1}{\sqrt[3]{1+x^2}(9+x^2)} \, dx$

3.160.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)} \, dx = \frac{1}{12}\arctan\left(\frac{x}{3}\right)+\frac{1}{12}\arctan\left(\frac{\left(1-\sqrt[3]{1+x^2}\right)^2}{3x}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+x^2}\right)}{x}\right)}{4\sqrt{3}}$$

output
```
1/12*arctan(1/3*x)+1/12*arctan(1/3*(1-(x^2+1)^(1/3))^2/x)-1/12*arctanh((1-
(x^2+1)^(1/3))*3^(1/2)/x)*3^(1/2)
```

3.160.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)} \, dx = -\frac{27x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,-\frac{x^2}{9}\right)}{\sqrt[3]{1+x^2}\,(9+x^2)\left(-27\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,-\frac{x^2}{9}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,-\frac{x^2}{9}\right)+3\,\text{Appe}\right.\right.}$$

input
```
Integrate[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

output
```
(-27*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -1/9*x^2])/((1 + x^2)^(1/3)*(9 + x
^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -1/9*x^2] + 2*x^2*(AppellF1[3/2,
 1/3, 2, 5/2, -x^2, -1/9*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2, -x^2, -1/9*x^
2])))
```

3.160.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {306}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{x^2+1}\,(x^2+9)}dx$$

$$\Big\downarrow \, 306$$

$$\frac{1}{12}\arctan\left(\frac{\left(1-\sqrt[3]{x^2+1}\right)^2}{3x}\right)+\frac{1}{12}\arctan\left(\frac{x}{3}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{x^2+1}\right)}{x}\right)}{4\sqrt{3}}$$

input
```
Int[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

output
```
ArcTan[x/3]/12 + ArcTan[(1 - (1 + x^2)^(1/3))^2/(3*x)]/12 - ArcTanh[(Sqrt[
3]*(1 - (1 + x^2)^(1/3)))/x]/(4*Sqrt[3])
```

3.160.3.1 Defintions of rubi rules used

rule 306
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

3.160.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.47 (sec) , antiderivative size = 623, normalized size of antiderivative = 8.90

method	result
trager	$144\,\mathrm{RootOf}\left(20736\_Z^4 - 144\_Z^2 + 1\right)^3 \ln\left(-\frac{497664\,\mathrm{RootOf}\left(20736\_\_Z^4-144\_\_Z^2+1\right)^5 (x^2+1)^{\frac{1}{3}} x-995328\,\mathrm{Ro}}{}\right.$

input
```
int(1/(x^2+1)^(1/3)/(x^2+9),x,method=_RETURNVERBOSE)
```

output
```
144*RootOf(20736*_Z^4-144*_Z^2+1)^3*ln(-(497664*RootOf(20736*_Z^4-144*_Z^2
+1)^5*(x^2+1)^(1/3)*x-995328*RootOf(20736*_Z^4-144*_Z^2+1)^5*x-6912*RootOf
(20736*_Z^4-144*_Z^2+1)^3*(x^2+1)^(1/3)*x+20736*RootOf(20736*_Z^4-144*_Z^2
+1)^3*x+144*RootOf(20736*_Z^4-144*_Z^2+1)^2*x^2-864*RootOf(20736*_Z^4-144*
_Z^2+1)^2*(x^2+1)^(1/3)-432*RootOf(20736*_Z^4-144*_Z^2+1)^2-96*RootOf(2073
6*_Z^4-144*_Z^2+1)*x+6*(x^2+1)^(2/3)-x^2+3)/(x^2+9))-RootOf(20736*_Z^4-144
*_Z^2+1)*ln(-(497664*RootOf(20736*_Z^4-144*_Z^2+1)^5*(x^2+1)^(1/3)*x-99532
8*RootOf(20736*_Z^4-144*_Z^2+1)^5*x-6912*RootOf(20736*_Z^4-144*_Z^2+1)^3*(
x^2+1)^(1/3)*x+20736*RootOf(20736*_Z^4-144*_Z^2+1)^3*x+144*RootOf(20736*_Z
^4-144*_Z^2+1)^2*x^2-864*RootOf(20736*_Z^4-144*_Z^2+1)^2*(x^2+1)^(1/3)-432
*RootOf(20736*_Z^4-144*_Z^2+1)^2-96*RootOf(20736*_Z^4-144*_Z^2+1)*x+6*(x^2
+1)^(2/3)-x^2+3)/(x^2+9))+RootOf(20736*_Z^4-144*_Z^2+1)*ln(-(82944*RootOf(
20736*_Z^4-144*_Z^2+1)^5*(x^2+1)^(1/3)*x-165888*RootOf(20736*_Z^4-144*_Z^2
+1)^5*x-1728*RootOf(20736*_Z^4-144*_Z^2+1)^3*(x^2+1)^(1/3)*x+2304*RootOf(2
0736*_Z^4-144*_Z^2+1)^3*x-24*RootOf(20736*_Z^4-144*_Z^2+1)^2*x^2+144*RootO
f(20736*_Z^4-144*_Z^2+1)^2*(x^2+1)^(1/3)+8*RootOf(20736*_Z^4-144*_Z^2+1)*(
x^2+1)^(1/3)*x+72*RootOf(20736*_Z^4-144*_Z^2+1)^2+(x^2+1)^(2/3)-(x^2+1)^(1
/3))/(x^2+9))
```

3.160.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 1059, normalized size of antiderivative = 15.13

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="fricas")
```

output
```
-1/144*sqrt(2)*sqrt(-I*sqrt(3) + 1)*log(-(42*x^5 - 828*x^3 + sqrt(2)*(x^6
- 315*x^4 + 567*x^2 + 243)*sqrt(-I*sqrt(3) + 1) + 9*(40*x^3 + (sqrt(3)*sqr
t(2)*(-I*x^4 + 36*I*x^2 - 27*I) + sqrt(2)*(x^4 - 36*x^2 + 27))*sqrt(-I*sqr
t(3) + 1) - 216*x)*(x^2 + 1)^(2/3) + 6*sqrt(3)*(7*I*x^5 - 138*I*x^3 - 81*I
*x) - 3*(2*x^5 - 156*x^3 - 2*sqrt(3)*(I*x^5 - 78*I*x^3 + 81*I*x) - 3*(sqrt
(3)*sqrt(2)*(-5*I*x^4 + 54*I*x^2 + 27*I) - sqrt(2)*(5*x^4 - 54*x^2 - 27))*
sqrt(-I*sqrt(3) + 1) + 162*x)*(x^2 + 1)^(1/3) - 486*x)/(x^6 + 27*x^4 + 243
*x^2 + 729)) + 1/144*sqrt(2)*sqrt(-I*sqrt(3) + 1)*log(-(42*x^5 - 828*x^3 -
 sqrt(2)*(x^6 - 315*x^4 + 567*x^2 + 243)*sqrt(-I*sqrt(3) + 1) + 9*(40*x^3
+ (sqrt(3)*sqrt(2)*(I*x^4 - 36*I*x^2 + 27*I) - sqrt(2)*(x^4 - 36*x^2 + 27)
)*sqrt(-I*sqrt(3) + 1) - 216*x)*(x^2 + 1)^(2/3) + 6*sqrt(3)*(7*I*x^5 - 138
*I*x^3 - 81*I*x) - 3*(2*x^5 - 156*x^3 - 2*sqrt(3)*(I*x^5 - 78*I*x^3 + 81*I
*x) - 3*(sqrt(3)*sqrt(2)*(5*I*x^4 - 54*I*x^2 - 27*I) + sqrt(2)*(5*x^4 - 54
*x^2 - 27))*sqrt(-I*sqrt(3) + 1) + 162*x)*(x^2 + 1)^(1/3) - 486*x)/(x^6 +
27*x^4 + 243*x^2 + 729)) - 1/144*sqrt(2)*sqrt(I*sqrt(3) + 1)*log(-(42*x^5
- 828*x^3 + 72*(5*x^3 - 27*x)*(x^2 + 1)^(2/3) + 6*sqrt(3)*(-7*I*x^5 + 138*
I*x^3 + 81*I*x) + (9*(sqrt(3)*sqrt(2)*(I*x^4 - 36*I*x^2 + 27*I) + sqrt(2)*
(x^4 - 36*x^2 + 27))*(x^2 + 1)^(2/3) + sqrt(2)*(x^6 - 315*x^4 + 567*x^2 +
243) + 9*(sqrt(3)*sqrt(2)*(5*I*x^4 - 54*I*x^2 - 27*I) - sqrt(2)*(5*x^4 - 5
4*x^2 - 27))*(x^2 + 1)^(1/3))*sqrt(I*sqrt(3) + 1) - 6*(x^5 - 78*x^3 - s...
```

3.160.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = \int \frac{1}{\sqrt[3]{x^2+1}\,(x^2+9)}\,dx$$

input
```
integrate(1/(x**2+1)**(1/3)/(x**2+9),x)
```

output
```
Integral(1/((x**2 + 1)**(1/3)*(x**2 + 9)), x)
```

3.160.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = \int \frac{1}{(x^2+9)(x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

3.160.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = \int \frac{1}{(x^2+9)(x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = \int \frac{1}{(x^2+1)^{1/3}\,(x^2+9)}\,dx$$

input
```
int(1/((x^2 + 1)^(1/3)*(x^2 + 9)),x)
```

output
```
int(1/((x^2 + 1)^(1/3)*(x^2 + 9)), x)
```

3.161 $\int \frac{1}{\sqrt[3]{1+bx^2}(9+bx^2)}\,dx$

3.161.1 Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{3}\right)}{12\sqrt{b}}+\frac{\arctan\left(\frac{\left(1-\sqrt[3]{1+bx^2}\right)^2}{3\sqrt{b}x}\right)}{12\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}\sqrt{b}}$$

output
```
1/12*arctan(1/3*(1-(b*x^2+1)^(1/3))^2/x/b^(1/2))/b^(1/2)+1/12*arctan(1/3*x
*b^(1/2))/b^(1/2)-1/12*arctanh((1-(b*x^2+1)^(1/3))*3^(1/2)/x/b^(1/2))*3^(1
/2)/b^(1/2)
```

3.161.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.79 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx=$$

$$-\frac{27x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-bx^2,-\frac{bx^2}{9}\right)}{\sqrt[3]{1+bx^2}\,(9+bx^2)\left(-27\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-bx^2,-\frac{bx^2}{9}\right)+2bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-bx^2,-\frac{bx^2}{9}\right)+\cdots\right.\right.}$$

input
```
Integrate[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
```

output
```
(-27*x*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2), -1/9*(b*x^2)])/((1 + b*x^2)^(1
/3)*(9 + b*x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2), -1/9*(b*x^2)] +
2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -(b*x^2), -1/9*(b*x^2)] + 3*AppellF1[3
/2, 4/3, 1, 5/2, -(b*x^2), -1/9*(b*x^2)])))
```

3.161.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {306}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{bx^2+1}\,(bx^2+9)}dx$$

$$\Big\downarrow \text{306}$$

$$\frac{\arctan\left(\frac{\left(1-\sqrt[3]{bx^2+1}\right)^2}{3\sqrt{b}x}\right)}{12\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{b}x}{3}\right)}{12\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{bx^2+1}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}\sqrt{b}}$$

input
```
Int[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
```

output
```
ArcTan[(Sqrt[b]*x)/3]/(12*Sqrt[b]) + ArcTan[(1 - (1 + b*x^2)^(1/3))^2/(3*S
qrt[b]*x)]/(12*Sqrt[b]) - ArcTanh[(Sqrt[3]*(1 - (1 + b*x^2)^(1/3)))/(Sqrt[
b]*x)]/(4*Sqrt[3]*Sqrt[b])
```

3.161.3.1 Defintions of rubi rules used

rule 306
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

3.161.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+1\right)^{\frac{1}{3}}\left(b\,x^{2}+9\right)}dx$$

input
```
int(1/(b*x^2+1)^(1/3)/(b*x^2+9),x)
```

output
```
int(1/(b*x^2+1)^(1/3)/(b*x^2+9),x)
```

3.161.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="fricas")
```

output
```
Timed out
```

3.161.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx = \int \frac{1}{\sqrt[3]{bx^2+1}\,(bx^2+9)}\,dx$$

input
```
integrate(1/(b*x**2+1)**(1/3)/(b*x**2+9),x)
```

output
```
Integral(1/((b*x**2 + 1)**(1/3)*(b*x**2 + 9)), x)
```

3.161.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx = \int \frac{1}{(bx^2+9)(bx^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)), x)
```

3.161.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx = \int \frac{1}{(bx^2+9)(bx^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)), x)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx = \int \frac{1}{(b\,x^2+1)^{1/3}\,(b\,x^2+9)}\,dx$$

input
```
int(1/((b*x^2 + 1)^(1/3)*(b*x^2 + 9)),x)
```

output
```
int(1/((b*x^2 + 1)^(1/3)*(b*x^2 + 9)), x)
```

3.162 $\int \frac{1}{\sqrt[3]{1-x^2}(9-x^2)}\,dx$

3.162.1 Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}+\frac{1}{12}\text{arctanh}\left(\frac{x}{3}\right)-\frac{1}{12}\text{arctanh}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)$$

output
```
1/12*arctanh(1/3*x)-1/12*arctanh(1/3*(1-(-x^2+1)^(1/3))^2/x)+1/12*arctan((
1-(-x^2+1)^(1/3))*3^(1/2)/x)*3^(1/2)
```

3.162.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 8.55 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx$$
$$=\frac{\sqrt[3]{\frac{-1+x}{-3+x}}\sqrt[3]{\frac{1+x}{-3+x}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{4}{-3+x},-\frac{2}{-3+x}\right)-\sqrt[3]{\frac{-1+x}{3+x}}\sqrt[3]{\frac{1+x}{3+x}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{2}{3+x},\right.}{4\sqrt[3]{1-x^2}}$$

input
```
Integrate[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

output
```
(((-1 + x)/(-3 + x))^(1/3)*((1 + x)/(-3 + x))^(1/3)*AppellF1[2/3, 1/3, 1/3
, 5/3, -4/(-3 + x), -2/(-3 + x)] - ((-1 + x)/(3 + x))^(1/3)*((1 + x)/(3 +
x))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, 2/(3 + x), 4/(3 + x)])/(4*(1 - x^2)
^(1/3))
```

3.162.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx$$

$$\Big\downarrow \; 307$$

$$\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}-\frac{1}{12}\text{arctanh}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)+\frac{1}{12}\text{arctanh}\left(\frac{x}{3}\right)$$

input
```
Int[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

output
```
ArcTan[(Sqrt[3]*(1 - (1 - x^2)^(1/3)))/x]/(4*Sqrt[3]) + ArcTanh[x/3]/12 -
ArcTanh[(1 - (1 - x^2)^(1/3))^2/(3*x)]/12
```

3.162.3.1 Defintions of rubi rules used

rule 307
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

3.162.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.63 (sec) , antiderivative size = 539, normalized size of antiderivative = 7.28

method	result
trager	$-\frac{\ln\left(\frac{288\left(-x^{2}+1\right)^{\frac{1}{3}} \mathrm{RootOf}\left(144\_Z^{2}+12\_Z+1\right)^{2} x+36 \mathrm{RootOf}\left(144\_Z^{2}+12\_Z+1\right)\left(-x^{2}+1\right)^{\frac{1}{3}} x-576 \mathrm{RootOf}\left(144\_Z^{2}+12\_Z+1\right)^{2}}{}\right.}{}$

input
```
int(1/(-x^2+1)^(1/3)/(-x^2+9),x,method=_RETURNVERBOSE)
```

output
```
-1/12*ln((288*(-x^2+1)^(1/3)*RootOf(144*_Z^2+12*_Z+1)^2*x+36*RootOf(144*_Z
^2+12*_Z+1)*(-x^2+1)^(1/3)*x-576*RootOf(144*_Z^2+12*_Z+1)^2*x-6*RootOf(144
*_Z^2+12*_Z+1)*x^2+3*(-x^2+1)^(2/3)-36*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(
1/3)+x*(-x^2+1)^(1/3)-24*RootOf(144*_Z^2+12*_Z+1)*x-3*(-x^2+1)^(1/3)-18*Ro
otOf(144*_Z^2+12*_Z+1))/(-3+x)/(3+x))-ln((288*(-x^2+1)^(1/3)*RootOf(144*_Z
^2+12*_Z+1)^2*x+36*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)*x-576*RootOf(14
4*_Z^2+12*_Z+1)^2*x-6*RootOf(144*_Z^2+12*_Z+1)*x^2+3*(-x^2+1)^(2/3)-36*Roo
tOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)+x*(-x^2+1)^(1/3)-24*RootOf(144*_Z^2+1
2*_Z+1)*x-3*(-x^2+1)^(1/3)-18*RootOf(144*_Z^2+12*_Z+1))/(-3+x)/(3+x))*Root
Of(144*_Z^2+12*_Z+1)+RootOf(144*_Z^2+12*_Z+1)*ln((576*(-x^2+1)^(1/3)*RootO
f(144*_Z^2+12*_Z+1)^2*x+24*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)*x-1152*
RootOf(144*_Z^2+12*_Z+1)^2*x+12*RootOf(144*_Z^2+12*_Z+1)*x^2+6*(-x^2+1)^(2
/3)+72*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)-144*RootOf(144*_Z^2+12*_Z+1
)*x+x^2+36*RootOf(144*_Z^2+12*_Z+1)-4*x+3)/(-3+x)/(3+x))
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(53) = 106$.

Time = 0.54 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.64

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx =$$
$$-\frac{1}{36}\sqrt{3}\arctan\left(\frac{36\sqrt{3}(x^4-32x^3-42x^2+9)(-x^2+1)^{\frac{2}{3}}+12\sqrt{3}(x^5+27x^4-210x^3-54x^2+81x}{3(x^6+108x^5-1647x^4-1080x}\right.$$
$$-\frac{1}{72}\log\left(\frac{x^3+33x^2+18(-x^2+1)^{\frac{2}{3}}(x+1)-6(x^2+6x-3)(-x^2+1)^{\frac{1}{3}}-9x-9}{x^3+9x^2+27x+27}\right)$$
$$+\frac{1}{36}\log\left(-\frac{x^3-33x^2+18(-x^2+1)^{\frac{2}{3}}(x-1)+6(x^2-6x-3)(-x^2+1)^{\frac{1}{3}}-9x+9}{x^3+9x^2+27x+27}\right)$$

input
```
integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="fricas")
```

output
```
-1/36*sqrt(3)*arctan(1/3*(36*sqrt(3)*(x^4 - 32*x^3 - 42*x^2 + 9)*(-x^2 + 1
)^(2/3) + 12*sqrt(3)*(x^5 + 27*x^4 - 210*x^3 - 54*x^2 + 81*x + 27)*(-x^2 +
 1)^(1/3) + sqrt(3)*(x^6 - 108*x^5 - 567*x^4 + 1080*x^3 + 459*x^2 - 972*x
- 405))/(x^6 + 108*x^5 - 1647*x^4 - 1080*x^3 + 891*x^2 + 972*x + 243)) - 1
/72*log((x^3 + 33*x^2 + 18*(-x^2 + 1)^(2/3)*(x + 1) - 6*(x^2 + 6*x - 3)*(-
x^2 + 1)^(1/3) - 9*x - 9)/(x^3 + 9*x^2 + 27*x + 27)) + 1/36*log(-(x^3 - 33
*x^2 + 18*(-x^2 + 1)^(2/3)*(x - 1) + 6*(x^2 - 6*x - 3)*(-x^2 + 1)^(1/3) -
9*x + 9)/(x^3 + 9*x^2 + 27*x + 27))
```

3.162.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx = -\int \frac{1}{x^2\sqrt[3]{1-x^2}-9\sqrt[3]{1-x^2}}\,dx$$

input
```
integrate(1/(-x**2+1)**(1/3)/(-x**2+9),x)
```

output
```
-Integral(1/(x**2*(1 - x**2)**(1/3) - 9*(1 - x**2)**(1/3)), x)
```

3.162.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx = \int -\frac{1}{(x^2-9)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="maxima")
```

output
```
-integrate(1/((x^2 - 9)*(-x^2 + 1)^(1/3)), x)
```

3.162.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx = \int -\frac{1}{(x^2-9)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="giac")
```

output
```
integrate(-1/((x^2 - 9)*(-x^2 + 1)^(1/3)), x)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx = -\int \frac{1}{(1-x^2)^{1/3}\,(x^2-9)}\,dx$$

input
```
int(-1/((1 - x^2)^(1/3)*(x^2 - 9)),x)
```

output
```
-int(1/((1 - x^2)^(1/3)*(x^2 - 9)), x)
```

3.163 $\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx$

3.163.1 Optimal result

Integrand size = 29, antiderivative size = 79

$$\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx=\frac{x\sqrt{-1+c^2x^2}}{2d\,(d-c^2dx^2)^{3/2}}+\frac{\sqrt{-1+c^2x^2}\text{arctanh}(cx)}{2cd^2\sqrt{d-c^2dx^2}}$$

output
```
1/2*x*(c^2*x^2-1)^(1/2)/d/(-c^2*d*x^2+d)^(3/2)+1/2*arctanh(c*x)*(c^2*x^2-1
)^(1/2)/c/d^2/(-c^2*d*x^2+d)^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx=\frac{-cx+(-1+c^2x^2)\,\text{arctanh}(cx)}{2cd^2\sqrt{-1+c^2x^2}\sqrt{d-c^2dx^2}}$$

input
```
Integrate[Sqrt[-1 + c^2*x^2]/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(-(c*x) + (-1 + c^2*x^2)*ArcTanh[c*x])/(2*c*d^2*Sqrt[-1 + c^2*x^2]*Sqrt[d
- c^2*d*x^2])
```

3.163.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {283, 215, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2-1}}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{283}$$

$$\frac{(c^2x^2-1)^{5/2}\int \frac{1}{(c^2x^2-1)^2}dx}{(d-c^2dx^2)^{5/2}}$$

$$\Big\downarrow \text{215}$$

$$\frac{(c^2x^2-1)^{5/2}\left(\frac{x}{2(1-c^2x^2)}-\frac{1}{2}\int \frac{1}{c^2x^2-1}dx\right)}{(d-c^2dx^2)^{5/2}}$$

$$\Big\downarrow \text{220}$$

$$\frac{(c^2x^2-1)^{5/2}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{(d-c^2dx^2)^{5/2}}$$

input
```
Int[Sqrt[-1 + c^2*x^2]/(d - c^2*d*x^2)^(5/2),x]
```

output
```
((-1 + c^2*x^2)^(5/2)*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(d - c^2
*d*x^2)^(5/2)
```

3.163.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

3.163.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\sqrt{-\left(c^{2} x^{2}-1\right) d}\, \left(\ln \left(c x -1\right) c^{2} x^{2}-\ln \left(c x +1\right) c^{2} x^{2}+2 c x -\ln \left(c x -1\right)+\ln \left(c x +1\right)\right)}{4 \sqrt{c^{2} x^{2}-1}\, d^{3} \left(c x -1\right) c \left(c x +1\right)}$	94
risch	$-\frac{x}{2 d^{2} \sqrt{c^{2} x^{2}-1}\, \sqrt{-\left(c^{2} x^{2}-1\right) d}}+\frac{\sqrt{c^{2} x^{2}-1}\, \ln \left(-c x -1\right)}{4 d^{2} \sqrt{-\left(c^{2} x^{2}-1\right) d}\, c}-\frac{\sqrt{c^{2} x^{2}-1}\, \ln \left(c x -1\right)}{4 d^{2} \sqrt{-\left(c^{2} x^{2}-1\right) d}\, c}$	112

input
```
int((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x-1)*c^2*x^2-ln(c*x+1)*
c^2*x^2+2*c*x-ln(c*x-1)+ln(c*x+1))/d^3/(c*x-1)/c/(c*x+1)
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.99

$$\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx = \left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}cx-(c^4x^4-2\,c^2x^2+1)\sqrt{-d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2-4}{c^6x^6}\right.}{8\,(c^5d^3x^4-2\,c^3d^3x^2+cd^3)}\right.$$

input
```
integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2
+ 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c
*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*
x^4 + 3*c^2*x^2 - 1)))/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3), 1/4*(2*sqrt(
-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*
arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d
)))/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)]
```

3.163.6 Sympy [F]

$$\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{\sqrt{(cx-1)\,(cx+1)}}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c**2*x**2-1)**(1/2)/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(sqrt((c*x - 1)*(c*x + 1))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{x}{2\left(c^2\sqrt{-d}d^2x^2-\sqrt{-d}d^2\right)} - \frac{\sqrt{-d}\log\left(cx+1\right)}{4\,cd^3} + \frac{\sqrt{-d}\log\left(cx-1\right)}{4\,cd^3}$$

input
```
integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/2*x/(c^2*sqrt(-d)*d^2*x^2 - sqrt(-d)*d^2) - 1/4*sqrt(-d)*log(c*x + 1)/(
c*d^3) + 1/4*sqrt(-d)*log(c*x - 1)/(c*d^3)
```

3.163.8 Giac [F]

$$\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{\sqrt{c^2x^2-1}}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 - 1)/(-c^2*d*x^2 + d)^(5/2), x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{\sqrt{c^2\,x^2-1}}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((c^2*x^2 - 1)^(1/2)/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((c^2*x^2 - 1)^(1/2)/(d - c^2*d*x^2)^(5/2), x)
```

3.164 $\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx$

3.164.1 Optimal result

Integrand size = 29, antiderivative size = 74

$$\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx = \frac{dx\sqrt{-1+c^2x^2}}{2(d-c^2dx^2)^{3/2}} + \frac{\sqrt{-1+c^2x^2}\text{arctanh}(cx)}{2c\sqrt{d-c^2dx^2}}$$

output
```
1/2*d*x*(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2)+1/2*arctanh(c*x)*(c^2*x^2-1
)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)
```

3.164.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx = \frac{-cx+(-1+c^2x^2)\,\text{arctanh}(cx)}{2c\sqrt{-1+c^2x^2}\sqrt{d-c^2dx^2}}$$

input
```
Integrate[1/((-1 + c^2*x^2)^(3/2)*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(-(c*x) + (-1 + c^2*x^2)*ArcTanh[c*x])/(2*c*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^
2*d*x^2])
```

3.164.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {283, 215, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2x^2-1)^{3/2}\sqrt{d-c^2dx^2}}dx$$
$$\Big\downarrow \ 283$$
$$\frac{\sqrt{c^2x^2-1}\int \frac{1}{(c^2x^2-1)^2}dx}{\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \ 215$$
$$\frac{\sqrt{c^2x^2-1}\left(\frac{x}{2(1-c^2x^2)}-\frac{1}{2}\int \frac{1}{c^2x^2-1}dx\right)}{\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \ 220$$
$$\frac{\sqrt{c^2x^2-1}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{\sqrt{d-c^2dx^2}}$$

input
```
Int[1/((-1 + c^2*x^2)^(3/2)*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(Sqrt[-1 + c^2*x^2]*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/Sqrt[d - c
^2*d*x^2]
```

3.164.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

3.164.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{\sqrt{-\left(c^{2} x^{2}-1\right) d}\,\left(\ln \left(c x -1\right) c^{2} x^{2}-\ln \left(c x +1\right) c^{2} x^{2}+2 c x -\ln \left(c x -1\right)+\ln \left(c x +1\right)\right)}{4 \sqrt{c^{2} x^{2}-1}\, d \left(c x -1\right) c \left(c x +1\right)}$	94
risch	$-\frac{x}{2 \sqrt{c^{2} x^{2}-1}\, \sqrt{-\left(c^{2} x^{2}-1\right) d}}+\frac{\sqrt{c^{2} x^{2}-1}\, \ln \left(-c x -1\right)}{4 \sqrt{-\left(c^{2} x^{2}-1\right) d}\, c}-\frac{\sqrt{c^{2} x^{2}-1}\, \ln \left(c x -1\right)}{4 \sqrt{-\left(c^{2} x^{2}-1\right) d}\, c}$	103

input
```
int(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x-1)*c^2*x^2-ln(c*x+1)*
c^2*x^2+2*c*x-ln(c*x-1)+ln(c*x+1))/d/(c*x-1)/c/(c*x+1)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 4.09

$$\int \frac{1}{\left(-1+c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}\,dx = \left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}cx-(c^4x^4-2\,c^2x^2+1)\sqrt{-d}\log\left(-\frac{c^6dx^6+5\,c}{}\right.}{8\,(c^5dx^4-2\,c^3dx^2+cd)}\right.$$

input
```
integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2
+ 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c
*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*
x^4 + 3*c^2*x^2 - 1)))/(c^5*d*x^4 - 2*c^3*d*x^2 + c*d), 1/4*(2*sqrt(-c^2*d
*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arctan
(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/(c
^5*d*x^4 - 2*c^3*d*x^2 + c*d)]
```

3.164.6 Sympy [F]

$$\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx=\int \frac{1}{((cx-1)(cx+1))^{\frac{3}{2}}\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate(1/(c**2*x**2-1)**(3/2)/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(1/(((c*x - 1)*(c*x + 1))**(3/2)*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

3.164.7 Maxima [F]

$$\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx=\int \frac{1}{\sqrt{-c^2dx^2+d}(c^2x^2-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c^2*d*x^2 + d)*(c^2*x^2 - 1)^(3/2)), x)
```

3.164.8 Giac [F]

$$\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx=\int \frac{1}{\sqrt{-c^2dx^2+d}(c^2x^2-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c^2*d*x^2 + d)*(c^2*x^2 - 1)^(3/2)), x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx = \int \frac{1}{\sqrt{d-c^2\,d\,x^2}\,(c^2\,x^2-1)^{3/2}}\,dx$$

input
```
int(1/((d - c^2*d*x^2)^(1/2)*(c^2*x^2 - 1)^(3/2)),x)
```

output
```
int(1/((d - c^2*d*x^2)^(1/2)*(c^2*x^2 - 1)^(3/2)), x)
```

3.165 $\int \frac{1}{\sqrt{-1+c^2x^2}(d-c^2dx^2)^{3/2}}\,dx$

3.165.1 Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = -\frac{x\sqrt{-1+c^2x^2}}{2\,(d-c^2dx^2)^{3/2}} - \frac{\sqrt{-1+c^2x^2}\,\text{arctanh}(cx)}{2cd\sqrt{d-c^2dx^2}}$$

output
```
-1/2*x*(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2)-1/2*arctanh(c*x)*(c^2*x^2-1)
^(1/2)/c/d/(-c^2*d*x^2+d)^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = \frac{cx+(1-c^2x^2)\,\text{arctanh}(cx)}{2cd\sqrt{-1+c^2x^2}\sqrt{d-c^2dx^2}}$$

input
```
Integrate[1/(Sqrt[-1 + c^2*x^2]*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
(c*x + (1 - c^2*x^2)*ArcTanh[c*x])/(2*c*d*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*
d*x^2])
```

3.165.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {283, 215, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c^2x^2-1}\,(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \ 283$$

$$\frac{(c^2x^2-1)^{3/2}\int \frac{1}{(c^2x^2-1)^2}dx}{(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \ 215$$

$$\frac{(c^2x^2-1)^{3/2}\left(\frac{x}{2(1-c^2x^2)}-\frac{1}{2}\int \frac{1}{c^2x^2-1}dx\right)}{(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \ 220$$

$$\frac{(c^2x^2-1)^{3/2}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{(d-c^2dx^2)^{3/2}}$$

input
```
Int[1/(Sqrt[-1 + c^2*x^2]*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
((-1 + c^2*x^2)^(3/2)*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(d - c^2
*d*x^2)^(3/2)
```

3.165.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

3.165.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\sqrt{-\left(c^{2} x^{2}-1\right) d}\, \left(\ln \left(c x -1\right) c^{2} x^{2}-\ln \left(c x +1\right) c^{2} x^{2}+2 c x -\ln \left(c x -1\right)+\ln \left(c x +1\right)\right)}{4 \sqrt{c^{2} x^{2}-1}\, d^{2} \left(c x -1\right) c \left(c x +1\right)}$	94
risch	$\frac{x}{2 d \sqrt{c^{2} x^{2}-1}\, \sqrt{-\left(c^{2} x^{2}-1\right) d}}-\frac{\sqrt{c^{2} x^{2}-1}\, \ln \left(-c x -1\right)}{4 d \sqrt{-\left(c^{2} x^{2}-1\right) d}\, c}+\frac{\sqrt{c^{2} x^{2}-1}\, \ln \left(c x -1\right)}{4 d \sqrt{-\left(c^{2} x^{2}-1\right) d}\, c}$	112

input
```
int(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x-1)*c^2*x^2-ln(c*x+1)
*c^2*x^2+2*c*x-ln(c*x-1)+ln(c*x+1))/d^2/(c*x-1)/c/(c*x+1)
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 314, normalized size of antiderivative = 4.13

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = \left[-\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}cx + (c^4x^4-2\,c^2x^2+1)\sqrt{-d}\log\left(-\frac{c^6dx^6+}{}\right.}{8\,(c^5d^2x^4-2\,c^3d^2x^2+c} \right.$$

$$\left. -\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}cx-(c^4x^4-2\,c^2x^2+1)\sqrt{d}\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}c\sqrt{d}x}{c^4dx^4-d}\right)}{4\,(c^5d^2x^4-2\,c^3d^2x^2+cd^2)}\right]$$

input
```
integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x + (c^4*x^4 - 2*c^2*x^2
 + 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 +
c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4
*x^4 + 3*c^2*x^2 - 1)))/(c^5*d^2*x^4 - 2*c^3*d^2*x^2 + c*d^2), -1/4*(2*sqr
t(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d
)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 -
 d)))/(c^5*d^2*x^4 - 2*c^3*d^2*x^2 + c*d^2)]
```

3.165.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{(cx-1)\,(cx+1)}\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c**2*x**2-1)**(1/2)/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(1/(sqrt((c*x - 1)*(c*x + 1))*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

3.165.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{3}{2}}\sqrt{c^2x^2-1}}\,dx$$

input
```
integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c^2*d*x^2 + d)^(3/2)*sqrt(c^2*x^2 - 1)), x)
```

3.165.8 Giac [F]

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{3}{2}}\sqrt{c^2x^2-1}}\,dx$$

input
```
integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-c^2*d*x^2 + d)^(3/2)*sqrt(c^2*x^2 - 1)), x)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+c^2x^2}\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{1}{{\left(d-c^2\,d\,x^2\right)}^{3/2}\,\sqrt{c^2\,x^2-1}}\,dx$$

input
```
int(1/((d - c^2*d*x^2)^(3/2)*(c^2*x^2 - 1)^(1/2)),x)
```

output
```
int(1/((d - c^2*d*x^2)^(3/2)*(c^2*x^2 - 1)^(1/2)), x)
```

3.166 $\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\,dx$

3.166.1 Optimal result

Integrand size = 23, antiderivative size = 328

$$\begin{aligned}\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\,dx &= \frac{\left(7ac-\frac{2bc^2}{d}+\frac{3a^2d}{b}\right)x\sqrt{a+bx^2}}{15\sqrt{c+dx^2}} \\ &- \frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d}+\frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d} \\ &+ \frac{\sqrt{c}(2b^2c^2-7abcd-3a^2d^2)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &- \frac{c^{3/2}(bc-9ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
1/5*b*x*(d*x^2+c)^(3/2)*(b*x^2+a)^(1/2)/d+1/15*(7*a*c-2*b*c^2/d+3*a^2*d/b)
*x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-1/15*c^(3/2)*(-9*a*d+b*c)*(1/(1+d*x^2/c
))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(d*x^2+c)^(1/2)+1/15*(-3*a^2*d^2-7*a*b*c*d+2*b^2*c^2)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a
/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/b/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(d*x^2+c)^(1/2)-2/15*(-3*a*d+b*c)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

3.166.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.74

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\,(c+dx^2)\,(6ad+b(c+3dx^2)) - ic(-2b^2c^2+7abcd+3a^2d^2)\sqrt{1+}}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(6*a*d + b*(c + 3*d*x^2)) - I*c*(-2
*b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(b^2*c^2 - 4*a*b*
c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])
```

3.166.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {318, 25, 403, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \text{318}$$

$$\frac{\int -\frac{\sqrt{dx^2+c}(2b(bc-3ad)x^2+a(bc-5ad))}{\sqrt{bx^2+a}}dx}{5d} + \frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d} - \frac{\int \frac{\sqrt{dx^2+c}(2b(bc-3ad)x^2+a(bc-5ad))}{\sqrt{bx^2+a}}dx}{5d}$$

$$\Big\downarrow \; 403$$

$$\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{\int \frac{b\left(\left(2b^2c^2-7abdc-3a^2d^2\right)x^2+ac(bc-9ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-3ad)}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\int \frac{\left(2b^2c^2-7abdc-3a^2d^2\right)x^2+ac(bc-9ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-3ad)}{5d}$$

$$\Big\downarrow \; 406$$

$$\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-$$
$$\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(bc-9ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-}{5d}$$

$$\Big\downarrow \; 320$$

$$\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-$$
$$\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{}}{5d}$$

$$\Big\downarrow \; 388$$

$$\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-$$
$$\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x}{5d}$$

$$\Big\downarrow \; 313$$

$$\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-$$
$$\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5d}$$

input

```
Int[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2],x]
```

3.166. $\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,dx$

output
```
(b*x*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(5*d) - ((2*(b*c - 3*a*d)*x*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])/3 + ((2*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*((x*Sq
rt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*c - 9*a*d)*Sqrt[a + b*
x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/3)/(5*d)
```

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.166.4 Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.26

method	result
risch	$\frac{x(3bd\,x^2+6ad+bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{15d}+\frac{\left(\frac{9a^2cd\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{b\,c^2a\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1\cdots}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{b\,x^3\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{5}+\frac{\left(2abd+b^2c-\frac{b(4ad+4bc)}{5}\right)x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3bd}+\frac{\left(a^2c-\frac{\left(2abd+b^2c-\frac{b(4ad+4bc)}{5}\right)\cdots}{3bd}\cdots\right.}{\sqrt{-\frac{b}{a}}\cdots}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(3\sqrt{-\frac{b}{a}}\,b^2d^3x^7+9\sqrt{-\frac{b}{a}}\,ab\,d^3x^5+4\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5+6\sqrt{-\frac{b}{a}}\,a^2d^3x^3+10\sqrt{-\frac{b}{a}}\,abc\,d^2x^3+\sqrt{-\frac{b}{a}}\,b^2c^2d\,x^3+6\sqrt{\frac{b\,x^2\cdots}{a}}\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*b*d*x^2+6*a*d+b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d+1/15/d*(9*a
^2*c*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-b*c^2
*a/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(3*a^2*d^
2+7*a*b*c*d-2*b^2*c^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))*((b*x
^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.71

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\,dx = \frac{(2\,b^2c^3-7\,abc^2d-3\,a^2cd^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)-(2\,b^2c^3-7\,abc^2d-\cdots}{\cdots}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/15*((2*b^2*c^3 - 7*a*b*c^2*d - 3*a^2*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*ellip
tic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (2*b^2*c^3 - 7*a*b*c^2*d - 9*a^2*
d^3 - (3*a^2 - a*b)*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-
c/d)/x), a*d/(b*c)) + (3*b^2*d^3*x^4 - 2*b^2*c^2*d + 7*a*b*c*d^2 + 3*a^2*d
^3 + (b^2*c*d^2 + 6*a*b*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^3*
x)
```

3.166.6 Sympy [F]

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\,dx = \int (a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2), x)
```

3.166.7 Maxima [F]

$$\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2}\,dx = \int \left(bx^2+a\right)^{\frac{3}{2}} \sqrt{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c), x)
```

3.166.8 Giac [F]

$$\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2}\,dx = \int \left(bx^2+a\right)^{\frac{3}{2}} \sqrt{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c), x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2}\,dx = \int \left(b\,x^2+a\right)^{3/2} \sqrt{d\,x^2+c}\,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2), x)
```

3.167 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$

3.167.1 Optimal result

Integrand size = 23, antiderivative size = 249

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \frac{(bc+ad)x\sqrt{a+bx^2}}{3b\sqrt{c+dx^2}}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2} - \frac{\sqrt{c}(bc+ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{2c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*(a*d+b*c)*x*(b*x^2+a)^(1/2)/b/(d*x^2+c)^(1/2)+2/3*c^(3/2)*(1/(1+d*x^2/
c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),
(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(d*x^2+c)^(1/2)-1/3*(a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ell
ipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x
^2+a)^(1/2)/b/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*
x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

3.167.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.80

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$$
$$=\frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)-ic(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic(-bc+ad)\sqrt{1+}}{3\sqrt{\frac{b}{a}}d\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - I*c*(b*c + a*d)*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*
c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])
```

3.167.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {319, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$$
$$\Big\downarrow \text{ 319}$$
$$\frac{2}{3}\int \frac{(bc+ad)x^2+2ac}{2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{1}{3}\int \frac{(bc+ad)x^2+2ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}$$
$$\Big\downarrow \text{ 406}$$

$$\frac{1}{3}\left(2ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(ad+bc)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

$$\Big\downarrow \text{320}$$

$$\frac{1}{3}\left((ad+bc)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

$$\Big\downarrow \text{388}$$

$$\frac{1}{3}\left((ad+bc)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{2c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{1}{3}\left(\frac{2c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(ad+bc)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.\right.$$
$$\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

input
```
Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/3 + ((b*c + a*d)*((x*Sqrt[a + b*x^2])/
(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2])) + (2*c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[c + d*x^2]))/3
```

3.167. $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$

3.167.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 319
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*(a + b*x^2)^p*((c + d*x^2)^q/(2*(p + q) + 1)), x] + Simp[2/(2*(p + q) +
 1)   Int[(a + b*x^2)^(p - 1)*(c + d*x^2)^(q - 1)*Simp[a*c*(p + q) + (q*(b*
c - a*d) + a*d*(p + q))*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 2, p, q,
x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.167.4 Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.12

method	result
risch	$\frac{x\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{3}+\frac{\left(\frac{2ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{(ad+bc)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\right.\right.}{3\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}\,d}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3}+\frac{2ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{\left(\frac{ad}{3}+\frac{bc}{3}\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3+ac\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{\right.\right.}{3(bd\,x^4+ad\,x^2+cb\,x^2+a}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)+(2/3*a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/3*(a*d+b*c)*c/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)
^(1/2)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.61

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx =$$

$$-\frac{(bc^2+acd)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(bc^2+acd+2\,ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-}{3\,bd^2x}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((b*c^2 + a*c*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/
x), a*d/(b*c)) - (b*c^2 + a*c*d + 2*a*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic
_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x
^2 + a)*sqrt(d*x^2 + c))/(b*d^2*x)
```

3.167.6 Sympy [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2), x)
```

3.167.7 Maxima [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \int \sqrt{bx^2+a}\sqrt{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c), x)
```

3.167.8 Giac [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \int \sqrt{bx^2+a}\sqrt{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c), x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

3.168 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$

3.168.1 Optimal result

Integrand size = 23, antiderivative size = 204

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
d*x*(b*x^2+a)^(1/2)/b/(d*x^2+c)^(1/2)+c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x
^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/
2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+
d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/b/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

input
```
Integrate[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*
d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

3.168.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{324}$$

$$c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{320}$$

$$d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{388}$$

$$d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{313}$$

$$\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)$$

input
```
Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

output
```
d*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

3.168.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.168.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

method	result
default	$\frac{\sqrt{d x^{2}+c}\,\sqrt{b x^{2}+a}\,c\sqrt{\frac{b x^{2}+a}{a}}\,\sqrt{\frac{d x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right)}{\left(b d x^{4}+a d x^{2}+c b x^{2}+a c\right)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{\left(b x^{2}+a\right)\left(d x^{2}+c\right)}\left(\frac{c\sqrt{1+\frac{b x^{2}}{a}}\,\sqrt{1+\frac{d x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{b d x^{4}+a d x^{2}+c b x^{2}+a c}}-\frac{c\sqrt{1+\frac{b x^{2}}{a}}\,\sqrt{1+\frac{d x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+a\right.\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b d x^{4}+a d x^{2}+c b x^{2}+a c}}}{\sqrt{b x^{2}+a}\,\sqrt{d x^{2}+c}}$

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-
b/a)^(1/2)
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx =$$
$$-\frac{\sqrt{bd}cx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{bd}(c+d)x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{bx^2+a}\sqrt{dx^2+c}d}{bdx}$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - s
qrt(b*d)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c))
- sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d)/(b*d*x)
```

3.168.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/sqrt(a + b*x**2), x)
```

3.168.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

3.168.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^(1/2),x)
```

output
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.169 $\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}} \, dx$

3.169.1 Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}} \, dx = \frac{\sqrt{c+dx^2}\, E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
(1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x
^2/a)^(1/2),(1-a*d/b/c)^(1/2))*(d*x^2+c)^(1/2)/a^(1/2)/b^(1/2)/(b*x^2+a)^(
1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

3.169.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}} \, dx = \frac{x(c+dx^2) + \frac{ic\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) - \text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}}}{a\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(3/2),x]
```

output
```
(x*(c + d*x^2) + (I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)]))/Sqrt[b/a])/(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.169.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 313$$

$$\frac{\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[Sqrt[c + d*x^2]/(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/
(Sqrt[a]*Sqrt[b]*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

3.169.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

3.169.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.15

method	result
default	$\frac{\sqrt{d\,x^{2}+c}\,\sqrt{b\,x^{2}+a}\,\left(\sqrt{-\frac{b}{a}}\,d\,x^{3}+\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)c-\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)c+\sqrt{-\frac{b}{a}}\,cx\right)}{(bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac)a\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}\,\left(\frac{\left(bd\,x^{2}+bc\right)x}{ba\sqrt{\left(x^{2}+\frac{a}{b}\right)\left(bd\,x^{2}+bc\right)}}+\frac{\left(\frac{d}{b}-\frac{ad-bc}{ba}-\frac{c}{a}\right)\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}+\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x \cdots\right.\right.}{a\sqrt{-\cdots}}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*((-b/a)^(1/2)*d*x^3+((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c-((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c+(-
b/a)^(1/2)*c*x)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/a/(-b/a)^(1/2)
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}ax-(bx^2+a)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})+(bx^2+a)\sqrt{ac}\sqrt{\cdots}}{a^2bx^2+a^3}$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*x - (b*x^2 + a)*sqrt(a*c)*sqrt(-b/a)*el
liptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (b*x^2 + a)*sqrt(a*c)*sqrt(-b/
a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a^2*b*x^2 + a^3)
```

3.169.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/(a + b*x**2)**(3/2), x)
```

3.169.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2), x)
```

3.169.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2), x)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^(3/2),x)
```

output
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^(3/2), x)
```

3.170 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx$

3.170.1 Optimal result

Integrand size = 23, antiderivative size = 237

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx = \frac{x\sqrt{c+dx^2}}{3a(a+bx^2)^{3/2}} + \frac{(2bc-ad)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c
^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a^2/(-
a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*x*(d*x^2+c)^(
1/2)/a/(b*x^2+a)^(3/2)+1/3*(-a*d+2*b*c)*(1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^
(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))*(d*
x^2+c)^(1/2)/a^(3/2)/(-a*d+b*c)/b^(1/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)
```

3.170.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(c+dx^2)\left(2a^2d-2b^2cx^2+ab(-3c+dx^2)\right)+ic(-2bc+ad)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{}}{3a^2\sqrt{\frac{b}{a}}(}$$

input
```
Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[b/a]*x*(c + d*x^2)*(2*a^2*d - 2*b^2*c*x^2 + a*b*(-3*c + d*x^2)) + I*
c*(-2*b*c + a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(-(b*c) + a*d)*(a + b*
x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*(-(b*c) + a*d)*(a + b*x^2)^(3/2)*Sqrt[
c + d*x^2])
```

3.170.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {314, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{314}$$

$$\frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{dx^2+2c}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{dx^2+2c}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{3a}+\frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{400}$$

$$\frac{\frac{(2bc-ad)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{cd\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a}+\frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\sqrt{c+dx^2}(2bc-ad)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{cd\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a}+\frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{\sqrt{c+dx^2}(2bc-ad)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a}+\frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}}$$

input
```
Int[Sqrt[c + d*x^2]/(a + b*x^2)^(5/2),x]
```

output
```
(x*Sqrt[c + d*x^2])/(3*a*(a + b*x^2)^(3/2)) + (((2*b*c - a*d)*Sqrt[c + d*x
^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[a]*Sqrt
[b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (
c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2]))/(3*a)
```

3.170.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

3.170.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.76

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{x\sqrt{bd x^4+ad x^2+cb x^2+ac}}{3b^2 a\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd x^2+bc\right)x(ad-2bc)}{3b a^2(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd x^2+bc\right)}}+\frac{\left(\frac{d}{3ab}-\frac{ad-2bc}{3b a^2}-\frac{c(ad-2bc)}{3a^2(ad-bc)}\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F(}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x^2+a}}\right.}{\sqrt{b x^2+a}\sqrt{d x^2+c}}$
default	$\frac{\sqrt{-\frac{b}{a}}\, ab d^2 x^5-2\sqrt{-\frac{b}{a}}\, b^2 cd x^5+2\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}\, F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abcd x^2-2\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}\, F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b^2c^2x^2-\sqrt{b}}{}$

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3/b^2/a*x*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+1/3*(b*d*x^2+b*c)/b/a^2/(a*
d-b*c)*x*(a*d-2*b*c)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(1/3/a/b*d-1/3*(a*d-2
*b*c)/b/a^2-1/3*c/a^2/(a*d-b*c)*(a*d-2*b*c))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/3*(a*d-2*b*c)/(a*d-b*c)/a^2*c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx =$$

$$-\frac{(2\,a^2b^2c - a^3bd + (2\,b^4c - ab^3d)x^4 + 2\,(2\,ab^3c - a^2b^2d)x^2)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - (2\,a^2b^2c +}{}$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*((2*a^2*b^2*c - a^3*b*d + (2*b^4*c - a*b^3*d)*x^4 + 2*(2*a*b^3*c - a^
2*b^2*d)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b
*c)) - (2*a^2*b^2*c + (2*b^4*c + (a^2*b^2 - a*b^3)*d)*x^4 + 2*(2*a*b^3*c +
 (a^3*b - a^2*b^2)*d)*x^2 + (a^4 - a^3*b)*d)*sqrt(a*c)*sqrt(-b/a)*elliptic
_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((2*a*b^3*c - a^2*b^2*d)*x^3 + (3*a^
2*b^2*c - 2*a^3*b*d)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^5*b^2*c - a^6*
b*d + (a^3*b^4*c - a^4*b^3*d)*x^4 + 2*(a^4*b^3*c - a^5*b^2*d)*x^2)
```

3.170.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(5/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/(a + b*x**2)**(5/2), x)
```

3.170.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2), x)
```

3.170.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2), x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^(5/2),x)
```

output
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^(5/2), x)
```

3.171 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx$

3.171.1 Optimal result

Integrand size = 23, antiderivative size = 309

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx = \frac{x\sqrt{c+dx^2}}{5a(a+bx^2)^{5/2}} + \frac{(4bc-3ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)(a+bx^2)^{3/2}} + \frac{(8b^2c^2-13abcd+3a^2d^2)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{b}(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{2c^{3/2}\sqrt{d}(2bc-3ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^3(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-2/15*c^(3/2)*(-3*a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellip
ticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2
+a)^(1/2)/a^3/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
+1/5*x*(d*x^2+c)^(1/2)/a/(b*x^2+a)^(5/2)+1/15*(-3*a*d+4*b*c)*x*(d*x^2+c)^(
1/2)/a^2/(-a*d+b*c)/(b*x^2+a)^(3/2)+1/15*(3*a^2*d^2-13*a*b*c*d+8*b^2*c^2)*
(1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x
^2/a)^(1/2),(1-a*d/b/c)^(1/2))*(d*x^2+c)^(1/2)/a^(5/2)/(-a*d+b*c)^2/b^(1/2
)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

3.171.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.87 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(c+dx^2)\left(3a^2(bc-ad)^2+a(-bc+ad)(-4bc+3ad)\left(a+bx^2\right)+\left(8b^2c^2-13abcd\right.\right.}{}$$

input
```
Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(7/2),x]
```

output
```
(Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + a*(-(b*c) + a*d)*(-4*b*c +
 3*a*d)*(a + b*x^2) + (8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)^2)
+ I*c*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*((8*b^2*c^2 -
13*a*b*c*d + 3*a^2*d^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (
-8*b^2*c^2 + 17*a*b*c*d - 9*a^2*d^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*
d)/(b*c)]))/(15*a^3*Sqrt[b/a]*(b*c - a*d)^2*(a + b*x^2)^(5/2)*Sqrt[c + d*x
^2])
```

3.171.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {314, 25, 402, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 314}$$

$$\frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}-\frac{\int -\frac{3dx^2+4c}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx}{5a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3dx^2+4c}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx}{5a}+\frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{x\sqrt{c+dx^2}(4bc-3ad)}{3a(a+bx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{d(4bc-3ad)x^2+c(8bc-9ad)}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}}{5a}+\frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{d(4bc-3ad)x^2+c(8bc-9ad)}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}+\frac{x\sqrt{c+dx^2}(4bc-3ad)}{3a(a+bx^2)^{3/2}(bc-ad)}}{5a}+\frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 400$$

$$\frac{\frac{\frac{\left(3a^2d^2-13abcd+8b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{2cd(2bc-3ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{x\sqrt{c+dx^2}(4bc-3ad)}{3a(a+bx^2)^{3/2}(bc-ad)}}{5a}+\frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\frac{\sqrt{c+dx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{2cd(2bc-3ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{x\sqrt{c+dx^2}(4bc-3ad)}{3a(a+bx^2)^{3/2}(bc-ad)}}{5a}+\frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{\frac{\sqrt{c+dx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{2c^{3/2}\sqrt{d}\sqrt{a+bx^2}(2bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}+\frac{x\sqrt{c+dx^2}(4bc-3ad)}{3a(a+bx^2)^{3/2}(bc-ad)}}{5a}+\frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}$$

input
```
Int[Sqrt[c + d*x^2]/(a + b*x^2)^(7/2),x]
```

3.171. $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx$

output
```
(x*Sqrt[c + d*x^2])/(5*a*(a + b*x^2)^(5/2)) + (((4*b*c - 3*a*d)*x*Sqrt[c +
 d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (((8*b^2*c^2 - 13*a*b*c*d +
 3*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*
d)/(b*c)])/(Sqrt[a]*Sqrt[b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2
))/(c*(a + b*x^2))]) - (2*c^(3/2)*Sqrt[d]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*a*(b*c - a*d)))/(
5*a)
```

3.171.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

3.171. $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx$

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.171.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.84

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{5b^3a\left(x^2+\frac{a}{b}\right)^3}+\frac{(3ad-4bc)x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{15(ad-bc)a^2b^2\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd\,x^2+bc\right)x\left(3a^2d^2-13abcd+8b^2c^2\right)}{15b\,a^3(ad-bc)^2\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(\frac{d(3ad-4}{15b(ad-bc}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5/b^3/a*x*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^3+1/15*(3*a*d-4*b*c)/(a*d-b*c
)/a^2/b^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+1/15*(b*d*x^2+
b*c)/b/a^3/(a*d-b*c)^2*x*(3*a^2*d^2-13*a*b*c*d+8*b^2*c^2)/((x^2+a/b)*(b*d*
x^2+b*c))^(1/2)+(1/15*d*(3*a*d-4*b*c)/b/(a*d-b*c)/a^2-1/15/(a*d-b*c)/b*(3*
a^2*d^2-13*a*b*c*d+8*b^2*c^2)/a^3-1/15*c/a^3/(a*d-b*c)^2*(3*a^2*d^2-13*a*b
*c*d+8*b^2*c^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))+1/15*(3*a^2*d^2-13*a*b*c*d+8*b^2*c^2)/(a*d-b*c)^2/a^3*c/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.171.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(291) = 582$.

Time = 0.11 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx =$$

$$-\frac{(8\,a^3b^3c^2 - 13\,a^4b^2cd + 3\,a^5bd^2 + (8\,b^6c^2 - 13\,ab^5cd + 3\,a^2b^4d^2)x^6 + 3\,(8\,ab^5c^2 - 13\,a^2b^4cd + 3\,a^3b^3d^2)x^4 +}{}$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*((8*a^3*b^3*c^2 - 13*a^4*b^2*c*d + 3*a^5*b*d^2 + (8*b^6*c^2 - 13*a*b
^5*c*d + 3*a^2*b^4*d^2)*x^6 + 3*(8*a*b^5*c^2 - 13*a^2*b^4*c*d + 3*a^3*b^3*
d^2)*x^4 + 3*(8*a^2*b^4*c^2 - 13*a^3*b^3*c*d + 3*a^4*b^2*d^2)*x^2)*sqrt(a*
c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (8*a^3*b^3*c^2
 + (8*b^6*c^2 + (4*a^2*b^4 - 13*a*b^5)*c*d - 3*(2*a^3*b^3 - a^2*b^4)*d^2)*
x^6 + 3*(8*a*b^5*c^2 + (4*a^3*b^3 - 13*a^2*b^4)*c*d - 3*(2*a^4*b^2 - a^3*b
^3)*d^2)*x^4 + (4*a^5*b - 13*a^4*b^2)*c*d - 3*(2*a^6 - a^5*b)*d^2 + 3*(8*a
^2*b^4*c^2 + (4*a^4*b^2 - 13*a^3*b^3)*c*d - 3*(2*a^5*b - a^4*b^2)*d^2)*x^2
)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((8*a
*b^5*c^2 - 13*a^2*b^4*c*d + 3*a^3*b^3*d^2)*x^5 + (20*a^2*b^4*c^2 - 33*a^3*
b^3*c*d + 9*a^4*b^2*d^2)*x^3 + (15*a^3*b^3*c^2 - 26*a^4*b^2*c*d + 9*a^5*b*
d^2)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^7*b^3*c^2 - 2*a^8*b^2*c*d + a^
9*b*d^2 + (a^4*b^6*c^2 - 2*a^5*b^5*c*d + a^6*b^4*d^2)*x^6 + 3*(a^5*b^5*c^2
 - 2*a^6*b^4*c*d + a^7*b^3*d^2)*x^4 + 3*(a^6*b^4*c^2 - 2*a^7*b^3*c*d + a^8
*b^2*d^2)*x^2)
```

3.171.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(7/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/(a + b*x**2)**(7/2), x)
```

3.171.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{7/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2), x)
```

3.171.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{7/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2), x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{7/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\left(b\,x^2+a\right)^{7/2}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^(7/2),x)
```

output
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^(7/2), x)
```

3.172 $\int (a+bx^2)^{3/2} (c+dx^2)^{3/2} \, dx$

3.172.1 Optimal result

Integrand size = 23, antiderivative size = 410

$$\begin{aligned}\int (a+bx^2)^{3/2} (c+dx^2)^{3/2} \, dx = & -\frac{2(bc+ad)(b^2c^2-6abcd+a^2d^2)x\sqrt{a+bx^2}}{35b^2d\sqrt{c+dx^2}} \\ & + \frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2} \\ & + \frac{2(4bc-ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35b} + \frac{dx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7b} \\ & + \frac{2\sqrt{c}(bc+ad)(b^2c^2-6abcd+a^2d^2)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^2d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ & - \frac{c^{3/2}(b^2c^2-18abcd+a^2d^2)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-2/35*(a*d+b*c)*(a^2*d^2-6*a*b*c*d+b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2/d/(d*x^2
+c)^(1/2)-1/35*c^(3/2)*(a^2*d^2-18*a*b*c*d+b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*
(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d
)^(1/2))*(b*x^2+a)^(1/2)/b/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+
c)^(1/2)+2/35*(a*d+b*c)*(a^2*d^2-6*a*b*c*d+b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*
(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d
)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/b^2/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(d*x^2+c)^(1/2)+2/35*(-a*d+4*b*c)*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b+1
/7*d*x*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/b+1/35*(9*a*c+b*c^2/d-2*a^2*d/b)*x*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

3.172.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.18 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.74

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(a^2d^2+abd(17c+8dx^2)+b^2(c^2+8cdx^2+5d^2x^4))+2ic(b^3c^3-5c$$

input
```
Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a^2*d^2 + a*b*d*(17*c + 8*d*x^2) +
 b^2*(c^2 + 8*c*d*x^2 + 5*d^2*x^4)) + (2*I)*c*(b^3*c^3 - 5*a*b^2*c^2*d - 5
*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(2*b^3*c^3 - 11*a*b^2*c^2*d + 8
*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(35*b*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])
```

3.172.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {318, 403, 27, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} dx$$

$$\Big\downarrow \; 318$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(2d(4bc-ad)x^2+c(7bc-ad))}{\sqrt{dx^2+c}}dx}{7b}+\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\int \frac{3d\sqrt{bx^2+a}\left(\left(b^2c^2+9abdc-2a^2d^2\right)x^2+ac(9bc-ad)\right)}{\sqrt{dx^2+c}}dx}{5d}+\frac{2}{5}x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(4bc-ad)}{7b}+\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$\Big\downarrow$ 27

$$\frac{\frac{3}{5}\int \frac{\sqrt{bx^2+a}\left(\left(b^2c^2+9abdc-2a^2d^2\right)x^2+ac(9bc-ad)\right)}{\sqrt{dx^2+c}}dx+\frac{2}{5}x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(4bc-ad)}{7b}+\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$\Big\downarrow$ 403

$$\frac{\frac{3}{5}\left(\frac{\int -\frac{2(bc+ad)\left(b^2c^2-6abdc+a^2d^2\right)x^2+ac\left(b^2c^2-18abdc+a^2d^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}\right)+\frac{2}{5}x\left(a+bx^2\right)^{3/2}\sqrt{c+}}{7b}+\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$\Big\downarrow$ 25

$$\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{\int \frac{2(bc+ad)\left(b^2c^2-6abdc+a^2d^2\right)x^2+ac\left(b^2c^2-18abdc+a^2d^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)+\frac{2}{5}x\left(a+bx^2\right)^{3/2}\sqrt{c+d}}{7b}+\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$\Big\downarrow$ 406

$$\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{ac\left(a^2d^2-18abcd+b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3d}\right.}{7b}+\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$\Big\downarrow$ 320

3.172. $\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\,dx$

$$\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-18abcd+b^2c^2\right)\text{EllipticF}\left(\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}\right.}{7b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$\big\downarrow$ 388

$$\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-18abcd+b^2c\right.}{\sqrt{d}\sqrt{c+dx^2}}}{3d}\right.}{7b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$\big\downarrow$ 313

$$\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(\right.}{}}{3d}\right.}{7b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

input
```
Int[(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x]
```

3.172. $\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\,dx$

output
```
(d*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*b) + ((2*(4*b*c - a*d)*x*(a + b
*x^2)^(3/2)*Sqrt[c + d*x^2])/5 + (3*(((b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2)*x*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - (2*(b*c + a*d)*(b^2*c^2 - 6*a*b*c
*d + a^2*d^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a +
 b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d
]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b^2*
c^2 - 18*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2]))/(3*d)))/5)/(7*b)
```

3.172.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.172.4 Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.38

method	result
risch	$\frac{x\left(5b^2d^2x^4+8x^2ab\,d^2+8x^2b^2cd+a^2d^2+17abcd+b^2c^2\right)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{35bd}-\frac{\left(\frac{a^3c\,d^2\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}+\frac{b^2c \cdots}{\cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{bd\,x^5\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{7}+\frac{\left(2ab\,d^2+2b^2cd-\frac{bd(6ad+6bc)}{7}\right)x^3\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{5bd}+\frac{\left(a^2d \cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(5\sqrt{-\frac{b}{a}}\,b^3d^4x^9+13\sqrt{-\frac{b}{a}}\,a\,b^2d^4x^7+13\sqrt{-\frac{b}{a}}\,b^3c\,d^3x^7+9\sqrt{-\frac{b}{a}}\,a^2b\,d^4x^5+38\sqrt{-\frac{b}{a}}\,a\,b^2c\,d^3x^5+9\sqrt{-\frac{b}{a}}\,b^3c^2d^2x^5+\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35/b/d*x*(5*b^2*d^2*x^4+8*a*b*d^2*x^2+8*b^2*c*d*x^2+a^2*d^2+17*a*b*c*d+b
^2*c^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)-1/35/b/d*(a^3*c*d^2/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+b^2*c^3*a/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellip
ticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-18*a^2*b*c^2*d/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*a^3*d^3-10*a^2*b*c*d^2
-10*a*b^2*c^2*d+2*b^3*c^3)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))*(
(b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.77

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} \, dx = \frac{2\left(b^3c^4-5\,ab^2c^3d-5\,a^2bc^2d^2+a^3cd^3\right)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\left(2\,b^3c^4-10\,ab^2c^3\right.}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/35*(2*(b^3*c^4 - 5*a*b^2*c^3*d - 5*a^2*b*c^2*d^2 + a^3*c*d^3)*sqrt(b*d)*
x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (2*b^3*c^4 - 10
*a*b^2*c^3*d + a^3*d^4 - (10*a^2*b - a*b^2)*c^2*d^2 + 2*(a^3 - 9*a^2*b)*c*
d^3)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) +
(5*b^3*d^4*x^6 - 2*b^3*c^3*d + 10*a*b^2*c^2*d^2 + 10*a^2*b*c*d^3 - 2*a^3*d
^4 + 8*(b^3*c*d^3 + a*b^2*d^4)*x^4 + (b^3*c^2*d^2 + 17*a*b^2*c*d^3 + a^2*b
*d^4)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^3*x)
```

3.172.6 Sympy [F]

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\,dx=\int \left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2), x)
```

3.172.7 Maxima [F]

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2), x)
```

3.172.8 Giac [F]

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} \, dx = \int \left(b\,x^2+a\right)^{3/2} \left(d\,x^2+c\right)^{3/2} dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2), x)
```

3.173 $\int \sqrt{a+bx^2}(c+dx^2)^{3/2}\,dx$

3.173.1 Optimal result

Integrand size = 23, antiderivative size = 336

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx=\frac{(3b^2c^2+7abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{15b^2\sqrt{c+dx^2}}+\frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}-\frac{\sqrt{c}(3b^2c^2+7abcd-2a^2d^2)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{c^{3/2}(9bc-ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/15*(-2*a^2*d^2+7*a*b*c*d+3*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2/(d*x^2+c)^(1/2
)+1/15*c^(3/2)*(-a*d+9*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)
/b/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*(-2*a^2*d^
2+7*a*b*c*d+3*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x
*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1
/2)/b^2/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/5*d*x*(b
*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b+2/15*(-a*d+3*b*c)*x*(b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/b
```

3.173.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.73

$$\int \sqrt{a+bx^2}(c+dx^2)^{3/2}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(6bc+ad+3bdx^2\right)+ic(-3b^2c^2-7abcd+2a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\cdots}}{15b\sqrt{\cdots}}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(6*b*c + a*d + 3*b*d*x^2) + I*c*(-3
*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-3*b^2*c^2 + 2*a*b*c
*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)])/(15*b*Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])
```

3.173.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {318, 403, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \ 318$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(2d(3bc-ad)x^2+c(5bc-ad)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$$\Big\downarrow \ 403$$

$$\frac{\frac{\int \frac{d\left(\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{3}\int \frac{(3b^2c^2+7abdc-2a^2d^2)x^2+ac(9bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$\big\downarrow$ 406

$$\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(9bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-}{5b}$$
$$\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$\big\downarrow$ 320

$$\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{}}{5b}$$
$$\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$\big\downarrow$ 388

$$\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x}{5b}$$
$$\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$\big\downarrow$ 313

$$\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5b}$$
$$\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2),x]
```

3.173. $\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx$

output
```
(d*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + ((2*(3*b*c - a*d)*x*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])/3 + ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*((x*Sq
rt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(9*b*c - a*d)*Sqrt[a + b*
x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/3)/(5*b)
```

3.173.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.173. $\int \sqrt{a+bx^2}(c+dx^2)^{3/2}\,dx$

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.173.4 Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.22

method	result
risch	$\frac{x(3bd\,x^2+ad+6bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{15b} - \frac{\left(\frac{a^2cd\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}} - \frac{9b\,c^2a\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{d\,x^3\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{5}+\frac{\left(a\,d^2+2bcd-\frac{d(4ad+4bc)}{5}\right)x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3bd}+\frac{\left(c^2a-\frac{\left(a\,d^2+2bcd-\frac{d(4ad+4bc)}{5}\right.}{3bd}\right.}{\sqrt{-\frac{b}{a}}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(3\sqrt{-\frac{b}{a}}\,b^2d^3x^7+4\sqrt{-\frac{b}{a}}\,ab\,d^3x^5+9\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5+\sqrt{-\frac{b}{a}}\,a^2d^3x^3+10\sqrt{-\frac{b}{a}}\,abc\,d^2x^3+6\sqrt{-\frac{b}{a}}\,b^2c^2d\,x^3+\sqrt{\frac{b\,x^2+}{a}}\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*b*d*x^2+a*d+6*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b-1/15/b*(a^2
*c*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-9*b*c^2
*a/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*a^2*d^
2-7*a*b*c*d-3*b^2*c^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))*((b*x
^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.69

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx =$$

$$-\frac{(3\,b^2c^3+7\,abc^2d-2\,a^2cd^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(3\,b^2c^3+7\,abc^2d-a^2d^3-(2\,a^2-9\,ab)cd}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*((3*b^2*c^3 + 7*a*b*c^2*d - 2*a^2*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elli
ptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b^2*c^3 + 7*a*b*c^2*d - a^2*d
^3 - (2*a^2 - 9*a*b)*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(
-c/d)/x), a*d/(b*c)) - (3*b^2*d^3*x^4 + 3*b^2*c^2*d + 7*a*b*c*d^2 - 2*a^2*
d^3 + (6*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d
^2*x)
```

3.173.6 Sympy [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx = \int \sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2), x)
```

3.173.7 Maxima [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2), x)
```

3.173.8 Giac [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2), x)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2), x)
```

3.174 $\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx$

3.174.1 Optimal result

Integrand size = 23, antiderivative size = 273

$$\begin{aligned}\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx &= \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3b^2\sqrt{c+dx^2}}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\\ &-\frac{2\sqrt{c}\sqrt{d}(2bc-ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\ &+\frac{c^{3/2}(3bc-ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
2/3*d*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/b^2/(d*x^2+c)^(1/2)+1/3*c^(3/2)*(-a*d
+3*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2
)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/a/b/d^(1/2)/(c*(b*x
^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2/3*(-a*d+2*b*c)*(1/(1+d*x^2/c))^
(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/b^2/(c*(b*x^2+a)/a/(d*x^2+c
))^(1/2)/(d*x^2+c)^(1/2)+1/3*d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b
```

3.174.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.73

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)+2ic(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{3b\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+}}$$

input
```
Integrate[(c + d*x^2)^(3/2)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + (2*I)*c*(-2*b*c + a*d)*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)] - I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])
```

3.174.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {318, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \ 318$$

$$\frac{\int \frac{2d(2bc-ad)x^2+c(3bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}$$

$$\Big\downarrow \ 406$$

$$\frac{c(3bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2d(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}$$

$$\Big\downarrow \ 320$$

$$\frac{\frac{2d(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}$$

$\downarrow$ 388

$$\frac{2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}$$

$\downarrow$ 313

$$\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}$$

input
```
Int[(c + d*x^2)^(3/2)/Sqrt[a + b*x^2],x]
```

output
```
(d*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b) + (2*d*(2*b*c - a*d)*((x*Sqrt[
a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(3*b*c - a*d)*Sqrt[a + b*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b)
```

3.174.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

3.174. $\int \frac{\left(c+dx^2\right)^{3/2}}{\sqrt{a+bx^2}}dx$

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.174.4 Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{d x\sqrt{b d x^4+a d x^2+c b x^2+a c}}{3b}+\frac{\left(c^2-\frac{d a c}{3b}\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{b d x^4+a d x^2+c b x^2+a c}}-\frac{\left(2cd-\frac{d(2ad+2bc)}{3b}\right)c\sqrt{1+\frac{b x^2}{a}}}{}\right.}{\sqrt{b x^2+a}\sqrt{d x^2+c}}$
default	$\frac{\sqrt{d x^2+c}\sqrt{b x^2+a}\left(\sqrt{-\frac{b}{a}}\,b d^2 x^5+\sqrt{-\frac{b}{a}}\,a d^2 x^3+\sqrt{-\frac{b}{a}}\,b c d x^3+a c\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right)d-\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}\,F\left(x\right.\right.}{3(b d x^4+a d x^2+c b x^2+}$
risch	$\frac{d x\sqrt{b x^2+a}\sqrt{d x^2+c}}{3b}-\frac{\left(\frac{a c d\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{b d x^4+a d x^2+c b x^2+a c}}-\frac{3b c^2\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{b d x^4+a d x^2+c b x^2+a c}}-\frac{\left(2a d^2-\right.}{}\right.}{3b\sqrt{b x^2+a}\sqrt{}}$

input
```
int((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*d/b*x*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(c^2-1/3*d/b*a*c)/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*c*d-1/3*d/b*(2*a*d+2*b*c))*c/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = -\frac{2\,(2\,bc^2-acd)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(4\,bc^2-(2\,a-3\,b)cd-ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right.}{3\,b^2dx}$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(2*b*c^2 - a*c*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c
/d)/x), a*d/(b*c)) - (4*b*c^2 - (2*a - 3*b)*c*d - a*d^2)*sqrt(b*d)*x*sqrt(
-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (b*d^2*x^2 + 4*b*c*d -
 2*a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d*x)
```

3.174.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)/sqrt(a + b*x**2), x)
```

3.174.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a), x)
```

3.174.8 Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a), x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c + d*x^2)^(3/2)/(a + b*x^2)^(1/2),x)
```

output
```
int((c + d*x^2)^(3/2)/(a + b*x^2)^(1/2), x)
```

3.175 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$

3.175.1 Optimal result

Integrand size = 23, antiderivative size = 267

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = -\frac{d(bc-2ad)x\sqrt{a+bx^2}}{ab^2\sqrt{c+dx^2}} + \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} + \frac{\sqrt{c}\sqrt{d}(bc-2ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{ab\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-d*(-2*a*d+b*c)*x*(b*x^2+a)^(1/2)/a/b^2/(d*x^2+c)^(1/2)+c^(3/2)*(1/(1+d*x^
2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2
),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a/b/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)/(d*x^2+c)^(1/2)+(-2*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2
)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)
*d^(1/2)*(b*x^2+a)^(1/2)/a/b^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(
1/2)+(-a*d+b*c)*x*(d*x^2+c)^(1/2)/a/b/(b*x^2+a)^(1/2)
```

3.175.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.72

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \frac{-ic(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(bc-ad)\left(\sqrt{\frac{b}{a}}x(c+dx\right.}{a^2\left(\frac{b}{a}\right)^{3/2}\sqrt{a+bx^2}\sqrt{c+dx}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x]
```

output
```
((-I)*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (b*c - a*d)*(Sqrt[b/a]*x*(c + d*x^
2) - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)]))/(a^2*(b/a)^(3/2)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.175.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {315, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{315}$$

$$\frac{\int \frac{d(ac-(bc-2ad)x^2)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{ac-(bc-2ad)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{d\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{320}$$

$$\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{388}$$

$$\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{313}$$

$$\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x]
```

output
```
((b*c - a*d)*x*Sqrt[c + d*x^2])/(a*b*Sqrt[a + b*x^2]) + (d*(-((b*c - 2*a*d
)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*Sqrt[a + b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*b)
```

3.175. $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$

3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.175. $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$

3.175.4 Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.24

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, a\, d^2 x^3+\sqrt{-\frac{b}{a}}\, bcd\, x^3-ac\sqrt{\frac{b\, x^2+a}{a}}\, \sqrt{\frac{d\, x^2+c}{c}}\, F\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) d+\sqrt{\frac{b\, x^2+a}{a}}\, \sqrt{\frac{d\, x^2+c}{c}}\, F\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) b\, c^2+2\sqrt{\frac{b\, x^2+a}{a}}\, \sqrt{}\right.}{b(bd\, x^4+ad\, x^2+cb\, x^2+c}$
elliptic	$\frac{\sqrt{(b\, x^2+a)(d\, x^2+c)}\left(-\frac{\left(bd\, x^2+bc\right)(ad-bc)x}{a\, b^2\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\, x^2+bc\right)}}+\frac{\left(-\frac{d(ad-2bc)}{b^2}+\frac{(ad-bc)^2}{b^2 a}+\frac{c(ad-bc)}{ba}\right)\sqrt{1+\frac{b\, x^2}{a}}\, \sqrt{1+\frac{d\, x^2}{c}}\, F\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^4+ad\, x^2+cb\, x^2+ac}}-\right.}{\sqrt{b\, x^2+a}\, \sqrt{d\, x^2+c}}$

input
```
int((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*a*d^2*x^3+(-b/a)^(1/2)*b*c*d*x^3-a*c*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d+((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^
2+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a*c*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/
a)^(1/2),(a*d/b/c)^(1/2))*b*c^2-(-b/a)^(1/2)*a*c*d*x+(-b/a)^(1/2)*b*c^2*x)
*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/b/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/a/(-b/a)^
(1/2)
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\, dx = \frac{((b^2c^2-2\, abcd)x^3+(abc^2-2\, a^2cd)x)\sqrt{bd}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc})-((b^2c^2-2\, ab}{}$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
(((b^2*c^2 - 2*a*b*c*d)*x^3 + (a*b*c^2 - 2*a^2*c*d)*x)*sqrt(b*d)*sqrt(-c/d
)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((b^2*c^2 - 2*a*b*c*d - a*
b*d^2)*x^3 + (a*b*c^2 - 2*a^2*c*d - a^2*d^2)*x)*sqrt(b*d)*sqrt(-c/d)*ellip
tic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (a*b*d^2*x^2 - a*b*c*d + 2*a^2*d^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^3*d*x^3 + a^2*b^2*d*x)
```

3.175.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)/(a + b*x**2)**(3/2), x)
```

3.175.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2), x)
```

3.175.8 Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2), x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x)
```

output
```
int((c + d*x^2)^(3/2)/(a + b*x^2)^(3/2), x)
```

3.176 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx$

3.176.1 Optimal result

Integrand size = 23, antiderivative size = 229

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)x\sqrt{c+dx^2}}{3ab(a+bx^2)^{3/2}} + \frac{2(bc+ad)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c
^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a^2/b/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*(-a*d+b*c)*x*(d*x^2+c)
^(1/2)/a/b/(b*x^2+a)^(3/2)+2/3*(a*d+b*c)*(1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)
^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))*(d
*x^2+c)^(1/2)/a^(3/2)/b^(3/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1
/2)
```

3.176.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.38 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(c+dx^2)\left(a^2d+2b^2cx^2+ab(3c+2dx^2)\right)+2ic(bc+ad)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\cdots}}{3a^3\left(\frac{b}{a}\cdots\right.}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[b/a]*x*(c + d*x^2)*(a^2*d + 2*b^2*c*x^2 + a*b*(3*c + 2*d*x^2)) + (2*
I)*c*(b*c + a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(2*b*c + a*d)*(a + b*x^2)*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)])/(3*a^3*(b/a)^(3/2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

3.176.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {315, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 315$$

$$\frac{\int \frac{d(bc+2ad)x^2+c(2bc+ad)}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{3ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 400$$

$$\frac{2(ad+bc)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{3/2}}dx-cd\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{2\sqrt{c+dx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-cd\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\frac{2\sqrt{c+dx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x]
```

output
```
((b*c - a*d)*x*Sqrt[c + d*x^2])/(3*a*b*(a + b*x^2)^(3/2)) + ((2*(b*c + a*d
)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])
/(Sqrt[a]*Sqrt[b]*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) -
 (c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(a*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
)/(3*a*b)
```

3.176.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.176. $\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx$

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

3.176.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.84

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{(ad-bc)x\sqrt{bd x^4+ad x^2+cb x^2+ac}}{3a b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{2\left(bd x^2+bc\right)x(ad+bc)}{3b^2a^2\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd x^2+bc\right)}}+\frac{\left(\frac{d^2}{b^2}-\frac{d(ad-bc)}{3b^2a}-\frac{2(ad+bc)(ad-bc)}{3b^2a^2}-\frac{2c(ad+bc)}{3b a^2}\right)\cdots}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad\cdots}}\right.}{\sqrt{b x^2+a}\,\sqrt{d\cdots}}$
default	$\frac{2\sqrt{-\frac{b}{a}}\,ab d^2x^5+2\sqrt{-\frac{b}{a}}\,b^2cd x^5+\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abcd x^2+2\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b^2c^2x^2-2\sqrt{\cdots}}{\cdots}$

input
```
int((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*(a*d-b*c
)/a/b^3*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+2/3*(b*d*x^2+b*c
)/b^2/a^2*x*(a*d+b*c)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(d^2/b^2-1/3/b^2*d*(
a*d-b*c)/a-2/3*(a*d+b*c)/b^2*(a*d-b*c)/a^2-2/3/b*c/a^2*(a*d+b*c))/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+2/3*(a*d+b*c)/b/a^2*
c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.28

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx =$$

$$-\frac{2\left(a^2b^2c+a^3bd+(b^4c+ab^3d)x^4+2\left(ab^3c+a^2b^2d\right)x^2\right)\sqrt{ac}\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}\right)-\left(2a^2b^2c+(2b\cdots\right.}{\cdots}$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(a^2*b^2*c + a^3*b*d + (b^4*c + a*b^3*d)*x^4 + 2*(a*b^3*c + a^2*b^
2*d)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c))
 - (2*a^2*b^2*c + (2*b^4*c + (a^2*b^2 + 2*a*b^3)*d)*x^4 + 2*(2*a*b^3*c + (
a^3*b + 2*a^2*b^2)*d)*x^2 + (a^4 + 2*a^3*b)*d)*sqrt(a*c)*sqrt(-b/a)*ellipt
ic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*(a*b^3*c + a^2*b^2*d)*x^3 + (3*
a^2*b^2*c + a^3*b*d)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*b^4*x^4 + 2*
a^4*b^3*x^2 + a^5*b^2)
```

3.176.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)/(a + b*x**2)**(5/2), x)
```

3.176.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2), x)
```

3.176.8 Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2), x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x)
```

output
```
int((c + d*x^2)^(3/2)/(a + b*x^2)^(5/2), x)
```

3.177 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx$

3.177.1 Optimal result

Integrand size = 23, antiderivative size = 315

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\,(a+bx^2)^{5/2}} + \frac{2(2bc+ad)x\sqrt{c+dx^2}}{15a^2b\,(a+bx^2)^{3/2}} + \frac{(8b^2c^2-3abcd-2a^2d^2)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}b^{3/2}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{c^{3/2}\sqrt{d}(4bc-ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^3b(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/15*c^(3/2)*(-a*d+4*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a
)^(1/2)/a^3/b/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1
/5*(-a*d+b*c)*x*(d*x^2+c)^(1/2)/a/b/(b*x^2+a)^(5/2)+2/15*(a*d+2*b*c)*x*(d*
x^2+c)^(1/2)/a^2/b/(b*x^2+a)^(3/2)+1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*(
1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^
2/a)^(1/2),(1-a*d/b/c)^(1/2))*(d*x^2+c)^(1/2)/a^(5/2)/b^(3/2)/(-a*d+b*c)/(
b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

3.177.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(c+dx^2)\left(3a^2(bc-ad)^2+2a(bc-ad)(2bc+ad)(a+bx^2)+(8b^2c^2-3abcd-2\right.}{}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x]
```

output
```
(Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + 2*a*(b*c - a*d)*(2*b*c + a
*d)*(a + b*x^2) + (8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*(a + b*x^2)^2) - I*c
*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*((-8*b^2*c^2 + 3*a*
b*c*d + 2*a^2*d^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (8*b^2
*c^2 - 7*a*b*c*d - a^2*d^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
))/(15*a^4*(b/a)^(3/2)*(b*c - a*d)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])
```

3.177.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {315, 402, 25, 27, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx$$

$$\Big\downarrow \ 315$$

$$\frac{\int \frac{d(3bc+2ad)x^2+c(4bc+ad)}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx}{5ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{5ab\,(a+bx^2)^{5/2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{2x\sqrt{c+dx^2}(ad+2bc)}{3a(a+bx^2)^{3/2}}-\frac{\int -\frac{(bc-ad)\left(2d(2bc+ad)x^2+c(8bc+ad)\right)}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}}{5ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{5ab\,(a+bx^2)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{(bc-ad)\left(2d(2bc+ad)x^2+c(8bc+ad)\right)}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}+\frac{2x\sqrt{c+dx^2}(ad+2bc)}{3a(a+bx^2)^{3/2}}}{5ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{2d(2bc+ad)x^2+c(8bc+ad)}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a}+\frac{2x\sqrt{c+dx^2}(ad+2bc)}{3a(a+bx^2)^{3/2}}}{5ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 400$$

$$\frac{\frac{\frac{\left(-2a^2d^2-3abcd+8b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{cd(4bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a}+\frac{2x\sqrt{c+dx^2}(ad+2bc)}{3a(a+bx^2)^{3/2}}}{5ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\frac{\sqrt{c+dx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{cd(4bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a}+\frac{2x\sqrt{c+dx^2}(ad+2bc)}{3a(a+bx^2)^{3/2}}}{5ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{\frac{\sqrt{c+dx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}(4bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a}+\frac{2x\sqrt{c+dx^2}(ad+2bc)}{3a(a+bx^2)^{3/2}}}{5ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{5ab\left(a+bx^2\right)^{5/2}}$$

input
```
Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x]
```

3.177. $\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{7/2}}dx$

output
```
((b*c - a*d)*x*Sqrt[c + d*x^2])/(5*a*b*(a + b*x^2)^(5/2)) + ((2*(2*b*c + a
*d)*x*Sqrt[c + d*x^2])/(3*a*(a + b*x^2)^(3/2)) + (((8*b^2*c^2 - 3*a*b*c*d
- 2*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a
*d)/(b*c)])/(Sqrt[a]*Sqrt[b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^
2))/(c*(a + b*x^2))]) - (c^(3/2)*Sqrt[d]*(4*b*c - a*d)*Sqrt[a + b*x^2]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*a))/(5*a*b)
```

3.177.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.177. $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx$

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.177.4 Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{(ad-bc)x\sqrt{bd x^4+ad x^2+cb x^2+ac}}{5ab^4\left(x^2+\frac{a}{b}\right)^3}+\frac{2(ad+2bc)x\sqrt{bd x^4+ad x^2+cb x^2+ac}}{15a^2b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd x^2+bc\right)x\left(2a^2d^2+3abcd-8b^2c^2\right)}{15b^2a^3(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd x^2+bc\right)}}+\right.}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*(a*d-b*c
)/a/b^4*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^3+2/15*(a*d+2*b*c)
/a^2/b^3*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+1/15*(b*d*x^2+b
*c)/b^2/a^3/(a*d-b*c)*x*(2*a^2*d^2+3*a*b*c*d-8*b^2*c^2)/((x^2+a/b)*(b*d*x^
2+b*c))^(1/2)+(2/15*d*(a*d+2*b*c)/a^2/b^2-1/15/b^2*(2*a^2*d^2+3*a*b*c*d-8*
b^2*c^2)/a^3-1/15/b*c/a^3/(a*d-b*c)*(2*a^2*d^2+3*a*b*c*d-8*b^2*c^2))/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/15/b*(2*a^2*d^2
+3*a*b*c*d-8*b^2*c^2)/(a*d-b*c)/a^3*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))))
```

3.177.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(297) = 594$.

Time = 0.10 (sec) , antiderivative size = 635, normalized size of antiderivative = 2.02

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx =$$

$$-\frac{(8\,a^3b^3c^2 - 3\,a^4b^2cd - 2\,a^5bd^2 + (8\,b^6c^2 - 3\,ab^5cd - 2\,a^2b^4d^2)x^6 + 3\,(8\,ab^5c^2 - 3\,a^2b^4cd - 2\,a^3b^3d^2)x^4 + 3\,(}{}$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*((8*a^3*b^3*c^2 - 3*a^4*b^2*c*d - 2*a^5*b*d^2 + (8*b^6*c^2 - 3*a*b^5
*c*d - 2*a^2*b^4*d^2)*x^6 + 3*(8*a*b^5*c^2 - 3*a^2*b^4*c*d - 2*a^3*b^3*d^2
)*x^4 + 3*(8*a^2*b^4*c^2 - 3*a^3*b^3*c*d - 2*a^4*b^2*d^2)*x^2)*sqrt(a*c)*s
qrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (8*a^3*b^3*c^2 + (
8*b^6*c^2 + (4*a^2*b^4 - 3*a*b^5)*c*d - (a^3*b^3 + 2*a^2*b^4)*d^2)*x^6 + 3
*(8*a*b^5*c^2 + (4*a^3*b^3 - 3*a^2*b^4)*c*d - (a^4*b^2 + 2*a^3*b^3)*d^2)*x
^4 + (4*a^5*b - 3*a^4*b^2)*c*d - (a^6 + 2*a^5*b)*d^2 + 3*(8*a^2*b^4*c^2 +
(4*a^4*b^2 - 3*a^3*b^3)*c*d - (a^5*b + 2*a^4*b^2)*d^2)*x^2)*sqrt(a*c)*sqrt
(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((8*a*b^5*c^2 - 3*a^2
*b^4*c*d - 2*a^3*b^3*d^2)*x^5 + 2*(10*a^2*b^4*c^2 - 4*a^3*b^3*c*d - 3*a^4*
b^2*d^2)*x^3 + (15*a^3*b^3*c^2 - 11*a^4*b^2*c*d - a^5*b*d^2)*x)*sqrt(b*x^2
 + a)*sqrt(d*x^2 + c))/(a^7*b^3*c - a^8*b^2*d + (a^4*b^6*c - a^5*b^5*d)*x^
6 + 3*(a^5*b^5*c - a^6*b^4*d)*x^4 + 3*(a^6*b^4*c - a^7*b^3*d)*x^2)
```

3.177.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(7/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)/(a + b*x**2)**(7/2), x)
```

3.177.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2), x)
```

3.177.8 Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2), x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)^{7/2}}\,dx$$

input
```
int((c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x)
```

output
```
int((c + d*x^2)^(3/2)/(a + b*x^2)^(7/2), x)
```

3.178 $\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx$

3.178.1 Optimal result

Integrand size = 23, antiderivative size = 235

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx = \frac{(3b+2d)x\sqrt{2+bx^2}}{3b\sqrt{3+dx^2}}+\frac{1}{3}x\sqrt{2+bx^2}\sqrt{3+dx^2} - \frac{\sqrt{2}(3b+2d)\sqrt{2+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3b\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} + \frac{2\sqrt{2}\sqrt{2+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

output
```
1/3*(3*b+2*d)*x*(b*x^2+2)^(1/2)/b/(d*x^2+3)^(1/2)-1/3*(3*b+2*d)*(1/(3*d*x^
2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2
),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/b/d^(1/2)/((b*x^2+2)/(d*x^2
+3))^(1/2)/(d*x^2+3)^(1/2)+2*(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*Ellip
ticF(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x
^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)+1/3*x*(b*x
^2+2)^(1/2)*(d*x^2+3)^(1/2)
```

3.178.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.54

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx$$
$$=\frac{\sqrt{b}dx\sqrt{2+bx^2}\sqrt{3+dx^2}-i\sqrt{3}(3b+2d)E\left(i\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)+i\sqrt{3}(3b-2d)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right.\right.}{3\sqrt{b}d}$$

input
```
Integrate[Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2],x]
```

output
```
(Sqrt[b]*d*x*Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2] - I*Sqrt[3]*(3*b + 2*d)*Ellip
ticE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)] + I*Sqrt[3]*(3*b - 2*d)*
EllipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)])/(3*Sqrt[b]*d)
```

3.178.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {319, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{bx^2+2}\sqrt{dx^2+3}\,dx$$

$$\Big\downarrow \text{319}$$

$$\frac{2}{3}\int \frac{(3b+2d)x^2+12}{2\sqrt{bx^2+2}\sqrt{dx^2+3}}dx+\frac{1}{3}x\sqrt{bx^2+2}\sqrt{dx^2+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(3b+2d)x^2+12}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx+\frac{1}{3}x\sqrt{bx^2+2}\sqrt{dx^2+3}$$

$$\Big\downarrow \text{406}$$

$$\frac{1}{3}\left(12\int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx+(3b+2d)\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx\right)+\frac{1}{3}x\sqrt{bx^2+2}\sqrt{dx^2+3}$$

$$\Big\downarrow \text{320}$$

$$\frac{1}{3}\left((3b+2d)\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx+\frac{6\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}\right)+\frac{1}{3}x\sqrt{bx^2+2}\sqrt{dx^2+3}$$

$\downarrow$ 388

$$\frac{1}{3}\left((3b+2d)\left(\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{3\int \frac{\sqrt{bx^2+2}}{(dx^2+3)^{3/2}}dx}{b}\right)+\frac{6\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}\right)+\frac{1}{3}x\sqrt{bx^2+2}\sqrt{dx^2+3}$$

$\downarrow$ 313

$$\frac{1}{3}\left(\frac{6\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+(3b+2d)\left(\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1\right.}{b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}\right.\right.$$
$$\frac{1}{3}x\sqrt{bx^2+2}\sqrt{dx^2+3}$$

input
```
Int[Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2],x]
```

output
```
(x*Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2])/3 + ((3*b + 2*d)*((x*Sqrt[2 + b*x^2])/
(b*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[3]], 1 - (3*b)/(2*d)])/(b*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqr
t[3 + d*x^2])) + (6*Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/S
qrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 +
 d*x^2]))/3
```

3.178.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 319
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*(a + b*x^2)^p*((c + d*x^2)^q/(2*(p + q) + 1)), x] + Simp[2/(2*(p + q) +
 1)   Int[(a + b*x^2)^(p - 1)*(c + d*x^2)^(q - 1)*Simp[a*c*(p + q) + (q*(b*
c - a*d) + a*d*(p + q))*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 2, p, q,
x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.178.4 Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.07

method	result
risch	$\frac{x\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}{3}+\frac{\left(\frac{2\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}-\frac{(3b+2d)\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\left(F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)-E\right.}{3\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}\,b}\right.}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}$
elliptic	$\frac{\sqrt{(b\,x^2+2)(d\,x^2+3)}\left(\frac{x\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}{3}+\frac{2\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}-\frac{\left(b+\frac{2d}{3}\right)\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\left(F\left(\frac{x\sqrt{-}}{3}\right.\right.}{\sqrt{-3d}\,\sqrt{b}}\right.}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}$
default	$\frac{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\left(b^2d\,x^5\sqrt{-d}+3b^2x^3\sqrt{-d}+2bd\,x^3\sqrt{-d}+3\sqrt{2}\,F\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)b\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}-2\sqrt{2}\,F\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}}{}\right.\right.}{3(bd\,x^4+3}$

input
```
int((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)+(2/(-3*d)^(1/2)*(3*d*x^2+9)^(1/2)*(2
*b*x^2+4)^(1/2)/(b*d*x^4+3*b*x^2+2*d*x^2+6)^(1/2)*EllipticF(1/3*x*(-3*d)^(
1/2),1/2*(-4+2*(3*b+2*d)/d)^(1/2))-1/3*(3*b+2*d)/(-3*d)^(1/2)*(3*d*x^2+9)^
(1/2)*(2*b*x^2+4)^(1/2)/(b*d*x^4+3*b*x^2+2*d*x^2+6)^(1/2)/b*(EllipticF(1/3
*x*(-3*d)^(1/2),1/2*(-4+2*(3*b+2*d)/d)^(1/2))-EllipticE(1/3*x*(-3*d)^(1/2)
,1/2*(-4+2*(3*b+2*d)/d)^(1/2))))*((b*x^2+2)*(d*x^2+3))^(1/2)/(b*x^2+2)^(1/
2)/(d*x^2+3)^(1/2)
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.63

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx =$$

$$-\frac{3\sqrt{3}\sqrt{bd}(3b+2d)x\sqrt{-\frac{1}{d}}E\left(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right) \mid \frac{2d}{3b}\right) - \sqrt{3}\sqrt{bd}(4d^2+9b+6d)x\sqrt{-\frac{1}{d}}F\left(\arcsin\left(\frac{\sqrt{3}\sqrt{-}}{x}\right.\right.}{3bd^2x}$$

input
```
integrate((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(3*sqrt(3)*sqrt(b*d)*(3*b + 2*d)*x*sqrt(-1/d)*elliptic_e(arcsin(sqrt(
3)*sqrt(-1/d)/x), 2/3*d/b) - sqrt(3)*sqrt(b*d)*(4*d^2 + 9*b + 6*d)*x*sqrt(
-1/d)*elliptic_f(arcsin(sqrt(3)*sqrt(-1/d)/x), 2/3*d/b) - (b*d^2*x^2 + 3*b
*d + 2*d^2)*sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3))/(b*d^2*x)
```

3.178.6 Sympy [F]

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx = \int \sqrt{bx^2+2}\sqrt{dx^2+3}\,dx$$

input
```
integrate((b*x**2+2)**(1/2)*(d*x**2+3)**(1/2),x)
```

output
```
Integral(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3), x)
```

3.178.7 Maxima [F]

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx = \int \sqrt{bx^2+2}\sqrt{dx^2+3}\,dx$$

input
```
integrate((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3), x)
```

3.178.8 Giac [F]

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx = \int \sqrt{bx^2+2}\sqrt{dx^2+3}\,dx$$

input
```
integrate((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3), x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx = \int \sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\,dx$$

input
```
int((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2),x)
```

output
```
int((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2), x)
```

3.179 $\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx$

3.179.1 Optimal result

Integrand size = 23, antiderivative size = 38

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx=\sqrt{\frac{2}{3}}x\sqrt{1-4x^4}+\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-1\right)}{\sqrt{3}}$$

output
```
2/3*EllipticF(x*2^(1/2),I)*3^(1/2)+1/3*x*6^(1/2)*(-4*x^4+1)^(1/2)
```

3.179.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx=\sqrt{6}x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},4x^4\right)$$

input
```
Integrate[Sqrt[3 - 6*x^2]*Sqrt[2 + 4*x^2],x]
```

output
```
Sqrt[6]*x*Hypergeometric2F1[-1/2, 1/4, 5/4, 4*x^4]
```

3.179.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {284, 748, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-6x^2}\sqrt{4x^2+2}\,dx$$

$$\Big\downarrow \text{284}$$

$$\int \sqrt{6-24x^4}dx$$

$$\Big\downarrow \text{748}$$

$$4\int \frac{1}{\sqrt{6-24x^4}}dx+\sqrt{\frac{2}{3}}\sqrt{1-4x^4}x$$

$$\Big\downarrow \text{762}$$

$$\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-1\right)}{\sqrt{3}}+\sqrt{\frac{2}{3}}\sqrt{1-4x^4}x$$

input
```
Int[Sqrt[3 - 6*x^2]*Sqrt[2 + 4*x^2],x]
```

output
```
Sqrt[2/3]*x*Sqrt[1 - 4*x^4] + (2*EllipticF[ArcSin[Sqrt[2]*x], -1])/Sqrt[3]
```

3.179.3.1 Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

3.179.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(29) = 58$.

Time = 2.75 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

method	result	size
default	$-\frac{\sqrt{-6x^2+3}\,\sqrt{2}\,\sqrt{2x^2+1}\left(\sqrt{2}\,\sqrt{3}\,\sqrt{-6x^2+3}\,\sqrt{2x^2+1}\,F\left(\sqrt{2}\,x,i\right)-12x^5+3x\right)}{9(4x^4-1)}$	75
elliptic	$-\frac{\sqrt{-6x^2+3}\,\sqrt{4x^2+2}\,\sqrt{-24x^4+6}\left(\frac{x\sqrt{-24x^4+6}}{3}+\frac{2\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{2x^2+1}\,F\left(\sqrt{2}\,x,i\right)}{\sqrt{-24x^4+6}}\right)}{6(4x^4-1)}$	92
risch	$-\frac{x(2x^2-1)(2x^2+1)\sqrt{(-6x^2+3)(4x^2+2)}\,\sqrt{6}}{3\sqrt{-(2x^2-1)(2x^2+1)}\,\sqrt{-6x^2+3}\,\sqrt{4x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{2x^2+1}\,F\left(\sqrt{2}\,x,i\right)\sqrt{(-6x^2+3)(4x^2+2)}\,\sqrt{6}}{3\sqrt{-4x^4+1}\,\sqrt{-6x^2+3}\,\sqrt{4x^2+2}}$	153

input
```
int((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-6*x^2+3)^(1/2)*2^(1/2)*(2*x^2+1)^(1/2)*(2^(1/2)*3^(1/2)*(-6*x^2+3)^
(1/2)*(2*x^2+1)^(1/2)*EllipticF(2^(1/2)*x,I)-12*x^5+3*x)/(4*x^4-1)
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx = \frac{1}{3}\sqrt{4\,x^2+2}\sqrt{-6\,x^2+3}x + \frac{1}{3}\sqrt{2}\sqrt{-6}F(\arcsin\left(\frac{\sqrt{2}}{2\,x}\right)\,|-1)$$

input
```
integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3)*x + 1/3*sqrt(2)*sqrt(-6)*elliptic_f(a
rcsin(1/2*sqrt(2)/x), -1)
```

3.179.6 Sympy [F]

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx = \sqrt{6}\int \sqrt{1-2x^2}\sqrt{2x^2+1}\,dx$$

input
```
integrate((-6*x**2+3)**(1/2)*(4*x**2+2)**(1/2),x)
```

output
```
sqrt(6)*Integral(sqrt(1 - 2*x**2)*sqrt(2*x**2 + 1), x)
```

3.179.7 Maxima [F]

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx = \int \sqrt{4\,x^2+2}\sqrt{-6\,x^2+3}\,dx$$

input
```
integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3), x)
```

3.179.8 Giac [F]

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx = \int \sqrt{4\,x^2+2}\sqrt{-6\,x^2+3}\,dx$$

input
```
integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3), x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx = \int \sqrt{4\,x^2+2}\,\sqrt{3-6\,x^2}\,dx$$

input
```
int((4*x^2 + 2)^(1/2)*(3 - 6*x^2)^(1/2),x)
```

output
```
int((4*x^2 + 2)^(1/2)*(3 - 6*x^2)^(1/2), x)
```

3.180 $\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx$

3.180.1 Optimal result

Integrand size = 23, antiderivative size = 20

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \sqrt{6}x + 2\sqrt{\frac{2}{3}}x^3$$

output
```
2/3*x^3*6^(1/2)+x*6^(1/2)
```

3.180.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \sqrt{6}\left(x + \frac{2x^3}{3}\right)$$

input
```
Integrate[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]
```

output
```
Sqrt[6]*(x + (2*x^3)/3)
```

3.180.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {282, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4x^2+2}\sqrt{6x^2+3}\,dx$$

$$\Big\downarrow \text{282}$$

$$\sqrt{\frac{3}{2}}\int \left(4x^2+2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\frac{3}{2}}\left(\frac{4x^3}{3}+2x\right)$$

input
```
Int[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]
```

output
```
Sqrt[3/2]*(2*x + (4*x^3)/3)
```

3.180.3.1 Defintions of rubi rules used

rule 282
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b, c
, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !SimplerQ[a + b*x
^n, c + d*x^n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.180.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 2 vs. order 1.

Time = 2.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

method	result	size
gosper	$\frac{x(2x^2+3)\sqrt{4x^2+2}}{\sqrt{6x^2+3}}$	38

input
```
int((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(2*x^2+3)*(4*x^2+2)^(1/2)*(6*x^2+3)^(1/2)/(2*x^2+1)
```

3.180.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(14) = 28.

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \frac{(2\,x^3+3\,x)\sqrt{6\,x^2+3}\sqrt{4\,x^2+2}}{3\,(2\,x^2+1)}$$

input
```
integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*x^3 + 3*x)*sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2)/(2*x^2 + 1)
```

3.180.6 Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \frac{2\sqrt{6}x^3}{3}+\sqrt{6}x$$

input
```
integrate((4*x**2+2)**(1/2)*(6*x**2+3)**(1/2),x)
```

output
```
2*sqrt(6)*x**3/3 + sqrt(6)*x
```

3.180.7 Maxima [F]

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \int \sqrt{6\,x^2+3}\sqrt{4\,x^2+2}\,dx$$

input
```
integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2), x)
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \frac{1}{3}\,\sqrt{3}\sqrt{2}\left(2\,x^3+3\,x\right)$$

input
```
integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*sqrt(2)*(2*x^3 + 3*x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \int \sqrt{4\,x^2+2}\,\sqrt{6\,x^2+3}\,dx$$

input
```
int((4*x^2 + 2)^(1/2)*(6*x^2 + 3)^(1/2),x)
```

output
```
int((4*x^2 + 2)^(1/2)*(6*x^2 + 3)^(1/2), x)
```

3.181 $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

3.181.1 Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} - \frac{\sqrt{2}\sqrt{2+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} + \frac{\sqrt{2}\sqrt{2+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

output
```
x*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2)-(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*
EllipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)
*(b*x^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)+(1/(3
*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticF(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)
^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*
x^2+3))^(1/2)/(d*x^2+3)^(1/2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx=\frac{\sqrt{2}E\left(\arcsin\left(\frac{\sqrt{-d}x}{\sqrt{3}}\right)|\frac{3b}{2d}\right)}{\sqrt{-d}}$$

input
```
Integrate[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

output
```
(Sqrt[2]*EllipticE[ArcSin[(Sqrt[-d]*x)/Sqrt[3]], (3*b)/(2*d)])/Sqrt[-d]
```

3.181.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}dx$$

$$\Big\downarrow \text{ 324}$$

$$2\int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx+b\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx$$

$$\Big\downarrow \text{ 320}$$

$$b\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx+\frac{\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

$$\Big\downarrow \text{ 388}$$

$$b\left(\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{3\int \frac{\sqrt{bx^2+2}}{(dx^2+3)^{3/2}}dx}{b}\right)+\frac{\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+b\left(\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}\right)$$

input
```
Int[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

output
```
b*((x*Sqrt[2 + b*x^2])/(b*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(b*Sqrt[d]*Sqrt[(2 +
b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])) + (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF
[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/
(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

3.181.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.181. $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

3.181.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.20

method	result
default	$\frac{E\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)\sqrt{2}}{\sqrt{-d}}$
elliptic	$\frac{\sqrt{(b\,x^2+2)(d\,x^2+3)}\left(\frac{\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}-\frac{\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\left(F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)-E\left(\frac{x\sqrt{-3d}}{3},\sqrt{-}\right.\right.}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}\right.}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}$

input
```
int((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))*2^(1/2
)/(-d)^(1/2)
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx =$$
$$-\frac{9\sqrt{3}\sqrt{bd}bx\sqrt{-\frac{1}{d}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right) \mid \frac{2d}{3b}) - \sqrt{3}\sqrt{bd}(2d^2+9b)x\sqrt{-\frac{1}{d}}F(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right) \mid \frac{2d}{3b}) - 3\sqrt{}}{3bd^2x}$$

input
```
integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(9*sqrt(3)*sqrt(b*d)*b*x*sqrt(-1/d)*elliptic_e(arcsin(sqrt(3)*sqrt(-1
/d)/x), 2/3*d/b) - sqrt(3)*sqrt(b*d)*(2*d^2 + 9*b)*x*sqrt(-1/d)*elliptic_f
(arcsin(sqrt(3)*sqrt(-1/d)/x), 2/3*d/b) - 3*sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3
)*b*d)/(b*d^2*x)
```

3.181.6 Sympy [F]

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

input
```
integrate((b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

output
```
Integral(sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

3.181.7 Maxima [F]

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

input
```
integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

3.181.8 Giac [F]

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

input
```
integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx=\int \frac{\sqrt{b\,x^2+2}}{\sqrt{d\,x^2+3}}\,dx$$

input
```
int((b*x^2 + 2)^(1/2)/(d*x^2 + 3)^(1/2),x)
```

output
```
int((b*x^2 + 2)^(1/2)/(d*x^2 + 3)^(1/2), x)
```

3.182 $\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx$

3.182.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{d\sqrt{1+\frac{dx^2}{c}}} + \frac{(c+4d)\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

output
```
-EllipticE(1/2*x,2*(-d/c)^(1/2))*(d*x^2+c)^(1/2)/d/(1+d*x^2/c)^(1/2)+(c+4*
d)*EllipticF(1/2*x,2*(-d/c)^(1/2))*(1+d*x^2/c)^(1/2)/d/(d*x^2+c)^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx = \frac{2\sqrt{\frac{c+dx^2}{c}}\,E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)\middle|-\frac{c}{4d}\right)}{\sqrt{-\frac{d}{c}}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[4 - x^2]/Sqrt[c + d*x^2],x]
```

output
```
(2*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -1/4*c/d])/(Sqrt[
-(d/c)]*Sqrt[c + d*x^2])
```

3.182.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {326, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{326}$$

$$\frac{(c+4d)\int \frac{1}{\sqrt{4-x^2}\sqrt{dx^2+c}}dx}{d}-\frac{\int \frac{\sqrt{dx^2+c}}{\sqrt{4-x^2}}dx}{d}$$

$$\Big\downarrow \text{323}$$

$$\frac{(c+4d)\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{4-x^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{dx^2+c}}{\sqrt{4-x^2}}dx}{d}$$

$$\Big\downarrow \text{321}$$

$$\frac{(c+4d)\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{dx^2+c}}{\sqrt{4-x^2}}dx}{d}$$

$$\Big\downarrow \text{330}$$

$$\frac{(c+4d)\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}-\frac{\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{4-x^2}}dx}{d\sqrt{\frac{dx^2}{c}+1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{(c+4d)\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}-\frac{\sqrt{c+dx^2}E\left(\arcsin\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}$$

input
```
Int[Sqrt[4 - x^2]/Sqrt[c + d*x^2],x]
```

output
```
-((Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 + (d*x^2)/c
])) + ((c + 4*d)*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/(d*
Sqrt[c + d*x^2])
```

3.182.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

3.182.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

method	result
default	$\frac{\left(cF\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)+4F\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)d-cE\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)\right)\sqrt{\frac{d\,x^{2}+c}{c}}}{\sqrt{d\,x^{2}+c}\,d}$
elliptic	$\frac{\sqrt{-(d\,x^{2}+c)(x^{2}-4)}\left(\frac{4\sqrt{-x^{2}+4}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)}{\sqrt{-d\,x^{4}-c\,x^{2}+4d\,x^{2}+4c}}+\frac{c\sqrt{-x^{2}+4}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)-E\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)\right)}{\sqrt{-d\,x^{4}-c\,x^{2}+4d\,x^{2}+4c}\,d}\right)}{\sqrt{-x^{2}+4}\,\sqrt{d\,x^{2}+c}}$

input
```
int((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*EllipticF(1/2*x,2*(-d/c)^(1/2))+4*EllipticF(1/2*x,2*(-d/c)^(1/2))*d-c*E
llipticE(1/2*x,2*(-d/c)^(1/2)))*((d*x^2+c)/c)^(1/2)/(d*x^2+c)^(1/2)/d
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx = \frac{2\left(4\,xE\left(\arcsin\left(\frac{2}{x}\right)\,|-\frac{c}{4\,d}\right)-3\,xF\left(\arcsin\left(\frac{2}{x}\right)\,|-\frac{c}{4\,d}\right)\right)\sqrt{-d}+\sqrt{dx^2+c}\sqrt{-x^2+4}}{dx}$$

input
```
integrate((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
(2*(4*x*elliptic_e(arcsin(2/x), -1/4*c/d) - 3*x*elliptic_f(arcsin(2/x), -1
/4*c/d))*sqrt(-d) + sqrt(d*x^2 + c)*sqrt(-x^2 + 4))/(d*x)
```

3.182.6 Sympy [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-(x-2)(x+2)}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 2)*(x + 2))/sqrt(c + d*x**2), x)
```

3.182.7 Maxima [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{-x^2+4}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c), x)
```

3.182.8 Giac [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{-x^2+4}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c), x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{4-x^2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((4 - x^2)^(1/2)/(c + d*x^2)^(1/2),x)
```

output
```
int((4 - x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.183 $\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx$

3.183.1 Optimal result

Integrand size = 21, antiderivative size = 150

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}} - \frac{\sqrt{c+dx^2}E\left(\arctan\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}} + \frac{4\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}$$

output
```
x*(d*x^2+c)^(1/2)/d/(x^2+4)^(1/2)-(1/(x^2+4))^(1/2)*EllipticE(x/(x^2+4)^(1
/2),(1-4*d/c)^(1/2))*(d*x^2+c)^(1/2)/d/((d*x^2+c)/c/(x^2+4))^(1/2)+4*(1/(x
^2+4))^(1/2)*EllipticF(x/(x^2+4)^(1/2),(1-4*d/c)^(1/2))*(d*x^2+c)^(1/2)/c/
((d*x^2+c)/c/(x^2+4))^(1/2)
```

3.183.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \frac{2\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{c}{4d}\right)}{\sqrt{-\frac{d}{c}}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[4 + x^2]/Sqrt[c + d*x^2],x]
```

output
```
(2*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], c/(4*d)])/(Sqrt[-
(d/c)]*Sqrt[c + d*x^2])
```

3.183.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 324}$$

$$4\int \frac{1}{\sqrt{x^2+4}\sqrt{dx^2+c}}dx+\int \frac{x^2}{\sqrt{x^2+4}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{ 320}$$

$$\int \frac{x^2}{\sqrt{x^2+4}\sqrt{dx^2+c}}dx+\frac{4\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

$$\Big\downarrow \text{ 388}$$

$$-\frac{4\int \frac{\sqrt{dx^2+c}}{(x^2+4)^{3/2}}dx}{d}+\frac{4\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}+\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{4\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}-\frac{\sqrt{c+dx^2}E\left(\arctan\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}+\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}$$

input
```
Int[Sqrt[4 + x^2]/Sqrt[c + d*x^2],x]
```

output
```
(x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*EllipticE[ArcTan[
x/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))]) + (4
*Sqrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*d)/c])/(c*Sqrt[4 + x^2]*Sqr
t[(c + d*x^2)/(c*(4 + x^2))])
```

3.183.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

3.183.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.35

method	result
default	$\frac{2E\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{\frac{c}{d}}}{2}\right)\sqrt{\frac{d\,x^{2}+c}{c}}}{\sqrt{d\,x^{2}+c}\,\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{(d\,x^{2}+c)(x^{2}+4)}\left(\frac{2\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{x^{2}+4}\,F\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{d\,x^{4}+c\,x^{2}+4d\,x^{2}+4c}}-\frac{2\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{x^{2}+4}\left(F\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right)-E\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{d\,x^{4}+c\,x^{2}+4d\,x^{2}+4c}}\right.}{\sqrt{d\,x^{2}+c}\,\sqrt{x^{2}+4}}$

input
```
int((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*EllipticE(x*(-d/c)^(1/2),1/2*(c/d)^(1/2))*((d*x^2+c)/c)^(1/2)/(d*x^2+c)^
(1/2)/(-d/c)^(1/2)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx =$$
$$-\frac{c^2\sqrt{d}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{4\,d}{c}) - (c^2+4\,d^2)\sqrt{d}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{4\,d}{c}) - \sqrt{dx^2+c}\sqrt{x^2+4}}{cd^2x}$$

input
```
integrate((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(c^2*sqrt(d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), 4*d/c) - (c^2
+ 4*d^2)*sqrt(d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), 4*d/c) - sq
rt(d*x^2 + c)*sqrt(x^2 + 4)*c*d)/(c*d^2*x)
```

3.183.6 Sympy [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + 4)/sqrt(c + d*x**2), x)
```

3.183.7 Maxima [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c), x)
```

3.183.8 Giac [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c), x)
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2 + 4)^(1/2)/(c + d*x^2)^(1/2),x)
```

output
```
int((x^2 + 4)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.184 $\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx$

3.184.1 Optimal result

Integrand size = 23, antiderivative size = 20

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticE(1/2*x*6^(1/2),1/3*6^(1/2))*3^(1/2)
```

3.184.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[1 - x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
EllipticE[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]
```

3.184.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx$$

$$\Big\downarrow \ 327$$

$$\frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Int[Sqrt[1 - x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
EllipticE[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]
```

3.184.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.184.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\sqrt{2}\left(F\left(x,\frac{\sqrt{6}}{2}\right)+2E\left(x,\frac{\sqrt{6}}{2}\right)\right)}{6}$	23
elliptic	$\frac{\sqrt{(3x^2-2)(x^2-1)}\left(\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,F\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4-5x^2+2}}-\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\left(F\left(x,\frac{\sqrt{6}}{2}\right)-E\left(x,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^4-5x^2+2}}\right)}{\sqrt{-x^2+1}\,\sqrt{-3x^2+2}}$	128

input
```
int((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*(EllipticF(x,1/2*6^(1/2))+2*EllipticE(x,1/2*6^(1/2)))
```

3.184.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{\sqrt{3}xE\left(\arcsin\left(\frac{1}{x}\right)\mid\frac{2}{3}\right)+\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}{3\,x}$$

input
```
integrate((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(3)*x*elliptic_e(arcsin(1/x), 2/3) + sqrt(-x^2 + 1)*sqrt(-3*x^2
+ 2))/x
```

3.184.6 Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \begin{cases} \frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{2}{3}\right)}{3} & \text{for}\: x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

input
```
integrate((-x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

output
```
Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 2/3)/3, (x > -sqrt(6)/3)
& (x < sqrt(6)/3)))
```

3.184.7 Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.184.8 Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{2-3\,x^2}}\,dx$$

input
```
int((1 - x^2)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

output
```
int((1 - x^2)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.185 $\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx$

3.185.1 Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = \frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

output
```
2/3*EllipticE(1/2*x*6^(1/2),1/6*6^(1/2))*3^(1/2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = \frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[4 - x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
(2*EllipticE[ArcSin[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

3.185.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx$$

$$\Big\downarrow \; 327$$

$$\frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

input
```
Int[Sqrt[4 - x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
(2*EllipticE[ArcSin[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

3.185.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.185.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2E\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)\sqrt{3}}{3}$	18
elliptic	$\frac{\sqrt{(3x^2-2)(x^2-4)}\left(\frac{\sqrt{6}\sqrt{-6x^2+4}\sqrt{-x^2+4}\,F\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)}{3\sqrt{3x^4-14x^2+8}}-\frac{\sqrt{6}\sqrt{-6x^2+4}\sqrt{-x^2+4}\left(F\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)\right)}{3\sqrt{3x^4-14x^2+8}}\right)}{\sqrt{-x^2+4}\sqrt{-3x^2+2}}$	149

input
```
int((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*EllipticE(1/2*x*6^(1/2),1/6*6^(1/2))*3^(1/2)
```

3.185.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.52

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{8\sqrt{3}xE\left(\arcsin\left(\frac{2}{x}\right)\mid\frac{1}{6}\right) - 6\sqrt{3}xF\left(\arcsin\left(\frac{2}{x}\right)\mid\frac{1}{6}\right) + \sqrt{-x^2+4}\sqrt{-3\,x^2+2}}{3\,x}$$

input
```
integrate((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(8*sqrt(3)*x*elliptic_e(arcsin(2/x), 1/6) - 6*sqrt(3)*x*elliptic_f(ar
csin(2/x), 1/6) + sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2))/x
```

3.185.6 Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = \begin{cases} \frac{2\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{1}{6}\right)}{3} & \text{for}\: x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

input
```
integrate((-x**2+4)**(1/2)/(-3*x**2+2)**(1/2),x)
```

output
```
Piecewise((2*sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 1/6)/3, (x > -sqrt(6)/3
) & (x < sqrt(6)/3)))
```

3.185.7 Maxima [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{-x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

3.185.8 Giac [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{-x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{4-x^2}}{\sqrt{2-3\,x^2}}\,dx$$

input
```
int((4 - x^2)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

output
```
int((4 - x^2)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.186 $\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx$

3.186.1 Optimal result

Integrand size = 23, antiderivative size = 20

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|\frac{8}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticE(1/2*x*6^(1/2),2/3*6^(1/2))*3^(1/2)
```

3.186.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|\frac{8}{3}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[1 - 4*x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
EllipticE[ArcSin[Sqrt[3/2]*x], 8/3]/Sqrt[3]
```

3.186.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx$$

$$\Big\downarrow \; 327$$

$$\frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

input
```
Int[Sqrt[1 - 4*x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
EllipticE[ArcSin[Sqrt[3/2]*x], 8/3]/Sqrt[3]
```

3.186.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.186.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\sqrt{2}\left(5F\left(2x,\frac{\sqrt{6}}{4}\right)-8E\left(2x,\frac{\sqrt{6}}{4}\right)\right)}{12}$	29
elliptic	$\frac{\sqrt{(3x^2-2)(4x^2-1)}\left(\frac{\sqrt{-4x^2+1}\,\sqrt{-6x^2+4}\,F\left(2x,\frac{\sqrt{6}}{4}\right)}{4\sqrt{12x^4-11x^2+2}}-\frac{2\sqrt{-4x^2+1}\,\sqrt{-6x^2+4}\left(F\left(2x,\frac{\sqrt{6}}{4}\right)-E\left(2x,\frac{\sqrt{6}}{4}\right)\right)}{3\sqrt{12x^4-11x^2+2}}\right)}{\sqrt{-4x^2+1}\,\sqrt{-3x^2+2}}$	136

input
```
int((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*2^(1/2)*(5*EllipticF(2*x,1/4*6^(1/2))-8*EllipticE(2*x,1/4*6^(1/2)))
```

3.186.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.30

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{16\,\sqrt{2}xE\left(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right)\mid\frac{3}{8}\right)-7\,\sqrt{2}xF\left(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right)\mid\frac{3}{8}\right)+12\,\sqrt{-3\,x^2+2}\sqrt{-4\,x^2+1}}{36\,x}$$

input
```
integrate((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/36*(16*sqrt(2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/x), 3/8) - 7*sqr
t(2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(2)/x), 3/8) + 12*sqrt(-3*x^2 + 2
)*sqrt(-4*x^2 + 1))/x
```

3.186.6 Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \begin{cases} \frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{8}{3}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

input
```
integrate((-4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

output
```
Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 8/3)/3, (x > -sqrt(6)/3)
& (x < sqrt(6)/3)))
```

3.186.7 Maxima [F]

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{-4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.186.8 Giac [F]

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{-4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{1-4\,x^2}}{\sqrt{2-3\,x^2}}\,dx$$

input
```
int((1 - 4*x^2)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

output
```
int((1 - 4*x^2)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.187 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx$

3.187.1 Optimal result

Integrand size = 21, antiderivative size = 4

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = E(\arcsin(x)|-1)$$

output
```
EllipticE(x,I)
```

3.187.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = E(\arcsin(x)|-1)$$

input
```
Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^2],x]
```

output
```
EllipticE[ArcSin[x], -1]
```

3.187.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \ 327$$

$$E(\arcsin(x)|-1)$$

input
```
Int[Sqrt[1 + x^2]/Sqrt[1 - x^2],x]
```

output
```
EllipticE[ArcSin[x], -1]
```

3.187.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.187.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$E(x, i)$	5
elliptic	$\frac{\sqrt{-x^4+1}\left(\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,F(x,i)}{\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{\sqrt{-x^4+1}}\right)}{\sqrt{x^2+1}\,\sqrt{-x^2+1}}$	96

input
```
int((x^2+1)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticE(x,I)
```

3.187.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(3) = 6$.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 10.25

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \frac{-i\,xE\left(\arcsin\left(\frac{1}{x}\right)|-1\right) + 2i\,xF\left(\arcsin\left(\frac{1}{x}\right)|-1\right) - \sqrt{x^2+1}\sqrt{-x^2+1}}{x}$$

input
```
integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(-I*x*elliptic_e(arcsin(1/x), -1) + 2*I*x*elliptic_f(arcsin(1/x), -1) - sq
rt(x^2 + 1)*sqrt(-x^2 + 1))/x
```

3.187.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(2) = 4$.

Time = 1.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \begin{cases} E(\operatorname{asin}(x)|-1) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

input
```
integrate((x**2+1)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Piecewise((elliptic_e(asin(x), -1), (x > -1) & (x < 1)))
```

3.187.7 Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

3.187.8 Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}\,dx$$

input
```
int((x^2 + 1)^(1/2)/(1 - x^2)^(1/2),x)
```

output
```
int((x^2 + 1)^(1/2)/(1 - x^2)^(1/2), x)
```

3.188 $\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx$

3.188.1 Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticE(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

3.188.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[1 + x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

3.188.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{2-3x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$\frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Int[Sqrt[1 + x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

3.188.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.188.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{E\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\sqrt{3}}{3}$	19
elliptic	$\frac{\sqrt{-(3x^2-2)(x^2+1)}\left(\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{-3x^4-x^2+2}}-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\right)}{6\sqrt{-3x^4-x^2+2}}\right)}{\sqrt{x^2+1}\,\sqrt{-3x^2+2}}$	147

input
```
int((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*EllipticE(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

3.188.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.80

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{4\sqrt{3}\sqrt{2}\sqrt{-3}xE\left(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)\middle|-\frac{3}{2}\right)-13\sqrt{3}\sqrt{2}\sqrt{-3}xF\left(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)\middle|-\frac{3}{2}\right)+18\sqrt{x^2+1}\sqrt{-3x^2+}}{54x}$$

input
```
integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/54*(4*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/
x), -3/2) - 13*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_f(arcsin(1/3*sqrt(3)*sq
rt(2)/x), -3/2) + 18*sqrt(x^2 + 1)*sqrt(-3*x^2 + 2))/x
```

3.188.6 Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx = \begin{cases} \frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{2}{3}\right)}{3} & \text{for } x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

input
```
integrate((x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

output
```
Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -2/3)/3, (x > -sqrt(6)/3)
 & (x < sqrt(6)/3)))
```

3.188.7 Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx=\int \frac{\sqrt{x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.188.8 Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx=\int \frac{\sqrt{x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx=\int \frac{\sqrt{x^2+1}}{\sqrt{2-3\,x^2}}\,dx$$

input
```
int((x^2 + 1)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

output
```
int((x^2 + 1)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.189 $\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx$

3.189.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx=\frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

output
```
2/3*EllipticE(1/2*x*6^(1/2),1/6*I*6^(1/2))*3^(1/2)
```

3.189.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx=\frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[4 + x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
(2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

3.189.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{2-3x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$\frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

input
```
Int[Sqrt[4 + x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
(2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

3.189.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.189.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2E\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)\sqrt{3}}{3}$	19
elliptic	$\frac{\sqrt{-(3x^2-2)(x^2+4)}\left(\frac{\sqrt{6}\sqrt{-6x^2+4}\sqrt{x^2+4}F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)}{3\sqrt{-3x^4-10x^2+8}}-\frac{\sqrt{6}\sqrt{-6x^2+4}\sqrt{x^2+4}\left(F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)\right)}{3\sqrt{-3x^4-10x^2+8}}\right)}{\sqrt{x^2+4}\sqrt{-3x^2+2}}$	147

input
```
int((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*EllipticE(1/2*x*6^(1/2),1/6*I*6^(1/2))*3^(1/2)
```

3.189.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.62

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{2\sqrt{3}\sqrt{2}\sqrt{-3}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)|-6) - 20\sqrt{3}\sqrt{2}\sqrt{-3}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)|-6) + 9\sqrt{x^2+4}\sqrt{-3x^2+}}{27x}$$

input
```
integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/27*(2*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/
x), -6) - 20*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt
(2)/x), -6) + 9*sqrt(x^2 + 4)*sqrt(-3*x^2 + 2))/x
```

3.189.6 Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx = \begin{cases} \frac{2\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{1}{6}\right)}{3} & \text{for } x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

input
```
integrate((x**2+4)**(1/2)/(-3*x**2+2)**(1/2),x)
```

output
```
Piecewise((2*sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -1/6)/3, (x > -sqrt(6)/
3) & (x < sqrt(6)/3)))
```

3.189.7 Maxima [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

3.189.8 Giac [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{2-3\,x^2}}\,dx$$

input
```
int((x^2 + 4)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

output
```
int((x^2 + 4)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.190 $\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx$

3.190.1 Optimal result

Integrand size = 23, antiderivative size = 20

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticE(1/2*x*6^(1/2),2/3*I*6^(1/2))*3^(1/2)
```

3.190.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[1 + 4*x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/Sqrt[3]
```

3.190.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2+1}}{\sqrt{2-3x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$\frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

input
```
Int[Sqrt[1 + 4*x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/Sqrt[3]
```

3.190.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.190.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{E\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)\sqrt{3}}{3}$	19
elliptic	$\frac{\sqrt{-(3x^2-2)(4x^2+1)}\left(\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{4x^2+1}\,F\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)}{6\sqrt{-12x^4+5x^2+2}}-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{4x^2+1}\left(F\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)\right)}{6\sqrt{-12x^4+5x^2+2}}\right)}{\sqrt{4x^2+1}\,\sqrt{-3x^2+2}}$	155

input
```
int((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*EllipticE(1/2*x*6^(1/2),2/3*I*6^(1/2))*3^(1/2)
```

3.190.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.90

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{16\sqrt{3}\sqrt{2}\sqrt{-3}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)|-\frac{3}{8}) - 25\sqrt{3}\sqrt{2}\sqrt{-3}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)|-\frac{3}{8}) + 36\sqrt{4x^2+1}\sqrt{-3x}}{108x}$$

input
```
integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/108*(16*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2
)/x), -3/8) - 25*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_f(arcsin(1/3*sqrt(3)*
sqrt(2)/x), -3/8) + 36*sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2))/x
```

3.190.6 Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = \begin{cases} \frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{8}{3}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

input
```
integrate((4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

output
```
Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -8/3)/3, (x > -sqrt(6)/3)
 & (x < sqrt(6)/3)))
```

3.190.7 Maxima [F]

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.190.8 Giac [F]

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{4\,x^2+1}}{\sqrt{2-3\,x^2}}\,dx$$

input
```
int((4*x^2 + 1)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

output
```
int((4*x^2 + 1)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.191 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx$

3.191.1 Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = -E(\arcsin(x)|-1) + 2\,\text{EllipticF}(\arcsin(x), -1)$$

output
```
-EllipticE(x,I)+2*EllipticF(x,I)
```

3.191.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = -iE(i\,\text{arcsinh}(x)|-1)$$

input
```
Integrate[Sqrt[1 - x^2]/Sqrt[1 + x^2],x]
```

output
```
(-I)*EllipticE[I*ArcSinh[x], -1]
```

3.191.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 326}$$

$$2\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+1}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 284}$$

$$2\int \frac{1}{\sqrt{1-x^4}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$2\int \frac{1}{\sqrt{1-x^4}}dx-E(\arcsin(x)|-1)$$

$$\Big\downarrow \text{ 762}$$

$$2\,\text{EllipticF}(\arcsin(x),-1)-E(\arcsin(x)|-1)$$

input
```
Int[Sqrt[1 - x^2]/Sqrt[1 + x^2],x]
```

output
```
-EllipticE[ArcSin[x], -1] + 2*EllipticF[ArcSin[x], -1]
```

3.191.3.1 Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

3.191.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-E(x,i)+2F(x,i)$	14
elliptic	$\frac{\sqrt{-x^4+1}\left(\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,F(x,i)}{\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{\sqrt{-x^4+1}}\right)}{\sqrt{x^2+1}\,\sqrt{-x^2+1}}$	95

input
```
int((-x^2+1)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-EllipticE(x,I)+2*EllipticF(x,I)
```

3.191.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \frac{i\,xE\left(\arcsin\left(\frac{1}{x}\right)\mid -1\right)+\sqrt{x^2+1}\sqrt{-x^2+1}}{x}$$

input
```
integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(I*x*elliptic_e(arcsin(1/x), -1) + sqrt(x^2 + 1)*sqrt(-x^2 + 1))/x
```

3.191.6 Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x**2 + 1), x)
```

3.191.7 Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

3.191.8 Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{x^2+1}}\,dx$$

input
```
int((1 - x^2)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((1 - x^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.192 $\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx$

3.192.1 Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{1}{3}\sqrt{2}E\left(\arcsin(x)\left|-\frac{3}{2}\right.\right)+\frac{5\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{3\sqrt{2}}$$

output
```
5/6*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)-1/3*EllipticE(x,1/2*I*6^(1/2))*2^(1
/2)
```

3.192.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{iE\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\,|-\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[1 - x^2]/Sqrt[2 + 3*x^2],x]
```

output
```
((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -2/3])/Sqrt[3]
```

3.192.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {326, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{ 326}$$

$$\frac{5}{3}\int \frac{1}{\sqrt{1-x^2}\sqrt{3x^2+2}}dx-\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\frac{5\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{3\sqrt{2}}-\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$\frac{5\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{3\sqrt{2}}-\frac{1}{3}\sqrt{2}E\left(\arcsin(x)\left|-\frac{3}{2}\right.\right)$$

input
```
Int[Sqrt[1 - x^2]/Sqrt[2 + 3*x^2],x]
```

output
```
-1/3*(Sqrt[2]*EllipticE[ArcSin[x], -3/2]) + (5*EllipticF[ArcSin[x], -3/2])
/(3*Sqrt[2])
```

3.192.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.192.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\left(5F\left(x,\frac{i\sqrt{6}}{2}\right)-2E\left(x,\frac{i\sqrt{6}}{2}\right)\right)\sqrt{2}}{6}$	27
elliptic	$\frac{\sqrt{-(3x^2+2)(x^2-1)}\left(\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,F\left(x,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{-3x^4+x^2+2}}+\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\left(F\left(x,\frac{i\sqrt{6}}{2}\right)-E\left(x,\frac{i\sqrt{6}}{2}\right)\right)}{3\sqrt{-3x^4+x^2+2}}\right)}{\sqrt{-x^2+1}\,\sqrt{3x^2+2}}$	128

input
```
int((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(5*EllipticF(x,1/2*I*6^(1/2))-2*EllipticE(x,1/2*I*6^(1/2)))*2^(1/2)
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx=\frac{\sqrt{-3}xE(\arcsin\left(\frac{1}{x}\right)\mid-\frac{2}{3})+\sqrt{3\,x^2+2}\sqrt{-x^2+1}}{3\,x}$$

input
```
integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(-3)*x*elliptic_e(arcsin(1/x), -2/3) + sqrt(3*x^2 + 2)*sqrt(-x^2
+ 1))/x
```

3.192.6 Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{3x^2+2}}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/sqrt(3*x**2 + 2), x)
```

3.192.7 Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.192.8 Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
int((1 - x^2)^(1/2)/(3*x^2 + 2)^(1/2),x)
```

output
```
int((1 - x^2)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.193 $\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx$

3.193.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{1}{3}\sqrt{2}E\left(\arcsin\left(\frac{x}{2}\right)\middle|-6\right)+\frac{7}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-6\right)$$

output
```
-1/3*EllipticE(1/2*x,I*6^(1/2))*2^(1/2)+7/3*EllipticF(1/2*x,I*6^(1/2))*2^(
1/2)
```

3.193.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{2iE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[4 - x^2]/Sqrt[2 + 3*x^2],x]
```

output
```
((-2*I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

3.193.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {326, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4-x^2}}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{326}$$

$$\frac{14}{3}\int \frac{1}{\sqrt{4-x^2}\sqrt{3x^2+2}}dx-\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{4-x^2}}dx$$

$$\Big\downarrow \text{321}$$

$$\frac{7}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-6\right)-\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{4-x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$\frac{7}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-6\right)-\frac{1}{3}\sqrt{2}E\left(\arcsin\left(\frac{x}{2}\right)\middle|-6\right)$$

input
```
Int[Sqrt[4 - x^2]/Sqrt[2 + 3*x^2],x]
```

output
```
-1/3*(Sqrt[2]*EllipticE[ArcSin[x/2], -6]) + (7*Sqrt[2]*EllipticF[ArcSin[x/
2], -6])/3
```

3.193.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.193.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(7F\left(\frac{x}{2},i\sqrt{6}\right)-E\left(\frac{x}{2},i\sqrt{6}\right)\right)\sqrt{2}}{3}$	31
elliptic	$\frac{\sqrt{-(3x^2+2)(x^2-4)}\left(\frac{2\sqrt{-x^2+4}\,\sqrt{6x^2+4}\,F\left(\frac{x}{2},i\sqrt{6}\right)}{\sqrt{-3x^4+10x^2+8}}+\frac{\sqrt{-x^2+4}\,\sqrt{6x^2+4}\,\left(F\left(\frac{x}{2},i\sqrt{6}\right)-E\left(\frac{x}{2},i\sqrt{6}\right)\right)}{3\sqrt{-3x^4+10x^2+8}}\right)}{\sqrt{-x^2+4}\,\sqrt{3x^2+2}}$	138

input
```
int((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(7*EllipticF(1/2*x,I*6^(1/2))-EllipticE(1/2*x,I*6^(1/2)))*2^(1/2)
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = \frac{8\sqrt{-3}xE(\arcsin\left(\frac{2}{x}\right)|-\frac{1}{6})-6\sqrt{-3}xF(\arcsin\left(\frac{2}{x}\right)|-\frac{1}{6})+\sqrt{3x^2+2}\sqrt{-x^2+4}}{3x}$$

input
```
integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(8*sqrt(-3)*x*elliptic_e(arcsin(2/x), -1/6) - 6*sqrt(-3)*x*elliptic_f(
arcsin(2/x), -1/6) + sqrt(3*x^2 + 2)*sqrt(-x^2 + 4))/x
```

3.193.6 Sympy [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-(x-2)(x+2)}}{\sqrt{3x^2+2}}\,dx$$

input
```
integrate((-x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 2)*(x + 2))/sqrt(3*x**2 + 2), x)
```

3.193.7 Maxima [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2), x)
```

3.193.8 Giac [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2), x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx=\int \frac{\sqrt{4-x^2}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
int((4 - x^2)^(1/2)/(3*x^2 + 2)^(1/2),x)
```

output
```
int((4 - x^2)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.194 $\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}} \, dx$

3.194.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}} \, dx = -\frac{2}{3}\sqrt{2}E\left(\arcsin(2x)\left|-\frac{3}{8}\right.\right) + \frac{11 \operatorname{EllipticF}\left(\arcsin(2x), -\frac{3}{8}\right)}{6\sqrt{2}}$$

output
```
11/12*EllipticF(2*x,1/4*I*6^(1/2))*2^(1/2)-2/3*EllipticE(2*x,1/4*I*6^(1/2)
)*2^(1/2)
```

3.194.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}} \, dx = -\frac{iE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[1 - 4*x^2]/Sqrt[2 + 3*x^2],x]
```

output
```
((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -8/3])/Sqrt[3]
```

3.194.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {326, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{ 326}$$

$$\frac{11}{3}\int \frac{1}{\sqrt{1-4x^2}\sqrt{3x^2+2}}dx-\frac{4}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{1-4x^2}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\frac{11\,\text{EllipticF}\left(\arcsin(2x),-\frac{3}{8}\right)}{6\sqrt{2}}-\frac{4}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{1-4x^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$\frac{11\,\text{EllipticF}\left(\arcsin(2x),-\frac{3}{8}\right)}{6\sqrt{2}}-\frac{2}{3}\sqrt{2}E\left(\arcsin(2x)\left|-\frac{3}{8}\right.\right)$$

input
```
Int[Sqrt[1 - 4*x^2]/Sqrt[2 + 3*x^2],x]
```

output
```
(-2*Sqrt[2]*EllipticE[ArcSin[2*x], -3/8])/3 + (11*EllipticF[ArcSin[2*x], -
3/8])/(6*Sqrt[2])
```

3.194.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.194.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(11F\left(2x,\frac{i\sqrt{6}}{4}\right)-8E\left(2x,\frac{i\sqrt{6}}{4}\right)\right)\sqrt{2}}{12}$	31
elliptic	$\frac{\sqrt{-(3x^2+2)(4x^2-1)}\left(\frac{\sqrt{-4x^2+1}\,\sqrt{6x^2+4}\,F\left(2x,\frac{i\sqrt{6}}{4}\right)}{4\sqrt{-12x^4-5x^2+2}}+\frac{2\sqrt{-4x^2+1}\,\sqrt{6x^2+4}\left(F\left(2x,\frac{i\sqrt{6}}{4}\right)-E\left(2x,\frac{i\sqrt{6}}{4}\right)\right)}{3\sqrt{-12x^4-5x^2+2}}\right)}{\sqrt{-4x^2+1}\,\sqrt{3x^2+2}}$	140

input
```
int((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(11*EllipticF(2*x,1/4*I*6^(1/2))-8*EllipticE(2*x,1/4*I*6^(1/2)))*2^(1
/2)
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = \frac{\sqrt{-3}xE(\arcsin\left(\frac{1}{2\,x}\right)\,|-\frac{8}{3})+3\,\sqrt{-3}xF(\arcsin\left(\frac{1}{2\,x}\right)\,|-\frac{8}{3})+4\,\sqrt{3\,x^2+2}\sqrt{-4\,x^2+1}}{12\,x}$$

input
```
integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(-3)*x*elliptic_e(arcsin(1/2/x), -8/3) + 3*sqrt(-3)*x*elliptic_f
(arcsin(1/2/x), -8/3) + 4*sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1))/x
```

3.194.6 Sympy [F]

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-(2x-1)(2x+1)}}{\sqrt{3x^2+2}}\,dx$$

input
```
integrate((-4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(-(2*x - 1)*(2*x + 1))/sqrt(3*x**2 + 2), x)
```

3.194.7 Maxima [F]

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.194.8 Giac [F]

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{1-4\,x^2}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
int((1 - 4*x^2)^(1/2)/(3*x^2 + 2)^(1/2),x)
```

output
```
int((1 - 4*x^2)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.195 $\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx$

3.195.1 Optimal result

Integrand size = 21, antiderivative size = 131

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx = \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}} - \frac{\sqrt{2}\sqrt{2+3x^2}E\left(\arctan(x)\,\middle|-\frac{1}{2}\right)}{3\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}} + \frac{\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

output
```
1/3*x*(3*x^2+2)^(1/2)/(x^2+1)^(1/2)+1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2
+1)^(1/2),1/2*I*2^(1/2))*(3*x^2+2)^(1/2)*2^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
-1/3*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)*(3
*x^2+2)^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

3.195.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{iE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\middle|\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[1 + x^2]/Sqrt[2 + 3*x^2],x]
```

output
```
((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3])/Sqrt[3]
```

3.195.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{ 324}$$

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{3x^2+2}}dx+\int \frac{x^2}{\sqrt{x^2+1}\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{ 320}$$

$$\int \frac{x^2}{\sqrt{x^2+1}\sqrt{3x^2+2}}dx+\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

$$\Big\downarrow \text{ 388}$$

$$-\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{(x^2+1)^{3/2}}dx+\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}+\frac{\sqrt{3x^2+2}x}{3\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}+\frac{\sqrt{3x^2+2}x}{3\sqrt{x^2+1}}$$

input
```
Int[Sqrt[1 + x^2]/Sqrt[2 + 3*x^2],x]
```

output
```
(x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE
[ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)]) + (Sqrt[2
 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2 + 3*x
^2)/(1 + x^2)])
```

3.195.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

3.195.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.23

method	result	size
default	$-\frac{i\left(F\left(ix,\frac{\sqrt{6}}{2}\right)+2E\left(ix,\frac{\sqrt{6}}{2}\right)\right)\sqrt{2}}{6}$	30
elliptic	$\frac{\sqrt{(3x^2+2)(x^2+1)}\left(-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,F\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(F\left(ix,\frac{\sqrt{6}}{2}\right)-E\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^4+5x^2+2}}\right)}{\sqrt{3x^2+2}\,\sqrt{x^2+1}}$	133

input
```
int((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*(EllipticF(I*x,1/2*6^(1/2))+2*EllipticE(I*x,1/2*6^(1/2)))*2^(1/2)
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{4\sqrt{-2}xE\left(\arcsin\left(\frac{\sqrt{3}\sqrt{-2}}{3x}\right) \mid \frac{3}{2}\right) - 13\sqrt{-2}xF\left(\arcsin\left(\frac{\sqrt{3}\sqrt{-2}}{3x}\right) \mid \frac{3}{2}\right) - 6\sqrt{3x^2+2}\sqrt{x^2+1}}{18x}$$

input
```
integrate((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/18*(4*sqrt(-2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(-2)/x), 3/2) - 13*s
qrt(-2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(-2)/x), 3/2) - 6*sqrt(3*x^2 +
 2)*sqrt(x^2 + 1))/x
```

3.195.6 Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{3x^2+2}}\,dx$$

input
```
integrate((x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + 1)/sqrt(3*x**2 + 2), x)
```

3.195.7 Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{3x^2+2}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.195.8 Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx=\int \frac{\sqrt{x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx=\int \frac{\sqrt{x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
int((x^2 + 1)^(1/2)/(3*x^2 + 2)^(1/2),x)
```

output
```
int((x^2 + 1)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.196 $\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx$

3.196.1 Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}} - \frac{\sqrt{2}\sqrt{2+3x^2}E\left(\arctan\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}} + \frac{2\sqrt{2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),-5\right)}{\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}$$

output
```
1/3*x*(3*x^2+2)^(1/2)/(x^2+4)^(1/2)-1/3*(1/(x^2+4))^(1/2)*EllipticE(x/(x^2
+4)^(1/2),I*5^(1/2))*2^(1/2)*(3*x^2+2)^(1/2)/((3*x^2+2)/(x^2+4))^(1/2)+2*(
1/(x^2+4))^(1/2)*EllipticF(x/(x^2+4)^(1/2),I*5^(1/2))*2^(1/2)*(3*x^2+2)^(1
/2)/((3*x^2+2)/(x^2+4))^(1/2)
```

3.196.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{2iE\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\middle|\frac{1}{6}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[4 + x^2]/Sqrt[2 + 3*x^2],x]
```

output
```
((-2*I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

3.196.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{324}$$

$$4\int \frac{1}{\sqrt{x^2+4}\sqrt{3x^2+2}}dx+\int \frac{x^2}{\sqrt{x^2+4}\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{320}$$

$$\int \frac{x^2}{\sqrt{x^2+4}\sqrt{3x^2+2}}dx+\frac{2\sqrt{2}\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),-5\right)}{\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}$$

$$\Big\downarrow \text{388}$$

$$-\frac{4}{3}\int \frac{\sqrt{3x^2+2}}{\left(x^2+4\right)^{3/2}}dx+\frac{2\sqrt{2}\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),-5\right)}{\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}+\frac{\sqrt{3x^2+2}x}{3\sqrt{x^2+4}}$$

$$\Big\downarrow \text{313}$$

$$\frac{2\sqrt{2}\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),-5\right)}{\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\arctan\left(\frac{x}{2}\right)|-5\right)}{3\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}+\frac{\sqrt{3x^2+2}x}{3\sqrt{x^2+4}}$$

input
```
Int[Sqrt[4 + x^2]/Sqrt[2 + 3*x^2],x]
```

output
```
(x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE
[ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4 + x^2)]) + (2*Sqrt
[2]*Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x/2], -5])/(Sqrt[4 + x^2]*Sqrt[(2 + 3
*x^2)/(4 + x^2)])
```

3.196.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.196.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.19

method	result	size
default	$-\frac{i\left(5F\left(\frac{ix}{2},\sqrt{6}\right)+E\left(\frac{ix}{2},\sqrt{6}\right)\right)\sqrt{2}}{3}$	26
elliptic	$\frac{\sqrt{(3x^2+2)(x^2+4)}\left(-\frac{2i\sqrt{x^2+4}\,\sqrt{6x^2+4}\,F\left(\frac{ix}{2},\sqrt{6}\right)}{\sqrt{3x^4+14x^2+8}}+\frac{i\sqrt{x^2+4}\,\sqrt{6x^2+4}\,\left(F\left(\frac{ix}{2},\sqrt{6}\right)-E\left(\frac{ix}{2},\sqrt{6}\right)\right)}{3\sqrt{3x^4+14x^2+8}}\right)}{\sqrt{3x^2+2}\,\sqrt{x^2+4}}$	127

input
```
int((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*(5*EllipticF(1/2*I*x,6^(1/2))+EllipticE(1/2*I*x,6^(1/2)))*2^(1/2)
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx$$
$$= -\frac{2\sqrt{-2}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{-2}}{3x}\right)|6) - 20\sqrt{-2}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{-2}}{3x}\right)|6) - 3\sqrt{3x^2+2}\sqrt{x^2+4}}{9x}$$

input
```
integrate((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(2*sqrt(-2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(-2)/x), 6) - 20*sqrt
(-2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(-2)/x), 6) - 3*sqrt(3*x^2 + 2)*s
qrt(x^2 + 4))/x
```

3.196.6 Sympy [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{3x^2+2}}\,dx$$

input
```
integrate((x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + 4)/sqrt(3*x**2 + 2), x)
```

3.196.7 Maxima [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{3x^2+2}}\,dx$$

input
```
integrate((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2), x)
```

3.196.8 Giac [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2), x)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
int((x^2 + 4)^(1/2)/(3*x^2 + 2)^(1/2),x)
```

output
```
int((x^2 + 4)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.197 $\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}} dx$

3.197.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}} dx = \frac{4x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}} - \frac{2\sqrt{2}\sqrt{2+3x^2}E\left(\arctan(2x)\,|\,\frac{5}{8}\right)}{3\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}} + \frac{\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(2x),\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}$$

output
```
4/3*x*(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2)+1/4*(1/(4*x^2+1))^(1/2)*EllipticF(2*
x/(4*x^2+1)^(1/2),1/4*10^(1/2))*(3*x^2+2)^(1/2)*2^(1/2)/((3*x^2+2)/(4*x^2+
1))^(1/2)-2/3*(1/(4*x^2+1))^(1/2)*EllipticE(2*x/(4*x^2+1)^(1/2),1/4*10^(1/
2))*2^(1/2)*(3*x^2+2)^(1/2)/((3*x^2+2)/(4*x^2+1))^(1/2)
```

3.197.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}} dx = -\frac{iE\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\,|\,\frac{8}{3}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[1 + 4*x^2]/Sqrt[2 + 3*x^2],x]
```

output
```
((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 8/3])/Sqrt[3]
```

3.197.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2+1}}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{ 324}$$

$$\int \frac{1}{\sqrt{3x^2+2}\sqrt{4x^2+1}}dx+4\int \frac{x^2}{\sqrt{3x^2+2}\sqrt{4x^2+1}}dx$$

$$\Big\downarrow \text{ 320}$$

$$4\int \frac{x^2}{\sqrt{3x^2+2}\sqrt{4x^2+1}}dx+\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan(2x),\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}$$

$$\Big\downarrow \text{ 388}$$

$$4\left(\frac{x\sqrt{3x^2+2}}{3\sqrt{4x^2+1}}-\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{(4x^2+1)^{3/2}}dx\right)+\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan(2x),\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan(2x),\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}+4\left(\frac{x\sqrt{3x^2+2}}{3\sqrt{4x^2+1}}-\frac{\sqrt{3x^2+2}E\left(\arctan(2x)\left|\frac{5}{8}\right.\right)}{3\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}\right)$$

input
```
Int[Sqrt[1 + 4*x^2]/Sqrt[2 + 3*x^2],x]
```

output
```
4*((x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (Sqrt[2 + 3*x^2]*EllipticE[Ar
cTan[2*x], 5/8])/(3*Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
) + (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[2*x], 5/8])/(2*Sqrt[2]*Sqrt[(2 + 3*x
^2)/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
```

3.197.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.197.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.14

method	result	size
default	$-\frac{i E\left(\frac{i x \sqrt{6}}{2}, \frac{2 \sqrt{6}}{3}\right) \sqrt{3}}{3}$	20
elliptic	$\frac{\sqrt{(3x^2+2)(4x^2+1)}\left(-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{4x^2+1}\,F\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)}{6\sqrt{12x^4+11x^2+2}}+\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{4x^2+1}\left(F\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)-E\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)\right)}{6\sqrt{12x^4+11x^2+2}}\right)}{\sqrt{3x^2+2}\,\sqrt{4x^2+1}}$	156

input
```
int((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*EllipticE(1/2*I*x*6^(1/2),2/3*6^(1/2))*3^(1/2)
```

3.197.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx = \frac{-i\sqrt{3}xE\left(\arcsin\left(\frac{i}{2x}\right)\mid\frac{8}{3}\right)+5i\sqrt{3}xF\left(\arcsin\left(\frac{i}{2x}\right)\mid\frac{8}{3}\right)+4\sqrt{4x^2+1}\sqrt{3x^2+2}}{12x}$$

input
```
integrate((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(-I*sqrt(3)*x*elliptic_e(arcsin(1/2*I/x), 8/3) + 5*I*sqrt(3)*x*ellipt
ic_f(arcsin(1/2*I/x), 8/3) + 4*sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2))/x
```

3.197.6 Sympy [F]

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{4x^2+1}}{\sqrt{3x^2+2}}\,dx$$

input
```
integrate((4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(4*x**2 + 1)/sqrt(3*x**2 + 2), x)
```

3.197.7 Maxima [F]

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{4x^2+1}}{\sqrt{3x^2+2}}\,dx$$

input
```
integrate((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.197.8 Giac [F]

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

input
```
int((4*x^2 + 1)^(1/2)/(3*x^2 + 2)^(1/2),x)
```

output
```
int((4*x^2 + 1)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.198 $\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx$

3.198.1 Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx=\frac{\sqrt{1-2x^2}E\left(\arcsin\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+2x^2}}$$

output
```
1/2*EllipticE(x*2^(1/2),1/2*2^(1/2))*(-2*x^2+1)^(1/2)*2^(1/2)/(2*x^2-1)^(1
/2)
```

3.198.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx=\frac{\sqrt{1-2x^2}E\left(\arcsin\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{-2+4x^2}}$$

input
```
Integrate[Sqrt[1 - x^2]/Sqrt[-1 + 2*x^2],x]
```

output
```
(Sqrt[1 - 2*x^2]*EllipticE[ArcSin[Sqrt[2]*x], 1/2])/Sqrt[-2 + 4*x^2]
```

3.198.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {331, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2x^2-1}}dx$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{1-2x^2}\int \frac{\sqrt{1-x^2}}{\sqrt{1-2x^2}}dx}{\sqrt{2x^2-1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{1-2x^2}E\left(\arcsin\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2x^2-1}}$$

input
```
Int[Sqrt[1 - x^2]/Sqrt[-1 + 2*x^2],x]
```

output
```
(Sqrt[1 - 2*x^2]*EllipticE[ArcSin[Sqrt[2]*x], 1/2])/(Sqrt[2]*Sqrt[-1 + 2*x
^2])
```

3.198.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.198.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\left(F\left(x,\sqrt{2}\right)+E\left(x,\sqrt{2}\right)\right)\sqrt{-2x^2+1}}{2\sqrt{2x^2-1}}$	32
elliptic	$\frac{\sqrt{-(2x^2-1)(x^2-1)}\left(\frac{\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\,F\left(x,\sqrt{2}\right)}{\sqrt{-2x^4+3x^2-1}}-\frac{\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\,\left(F\left(x,\sqrt{2}\right)-E\left(x,\sqrt{2}\right)\right)}{2\sqrt{-2x^4+3x^2-1}}\right)}{\sqrt{-x^2+1}\,\sqrt{2x^2-1}}$	122

input
```
int((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(EllipticF(x,2^(1/2))+EllipticE(x,2^(1/2)))*(-2*x^2+1)^(1/2)/(2*x^2-1)
^(1/2)
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{-2}xE\left(\arcsin\left(\frac{1}{x}\right) \mid \frac{1}{2}\right) + \sqrt{2\,x^2-1}\sqrt{-x^2+1}}{2\,x}$$

input
```
integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-2)*x*elliptic_e(arcsin(1/x), 1/2) + sqrt(2*x^2 - 1)*sqrt(-x^2 +
 1))/x
```

3.198.6 Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx = \int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{\sqrt{2x^2-1}}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/sqrt(2*x**2 - 1), x)
```

3.198.7 Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{2\,x^2-1}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1), x)
```

3.198.8 Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{2\,x^2-1}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1), x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{2\,x^2-1}}\,dx$$

input
```
int((1 - x^2)^(1/2)/(2*x^2 - 1)^(1/2),x)
```

output
```
int((1 - x^2)^(1/2)/(2*x^2 - 1)^(1/2), x)
```

3.199 $\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx$

3.199.1 Optimal result

Integrand size = 23, antiderivative size = 423

$$\begin{aligned}\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx &= -\frac{8(bc-2ad)\left(6b^2c^2-11abcd+11a^2d^2\right)x\sqrt{a+bx^2}}{105d^3\sqrt{c+dx^2}} \\ &+ \frac{b(24b^2c^2-71abcd+71a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} \\ &- \frac{6b(bc-2ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35d^2} + \frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d} \\ &+ \frac{8\sqrt{c}(bc-2ad)\left(6b^2c^2-11abcd+11a^2d^2\right)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &- \frac{\sqrt{c}(3bc-7ad)\left(8b^2c^2-11abcd+15a^2d^2\right)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-8/105*(-2*a*d+b*c)*(11*a^2*d^2-11*a*b*c*d+6*b^2*c^2)*x*(b*x^2+a)^(1/2)/d^
3/(d*x^2+c)^(1/2)+8/105*(-2*a*d+b*c)*(11*a^2*d^2-11*a*b*c*d+6*b^2*c^2)*(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(7/2)/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/105*(-7*a*d+3*b*c)*(15*a^2*d^2-11*a*b*
c*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)
/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(7
/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-6/35*b*(-2*a*d+b*c)*x*
(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/d^2+1/7*b*x*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2
)/d+1/105*b*(71*a^2*d^2-71*a*b*c*d+24*b^2*c^2)*x*(b*x^2+a)^(1/2)*(d*x^2+c)
^(1/2)/d^3
```

3.199.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.65 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(122a^2d^2+abd(-89c+66dx^2)+3b^2(8c^2-6cdx^2+5d^2x^4)\right)}{}$$

input
```
Integrate[(a + b*x^2)^(7/2)/Sqrt[c + d*x^2],x]
```

output
```
(b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(122*a^2*d^2 + a*b*d*(-89*c + 66*
d*x^2) + 3*b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) - (8*I)*b*c*(-6*b^3*c^3 +
23*a*b^2*c^2*d - 33*a^2*b*c*d^2 + 22*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(48*b^4*c^4
 - 208*a*b^3*c^3*d + 353*a^2*b^2*c^2*d^2 - 298*a^3*b*c*d^3 + 105*a^4*d^4)*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)])/(105*Sqrt[b/a]*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.199.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {318, 25, 403, 25, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{7/2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \; 318$$

$$\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(6b(bc-2ad)x^2+a(bc-7ad)\right)}{\sqrt{dx^2+c}}dx}{7d}+\frac{bx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(6b(bc-2ad)x^2+a(bc-7ad)\right)}{\sqrt{dx^2+c}}dx}{7d}$$

$$\Big\downarrow \; 403$$

$$\frac{bx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{\int -\frac{\sqrt{bx^2+a}\left(b\left(24b^2c^2-71abdc+71a^2d^2\right)x^2+a\left(6b^2c^2-17abdc+35a^2d^2\right)\right)}{\sqrt{dx^2+c}}dx}{5d}+\frac{6bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-2ad)}{5d}}{7d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{6bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-2ad)}{5d}-\frac{\int \frac{\sqrt{bx^2+a}\left(b\left(24b^2c^2-71abdc+71a^2d^2\right)x^2+a\left(6b^2c^2-17abdc+35a^2d^2\right)\right)}{\sqrt{dx^2+c}}dx}{5d}}{7d}$$

$$\Big\downarrow \; 403$$

$$\frac{bx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{6bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-2ad)}{5d}-\frac{\frac{\int -\frac{8b(bc-2ad)\left(6b^2c^2-11abdc+11a^2d^2\right)x^2+a(3bc-7ad)\left(8b^2c^2-11abdc+15a^2d^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(71a^2d^2-71\right.}{3d}}{5d}}{7d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{6bx(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-2ad)}{5d}-\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(71a^2d^2-71abcd+24b^2c^2)}{3d}-\frac{\int\frac{8b(bc-2ad)(6b^2c^2-11abdc+11a^2d^2)x^2+a(3bc-7ad)(8b^2c^2-11abdc}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3d}}{5d}}{7d}$$

$\downarrow$ 406

$$\frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{6bx(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-2ad)}{5d}-\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(71a^2d^2-71abcd+24b^2c^2)}{3d}-\frac{a(3bc-7ad)(15a^2d^2-11abcd+8b^2c^2)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+8b(bc-2a}{3d}}{5d}}{7d}$$

$\downarrow$ 320

$$\frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{6bx(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-2ad)}{5d}-\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(71a^2d^2-71abcd+24b^2c^2)}{3d}-\frac{8b(bc-2ad)(11a^2d^2-11abcd+6b^2c^2)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+b}}{}}{}}{5d}}{7d}$$

$\downarrow$ 388

$$\frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{6bx(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-2ad)}{5d}-\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(71a^2d^2-71abcd+24b^2c^2)}{3d}-\frac{8b(bc-2ad)(11a^2d^2-11abcd+6b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right.}{}}{5d}}{7d}$$

$\downarrow$ 313

$$\frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{6bx(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-2ad)}{5d}-\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(71a^2d^2-71abcd+24b^2c^2)}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(3bc-7ad)(15a^2d^2-11abcd+8b^2c^2)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{}}{5d}}{7d}$$

input
```
Int[(a + b*x^2)^(7/2)/Sqrt[c + d*x^2],x]
```

3.199. $\int\frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx$

output
```
(b*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*d) - ((6*b*(b*c - 2*a*d)*x*(a +
 b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*d) - ((b*(24*b^2*c^2 - 71*a*b*c*d + 71*a
^2*d^2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - (8*b*(b*c - 2*a*d)*(6*b
^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2])
 - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
])) + (Sqrt[c]*(3*b*c - 7*a*d)*(8*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[
a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[
d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d))/(5*d))/(
7*d)
```

3.199.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.199. $\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx$

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.199.4 Maple [A] (verified)

Time = 7.29 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.48

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{b^3x^5\sqrt{bdx^4+adx^2+cbx^2+ac}}{7d}+\frac{\left(4ab^3-\frac{b^3(6ad+6bc)}{7d}\right)x^3\sqrt{bdx^4+adx^2+cbx^2+ac}}{5bd}+\frac{\left(6a^2b^2-\frac{5ab^3c}{7d}-\frac{\left(4ab^3-\frac{b^3(6ad\cdots}{7d}\right)\cdots}{5\cdots}\right)\cdots}{\cdots}\cdots\right.}{\cdots}$
risch	$\frac{bx(15b^2d^2x^4+66x^2abd^2-18x^2b^2cd+122a^2d^2-89abcd+24b^2c^2)\sqrt{bx^2+a}\sqrt{dx^2+c}}{105d^3}+\frac{\left(\frac{105a^4d^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\cdots}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+cbx^2+ac}}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(15\sqrt{-\frac{b}{a}}b^4d^4x^9+81\sqrt{-\frac{b}{a}}ab^3d^4x^7-3\sqrt{-\frac{b}{a}}b^4cd^3x^7+188\sqrt{-\frac{b}{a}}a^2b^2d^4x^5-26\sqrt{-\frac{b}{a}}ab^3cd^3x^5+6\sqrt{-\frac{b}{a}}b^4c^2d^2x\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*b^3/d*x^5
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(4*a*b^3-1/7*b^3/d*(6*a*d+6*b*c))
/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(6*a^2*b^2-5/7*a*b^3*c/d-
1/5*(4*a*b^3-1/7*b^3/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)+(a^4-1/3*(6*a^2*b^2-5/7*a*b^3*c/d-1/5*(4*a*b^3-1/
7*b^3/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(4*a^3*b-3/5*(4*a*b^3-1/7*b^3/d*(6*
a*d+6*b*c))/b/d*a*c-1/3*(6*a^2*b^2-5/7*a*b^3*c/d-1/5*(4*a*b^3-1/7*b^3/d*(6
*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \frac{8\left(6\,b^4c^5-23\,ab^3c^4d+33\,a^2b^2c^3d^2-22\,a^3bc^2d^3\right)\sqrt{bd}x\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)-(4\cdots}{\cdots}$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(8*(6*b^4*c^5 - 23*a*b^3*c^4*d + 33*a^2*b^2*c^3*d^2 - 22*a^3*b*c^2*d
^3)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (
48*b^4*c^5 - 184*a*b^3*c^4*d + 122*a^3*b*c*d^4 - 105*a^4*d^5 + 24*(11*a^2*
b^2 + a*b^3)*c^3*d^2 - (176*a^3*b + 89*a^2*b^2)*c^2*d^3)*sqrt(b*d)*x*sqrt(
-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (15*b^4*c*d^4*x^6 - 48
*b^4*c^4*d + 184*a*b^3*c^3*d^2 - 264*a^2*b^2*c^2*d^3 + 176*a^3*b*c*d^4 - 6
*(3*b^4*c^2*d^3 - 11*a*b^3*c*d^4)*x^4 + (24*b^4*c^3*d^2 - 89*a*b^3*c^2*d^3
 + 122*a^2*b^2*c*d^4)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^5*x)
```

3.199.6 Sympy [F]

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{7}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(7/2)/sqrt(c + d*x**2), x)
```

3.199.7 Maxima [F]

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c), x)
```

3.199.8 Giac [F]

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c), x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{7/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(7/2)/(c + d*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(7/2)/(c + d*x^2)^(1/2), x)
```

3.200 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

3.200.1 Optimal result

Integrand size = 23, antiderivative size = 344

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx &= \frac{(8b^2c^2-23abcd+23a^2d^2)\,x\sqrt{a+bx^2}}{15d^2\sqrt{c+dx^2}} \\ &- \frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d} \\ &- \frac{\sqrt{c}(8b^2c^2-23abcd+23a^2d^2)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &+ \frac{\sqrt{c}(4b^2c^2-11abcd+15a^2d^2)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
1/15*(23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/d^2/(d*x^2+c)^(1/
2)-1/15*(23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c
)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c
^(1/2)*(b*x^2+a)^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(
1/2)+1/15*(15*a^2*d^2-11*a*b*c*d+4*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2
/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
*c^(1/2)*(b*x^2+a)^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)
^(1/2)+1/5*b*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/d-4/15*b*(-2*a*d+b*c)*x*(b*
x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^2
```

3.200.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{\frac{b}{a}}dx(a+bx^2)\,(c+dx^2)\,(-4bc+11ad+3bdx^2)-ibc(8b^2c^2-23abcd+23a^2d^2)\,\sqrt{1}}{}$$

input

```
Integrate[(a + b*x^2)^(5/2)/Sqrt[c + d*x^2],x]
```

output

```
(b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + 11*a*d + 3*b*d*x^2) - I
*b*c*(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(-8*b^3*c^3 +
27*a*b^2*c^2*d - 34*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*
d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.200.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {318, 25, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 318$$

$$\frac{\int -\frac{\sqrt{bx^2+a}\,(4b(bc-2ad)x^2+a(bc-5ad))}{\sqrt{dx^2+c}}dx}{5d}+\frac{bx\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d}$$

$$\Big\downarrow \ 25$$

$$\frac{bx\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{\int \frac{\sqrt{bx^2+a}\,(4b(bc-2ad)x^2+a(bc-5ad))}{\sqrt{dx^2+c}}dx}{5d}$$

$$\Big\downarrow \ 403$$

$$\frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{\frac{\int -\frac{b\left(8b^2c^2-23abdc+23a^2d^2\right)x^2+a\left(4b^2c^2-11abdc+15a^2d^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{3d}}{5d}$$

$\downarrow$ 25

$$\frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{3d}-\frac{\int \frac{b\left(8b^2c^2-23abdc+23a^2d^2\right)x^2+a\left(4b^2c^2-11abdc+15a^2d^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{5d}$$

$\downarrow$ 406

$$\frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{3d}-\frac{a\left(15a^2d^2-11abcd+4b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\left(23a^2d^2-23abcd+8b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{5d}$$

$\downarrow$ 320

$$\frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{3d}-\frac{b\left(23a^2d^2-23abcd+8b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}}{5d}$$

$\downarrow$ 388

$$\frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{3d}-\frac{b\left(23a^2d^2-23abcd+8b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)\text{EllipticF}\left(\arctan\left(\cdots\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}}{5d}$$

$\downarrow$ 313

$$\frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(23a^2d^2-23abcd+8b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a\cdots}}{\cdots}\right.}{3d}}{5d}$$

3.200. $\int \frac{\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}dx$

input
```
Int[(a + b*x^2)^(5/2)/Sqrt[c + d*x^2],x]
```

output
```
(b*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*d) - ((4*b*(b*c - 2*a*d)*x*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - (b*(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d
^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(4*b^2*c^2 - 11
*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[
c + d*x^2]))/(3*d))/(5*d)
```

3.200.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.200.4 Maple [A] (verified)

Time = 6.43 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.26

method	result
elliptic	$\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{b^2 x^3 \sqrt{bd x^4+ad x^2+cb x^2+ac}}{5d}+\frac{\left(3a b^2-\frac{b^2(4ad+4bc)}{5d}\right) x \sqrt{bd x^4+ad x^2+cb x^2+ac}}{3bd}+\frac{\left(a^3-\frac{\left(3a b^2-\frac{b^2(4ad+4bc)}{5d}\right)ac}{3bd}\right)\sqrt{\cdots}}{\sqrt{-\frac{b}{a}}\sqrt{bd \cdots}}\cdots$
risch	$\frac{bx(3bd x^2+11ad-4bc)\sqrt{b x^2+a}\sqrt{d x^2+c}}{15d^2}+\frac{\left(\frac{15a^3 d^2\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x^2+ac}}+\frac{4a b^2 c^2\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\cdots}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x^2 \cdots}}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{b x^2+a}\sqrt{d x^2+c}\left(3\sqrt{-\frac{b}{a}}b^3 d^3 x^7+14\sqrt{-\frac{b}{a}}a b^2 d^3 x^5-\sqrt{-\frac{b}{a}}b^3 c d^2 x^5+11\sqrt{-\frac{b}{a}}a^2 b d^3 x^3+10\sqrt{-\frac{b}{a}}a b^2 c d^2 x^3-4\sqrt{-\frac{b}{a}}b^3 c^2 d x^3+15\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

3.200. $\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*b^2/d*x^3
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(3*a*b^2-1/5*b^2/d*(4*a*d+4*b*c))
/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a^3-1/3*(3*a*b^2-1/5*b^2/d*(4*
a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))-(3*a^2*b-3/5*a*b^2*c/d-1/3*(3*a*b^2-1/5*b^2/d*(4*a*d+4*b*c))/b/d*
(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(8\,b^3c^4-23\,ab^2c^3d+23\,a^2bc^2d^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(8\,b^3c^4-23\,ab^2c^3d-11\,a^2bcd^3+15}{}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((8*b^3*c^4 - 23*a*b^2*c^3*d + 23*a^2*b*c^2*d^2)*sqrt(b*d)*x*sqrt(-c
/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^3*c^4 - 23*a*b^2*c^
3*d - 11*a^2*b*c*d^3 + 15*a^3*d^4 + (23*a^2*b + 4*a*b^2)*c^2*d^2)*sqrt(b*d
)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b^3*c*d^3*
x^4 + 8*b^3*c^3*d - 23*a*b^2*c^2*d^2 + 23*a^2*b*c*d^3 - (4*b^3*c^2*d^2 - 1
1*a*b^2*c*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^4*x)
```

3.200.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

3.200.7 Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c), x)
```

3.200.8 Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x^2)^(1/2), x)
```

3.201 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

3.201.1 Optimal result

Integrand size = 23, antiderivative size = 260

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = -\frac{2(bc-2ad)x\sqrt{a+bx^2}}{3d\sqrt{c+dx^2}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} + \frac{2\sqrt{c}(bc-2ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{\sqrt{c}(bc-3ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-2/3*(-2*a*d+b*c)*x*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(1/2)+2/3*(-2*a*d+b*c)*(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*(-3*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(
1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)
^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d
*x^2+c)^(1/2)+1/3*b*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

3.201.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)-2ibc(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{a}{b}\right.}{3\sqrt{\frac{b}{a}}d^2\sqrt{a+bx}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/Sqrt[c + d*x^2],x]
```

output
```
(b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - (2*I)*b*c*(-(b*c) + 2*a*d)*Sqrt
[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)] - I*(2*b^2*c^2 - 5*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a
]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.201.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {318, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 318$$

$$\frac{\int -\frac{2b(bc-2ad)x^2+a(bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\int \frac{2b(bc-2ad)x^2+a(bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}$$

$$\Big\downarrow \; 406$$

$$\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{a(bc-3ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2b(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}$$

$$\Big\downarrow \ 320$$

$$\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{2b(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}$$

$$\Big\downarrow \ 388$$

$$\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{2b(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}$$

$$\Big\downarrow \ 313$$

$$\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2b(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3d}$$

input
```
Int[(a + b*x^2)^(3/2)/Sqrt[c + d*x^2],x]
```

output
```
(b*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - (2*b*(b*c - 2*a*d)*((x*Sqrt[
a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*c - 3*a*d)*Sqrt[a + b*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d)
```

3.201. $\int \frac{\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}dx$

3.201.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.201. $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

3.201.4 Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{bx\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3d}+\frac{\left(a^2-\frac{bac}{3d}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{\left(2ab-\frac{b(2ad+2bc)}{3d}\right)c\sqrt{1+\frac{b\,x^2}{a}}}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
risch	$\frac{bx\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{3d}+\frac{\left(\frac{3a^2d\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{abc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{(4abd-2}{}\right.}{3d\sqrt{b\,x^2+a}\,\sqrt{d}}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(\sqrt{-\frac{b}{a}}\,b^2d^2x^5+\sqrt{-\frac{b}{a}}\,ab\,d^2x^3+\sqrt{-\frac{b}{a}}\,b^2cd\,x^3+3\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2d^2-5\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\right.}{}$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*b/d*x*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a^2-1/3*b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*a*b-1/3*b/d*(2*a*d+2*b*c))*c/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{2\,(b^2c^3-2\,abc^2d)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(2\,b^2c^3-4\,abc^2d+abcd^2-3\,a^2d}{3\,bc}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*(b^2*c^3 - 2*a*b*c^2*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sq
rt(-c/d)/x), a*d/(b*c)) - (2*b^2*c^3 - 4*a*b*c^2*d + a*b*c*d^2 - 3*a^2*d^3
)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (b^
2*c*d^2*x^2 - 2*b^2*c^2*d + 4*a*b*c*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/
(b*c*d^3*x)
```

3.201.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

3.201.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c), x)
```

3.201.8 Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c), x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

input

```
int((a + b*x^2)^(3/2)/(c + d*x^2)^(1/2),x)
```

output

```
int((a + b*x^2)^(3/2)/(c + d*x^2)^(1/2), x)
```

3.202 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.202.1 Optimal result

Integrand size = 23, antiderivative size = 194

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(
b*x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx=\frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*
c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2])
```

3.202.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{324}$$

$$a\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{320}$$

$$b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{388}$$

$$b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+$$

$$b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)$$

input
```
Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

output
```
b*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

3.202.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.202.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

method	result
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\left(aF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bcF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bcE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)}{\left(bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(\frac{a\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}-\frac{bc\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\cdots\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}\,d}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(a
*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx =$$
$$-\frac{\sqrt{bd}bc^2x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bx^2+a}\sqrt{dx^2+c}bcd - (bc^2+ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right.\cdots}{bcd^2x}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*b*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
 - sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*c*d - (b*c^2 + a*d^2)*sqrt(b*d)*x*sqr
t(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)
```

3.202.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

3.202.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

3.202.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c + d*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.203 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.203.1 Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x
^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2
+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right),\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
 (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.203.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

3.203.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.203.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+cb\,x^2+ac)}$	100
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}$	122

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a
)^(1/2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{ac}\sqrt{-\frac{b}{a}}F\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right)}{bc}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c))/(b*c)
```

3.203.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.203.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.203.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.204 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

3.204.1 Optimal result

Integrand size = 23, antiderivative size = 273

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{dx\sqrt{a+bx^2}}{a(bc-ad)\sqrt{c+dx^2}} + \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-d*x*(b*x^2+a)^(1/2)/a/(-a*d+b*c)/(d*x^2+c)^(1/2)+(1/(1+d*x^2/c))^(1/2)*(1
+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^
(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+
c))^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Elliptic
F(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(
b*x^2+a)^(1/2)/a/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2
)+b*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.41

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx=\frac{-bx(c+dx^2)+\frac{ad\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}}}{a(-bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(b*x*(c + d*x^2)) + (a*d*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[ArcSin[Sqrt[-(d/c)]*x], (b*c)/(a*d)])/Sqrt[-(d/c)])/(a*(-(b*c) + a*d)*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.204.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {316, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 316}$$

$$\frac{bx\sqrt{c+dx^2}}{a\sqrt{a+bx^2}(bc-ad)}-\frac{\int \frac{d\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bx\sqrt{c+dx^2}}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{ 324}$$

$$\frac{bx\sqrt{c+dx^2}}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d\left(a\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{a(bc-ad)}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{bx\sqrt{c+dx^2}}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d\left(b\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{a(bc-ad)}$$

$$\Big\downarrow \text{388}$$

$$\frac{bx\sqrt{c+dx^2}}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d\left(b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{a(bc-ad)}$$

$$\Big\downarrow \text{313}$$

$$\frac{bx\sqrt{c+dx^2}}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{a(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(b*x*Sqrt[c + d*x^2])/(a*(b*c - a*d)*Sqrt[a + b*x^2]) - (d*(b*((x*Sqrt[a +
 b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*(b*c - a*d))
```

3.204.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

3.204. $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.204.4 Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.91

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, b d\, x^{3}+a \sqrt{\frac{b\,x^{2}+a}{a}}\, \sqrt{\frac{d\,x^{2}+c}{c}}\, F\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) d-\sqrt{\frac{b\,x^{2}+a}{a}}\, \sqrt{\frac{d\,x^{2}+c}{c}}\, F\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) b c+\sqrt{\frac{b\,x^{2}+a}{a}}\, \sqrt{\frac{d\,x^{2}+c}{c}}\, E\left(x \sqrt{-\frac{b}{a}},\right.\right.}{a \sqrt{-\frac{b}{a}}\, (a d-b c)(b d\,x^{4}+a d\,x^{2}+c b\,x^{2}+a c)}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}\left(-\frac{\left(b d\,x^{2}+b c\right) x}{a(a d-b c)\sqrt{\left(x^{2}+\frac{a}{b}\right)\left(b d\,x^{2}+b c\right)}}+\frac{\left(\frac{1}{a}+\frac{b c}{a(a d-b c)}\right)\sqrt{1+\frac{b\,x^{2}}{a}}\, \sqrt{1+\frac{d\,x^{2}}{c}}\, F\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\, \sqrt{b d\,x^{4}+a d\,x^{2}+c b\,x^{2}+a c}}-\frac{b c \sqrt{1+\frac{b\,x^{2}}{a}}\, \sqrt{1+d}}{}\right.}{\sqrt{b\,x^{2}+a}\, \sqrt{d\,x^{2}+c}}$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*d*x^3+a*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c+((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c-(-b/a)^(1/2)*b*
c*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/a/(-b/a)^(1/2)/(a*d-b*c)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}ab^2cx-(b^3cx^2+ab^2c)\sqrt{ac}\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right) \cdots}{a^3b^2c^2-a^4bcd+(a^2b^3c^2 \cdots}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*b^2*c*x - (b^3*c*x^2 + a*b^2*c)*sqrt(a*
c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (a*b^2*c + a^3
*d + (b^3*c + a^2*b*d)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(
-b/a)), a*d/(b*c)))/(a^3*b^2*c^2 - a^4*b*c*d + (a^2*b^3*c^2 - a^3*b^2*c*d)
*x^2)
```

3.204.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

3.204.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

3.204.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

3.205 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

3.205.1 Optimal result

Integrand size = 23, antiderivative size = 255

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{3a(bc-ad)(a+bx^2)^{3/2}} + \frac{2\sqrt{b}(bc-2ad)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{c}\sqrt{d}(bc-3ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/3*(-3*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)
^(1/2)/a^2/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/
3*b*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(3/2)+2/3*(-2*a*d+b*c)*(1/(1+
b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^
(1/2),(1-a*d/b/c)^(1/2))*b^(1/2)*(d*x^2+c)^(1/2)/a^(3/2)/(-a*d+b*c)^2/(b*x
^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

3.205.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{\frac{b}{a}}x(c+dx^2)\left(-5a^2d+2b^2cx^2+ab(3c-4dx^2)\right)-2ibc(-bc+2ad)\left(a+bx\right.}{}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
(b*Sqrt[b/a]*x*(c + d*x^2)*(-5*a^2*d + 2*b^2*c*x^2 + a*b*(3*c - 4*d*x^2))
- (2*I)*b*c*(-(b*c) + 2*a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(2*b^2*c^2 - 5*a
*b*c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*(b*c - a*d)
^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

3.205.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {316, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{ 316}$$

$$\frac{bx\sqrt{c+dx^2}}{3a(a+bx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{bdx^2+2bc-3ad}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{bdx^2+2bc-3ad}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{3a(a+bx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{ 400}$$

$$\frac{\frac{2b(bc-2ad)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{3/2}}dx}{bc-ad}-\frac{d(bc-3ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{3a(a+bx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{2\sqrt{b}\sqrt{c+dx^2}(bc-2ad)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{d(bc-3ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{3a(a+bx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{2\sqrt{b}\sqrt{c+dx^2}(bc-2ad)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{3a(a+bx^2)^{3/2}(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
(b*x*Sqrt[c + d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + ((2*Sqrt[b]*(b
*c - 2*a*d)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*
d)/(b*c)])/(Sqrt[a]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a
 + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*a*(b*c - a*d))
```

3.205.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

3.205.4 Maple [A] (verified)

Time = 5.63 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{x\sqrt{bd x^4+ad x^2+cb x^2+ac}}{3ba(ad-bc)\left(x^2+\frac{a}{b}\right)^2}-\frac{2\left(bd x^2+bc\right)x(2ad-bc)}{3a^2(ad-bc)^2\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd x^2+bc\right)}}+\frac{\left(-\frac{d}{3(ad-bc)a}+\frac{\frac{4ad}{3}-\frac{2bc}{3}}{a^2(ad-bc)}+\frac{2bc(2ad-bc)}{3a^2(ad-bc)^2}\right)\sqrt{1+\frac{b}{}}}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+}}\right.}{\sqrt{b x^2+a}\sqrt{d x}}$
default	$\frac{-4\sqrt{-\frac{b}{a}}\,a b^2 d^2 x^5+2\sqrt{-\frac{b}{a}}\,b^3 cd x^5+3\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2 b d^2 x^2-5\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a b^2 cd x}{}$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/b/a/(a*d
-b*c)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2-2/3*(b*d*x^2+b*c)/
a^2/(a*d-b*c)^2*x*(2*a*d-b*c)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(-1/3*d/(a*d
-b*c)/a+2/3*(2*a*d-b*c)/(a*d-b*c)/a^2+2/3*b*c/a^2/(a*d-b*c)^2*(2*a*d-b*c))
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-2/3*b*(2*a*
d-b*c)/(a*d-b*c)^2/a^2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.73

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{2\left(a^2b^3c^2-2\,a^3b^2cd+(b^5c^2-2\,ab^4cd)x^4+2\,(ab^4c^2-2\,a^2b^3cd)x^2\right)\sqrt{ac}\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}\right)-(2}{}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + (b^5*c^2 - 2*a*b^4*c*d)*x^4 + 2*(a*
b^4*c^2 - 2*a^2*b^3*c*d)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqr
t(-b/a)), a*d/(b*c)) - (2*a^2*b^3*c^2 - 3*a^5*d^2 + (2*b^5*c^2 - 3*a^3*b^2
*d^2 + (a^2*b^3 - 4*a*b^4)*c*d)*x^4 + (a^4*b - 4*a^3*b^2)*c*d + 2*(2*a*b^4
*c^2 - 3*a^4*b*d^2 + (a^3*b^2 - 4*a^2*b^3)*c*d)*x^2)*sqrt(a*c)*sqrt(-b/a)*
elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*(a*b^4*c^2 - 2*a^2*b^3*c*
d)*x^3 + (3*a^2*b^3*c^2 - 5*a^3*b^2*c*d)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c
))/(a^5*b^3*c^3 - 2*a^6*b^2*c^2*d + a^7*b*c*d^2 + (a^3*b^5*c^3 - 2*a^4*b^4
*c^2*d + a^5*b^3*c*d^2)*x^4 + 2*(a^4*b^4*c^3 - 2*a^5*b^3*c^2*d + a^6*b^2*c
*d^2)*x^2)
```

3.205.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

3.205.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

3.205.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)), x)
```

3.206 $\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

3.206.1 Optimal result

Integrand size = 23, antiderivative size = 334

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)(a+bx^2)^{5/2}} + \frac{4b(bc-2ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)^2(a+bx^2)^{3/2}} + \frac{\sqrt{b}(8b^2c^2-23abcd+23a^2d^2)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}(bc-ad)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{c}\sqrt{d}(4b^2c^2-11abcd+15a^2d^2)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^3(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/15*(15*a^2*d^2-11*a*b*c*d+4*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(
1/2)*d^(1/2)*(b*x^2+a)^(1/2)/a^3/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(d*x^2+c)^(1/2)+1/5*b*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(5/2)+4
/15*b*(-2*a*d+b*c)*x*(d*x^2+c)^(1/2)/a^2/(-a*d+b*c)^2/(b*x^2+a)^(3/2)+1/15
*(23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)*(1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)
*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))*b^(1/2)*
(d*x^2+c)^(1/2)/a^(5/2)/(-a*d+b*c)^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2
+a))^(1/2)
```

3.206.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{\frac{b}{a}}x(c+dx^2)\left(3a^2(bc-ad)^2+4a(bc-2ad)(bc-ad)\left(a+bx^2\right)+(8b^2c^2-\right.}{}$$

input
```
Integrate[1/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

output
```
(b*Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + 4*a*(b*c - 2*a*d)*(b*c -
 a*d)*(a + b*x^2) + (8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*(a + b*x^2)^2) +
 I*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(b*c*(8*b^2*c^2 -
 23*a*b*c*d + 23*a^2*d^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
 (-8*b^3*c^3 + 27*a*b^2*c^2*d - 34*a^2*b*c*d^2 + 15*a^3*d^3)*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*a^3*Sqrt[b/a]*(b*c - a*d)^3*(a + b
*x^2)^(5/2)*Sqrt[c + d*x^2])
```

3.206.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {316, 25, 402, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \; 316$$

$$\frac{bx\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}(bc-ad)} - \frac{\int -\frac{3bdx^2+4bc-5ad}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx}{5a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3bdx^2+4bc-5ad}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx}{5a(bc-ad)} + \frac{bx\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{4bx\sqrt{c+dx^2}(bc-2ad)}{3a(a+bx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{8b^2c^2-19abdc+15a^2d^2+4bd(bc-2ad)x^2}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}}{5a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{8b^2c^2-19abdc+15a^2d^2+4bd(bc-2ad)x^2}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}+\frac{4bx\sqrt{c+dx^2}(bc-2ad)}{3a(a+bx^2)^{3/2}(bc-ad)}}{5a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 400$$

$$\frac{\frac{\frac{b\left(23a^2d^2-23abcd+8b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{d\left(15a^2d^2-11abcd+4b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{4bx\sqrt{c+dx^2}(bc-2ad)}{3a(a+bx^2)^{3/2}(bc-ad)}}{5a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{d\left(15a^2d^2-11abcd+4b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{4bx\sqrt{c+dx^2}(bc-2ad)}{3a(a+bx^2)^{3/2}(bc-ad)}}{5a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}+\frac{4bx\sqrt{c+dx^2}(bc-2ad)}{3a(a+bx^2)^{3/2}(bc-ad)}}{5a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}(bc-ad)}$$

input

```
Int[1/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

3.206. $\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

output
```
(b*x*Sqrt[c + d*x^2])/(5*a*(b*c - a*d)*(a + b*x^2)^(5/2)) + ((4*b*(b*c - 2
*a*d)*x*Sqrt[c + d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + ((Sqrt[b]*(
8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqr
t[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[a]*(b*c - a*d)*Sqrt[a + b*x^2]*S
qrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(4*b^2*c^2 - 11*a
*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2]))/(3*a*(b*c - a*d)))/(5*a*(b*c - a*d))
```

3.206.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.206.4 Maple [A] (verified)

Time = 6.83 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.72

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{x\sqrt{b d\,x^4+a d\,x^2+c b\,x^2+a c}}{5b^2 a(a d-b c)\left(x^2+\frac{a}{b}\right)^3}-\frac{4(2a d-b c)x\sqrt{b d\,x^4+a d\,x^2+c b\,x^2+a c}}{15b(a d-b c)^2 a^2\left(x^2+\frac{a}{b}\right)^2}-\frac{\left(b d\,x^2+b c\right)x\left(23a^2d^2-23abcd+8b^2c^2\right)}{15a^3(a d-b c)^3\sqrt{\left(x^2+\frac{a}{b}\right)\left(b d\,x^2+b c\right)}}+\frac{\left(-\frac{4d(2}{15a^2(}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5/b^2/a/(a
*d-b*c)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^3-4/15*(2*a*d-b*c)
/b/(a*d-b*c)^2/a^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2-1/15*
(b*d*x^2+b*c)/a^3/(a*d-b*c)^3*x*(23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)/((x^2+a/
b)*(b*d*x^2+b*c))^(1/2)+(-4/15*d*(2*a*d-b*c)/a^2/(a*d-b*c)^2+1/15/(a*d-b*c
)^2*(23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)/a^3+1/15*b*c/a^3/(a*d-b*c)^3*(23*a^2
*d^2-23*a*b*c*d+8*b^2*c^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))-1/15*b*(23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)/(a*d-b*c)^3/a^3*c
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.206.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(316) = 632$.

Time = 0.11 (sec) , antiderivative size = 845, normalized size of antiderivative = 2.53

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(8\,a^3b^4c^3 - 23\,a^4b^3c^2d + 23\,a^5b^2cd^2 + (8\,b^7c^3 - 23\,ab^6c^2d + 23\,a^2b^5cd^2)x^6 + 3\,(8\,ab^6c^3 - 23\,a^2b^5c^2d + 23\,a^3$$

input
```
integrate(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((8*a^3*b^4*c^3 - 23*a^4*b^3*c^2*d + 23*a^5*b^2*c*d^2 + (8*b^7*c^3 -
 23*a*b^6*c^2*d + 23*a^2*b^5*c*d^2)*x^6 + 3*(8*a*b^6*c^3 - 23*a^2*b^5*c^2*
d + 23*a^3*b^4*c*d^2)*x^4 + 3*(8*a^2*b^5*c^3 - 23*a^3*b^4*c^2*d + 23*a^4*b
^3*c*d^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(
b*c)) - (8*a^3*b^4*c^3 + 15*a^7*d^3 + (8*b^7*c^3 + 15*a^4*b^3*d^3 + (4*a^2
*b^5 - 23*a*b^6)*c^2*d - (11*a^3*b^4 - 23*a^2*b^5)*c*d^2)*x^6 + 3*(8*a*b^6
*c^3 + 15*a^5*b^2*d^3 + (4*a^3*b^4 - 23*a^2*b^5)*c^2*d - (11*a^4*b^3 - 23*
a^3*b^4)*c*d^2)*x^4 + (4*a^5*b^2 - 23*a^4*b^3)*c^2*d - (11*a^6*b - 23*a^5*
b^2)*c*d^2 + 3*(8*a^2*b^5*c^3 + 15*a^6*b*d^3 + (4*a^4*b^3 - 23*a^3*b^4)*c^
2*d - (11*a^5*b^2 - 23*a^4*b^3)*c*d^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_
f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((8*a*b^6*c^3 - 23*a^2*b^5*c^2*d + 23
*a^3*b^4*c*d^2)*x^5 + 2*(10*a^2*b^5*c^3 - 29*a^3*b^4*c^2*d + 27*a^4*b^3*c*
d^2)*x^3 + (15*a^3*b^4*c^3 - 41*a^4*b^3*c^2*d + 34*a^5*b^2*c*d^2)*x)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c))/(a^7*b^4*c^4 - 3*a^8*b^3*c^3*d + 3*a^9*b^2*c^2
*d^2 - a^10*b*c*d^3 + (a^4*b^7*c^4 - 3*a^5*b^6*c^3*d + 3*a^6*b^5*c^2*d^2 -
 a^7*b^4*c*d^3)*x^6 + 3*(a^5*b^6*c^4 - 3*a^6*b^5*c^3*d + 3*a^7*b^4*c^2*d^2
 - a^8*b^3*c*d^3)*x^4 + 3*(a^6*b^5*c^4 - 3*a^7*b^4*c^3*d + 3*a^8*b^3*c^2*d
^2 - a^9*b^2*c*d^3)*x^2)
```

3.206.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

3.206.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)), x)
```

3.206.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{7/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)), x)
```

3.207 $\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx$

3.207.1 Optimal result

Integrand size = 23, antiderivative size = 445

$$\begin{aligned}\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx &= \frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\,x\sqrt{a+bx^2}}{15cd^3\sqrt{c+dx^2}} \\ &- \frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b(24b^2c^2-43abcd+15a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15cd^3} \\ &+ \frac{b(6bc-5ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5cd^2} \\ &- \frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &+ \frac{b\sqrt{c}(24b^2c^2-61abcd+45a^2d^2)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-(-a*d+b*c)*x*(b*x^2+a)^(5/2)/c/d/(d*x^2+c)^(1/2)+1/15*(-15*a^3*d^3+103*a^
2*b*c*d^2-128*a*b^2*c^2*d+48*b^3*c^3)*x*(b*x^2+a)^(1/2)/c/d^3/(d*x^2+c)^(1
/2)-1/15*(-15*a^3*d^3+103*a^2*b*c*d^2-128*a*b^2*c^2*d+48*b^3*c^3)*(1/(1+d*
x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1
/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(7/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/15*b*(45*a^2*d^2-61*a*b*c*d+24*b^2*c^2)*(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(7/2)/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/5*b*(-5*a*d+6*b*c)*x*(b*x^2+a)^(3/2)*(
d*x^2+c)^(1/2)/c/d^2-1/15*b*(15*a^2*d^2-43*a*b*c*d+24*b^2*c^2)*x*(b*x^2+a)
^(1/2)*(d*x^2+c)^(1/2)/c/d^3
```

3.207.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.17 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(-45a^2bcd^2+15a^3d^3+ab^2cd(61c+16dx^2)-3b^3c(8c^2+2cdx^2-d^2x^4\right.}{}$$

input
```
Integrate[(a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(-45*a^2*b*c*d^2 + 15*a^3*d^3 + a*b^2*c*d*(61*c
 + 16*d*x^2) - 3*b^3*c*(8*c^2 + 2*c*d*x^2 - d^2*x^4)) + I*b*c*(-48*b^3*c^3
 + 128*a*b^2*c^2*d - 103*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (4*I)*b*
c*(12*b^3*c^3 - 38*a*b^2*c^2*d + 41*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
)/(15*Sqrt[b/a]*c*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.207.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {315, 27, 403, 25, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{7/2}}{\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\int \frac{b\left(bx^2+a\right)^{3/2}\left((6bc-5ad)x^2+ac\right)}{\sqrt{dx^2+c}}dx}{cd}-\frac{x\left(a+bx^2\right)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\int \frac{\left(bx^2+a\right)^{3/2}\left((6bc-5ad)x^2+ac\right)}{\sqrt{dx^2+c}}dx}{cd}-\frac{x\left(a+bx^2\right)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{b\left(\frac{\int -\frac{\sqrt{bx^2+a}\left(\left(24b^2c^2-43abdc+15a^2d^2\right)x^2+2ac(3bc-5ad)\right)}{\sqrt{dx^2+c}}dx}{5d}+\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(6bc-5ad)}{5d}\right)}{cd}-\frac{x\left(a+bx^2\right)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(6bc-5ad)}{5d}-\frac{\int \frac{\sqrt{bx^2+a}\left(\left(24b^2c^2-43abdc+15a^2d^2\right)x^2+2ac(3bc-5ad)\right)}{\sqrt{dx^2+c}}dx}{5d}\right)}{cd}-\frac{x\left(a+bx^2\right)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{b\left(\frac{x(a+bx^2)^{3/2}\sqrt{c+dx^2}(6bc-5ad)}{5d}-\frac{\frac{\int -\frac{\left(48b^3c^3-128ab^2dc^2+103a^2bd^2c-15a^3d^3\right)x^2+ac\left(24b^2c^2-61abdc+45a^2d^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(15a^2d^2-43\right.}{3d}}{5d}\right.}{cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\downarrow$ 25

$$\frac{b\left(\frac{x(a+bx^2)^{3/2}\sqrt{c+dx^2}(6bc-5ad)}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(15a^2d^2-43abcd+24b^2c^2\right)}{3d}-\frac{\int\frac{\left(48b^3c^3-128ab^2dc^2+103a^2bd^2c-15a^3d^3\right)x^2+ac\left(24b^2c^2-61abdc\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3d}}{5d}\right.}{cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\downarrow$ 406

$$\frac{b\left(\frac{x(a+bx^2)^{3/2}\sqrt{c+dx^2}(6bc-5ad)}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(15a^2d^2-43abcd+24b^2c^2\right)}{3d}-\frac{ac\left(45a^2d^2-61abcd+24b^2c^2\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(-15a^3d^3+103\right.}{3d}}{5d}\right.}{cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\downarrow$ 320

$$\frac{b\left(\frac{x(a+bx^2)^{3/2}\sqrt{c+dx^2}(6bc-5ad)}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(15a^2d^2-43abcd+24b^2c^2\right)}{3d}-\frac{\left(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+}{}}{5d}\right.}{cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\downarrow$ 388

3.207. $\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx$

$$\frac{b\left(\frac{x(a+bx^2)^{3/2}\sqrt{c+dx^2}(6bc-5ad)}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(15a^2d^2-43abcd+24b^2c^2\right)}{3d}-\frac{\left(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{b}}{(dx^2}\cdots}{\cdots}\right)}{\cdots}}{5d}\right)}{cd}$$

$$\frac{x(a+bx^2)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\downarrow$ 313

$$\frac{b\left(\frac{x(a+bx^2)^{3/2}\sqrt{c+dx^2}(6bc-5ad)}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(15a^2d^2-43abcd+24b^2c^2\right)}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\cdots\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\cdots}{\cdots}}{5d}\right)}{cd}$$

$$\frac{x(a+bx^2)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

input
```
Int[(a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x]
```

output
```
-(((b*c - a*d)*x*(a + b*x^2)^(5/2))/(c*d*Sqrt[c + d*x^2])) + (b*(((6*b*c -
 5*a*d)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*d) - (((24*b^2*c^2 - 43*a*
b*c*d + 15*a^2*d^2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - ((48*b^3*c^
3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*((x*Sqrt[a + b*x^2])/(
b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2])) + (c^(3/2)*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^2)*Sq
rt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d))/(5*d)
))/(c*d)
```

3.207. $\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx$

3.207.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.207. $\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx$

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.207.4 Maple [A] (verified)

Time = 9.53 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.58

method	result
risch	$\frac{b^2 x\left(3bd\,x^2+16ad-9bc\right)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{15d^3}+\frac{\left(-\frac{b^2\left(58a^2d^2-83abcd+33b^2c^2\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}} \ldots\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}\,d} \ldots\right)}{\ldots}$
elliptic	$\frac{\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\left(\frac{\left(bd\,x^2+ad\right)\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{c\,d^4\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{b^3x^3\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{5d^2}+\frac{\left(\frac{b^3(4ad-bc)}{d^2}-\frac{b^3(4ad+4bc)}{5d^2}\right)x\sqrt{bd\,x \ldots}}{3bd} \ldots\right)}{\ldots}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(3\sqrt{-\frac{b}{a}}\,b^4c\,d^3x^7+19\sqrt{-\frac{b}{a}}\,a\,b^3c\,d^3x^5-6\sqrt{-\frac{b}{a}}\,b^4c^2d^2x^5+15\sqrt{-\frac{b}{a}}\,a^3b\,d^4x^3-29\sqrt{-\frac{b}{a}}\,a^2b^2c\,d^3x^3+55\sqrt{-\frac{b}{a}}\,a\,b^3c \ldots\right.}{\ldots}$

input
```
int((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*b^2*x*(3*b*d*x^2+16*a*d-9*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3+1/
15/d^3*(-b^2*(58*a^2*d^2-83*a*b*c*d+33*b^2*c^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2)))+b*(60*a^3*d^3-106*a^2*b*c*d^2+69*a*b^2*c^2*d-15*b^3*c^3
)/d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+(15*a^4*
d^4-60*a^3*b*c*d^3+90*a^2*b^2*c^2*d^2-60*a*b^3*c^3*d+15*b^4*c^4)/d*((b*d*x
^2+a*d)/c/(a*d-b*c)*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(1/c-1/c/(a*d-b*c)*a
*d)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+b/(a*d-b
*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Ellipti
cE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))))*((b*x^2+a)*(d*x^2+c))^(1/2)
/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{((48\,b^3c^4d-128\,ab^2c^3d^2+103\,a^2bc^2d^3-15\,a^3cd^4)x^3+(48\,b^3c^5-128\,ab^2c^4d+103\,a^2bc^3d^2-15\,a^3c^2d^3)x)}{}$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(((48*b^3*c^4*d - 128*a*b^2*c^3*d^2 + 103*a^2*b*c^2*d^3 - 15*a^3*c*d
^4)*x^3 + (48*b^3*c^5 - 128*a*b^2*c^4*d + 103*a^2*b*c^3*d^2 - 15*a^3*c^2*d
^3)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
((48*b^3*c^4*d - 128*a*b^2*c^3*d^2 + 45*a^3*d^5 + (103*a^2*b + 24*a*b^2)*c
^2*d^3 - (15*a^3 + 61*a^2*b)*c*d^4)*x^3 + (48*b^3*c^5 - 128*a*b^2*c^4*d +
45*a^3*c*d^4 + (103*a^2*b + 24*a*b^2)*c^3*d^2 - (15*a^3 + 61*a^2*b)*c^2*d^
3)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (
3*b^3*c*d^4*x^6 + 48*b^3*c^4*d - 128*a*b^2*c^3*d^2 + 103*a^2*b*c^2*d^3 - 1
5*a^3*c*d^4 - 2*(3*b^3*c^2*d^3 - 8*a*b^2*c*d^4)*x^4 + (24*b^3*c^3*d^2 - 67
*a*b^2*c^2*d^3 + 58*a^2*b*c*d^4)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*
d^6*x^3 + c^2*d^5*x)
```

3.207.6 Sympy [F]

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{7}{2}}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(7/2)/(c + d*x**2)**(3/2), x)
```

3.207.7 Maxima [F]

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2), x)
```

3.207.8 Giac [F]

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{7/2}}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)^(7/2)/(c + d*x^2)^(3/2), x)
```

3.208 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx$

3.208.1 Optimal result

Integrand size = 23, antiderivative size = 346

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx &= -\frac{(8b^2c^2-13abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{3cd^2\sqrt{c+dx^2}} \\ &- \frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{b(4bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd^2} \\ &+ \frac{(8b^2c^2-13abcd+3a^2d^2)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &- \frac{2b\sqrt{c}(2bc-3ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-(-a*d+b*c)*x*(b*x^2+a)^(3/2)/c/d/(d*x^2+c)^(1/2)-1/3*(3*a^2*d^2-13*a*b*c*
d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/c/d^2/(d*x^2+c)^(1/2)+1/3*(3*a^2*d^2-13*a*b
*c*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2
)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(5/2)/c^(
1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2/3*b*(-3*a*d+2*b*c)*
(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x
^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(5/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*b*(-3*a*d+4*b*c)*x*(b*x^2+a)^(1/2
)*(d*x^2+c)^(1/2)/c/d^2
```

3.208.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.37 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(-6abcd+3a^2d^2+b^2c(4c+dx^2)\right)+ibc(8b^2c^2-13abcd+3a^2d^2)\sqrt{1-}}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(-6*a*b*c*d + 3*a^2*d^2 + b^2*c*(4*c + d*x^2))
+ I*b*c*(8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(8*b^2*c
^2 - 17*a*b*c*d + 9*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c*d^3*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])
```

3.208.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {315, 27, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{315}$$

$$\frac{\int \frac{b\sqrt{bx^2+a}((4bc-3ad)x^2+ac)}{\sqrt{dx^2+c}}dx}{cd} - \frac{x(a+bx^2)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}((4bc-3ad)x^2+ac)}{\sqrt{dx^2+c}}dx}{cd} - \frac{x(a+bx^2)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{403}$$

$$\frac{b\left(\frac{\int -\frac{\left(8b^2c^2-13abdc+3a^2d^2\right)x^2+2ac(2bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3d}\right)}{cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\downarrow$ 25

$$\frac{b\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3d}-\frac{\int \frac{\left(8b^2c^2-13abdc+3a^2d^2\right)x^2+2ac(2bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)}{cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\downarrow$ 406

$$\frac{b\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3d}-\frac{(3a^2d^2-13abcd+8b^2c^2)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2ac(2bc-3ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)}{cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\downarrow$ 320

$$\frac{b\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3d}-\frac{(3a^2d^2-13abcd+8b^2c^2)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2c^{3/2}\sqrt{a+bx^2}(2bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}\right)}{cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\downarrow$ 388

$$\frac{b\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3d}-\frac{(3a^2d^2-13abcd+8b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{2c^{3/2}\sqrt{a+bx^2}(2bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\ldots\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}\ldots\right)}{cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\downarrow$ 313

3.208. $\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{3/2}}\,dx$

$$\frac{b\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3d}-\frac{(3a^2d^2-13abcd+8b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{2c^{3/2}\sqrt{a+bx^2}(2bc-3ad)\,\mathrm{EllipticF}\left(\cdots\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+\cdots)}{a(c+\cdots)}}}}{3d}\cdots\right)}{cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x]
```

output
```
-(((b*c - a*d)*x*(a + b*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2])) + (b*(((4*b*c -
 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - ((8*b^2*c^2 - 13*a*b*c*
d + 3*a^2*d^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a
+ b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[
d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (2*c^(3/2)*(2
*b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
]))/(3*d)))/(c*d)
```

3.208.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.208. $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}} \, dx$

3.208.4 Maple [A] (verified)

Time = 9.22 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.47

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{(bd\,x^2+ad)(a^2d^2-2abcd+b^2c^2)x}{c\,d^3\sqrt{\left(x^2+\frac{c}{d}\right)(bd\,x^2+ad)}}+\frac{b^2x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3d^2}+\frac{\left(\frac{b(3a^2d^2-3abcd+b^2c^2)}{d^3}+\frac{(a^2d^2-2abcd+b^2c^2)(ad}{d^3c}\right.}{}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(\sqrt{-\frac{b}{a}}\,b^3c\,d^2x^5+3\sqrt{-\frac{b}{a}}\,a^2b\,d^3x^3-5\sqrt{-\frac{b}{a}}\,a\,b^2c\,d^2x^3+4\sqrt{-\frac{b}{a}}\,b^3c^2d\,x^3+9\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)c\right.}{}$
risch	$\frac{b^2x\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{3d^2}+\frac{\left(\frac{(3a^3d^3-9a^2bc\,d^2+9a\,b^2c^2d-3b^3c^3)\left(\frac{(bd\,x^2+ad)x}{c(ad-bc)\sqrt{\left(x^2+\frac{c}{d}\right)(bd\,x^2+ad)}}+\frac{\left(\frac{1}{c}-\frac{ad}{c(ad-bc)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F(x}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x}}\right.}{d}\right.}{}$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+a*d)
*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c/d^3*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/3*b
^2/d^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(b*(3*a^2*d^2-3*a*b*c*d+b^2*c
^2)/d^3+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*(a*d-b*c)/c-a/d^2*(a^2*d^2-2*a*b*c
*d+b^2*c^2)/c-1/3*b^2/d^2*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))-(b^2/d^2*(3*a*d-b*c)-(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2*b/
c-1/3*b^2/d^2*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx = \frac{((8\,b^2c^3d-13\,abc^2d^2+3\,a^2cd^3)x^3+(8\,b^2c^4-13\,abc^3d+3\,a^2c^2d^2)x)\sqrt{bd}\sqrt{-\frac{c}{d}}E(\mathrm{arcsin}}{}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(((8*b^2*c^3*d - 13*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^3 + (8*b^2*c^4 - 13*a
*b*c^3*d + 3*a^2*c^2*d^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-
c/d)/x), a*d/(b*c)) - ((8*b^2*c^3*d - 13*a*b*c^2*d^2 - 6*a^2*d^4 + (3*a^2
+ 4*a*b)*c*d^3)*x^3 + (8*b^2*c^4 - 13*a*b*c^3*d - 6*a^2*c*d^3 + (3*a^2 + 4
*a*b)*c^2*d^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*
d/(b*c)) + (b^2*c*d^3*x^4 - 8*b^2*c^3*d + 13*a*b*c^2*d^2 - 3*a^2*c*d^3 - (
4*b^2*c^2*d^2 - 7*a*b*c*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d^5*
x^3 + c^2*d^4*x)
```

3.208.6 Sympy [F]

$$\int \frac{(a + bx^2)^{5/2}}{(c + dx^2)^{3/2}}\, dx = \int \frac{(a + bx^2)^{\frac{5}{2}}}{(c + dx^2)^{\frac{3}{2}}}\, dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)/(c + d*x**2)**(3/2), x)
```

3.208.7 Maxima [F]

$$\int \frac{(a + bx^2)^{5/2}}{(c + dx^2)^{3/2}}\, dx = \int \frac{(bx^2 + a)^{\frac{5}{2}}}{(dx^2 + c)^{\frac{3}{2}}}\, dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2), x)
```

3.208.8 Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2), x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x^2)^(3/2), x)
```

3.209 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$

3.209.1 Optimal result

Integrand size = 23, antiderivative size = 258

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} + \frac{(2bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} - \frac{(2bc-ad)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{b\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/d/(d*x^2+c)^(1/2)+(-a*d+2*b*c)*x*(b*x^2+a)
^(1/2)/c/d/(d*x^2+c)^(1/2)-(-a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*
x^2+a)^(1/2)/d^(3/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/
2)+b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.209.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.73 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \frac{ibc(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(-bc+ad)\left(\sqrt{\frac{b}{a}}dx(a+\right.}{\sqrt{\frac{b}{a}}cd^2\sqrt{a+bx^2}\sqrt{c+d}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x]
```

output
```
(I*b*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-(b*c) + a*d)*(Sqrt[b/a]*d*x*(a + b*
x^2) - (2*I)*b*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*c*d^2*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])
```

3.209.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {315, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{315}$$

$$\frac{\int \frac{b((2bc-ad)x^2+ac)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd} - \frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{(2bc-ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd} - \frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{b\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{b\left((2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{b\left((2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

input
```
Int[(a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x]
```

output
```
-(((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2])) + (b*((2*b*c - a*
d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(c*d)
```

3.209. $\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx$

3.209.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.209. $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$

3.209.4 Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.34

method	result
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(\sqrt{-\frac{b}{a}}\,ab\,d^2x^3-\sqrt{-\frac{b}{a}}\,b^2cd\,x^3+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abcd-2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\cdots}\right.\right.}{(bd\,x^4+ad\,x^2+cb\,x^2\cdots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{\left(bd\,x^2+ad\right)(ad-bc)x}{c\,d^2\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\left(\frac{b(2ad-bc)}{d^2}+\frac{(ad-bc)^2}{d^2c}-\frac{a(ad-bc)}{dc}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\left(\frac{b^2}{d}\cdots\right.\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*a*b*d^2*x^3-(-b/a)^(1/2)*b^2
*c*d*x^3+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*a*b*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+2*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2
*c^2+(-b/a)^(1/2)*a^2*d^2*x-(-b/a)^(1/2)*a*b*c*d*x)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)/d^2/c/(-b/a)^(1/2)
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{((2\,bc^2d-acd^2)x^3+(2\,bc^3-ac^2d)x)\sqrt{bd}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-((2\,bc^2d-acd^2+ad^3)x^3+(2\,bc^3\cdots}{cd^4x^3+c^2d\cdots}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-(((2*b*c^2*d - a*c*d^2)*x^3 + (2*b*c^3 - a*c^2*d)*x)*sqrt(b*d)*sqrt(-c/d)
*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((2*b*c^2*d - a*c*d^2 + a*d
^3)*x^3 + (2*b*c^3 - a*c^2*d + a*c*d^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f
(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (b*c*d^2*x^2 + 2*b*c^2*d - a*c*d^2)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d^4*x^3 + c^2*d^3*x)
```

3.209.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(c + d*x**2)**(3/2), x)
```

3.209.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2), x)
```

3.209.8 Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2), x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x^2)^(3/2), x)
```

3.210 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx$

3.210.1 Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx=\frac{\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x
^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/c^(1/2)/d^(1/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.210.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx=\frac{\frac{x(a+bx^2)}{c}+\frac{ia\sqrt{\frac{b}{a}}\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{d}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2),x]
```

output
```
((x*(a + b*x^2))/c + (I*a*Sqrt[b/a]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)]))/d)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.210.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{a+bx^2} E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

3.210.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

3.210.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.24

method	result
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(\sqrt{-\frac{b}{a}}\,bd\,x^{3}+\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bc-\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bc+\sqrt{-\frac{b}{a}}\,adx\right)}{\left(bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac\right)dc\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(\frac{\left(bd\,x^{2}+ad\right)x}{dc\sqrt{\left(x^{2}+\frac{c}{d}\right)\left(bd\,x^{2}+ad\right)}}+\frac{\left(\frac{b}{d}+\frac{ad-bc}{dc}-\frac{a}{c}\right)\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}+\cdots\right)}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*b*d*x^3+((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c-((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
b*c+(-b/a)^(1/2)*a*d*x)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/d/c/(-b/a)^(1/2)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,cadx - (bdx^2+bc)\sqrt{ac}\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right) + (bdx^2+bc)\sqrt{\cdots}}{acd^2x^2+ac^2d}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*d*x - (b*d*x^2 + b*c)*sqrt(a*c)*sqrt(-b
/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (b*d*x^2 + b*c)*sqrt(a*c
)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a*c*d^2*x^2 + a
*c^2*d)
```

3.210.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/(c + d*x**2)**(3/2), x)
```

3.210.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2), x)
```

3.210.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\left(d\,x^2+c\right)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c + d*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(c + d*x^2)^(3/2), x)
```

3.211 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}\,dx$

3.211.1 Optimal result

Integrand size = 23, antiderivative size = 194

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}\,dx=-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{b\sqrt{c}\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d
*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)/d^(1
/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*
(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d
)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/(-a*d+b*c)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+
c))^(1/2)/(d*x^2+c)^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{-dx(a+bx^2)+\frac{bc\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}}}{c(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
(-(d*x*(a + b*x^2)) + (b*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/Sqrt[-(b/a)])/(c*(b*c - a*d)*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])
```

3.211.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {316, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{b\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{324}$$

$$\frac{b\left(c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \ 388$$

$$\frac{b\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \ 313$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{c(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
-((d*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*(d*((x*Sqrt[
a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(c*(b*c - a*d))
```

3.211.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.211.4 Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.74

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, b d\, x^{3}-\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{d\, x^{2}+c}{c}}\, E\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) b c+\sqrt{-\frac{b}{a}}\, a d x\right) \sqrt{d\, x^{2}+c}\, \sqrt{b\, x^{2}+a}}{c \sqrt{-\frac{b}{a}}\, (a d-b c)(b d\, x^{4}+a d\, x^{2}+c b\, x^{2}+a c)}$
elliptic	$\frac{\sqrt{(b\, x^{2}+a)(d\, x^{2}+c)}\, \left(\frac{\left(b d\, x^{2}+a d\right) x}{c(a d-b c) \sqrt{\left(x^{2}+\frac{c}{d}\right)\left(b d\, x^{2}+a d\right)}}+\frac{\left(\frac{1}{c}-\frac{a d}{c(a d-b c)}\right) \sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, F\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\, \sqrt{b d\, x^{4}+a d\, x^{2}+c b\, x^{2}+a c}}+\frac{b \sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}} \cdots}{(\cdots}\right.}{\sqrt{b\, x^{2}+a}\, \sqrt{d\, x^{2}+c}}$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-b/a)^(1/2)*b*d*x^3-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*
(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c+(-b/a)^(1/2)*a*d*x)*(d*x^2+c)^(1/2)*(b*x
^2+a)^(1/2)/c/(-b/a)^(1/2)/(a*d-b*c)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = -\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}adx-(bdx^2+bc)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})+((a+b)dx^2+(a+b)c)\sqrt{ac}\sqrt{-}}{abc^3-a^2c^2d+(abc^2d-a^2cd^2)x^2}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*d*x - (b*d*x^2 + b*c)*sqrt(a*c)*sqrt(-
b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + ((a + b)*d*x^2 + (a + b
)*c)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a*
b*c^3 - a^2*c^2*d + (a*b*c^2*d - a^2*c*d^2)*x^2)
```

3.211.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.211.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.211.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

3.212 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$

3.212.1 Optimal result

Integrand size = 23, antiderivative size = 242

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \frac{bx}{a(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{\sqrt{d}(bc+ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{2b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
b*x/a/(-a*d+b*c)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+(a*d+b*c)*(1/(1+d*x^2/c))
^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-
b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)^2/c^(1/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2*b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)
^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^
(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(d*x^2+c)^(1/2)
```

3.212.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.93

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}x(a^2d^2+abd^2x^2+b^2c(c+dx^2))+ibc(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{d\cdots}{\cdots}}\right.}{bc(b\cdots}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*x*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c + d*x^2)) + I*b*
c*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c*(b*c
 - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.212.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {316, 27, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 316$$

$$\frac{bx}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int \frac{d(a-bx^2)}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{a(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{bx}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{d\int \frac{a-bx^2}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{a(bc-ad)}$$

$$\Big\downarrow \ 400$$

$$\frac{bx}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{d\left(\frac{2ab\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(ad+bc)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{a(bc-ad)}$$

$$\Big\downarrow \text{313}$$

$$\frac{bx}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{d\left(\frac{2ab\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{a(bc-ad)}$$

$$\Big\downarrow \text{320}$$

$$\frac{bx}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{d\left(\frac{2b\sqrt{c}\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{a(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) - (d*(-(((b*c + a*d)
*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2])) + (2*b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2])))/(a*(b*c - a*d))
```

3.212.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

3.212.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.46

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, ab\, d^2 x^3+\sqrt{-\frac{b}{a}}\, b^2 c d\, x^3-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\, F\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) abcd+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\, F\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) b^2c^2-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{}\right.}{c\sqrt{-\frac{b}{a}}\, a(ad-bc)^2(bd\,x^4+}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{2bd\left(-\frac{(ad+bc)x^3}{2ac\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{\left(a^2d^2+b^2c^2\right)x}{2ac\left(a^2d^2-2abcd+b^2c^2\right)bd}\right)}{\sqrt{\left(x^4+\frac{(ad+bc)x^2}{bd}+\frac{ac}{bd}\right)bd}}+\frac{\left(\frac{1}{ac}-\frac{a^2d^2+b^2c^2}{ac\left(a^2d^2-2abcd+b^2c^2\right)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-b/a)^(1/2)*a*b*d^2*x^3+(-b/a)^(1/2)*b^2*c*d*x^3-((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*b^2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*a*b*c*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+(-b/a)^(1/2)*a^2*d^2*x+(-b/a)^(1/
2)*b^2*c^2*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/c/(-b/a)^(1/2)/a/(a*d-b*c)^2
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.68

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = -\frac{(ab^2c^2+a^2bcd+(b^3cd+ab^2d^2)x^4+(b^3c^2+2\,ab^2cd+a^2bd^2)x^2)\sqrt{ac}\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right)-(ab^2 \cdots}{a^3b^2c^4-2 \cdots}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-((a*b^2*c^2 + a^2*b*c*d + (b^3*c*d + a*b^2*d^2)*x^4 + (b^3*c^2 + 2*a*b^2*
c*d + a^2*b*d^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a))
, a*d/(b*c)) - (a*b^2*c^2 + (b^3*c*d + (2*a^2*b + a*b^2)*d^2)*x^4 + (2*a^3
 + a^2*b)*c*d + (b^3*c^2 + 2*(a^2*b + a*b^2)*c*d + (2*a^3 + a^2*b)*d^2)*x^
2)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((a*
b^2*c*d + a^2*b*d^2)*x^3 + (a*b^2*c^2 + a^3*d^2)*x)*sqrt(b*x^2 + a)*sqrt(d
*x^2 + c))/(a^3*b^2*c^4 - 2*a^4*b*c^3*d + a^5*c^2*d^2 + (a^2*b^3*c^3*d - 2
*a^3*b^2*c^2*d^2 + a^4*b*c*d^3)*x^4 + (a^2*b^3*c^4 - a^3*b^2*c^3*d - a^4*b
*c^2*d^2 + a^5*c*d^3)*x^2)
```

3.212.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)), x)
```

3.212.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

3.212.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)), x)
```

3.213 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$

3.213.1 Optimal result

Integrand size = 23, antiderivative size = 323

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \frac{bx}{3a(bc-ad)(a+bx^2)^{3/2}\sqrt{c+dx^2}} + \frac{2b(bc-3ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{\sqrt{d}(2b^2c^2-7abcd-3a^2d^2)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{b\sqrt{c}\sqrt{d}(bc-9ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*b*x/a/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)+2/3*b*(-3*a*d+b*c)*x/
a^2/(-a*d+b*c)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*(-3*a^2*d^2-7*a*b*c*d
+2*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a^2/(-a
*d+b*c)^3/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*b*(-
9*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(
1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/
a^2/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.213.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.40 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(3a^4d^3+6a^3bd^3x^2-2b^4c^2x^2(c+dx^2)+a^2b^2d(8c^2+8cdx^2+3d^2x^4)}{}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*x*(3*a^4*d^3 + 6*a^3*b*d^3*x^2 - 2*b^4*c^2*x^2*(c + d*x^2) + a^
2*b^2*d*(8*c^2 + 8*c*d*x^2 + 3*d^2*x^4) + a*b^3*c*(-3*c^2 + 4*c*d*x^2 + 7*
d^2*x^4)) + I*b*c*(-2*b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1
+ (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)] + (2*I)*b*c*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(3*a^2*Sqrt[b/a]*c*(-(b*c) + a*d)^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

3.213.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {316, 25, 402, 25, 27, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 316$$

$$\frac{bx}{3a(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int -\frac{3bdx^2+2bc-3ad}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}dx}{3a(bc-ad)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{3bdx^2+2bc-3ad}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}dx}{3a(bc-ad)}+\frac{bx}{3a(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{2bx(bc-3ad)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int -\frac{d\left(2b(bc-3ad)x^2+a(bc+3ad)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{d\left(2b(bc-3ad)x^2+a(bc+3ad)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}+\frac{2bx(bc-3ad)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\int \frac{2b(bc-3ad)x^2+a(bc+3ad)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}+\frac{2bx(bc-3ad)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{400}$$

$$\frac{\frac{d\left(\frac{\left(-3a^2d^2-7abcd+2b^2c^2\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}-\frac{ab(bc-9ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{a(bc-ad)}+\frac{2bx(bc-3ad)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{d\left(\frac{\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{ab(bc-9ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{a(bc-ad)}+\frac{2bx(bc-3ad)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{320}$$

3.213. $\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}\,dx$

$$\frac{\frac{d\left(\frac{\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(bc-9ad)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{a(bc-ad)}+\frac{2bx(bc-3ad)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]) + ((2*b*(b*c - 3
*a*d)*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (d*(((2*b^2*c^2
 - 7*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*(b*c - 9*a*d)*Sqrt[a + b*x
^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*(b*c
 - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*(b*c -
 a*d)))/(3*a*(b*c - a*d))
```

3.213.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.213.4 Maple [A] (verified)

Time = 6.71 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3a(ad-bc)^2\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd\,x^2+bc\right)bx(7ad-2bc)}{3a^2(ad-bc)^3\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(bd\,x^2+ad\right)d^2x}{c(ad-bc)^3\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\left(\frac{bd}{3(ad-bc)^2a}-\frac{b}{3(}\right.}{}\right.}{}$
default	$-\frac{-3\sqrt{-\frac{b}{a}}\,a^2b^2d^3x^5-7\sqrt{-\frac{b}{a}}\,a\,b^3c\,d^2x^5+2\sqrt{-\frac{b}{a}}\,b^4c^2d\,x^5+6\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2b^2c\,d^2x^2-8\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}}{}$

3.213. $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3/a/(a*d-b*
c)^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+1/3*(b*d*x^2+b*c)*b
/a^2/(a*d-b*c)^3*x*(7*a*d-2*b*c)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(b*d*x^2+
a*d)*d^2/c/(a*d-b*c)^3*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(1/3*b*d/(a*d-b*c
)^2/a-1/3/(a*d-b*c)^2*b*(7*a*d-2*b*c)/a^2-1/3*b^2*c/a^2/(a*d-b*c)^3*(7*a*d
-2*b*c)+d^2/(a*d-b*c)^2/c-a*d^3/c/(a*d-b*c)^3)/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-1/3*b^2*d*(7*a*d-2*b*c)/(a*d-b*c)^3/a
^2-b*d^3/(a*d-b*c)^3/c)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+
b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.213.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 856 vs. $2(305) = 610$.

Time = 0.13 (sec) , antiderivative size = 856, normalized size of antiderivative = 2.65

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{(2\,a^2b^3c^3 - 7\,a^3b^2c^2d - 3\,a^4bcd^2 + (2\,b^5c^2d - 7\,ab^4cd^2 - 3\,a^2b^3d^3)x^6 + (2\,b^5c^3 - 3\,ab^4c^2d - 17\,a^2b^3cd^2 - 6\,a}{}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((2*a^2*b^3*c^3 - 7*a^3*b^2*c^2*d - 3*a^4*b*c*d^2 + (2*b^5*c^2*d - 7*
a*b^4*c*d^2 - 3*a^2*b^3*d^3)*x^6 + (2*b^5*c^3 - 3*a*b^4*c^2*d - 17*a^2*b^3
*c*d^2 - 6*a^3*b^2*d^3)*x^4 + (4*a*b^4*c^3 - 12*a^2*b^3*c^2*d - 13*a^3*b^2
*c*d^2 - 3*a^4*b*d^3)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-
b/a)), a*d/(b*c)) - (2*a^2*b^3*c^3 + (2*b^5*c^2*d + (a^2*b^3 - 7*a*b^4)*c*
d^2 - 3*(3*a^3*b^2 + a^2*b^3)*d^3)*x^6 + (2*b^5*c^3 + (a^2*b^3 - 3*a*b^4)*
c^2*d - (7*a^3*b^2 + 17*a^2*b^3)*c*d^2 - 6*(3*a^4*b + a^3*b^2)*d^3)*x^4 +
(a^4*b - 7*a^3*b^2)*c^2*d - 3*(3*a^5 + a^4*b)*c*d^2 + (4*a*b^4*c^3 + 2*(a^
3*b^2 - 6*a^2*b^3)*c^2*d - (17*a^4*b + 13*a^3*b^2)*c*d^2 - 3*(3*a^5 + a^4*
b)*d^3)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*
c)) - ((2*a*b^4*c^2*d - 7*a^2*b^3*c*d^2 - 3*a^3*b^2*d^3)*x^5 + 2*(a*b^4*c^
3 - 2*a^2*b^3*c^2*d - 4*a^3*b^2*c*d^2 - 3*a^4*b*d^3)*x^3 + (3*a^2*b^3*c^3
- 8*a^3*b^2*c^2*d - 3*a^5*d^3)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^5*b^
3*c^5 - 3*a^6*b^2*c^4*d + 3*a^7*b*c^3*d^2 - a^8*c^2*d^3 + (a^3*b^5*c^4*d -
 3*a^4*b^4*c^3*d^2 + 3*a^5*b^3*c^2*d^3 - a^6*b^2*c*d^4)*x^6 + (a^3*b^5*c^5
 - a^4*b^4*c^4*d - 3*a^5*b^3*c^3*d^2 + 5*a^6*b^2*c^2*d^3 - 2*a^7*b*c*d^4)*
x^4 + (2*a^4*b^4*c^5 - 5*a^5*b^3*c^4*d + 3*a^6*b^2*c^3*d^2 + a^7*b*c^2*d^3
 - a^8*c*d^4)*x^2)
```

3.213.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**(3/2)), x)
```

3.213.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

3.213.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)), x)
```

3.214 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.214.1 Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x
^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2
+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right),\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
 (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.214.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

3.214.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.214.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+cb\,x^2+ac)}$	100
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}$	122

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a
)^(1/2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{ac}\sqrt{-\frac{b}{a}}F\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right)}{bc}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c))/(b*c)
```

3.214.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.214.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.214.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.215 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

3.215.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
EllipticF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.215.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right),-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[(a - b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[b/a]*x], -(
(a*d)/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

3.215.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {323, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

input
```
Int[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]
*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

3.215.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.215.4 Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{F\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{-b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{\sqrt{\frac{b}{a}}\,\left(-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac\right)}$	103
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}}$	127

input
```
int(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/
a)^(1/2)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b/a)^(1/2)/(-b*d*x^4+a*d*x^2-b*
c*x^2+a*c)
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{ac}\sqrt{\frac{b}{a}}F(\arcsin\left(x\sqrt{\frac{b}{a}}\right)\mid -\frac{ad}{bc})}{bc}$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*c)*sqrt(b/a)*elliptic_f(arcsin(x*sqrt(b/a)), -a*d/(b*c))/(b*c)
```

3.215.6 Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

3.215.7 Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.215.8 Giac [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{1}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.216 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$

3.216.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx=\frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

output
```
EllipticF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1
-d*x^2/c)^(1/2)/d^(1/2)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx=\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right),-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
 -((a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

3.216.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {323, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{bx^2+a}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
(Sqrt[c]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]
*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

3.216.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.216.4 Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{F\left(x\sqrt{\frac{d}{c}}, \sqrt{-\frac{bc}{ad}}\right)\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2+c}}{\sqrt{\frac{d}{c}}\,(-bd\,x^4-ad\,x^2+cb\,x^2+ac)}$	103
elliptic	$\frac{\sqrt{(b\,x^2+a)(-d\,x^2+c)}\,\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,F\left(x\sqrt{\frac{d}{c}}, \sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2+c}\,\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+cb\,x^2+ac}}$	127

input
```
int(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/
c)^(1/2)*(b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/(-b*d*x^4-a*d*x^2+b*
c*x^2+a*c)
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{ac}\sqrt{\frac{d}{c}}F\left(\arcsin\left(x\sqrt{\frac{d}{c}}\right)\mid -\frac{bc}{ad}\right)}{ad}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*c)*sqrt(d/c)*elliptic_f(arcsin(x*sqrt(d/c)), -b*c/(a*d))/(a*d)
```

3.216.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c - d*x**2)), x)
```

3.216.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{-dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

3.216.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{-dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

3.217 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

3.217.1 Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

output
```
EllipticF(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(1-b*x^2/a)^(1/2)*(1-
d*x^2/c)^(1/2)/d^(1/2)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

input
```
Integrate[1/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
(Sqrt[(a - b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[b/a]*x], (a
*d)/(b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

3.217.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {323, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

input
```
Int[1/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
(Sqrt[c]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]
*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

3.217.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.217.4 Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{F\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{-d\,x^{2}+c}\,\sqrt{-b\,x^{2}+a}}{\sqrt{\frac{d}{c}}\,(bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac)}$	104
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(-d\,x^{2}+c)}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}\,\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}}$	131

input
```
int(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/
c)^(1/2)*(-d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/(d/c)^(1/2)/(b*d*x^4-a*d*x^2-b*
c*x^2+a*c)
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx=\frac{\sqrt{ac}\sqrt{\frac{d}{c}}F\left(\arcsin\left(x\sqrt{\frac{d}{c}}\right)\mid\frac{bc}{ad}\right)}{ad}$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*c)*sqrt(d/c)*elliptic_f(arcsin(x*sqrt(d/c)), b*c/(a*d))/(a*d)
```

3.217.6 Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

3.217.7 Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

3.217.8 Giac [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{a-b\,x^2}\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int(1/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

output
```
int(1/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

3.218 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx$

3.218.1 Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{5}{2}\right)}{\sqrt{2}}$$

output
```
1/2*EllipticF(x,1/2*I*10^(1/2))*2^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{5}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

output
```
EllipticF[ArcSin[x], -5/2]/Sqrt[2]
```

3.218.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{5x^2+2}}\,dx$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\text{EllipticF}\left(\arcsin(x),-\frac{5}{2}\right)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

output
```
EllipticF[ArcSin[x], -5/2]/Sqrt[2]
```

3.218.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.218.4 Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{F\left(x,\frac{i\sqrt{10}}{2}\right)\sqrt{2}}{2}$	14
elliptic	$\frac{\sqrt{-(x^2-1)(5x^2+2)}\,\sqrt{10x^2+4}\,F\left(x,\frac{i\sqrt{10}}{2}\right)}{2\sqrt{5x^2+2}\,\sqrt{-5x^4+3x^2+2}}$	59

input
```
int(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,1/2*I*10^(1/2))*2^(1/2)
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin(x)\mid -\frac{5}{2})$$

input
```
integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), -5/2)
```

3.218.6 Sympy [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|-\frac{5}{2}\right)}{2} & \text{for } x > -1 \wedge x < 1 \end{cases}$$

input
```
integrate(1/(-x**2+1)**(1/2)/(5*x**2+2)**(1/2),x)
```

output
```
Piecewise((sqrt(2)*elliptic_f(asin(x), -5/2)/2, (x > -1) & (x < 1)))
```

3.218.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.218.8 Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{5\,x^2+2}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(5*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(5*x^2 + 2)^(1/2)), x)
```

3.219 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}} \, dx$

3.219.1 Optimal result

Integrand size = 23, antiderivative size = 10

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}} \, dx = \frac{\text{EllipticF}(\arcsin(x), -2)}{\sqrt{2}}$$

output
```
1/2*EllipticF(x,I*2^(1/2))*2^(1/2)
```

3.219.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 5.80

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}} \, dx = -\frac{i\sqrt{1-x^2}\sqrt{1+2x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}x\right), -\frac{1}{2}\right)}{2\sqrt{1+x^2-2x^4}}$$

input
```
Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

output
```
((-1/2*I)*Sqrt[1 - x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], -1
/2])/Sqrt[1 + x^2 - 2*x^4]
```

3.219.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{4x^2+2}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\text{EllipticF}(\arcsin(x),-2)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

output
```
EllipticF[ArcSin[x], -2]/Sqrt[2]
```

3.219.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.219.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

method	result	size
default	$\frac{F\left(x,i\sqrt{2}\right)\sqrt{2}}{2}$	14
elliptic	$\frac{\sqrt{-(x^2-1)(2x^2+1)}\,F\left(x,i\sqrt{2}\right)}{\sqrt{-4x^4+2x^2+2}}$	40

input
```
int(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,I*2^(1/2))*2^(1/2)
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin(x)\,|-2)$$

input
```
integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), -2)
```

3.219.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2x^2+1}}\,dx}{2}$$

input
```
integrate(1/(-x**2+1)**(1/2)/(4*x**2+2)**(1/2),x)
```

output
```
sqrt(2)*Integral(1/(sqrt(1 - x**2)*sqrt(2*x**2 + 1)), x)/2
```

3.219.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.219.8 Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{4\,x^2+2}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(4*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(4*x^2 + 2)^(1/2)), x)
```

3.220 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx$

3.220.1 Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}}$$

output
```
1/2*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)
```

3.220.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

3.220.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \; 321$$

$$\frac{\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

3.220.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.220.4 Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{F\left(x,\frac{i\sqrt{6}}{2}\right)\sqrt{2}}{2}$	14
elliptic	$\frac{\sqrt{-\left(3x^{2}+2\right)\left(x^{2}-1\right)}\,\sqrt{6x^{2}+4}\,F\left(x,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{3x^{2}+2}\,\sqrt{-3x^{4}+x^{2}+2}}$	57

input
```
int(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \frac{1}{2}\sqrt{2}F\left(\arcsin\left(x\right)\,|-\frac{3}{2}\right)$$

input
```
integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), -3/2)
```

3.220.6 Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}\left(x\right)\middle|-\frac{3}{2}\right)}{2} & \text{for}\: x > -1 \wedge x < 1 \end{cases}$$

input
```
integrate(1/(-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Piecewise((sqrt(2)*elliptic_f(asin(x), -3/2)/2, (x > -1) & (x < 1)))
```

3.220.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.220.8 Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{3\,x^2+2}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.221 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx$

3.221.1 Optimal result

Integrand size = 23, antiderivative size = 10

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}}$$

output
```
1/2*EllipticF(x,I)*2^(1/2)
```

3.221.2 Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

output
```
EllipticF[ArcSin[x], -1]/Sqrt[2]
```

3.221.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {284, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2x^2+2}}dx$$

$$\Big\downarrow \ 284$$

$$\int \frac{1}{\sqrt{2-2x^4}}dx$$

$$\Big\downarrow \ 762$$

$$\frac{\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

output
```
EllipticF[ArcSin[x], -1]/Sqrt[2]
```

3.221.3.1 Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

3.221.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{F(x,i)\sqrt{2}}{2}$	10
elliptic	$\frac{\sqrt{-x^4+1}\,F(x,i)}{\sqrt{-2x^4+2}}$	24

input
```
int(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,I)*2^(1/2)
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin(x)\,|-1)$$

input
```
integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), -1)
```

3.221.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(8) = 16.

Time = 11.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 7.30

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = -\frac{\sqrt{2}G_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4} \\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0 \end{matrix}\middle|\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}} + \frac{\sqrt{2}G_{6,6}^{3,5}\left(\begin{matrix} -\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1 \\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4} \end{matrix}\middle|\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

input
```
integrate(1/(-x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
```

output
```
-sqrt(2)*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4),
 (0,)), exp_polar(-2*I*pi)/x**4)/(16*pi**(3/2)) + sqrt(2)*meijerg(((-1/4,
0, 1/4, 1/2, 3/4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), x**(-4))/(16*pi
**(3/2))
```

3.221.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.221.8 Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2\,x^2+2}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(2*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

3.222 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx$

3.222.1 Optimal result

Integrand size = 21, antiderivative size = 12

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx=\frac{\text{EllipticF}\left(\arcsin(x),-\frac{1}{2}\right)}{\sqrt{2}}$$

output
```
1/2*EllipticF(x,1/2*I*2^(1/2))*2^(1/2)
```

3.222.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx=-i\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

input
```
Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + x^2]),x]
```

output
```
(-I)*EllipticF[I*ArcSinh[x/Sqrt[2]], -2]
```

3.222.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+2}}\,dx$$
$$\Big\downarrow \; 321$$
$$\frac{\text{EllipticF}\left(\arcsin(x),-\frac{1}{2}\right)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[1 - x^2]*Sqrt[2 + x^2]),x]
```

output
```
EllipticF[ArcSin[x], -1/2]/Sqrt[2]
```

3.222.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.222.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{F\left(x, \frac{i\sqrt{2}}{2}\right)\sqrt{2}}{2}$	14
elliptic	$\frac{\sqrt{-(x^2-1)(x^2+2)}\,\sqrt{2x^2+4}\,F\left(x, \frac{i\sqrt{2}}{2}\right)}{2\sqrt{x^2+2}\,\sqrt{-x^4-x^2+2}}$	55

input
```
int(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,1/2*I*2^(1/2))*2^(1/2)
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin{(x)}\mid-\frac{1}{2})$$

input
```
integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), -1/2)
```

3.222.6 Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}{(x)}\middle|-\frac{1}{2}\right)}{2} & \text{for}\: x > -1 \wedge x < 1 \end{cases}$$

input
```
integrate(1/(-x**2+1)**(1/2)/(x**2+2)**(1/2),x)
```

output
```
Piecewise((sqrt(2)*elliptic_f(asin(x), -1/2)/2, (x > -1) & (x < 1)))
```

3.222.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.222.8 Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{x^2+2}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(x^2 + 2)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(x^2 + 2)^(1/2)), x)
```

3.223 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx$

3.223.1 Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),\frac{1}{2}\right)}{\sqrt{2}}$$

output
```
1/2*EllipticF(x,1/2*2^(1/2))*2^(1/2)
```

3.223.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),\frac{1}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 - x^2]),x]
```

output
```
EllipticF[ArcSin[x], 1/2]/Sqrt[2]
```

3.223.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx$$
$$\Big\downarrow \; 321$$
$$\frac{\text{EllipticF}\left(\arcsin(x),\frac{1}{2}\right)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[1 - x^2]*Sqrt[2 - x^2]),x]
```

output
```
EllipticF[ArcSin[x], 1/2]/Sqrt[2]
```

3.223.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.223.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{F\left(x,\frac{\sqrt{2}}{2}\right)\sqrt{2}}{2}$	13
elliptic	$\frac{\sqrt{(x^2-1)(x^2-2)}\,\sqrt{-2x^2+4}\,F\left(x,\frac{\sqrt{2}}{2}\right)}{2\sqrt{-x^2+2}\,\sqrt{x^4-3x^2+2}}$	53

input
```
int(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,1/2*2^(1/2))*2^(1/2)
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \frac{1}{2}\sqrt{2}F\big(\arcsin{(x)}\,|\,\frac{1}{2}\big)$$

input
```
integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), 1/2)
```

3.223.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{2-x^2}}\,dx$$

input
```
integrate(1/(-x**2+1)**(1/2)/(-x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(2 - x**2)), x)
```

3.223.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.223.8 Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-x^2}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(2 - x^2)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(2 - x^2)^(1/2)), x)
```

3.224 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx$

3.224.1 Optimal result

Integrand size = 23, antiderivative size = 8

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = \frac{\operatorname{arctanh}(x)}{\sqrt{2}}$$

output
```
1/2*arctanh(x)*2^(1/2)
```

3.224.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 26 vs. 2(8) = 16.

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 3.25

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = -\frac{\frac{1}{2}\log(1-x)-\frac{1}{2}\log(1+x)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]
```

output
```
-((Log[1 - x]/2 - Log[1 + x]/2)/Sqrt[2])
```

3.224.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {282, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 282}$$

$$\frac{\int \frac{1}{1-x^2}dx}{\sqrt{2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\operatorname{arctanh}(x)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]
```

output
```
ArcTanh[x]/Sqrt[2]
```

3.224.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 282
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b, c
, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !SimplerQ[a + b*x
^n, c + d*x^n]
```

3.224.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

method	result	size
meijerg	$\frac{\operatorname{arctanh}(x)\sqrt{2}}{2}$	8

input
```
int(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(x)*2^(1/2)
```

3.224.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(7) = 14$.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 8.50

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{x^6+5\,x^4-2\sqrt{2}(x^3+x)\sqrt{-x^2+1}\sqrt{-2\,x^2+2}-5\,x^2-1}{x^6-3\,x^4+3\,x^2-1}\right)$$

input
```
integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-(x^6 + 5*x^4 - 2*sqrt(2)*(x^3 + x)*sqrt(-x^2 + 1)*sqrt(-2
*x^2 + 2) - 5*x^2 - 1)/(x^6 - 3*x^4 + 3*x^2 - 1))
```

3.224.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.95 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = -\sqrt{2}\left(\frac{\log(x-1)}{4}-\frac{\log(x+1)}{4}\right)$$

input
```
integrate(1/(-2*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
-sqrt(2)*(log(x - 1)/4 - log(x + 1)/4)
```

3.224.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

input
```
integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

3.224.8 Giac [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

input
```
integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-2\,x^2}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(2 - 2*x^2)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

3.225 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx$

3.225.1 Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x), \frac{3}{2}\right)}{\sqrt{2}}$$

output
```
1/2*EllipticF(x,1/2*6^(1/2))*2^(1/2)
```

3.225.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x), \frac{3}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

output
```
EllipticF[ArcSin[x], 3/2]/Sqrt[2]
```

3.225.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx$$

$$\Big\downarrow \; 321$$

$$\frac{\text{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

output
```
EllipticF[ArcSin[x], 3/2]/Sqrt[2]
```

3.225.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.225.4 Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{F\left(x,\frac{\sqrt{6}}{2}\right)\sqrt{2}}{2}$	13
elliptic	$\frac{\sqrt{(3x^2-2)(x^2-1)}\,\sqrt{-6x^2+4}\,F\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{-3x^2+2}\,\sqrt{3x^4-5x^2+2}}$	57

input
```
int(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,1/2*6^(1/2))*2^(1/2)
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin{(x)} \mid \frac{3}{2})$$

input
```
integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), 3/2)
```

3.225.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.

Time = 1.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \begin{cases} \frac{\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{2}{3}\right)}{3} & \text{for}\: x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

input
```
integrate(1/(-3*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Piecewise((sqrt(3)*elliptic_f(asin(sqrt(6)*x/2), 2/3)/3, (x > -sqrt(6)/3)
& (x < sqrt(6)/3)))
```

3.225.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.225.8 Giac [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-3\,x^2}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.226 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx$

3.226.1 Optimal result

Integrand size = 23, antiderivative size = 10

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),2)}{\sqrt{2}}$$

output
```
1/2*EllipticF(x,2^(1/2))*2^(1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),2)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[1 - x^2]),x]
```

output
```
EllipticF[ArcSin[x], 2]/Sqrt[2]
```

3.226.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx$$
$$\Big\downarrow \ 321$$
$$\frac{\text{EllipticF}(\arcsin(x),2)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[2 - 4*x^2]*Sqrt[1 - x^2]),x]
```

output
```
EllipticF[ArcSin[x], 2]/Sqrt[2]
```

3.226.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.226.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{F\left(x,\sqrt{2}\right)\sqrt{2}}{2}$	11
elliptic	$\frac{\sqrt{(2x^2-1)(x^2-1)}\,F\left(x,\sqrt{2}\right)}{\sqrt{4x^4-6x^2+2}}$	36

input
```
int(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,2^(1/2))*2^(1/2)
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \frac{1}{2}\,\sqrt{2}F(\arcsin{(x)}\,|2)$$

input
```
integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), 2)
```

3.226.6 Sympy [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.90

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \frac{\sqrt{2}\left(\begin{cases}\frac{\sqrt{2}F\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|\frac{1}{2}\right)}{2} & \text{for}\: x>-\frac{\sqrt{2}}{2}\wedge x<\frac{\sqrt{2}}{2}\end{cases}\right)}{2}$$

input
```
integrate(1/(-4*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
sqrt(2)*Piecewise((sqrt(2)*elliptic_f(asin(sqrt(2)*x), 1/2)/2, (x > -sqrt(
2)/2) & (x < sqrt(2)/2)))/2
```

3.226.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

input
```
integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

3.226.8 Giac [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

input
```
integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-4\,x^2}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(2 - 4*x^2)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(2 - 4*x^2)^(1/2)), x)
```

3.227 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx$

3.227.1 Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),\frac{5}{2}\right)}{\sqrt{2}}$$

output
```
1/2*EllipticF(x,1/2*10^(1/2))*2^(1/2)
```

3.227.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),\frac{5}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[1 - x^2]),x]
```

output
```
EllipticF[ArcSin[x], 5/2]/Sqrt[2]
```

3.227.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx$$

$$\Big\downarrow \; 321$$

$$\frac{\text{EllipticF}\left(\arcsin(x),\frac{5}{2}\right)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[2 - 5*x^2]*Sqrt[1 - x^2]),x]
```

output
```
EllipticF[ArcSin[x], 5/2]/Sqrt[2]
```

3.227.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.227.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{F\left(x,\frac{\sqrt{10}}{2}\right)\sqrt{2}}{2}$	13
elliptic	$\frac{\sqrt{(5x^2-2)(x^2-1)}\,\sqrt{-10x^2+4}\,F\left(x,\frac{\sqrt{10}}{2}\right)}{2\sqrt{-5x^2+2}\,\sqrt{5x^4-7x^2+2}}$	57

input
```
int(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,1/2*10^(1/2))*2^(1/2)
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{2}F\left(\arcsin\left(x\right) \mid \frac{5}{2}\right)$$

input
```
integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), 5/2)
```

3.227.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.

Time = 1.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \begin{cases} \frac{\sqrt{5}F\left(\operatorname{asin}\left(\frac{\sqrt{10}x}{2}\right)\middle|\frac{2}{5}\right)}{5} & \text{for}\: x > -\frac{\sqrt{10}}{5} \wedge x < \frac{\sqrt{10}}{5} \end{cases}$$

input
```
integrate(1/(-5*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Piecewise((sqrt(5)*elliptic_f(asin(sqrt(10)*x/2), 2/5)/5, (x > -sqrt(10)/5
) & (x < sqrt(10)/5)))
```

3.227.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

input
```
integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

3.227.8 Giac [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

input
```
integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-5\,x^2}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(2 - 5*x^2)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(2 - 5*x^2)^(1/2)), x)
```

3.228 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx$

3.228.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = \frac{\sqrt{2+5x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+5x^2}{1+x^2}}}$$

output
```
1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*6^(1/2))*(5*x^2+2)^(
1/2)*2^(1/2)/((5*x^2+2)/(x^2+1))^(1/2)
```

3.228.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = -\frac{i\,\text{EllipticF}\left(i\text{arcsinh}(x),\frac{5}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

output
```
((-I)*EllipticF[I*ArcSinh[x], 5/2])/Sqrt[2]
```

3.228.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{5x^2+2}}dx$$
$$\Big\downarrow \text{320}$$
$$\frac{\sqrt{5x^2+2}\,\text{EllipticF}\left(\arctan(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{5x^2+2}{x^2+1}}}$$

input
```
Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

output
```
(Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(
2 + 5*x^2)/(1 + x^2)])
```

3.228.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.228.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.33

method	result	size
default	$-\frac{i F\left(i x, \frac{\sqrt{10}}{2}\right) \sqrt{2}}{2}$	17
elliptic	$-\frac{i \sqrt{\left(x^{2}+1\right)\left(5 x^{2}+2\right)}\, \sqrt{10 x^{2}+4}\, F\left(i x, \frac{\sqrt{10}}{2}\right)}{2 \sqrt{5 x^{2}+2}\, \sqrt{5 x^{4}+7 x^{2}+2}}$	61

input
```
int(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*EllipticF(I*x,1/2*10^(1/2))*2^(1/2)
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = -\frac{1}{2}i\,\sqrt{2}F(\arcsin{(i\,x)}\mid\frac{5}{2})$$

input
```
integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*I*sqrt(2)*elliptic_f(arcsin(I*x), 5/2)
```

3.228.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{5x^2+2}}\,dx$$

input
```
integrate(1/(x**2+1)**(1/2)/(5*x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(x**2 + 1)*sqrt(5*x**2 + 2)), x)
```

3.228.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.228.8 Giac [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{5\,x^2+2}}\,dx$$

input
```
int(1/((x^2 + 1)^(1/2)*(5*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)^(1/2)*(5*x^2 + 2)^(1/2)), x)
```

3.229 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx$

3.229.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{1+2x^2}\,\text{EllipticF}(\arctan(x),-1)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{1+2x^2}{1+x^2}}}$$

output
```
1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),I)*(2*x^2+1)^(1/2)*2^(1/2)
/((2*x^2+1)/(x^2+1))^(1/2)
```

3.229.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = -\frac{i\,\text{EllipticF}(i\text{arcsinh}(x),2)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

output
```
((-I)*EllipticF[I*ArcSinh[x], 2])/Sqrt[2]
```

3.229.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{4x^2+2}}\,dx$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{2x^2+1}\,\text{EllipticF}(\arctan(x),-1)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

input
```
Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

output
```
(Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(1
+ 2*x^2)/(1 + x^2)])
```

3.229.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.229.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.31

method	result	size
default	$-\frac{i F\left(i x, \sqrt{2}\right) \sqrt{2}}{2}$	15
elliptic	$-\frac{i \sqrt{\left(x^{2}+1\right)\left(2 x^{2}+1\right)}\, F\left(i x, \sqrt{2}\right)}{\sqrt{4 x^{4}+6 x^{2}+2}}$	41

input
```
int(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*EllipticF(I*x,2^(1/2))*2^(1/2)
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = -\frac{1}{2}i\,\sqrt{2}F(\arcsin{(i\,x)}\,|2)$$

input
```
integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*I*sqrt(2)*elliptic_f(arcsin(I*x), 2)
```

3.229.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{2}\int\frac{1}{\sqrt{x^2+1}\sqrt{2x^2+1}}\,dx}{2}$$

input
```
integrate(1/(x**2+1)**(1/2)/(4*x**2+2)**(1/2),x)
```

output
```
sqrt(2)*Integral(1/(sqrt(x**2 + 1)*sqrt(2*x**2 + 1)), x)/2
```

3.229.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.229.8 Giac [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{4\,x^2+2}}\,dx$$

input
```
int(1/((x^2 + 1)^(1/2)*(4*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)^(1/2)*(4*x^2 + 2)^(1/2)), x)
```

3.230 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx$

3.230.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \frac{\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

output
```
1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*(3*x^2+2)^(
1/2)*2^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

3.230.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = -\frac{i\,\text{EllipticF}\left(i\text{arcsinh}(x),\frac{3}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
((-I)*EllipticF[I*ArcSinh[x], 3/2])/Sqrt[2]
```

3.230.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

input
```
Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
(Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(
2 + 3*x^2)/(1 + x^2)])
```

3.230.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.230.4 Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.33

method	result	size
default	$-\frac{iF\left(ix,\frac{\sqrt{6}}{2}\right)\sqrt{2}}{2}$	17
elliptic	$-\frac{i\sqrt{(3x^2+2)(x^2+1)}\,\sqrt{6x^2+4}\,F\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^2+2}\,\sqrt{3x^4+5x^2+2}}$	61

input
```
int(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*EllipticF(I*x,1/2*6^(1/2))*2^(1/2)
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = -\frac{1}{2}i\sqrt{2}F\left(\arcsin\left(i\,x\right)\mid\frac{3}{2}\right)$$

input
```
integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*I*sqrt(2)*elliptic_f(arcsin(I*x), 3/2)
```

3.230.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{3x^2+2}}\,dx$$

input
```
integrate(1/(x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(x**2 + 1)*sqrt(3*x**2 + 2)), x)
```

3.230.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.230.8 Giac [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{3\,x^2+2}}\,dx$$

input
```
int(1/((x^2 + 1)^(1/2)*(3*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.231 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx$

3.231.1 Optimal result

Integrand size = 21, antiderivative size = 8

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \frac{\arctan(x)}{\sqrt{2}}$$

output
```
1/2*arctan(x)*2^(1/2)
```

3.231.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \frac{\arctan(x)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

output
```
ArcTan[x]/Sqrt[2]
```

3.231.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{2x^2+2}}dx$$

$$\Big\downarrow \text{ 282}$$

$$\frac{\int \frac{1}{x^2+1}dx}{\sqrt{2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan(x)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

output
```
ArcTan[x]/Sqrt[2]
```

3.231.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 282
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b, c
, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !SimplerQ[a + b*x
^n, c + d*x^n]
```

3.231.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

method	result	size
meijerg	$\frac{\arctan(x)\sqrt{2}}{2}$	8

input
```
int(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)*2^(1/2)
```

3.231.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(7) = 14$.

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 4.25

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{2\,x^2+2}\sqrt{x^2+1}x}{x^4-1}\right)$$

input
```
integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(sqrt(2)*sqrt(2*x^2 + 2)*sqrt(x^2 + 1)*x/(x^4 - 1))
```

3.231.6 Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\,(x)}{2}$$

input
```
integrate(1/(x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
```

output
```
sqrt(2)*atan(x)/2
```

3.231.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.231.8 Giac [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{2\,x^2+2}}\,dx$$

input
```
int(1/((x^2 + 1)^(1/2)*(2*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

3.232 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx$

3.232.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx=\frac{\sqrt{2+x^2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

output
```
1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*(x^2+2)^(1/2)
*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)
```

3.232.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx=-\frac{i\,\text{EllipticF}\left(i\text{arcsinh}(x),\frac{1}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + x^2]),x]
```

output
```
((-I)*EllipticF[I*ArcSinh[x], 1/2])/Sqrt[2]
```

3.232.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}dx$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

input
```
Int[1/(Sqrt[1 + x^2]*Sqrt[2 + x^2]),x]
```

output
```
(Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2 +
 x^2)/(1 + x^2)])
```

3.232.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.232.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.32

method	result	size
default	$-iF\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)$	15
elliptic	$-\frac{i\sqrt{(x^2+1)(x^2+2)}\,\sqrt{2}\,\sqrt{2x^2+4}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2\sqrt{x^2+2}\,\sqrt{x^4+3x^2+2}}$	59

input
```
int(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))
```

3.232.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx = -i\,F\left(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2\right)$$

input
```
integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-I*elliptic_f(arcsin(1/2*I*sqrt(2)*x), 2)
```

3.232.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}\,dx$$

input
```
integrate(1/(x**2+1)**(1/2)/(x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(x**2 + 1)*sqrt(x**2 + 2)), x)
```

3.232.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.232.8 Giac [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{x^2+2}}\,dx$$

input
```
int(1/((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2)), x)
```

3.233 $\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx$

3.233.1 Optimal result

Integrand size = 21, antiderivative size = 10

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = \text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

3.233.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = -\frac{i\,\text{EllipticF}\left(i\text{arcsinh}(x),-\frac{1}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[2 - x^2]*Sqrt[1 + x^2]),x]
```

output
```
((-I)*EllipticF[I*ArcSinh[x], -1/2])/Sqrt[2]
```

3.233.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{321}$$

$$\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

input
```
Int[1/(Sqrt[2 - x^2]*Sqrt[1 + x^2]),x]
```

output
```
EllipticF[ArcSin[x/Sqrt[2]], -2]
```

3.233.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.233.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

method	result	size
default	$F\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)$	14
elliptic	$\frac{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\,\sqrt{2}\,\sqrt{-2x^2+4}\,F\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{2\sqrt{-x^2+2}\,\sqrt{-x^4+x^2+2}}$	63

input
```
int(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticF(1/2*2^(1/2)*x,I*2^(1/2))
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)|-2\right)$$

input
```
integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
elliptic_f(arcsin(1/2*sqrt(2)*x), -2)
```

3.233.6 Sympy [F]

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(-x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(2 - x**2)*sqrt(x**2 + 1)), x)
```

3.233.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-x^2+2}}\,dx$$

input
```
integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)), x)
```

3.233.8 Giac [F]

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-x^2+2}}\,dx$$

input
```
integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-x^2}}\,dx$$

input
```
int(1/((x^2 + 1)^(1/2)*(2 - x^2)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)^(1/2)*(2 - x^2)^(1/2)), x)
```

3.234 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx$

3.234.1 Optimal result

Integrand size = 21, antiderivative size = 10

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}}$$

output
```
1/2*EllipticF(x,I)*2^(1/2)
```

3.234.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[1 + x^2]),x]
```

output
```
EllipticF[ArcSin[x], -1]/Sqrt[2]
```

3.234.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {284, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 284}$$

$$\int \frac{1}{\sqrt{2-2x^4}}dx$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}}$$

input
```
Int[1/(Sqrt[2 - 2*x^2]*Sqrt[1 + x^2]),x]
```

output
```
EllipticF[ArcSin[x], -1]/Sqrt[2]
```

3.234.3.1 Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

3.234.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{F(x,i)\sqrt{2}}{2}$	10
elliptic	$\frac{\sqrt{-x^4+1}\,F(x,i)}{\sqrt{-2x^4+2}}$	24

input
```
int(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,I)*2^(1/2)
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin{(x)}\mid -1)$$

input
```
integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), -1)
```

3.234.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(8) = 16.

Time = 11.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 7.60

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \frac{\sqrt{2}iG_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{2}, 1, 1 & \frac{3}{4}, \frac{3}{4}, \frac{5}{4} \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4} & 0 \end{matrix} \middle| {\frac{1}{x^4}} \right)}{16\pi^{\frac{3}{2}}} - \frac{\sqrt{2}iG_{6,6}^{3,5}\left(\begin{matrix} -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} & 1 \\ 0, \frac{1}{2}, 0 & -\frac{1}{4}, \frac{1}{4}, \frac{1}{4} \end{matrix} \middle| {\frac{e^{-2i\pi}}{x^4}} \right)}{16\pi^{\frac{3}{2}}}$$

input
```
integrate(1/(-2*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
sqrt(2)*I*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4)
, (0,)), x**(-4))/(16*pi**(3/2)) - sqrt(2)*I*meijerg(((-1/4, 0, 1/4, 1/2,
3/4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), exp_polar(-2*I*pi)/x**4)/(16
*pi**(3/2))
```

3.234.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

input
```
integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

3.234.8 Giac [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

input
```
integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-2\,x^2}}\,dx$$

input
```
int(1/((x^2 + 1)^(1/2)*(2 - 2*x^2)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

3.235 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx$

3.235.1 Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

3.235.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

output
```
EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

3.235.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{x^2+1}}\,dx$$
$$\Big\downarrow \text{321}$$
$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Int[1/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

output
```
EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

3.235.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.235.4 Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\sqrt{3}}{3}$	19
elliptic	$\frac{\sqrt{-(3x^2-2)(x^2+1)}\,\sqrt{6}\,\sqrt{-6x^2+4}\,F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{-3x^2+2}\,\sqrt{-3x^4-x^2+2}}$	67

input
```
int(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \frac{1}{3}\sqrt{3}F(\arcsin\left(\frac{1}{2}\sqrt{3}\sqrt{2}x\right) \mid -\frac{2}{3})$$

input
```
integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*elliptic_f(arcsin(1/2*sqrt(3)*sqrt(2)*x), -2/3)
```

3.235.6 Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \begin{cases} \frac{\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle| -\frac{2}{3}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

input
```
integrate(1/(-3*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Piecewise((sqrt(3)*elliptic_f(asin(sqrt(6)*x/2), -2/3)/3, (x > -sqrt(6)/3)
 & (x < sqrt(6)/3)))
```

3.235.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.235.8 Giac [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-3\,x^2}}\,dx$$

input
```
int(1/((x^2 + 1)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.236 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx$

3.236.1 Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \frac{1}{2}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{2}\right)$$

output
```
1/2*EllipticF(x*2^(1/2),1/2*I*2^(1/2))
```

3.236.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \frac{1}{2}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{2}\right)$$

input
```
Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[1 + x^2]),x]
```

output
```
EllipticF[ArcSin[Sqrt[2]*x], -1/2]/2
```

3.236.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{x^2+1}}\,dx$$

$$\Big\downarrow \ 321$$

$$\frac{1}{2}\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{2}\right)$$

input
```
Int[1/(Sqrt[2 - 4*x^2]*Sqrt[1 + x^2]),x]
```

output
```
EllipticF[ArcSin[Sqrt[2]*x], -1/2]/2
```

3.236.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.236.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{F\left(\sqrt{2}\,x,\frac{i\sqrt{2}}{2}\right)}{2}$	15
elliptic	$\frac{\sqrt{-\left(2x^2-1\right)\left(x^2+1\right)}\,\sqrt{2}\,F\left(\sqrt{2}\,x,\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-4x^4-2x^2+2}}$	48

input
```
int(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(2^(1/2)*x,1/2*I*2^(1/2))
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \frac{1}{2}\,F(\arcsin\left(\sqrt{2}x\right)\,|-\frac{1}{2})$$

input
```
integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*elliptic_f(arcsin(sqrt(2)*x), -1/2)
```

3.236.6 Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \frac{\sqrt{2}\left(\begin{cases}\frac{\sqrt{2}F\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|-\frac{1}{2}\right)}{2} & \text{for}\: x > -\frac{\sqrt{2}}{2} \wedge x < \frac{\sqrt{2}}{2}\end{cases}\right)}{2}$$

input
```
integrate(1/(-4*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
sqrt(2)*Piecewise((sqrt(2)*elliptic_f(asin(sqrt(2)*x), -1/2)/2, (x > -sqrt
(2)/2) & (x < sqrt(2)/2)))/2
```

3.236.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

input
```
integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

3.236.8 Giac [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

input
```
integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-4\,x^2}}\,dx$$

input
```
int(1/((x^2 + 1)^(1/2)*(2 - 4*x^2)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)^(1/2)*(2 - 4*x^2)^(1/2)), x)
```

3.237 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx$

3.237.1 Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \frac{\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}}$$

output
```
1/5*EllipticF(1/2*x*10^(1/2),1/5*I*10^(1/2))*5^(1/2)
```

3.237.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \frac{\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}}$$

input
```
Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[1 + x^2]),x]
```

output
```
EllipticF[ArcSin[Sqrt[5/2]*x], -2/5]/Sqrt[5]
```

3.237.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{x^2+1}}\,dx$$
$$\Big\downarrow \; 321$$
$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}}$$

input
```
Int[1/(Sqrt[2 - 5*x^2]*Sqrt[1 + x^2]),x]
```

output
```
EllipticF[ArcSin[Sqrt[5/2]*x], -2/5]/Sqrt[5]
```

3.237.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.237.4 Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{F\left(\frac{\sqrt{10}\,x}{2},\frac{i\sqrt{10}}{5}\right)\sqrt{5}}{5}$	19
elliptic	$\frac{\sqrt{-(5x^2-2)(x^2+1)}\,\sqrt{10}\,\sqrt{-10x^2+4}\,F\left(\frac{\sqrt{10}\,x}{2},\frac{i\sqrt{10}}{5}\right)}{10\sqrt{-5x^2+2}\,\sqrt{-5x^4-3x^2+2}}$	67

input
```
int(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*EllipticF(1/2*10^(1/2)*x,1/5*I*10^(1/2))*5^(1/2)
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \frac{1}{5}\sqrt{5}F(\arcsin\left(\frac{1}{2}\sqrt{5}\sqrt{2}x\right)|-\frac{2}{5})$$

input
```
integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/5*sqrt(5)*elliptic_f(arcsin(1/2*sqrt(5)*sqrt(2)*x), -2/5)
```

3.237.6 Sympy [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \begin{cases} \frac{\sqrt{5}F\left(\operatorname{asin}\left(\frac{\sqrt{10}x}{2}\right)\middle|-\frac{2}{5}\right)}{5} & \text{for}\ x > -\frac{\sqrt{10}}{5} \wedge x < \frac{\sqrt{10}}{5} \end{cases}$$

input
```
integrate(1/(-5*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Piecewise((sqrt(5)*elliptic_f(asin(sqrt(10)*x/2), -2/5)/5, (x > -sqrt(10)/
5) & (x < sqrt(10)/5)))
```

3.237.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

input
```
integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

3.237.8 Giac [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

input
```
integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-5\,x^2}}\,dx$$

input
```
int(1/((x^2 + 1)^(1/2)*(2 - 5*x^2)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)^(1/2)*(2 - 5*x^2)^(1/2)), x)
```

3.238 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx$

3.238.1 Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx=\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{5}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

output
```
1/2*EllipticF(x,1/2*I*10^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

3.238.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx=\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{5}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

input
```
Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], -5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

3.238.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{5x^2+2}}\,dx$$
$$\Big\downarrow \; 323$$
$$\frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{5x^2+2}}dx}{\sqrt{x^2-1}}$$
$$\Big\downarrow \; 321$$
$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

input
```
Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], -5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

3.238.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

3.238.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{i F\left(\frac{i x \sqrt{10}}{2}, \frac{i \sqrt{10}}{5}\right) \sqrt{-x^{2}+1}\, \sqrt{5}}{5 \sqrt{x^{2}-1}}$	37
elliptic	$-\frac{i \sqrt{\left(x^{2}-1\right)\left(5 x^{2}+2\right)}\, \sqrt{10}\, \sqrt{10 x^{2}+4}\, \sqrt{-x^{2}+1}\, F\left(\frac{i x \sqrt{10}}{2}, \frac{i \sqrt{10}}{5}\right)}{10 \sqrt{x^{2}-1}\, \sqrt{5 x^{2}+2}\, \sqrt{5 x^{4}-3 x^{2}-2}}$	84

input
```
int(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*I*EllipticF(1/2*I*x*10^(1/2),1/5*I*10^(1/2))*(-x^2+1)^(1/2)*5^(1/2)/(
x^2-1)^(1/2)
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = -\frac{1}{2}\,\sqrt{-2}F(\arcsin{(x)}\mid -\frac{5}{2})$$

input
```
integrate(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-2)*elliptic_f(arcsin(x), -5/2)
```

3.238.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{(x-1)\,(x+1)}\sqrt{5x^2+2}}\,dx$$

input
```
integrate(1/(x**2-1)**(1/2)/(5*x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(5*x**2 + 2)), x)
```

3.238.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.238.8 Giac [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{5\,x^2+2}}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(5*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(5*x^2 + 2)^(1/2)), x)
```

3.239 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx$

3.239.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),-2)}{\sqrt{2}\sqrt{-1+x^2}}$$

output
```
1/2*EllipticF(x,I*2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

3.239.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),-2)}{\sqrt{2}\sqrt{-1+x^2}}$$

input
```
Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], -2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

3.239.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {323, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{4x^2+2}}dx$$
$$\Big\downarrow \text{ 323}$$
$$\frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2}\sqrt{1-x^2}\sqrt{2x^2+1}}dx}{\sqrt{x^2-1}}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2x^2+1}}dx}{\sqrt{2}\sqrt{x^2-1}}$$
$$\Big\downarrow \text{ 321}$$
$$\frac{\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),-2)}{\sqrt{2}\sqrt{x^2-1}}$$

input
```
Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], -2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

3.239.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.239.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

method	result	size
default	$-\frac{i F\left(i x \sqrt{2}, \frac{i \sqrt{2}}{2}\right) \sqrt{-x^{2}+1}}{2 \sqrt{x^{2}-1}}$	34
elliptic	$-\frac{i \sqrt{\left(x^{2}-1\right)\left(2 x^{2}+1\right)}\, \sqrt{2}\, \sqrt{-x^{2}+1}\, F\left(i x \sqrt{2}, \frac{i \sqrt{2}}{2}\right)}{2 \sqrt{x^{2}-1}\, \sqrt{4 x^{4}-2 x^{2}-2}}$	66

input
```
int(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*EllipticF(I*2^(1/2)*x,1/2*I*2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = -\frac{1}{2}\,\sqrt{-2}F(\arcsin{(x)}\,|{-2})$$

input
```
integrate(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-2)*elliptic_f(arcsin(x), -2)
```

3.239.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{2}\int \frac{1}{\sqrt{x^2-1}\sqrt{2x^2+1}}\,dx}{2}$$

input
```
integrate(1/(x**2-1)**(1/2)/(4*x**2+2)**(1/2),x)
```

output
```
sqrt(2)*Integral(1/(sqrt(x**2 - 1)*sqrt(2*x**2 + 1)), x)/2
```

3.239.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.239.8 Giac [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{4\,x^2+2}}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(4*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(4*x^2 + 2)^(1/2)), x)
```

3.240 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx$

3.240.1 Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

output
```
1/2*EllipticF(x,1/2*I*6^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

input
```
Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], -3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

3.240.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{3x^2+2}}dx$$
$$\Big\downarrow \text{323}$$
$$\frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{3x^2+2}}dx}{\sqrt{x^2-1}}$$
$$\Big\downarrow \text{321}$$
$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

input
```
Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], -3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

3.240.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

3.240.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{iF\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\sqrt{-x^{2}+1}\,\sqrt{3}}{3\sqrt{x^{2}-1}}$	37
elliptic	$-\frac{i\sqrt{(3x^{2}+2)(x^{2}-1)}\,\sqrt{6}\,\sqrt{6x^{2}+4}\,\sqrt{-x^{2}+1}\,F\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{3x^{2}+2}\,\sqrt{x^{2}-1}\,\sqrt{3x^{4}-x^{2}-2}}$	84

input
```
int(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*EllipticF(1/2*I*x*6^(1/2),1/3*I*6^(1/2))*(-x^2+1)^(1/2)*3^(1/2)/(x^
2-1)^(1/2)
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin{(x)}\mid -\frac{3}{2})$$

input
```
integrate(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-2)*elliptic_f(arcsin(x), -3/2)
```

3.240.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{3x^2+2}}\,dx$$

input
```
integrate(1/(x**2-1)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(3*x**2 + 2)), x)
```

3.240.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.240.8 Giac [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{3\,x^2+2}}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(3*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.241 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx$

3.241.1 Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \frac{1}{2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right),\frac{1}{2}\right)$$

output
```
1/2*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1/2))
```

3.241.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \frac{x\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)}{\sqrt{-1+x^2}\sqrt{2+2x^2}}$$

input
```
Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

output
```
(x*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4])/(Sqrt[-1 + x^2]*Sq
rt[2 + 2*x^2])
```

3.241.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {287}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{2x^2+2}}dx$$
$$\Big\downarrow \text{287}$$
$$\frac{1}{2}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)$$

input
```
Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

output
```
EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2]/2
```

3.241.3.1 Defintions of rubi rules used

rule 287
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/Sqrt[2*a*d])*EllipticF[ArcSin[Sqrt[2*d]*(x/Sqrt[c + d*x^2])], 1/2],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && GtQ[a, 0] && GtQ[d, 0]
```

3.241.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.53 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{iF(ix,i)\sqrt{-x^2+1}\,\sqrt{2}}{2\sqrt{x^2-1}}$	30
elliptic	$-\frac{i\sqrt{x^4-1}\,\sqrt{-x^2+1}\,F(ix,i)}{\sqrt{x^2-1}\,\sqrt{2x^4-2}}$	43

input
```
int(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*EllipticF(I*x,I)*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin{(x)}\,|-1)$$

input
```
integrate(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-2)*elliptic_f(arcsin(x), -1)
```

3.241.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.87 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \frac{\sqrt{2}iG_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{2}, 1, 1 & \frac{3}{4}, \frac{3}{4}, \frac{5}{4} \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4} & 0 \end{matrix}\middle| \frac{e^{2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}} - \frac{\sqrt{2}iG_{6,6}^{3,5}\left(\begin{matrix} -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} & 1 \\ 0, \frac{1}{2}, 0 & -\frac{1}{4}, \frac{1}{4}, \frac{1}{4} \end{matrix}\middle| \frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

input
```
integrate(1/(x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
```

output
```
sqrt(2)*I*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4)
, (0,)), exp_polar(2*I*pi)/x**4)/(16*pi**(3/2)) - sqrt(2)*I*meijerg(((-1/4
, 0, 1/4, 1/2, 3/4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), x**(-4))/(16*
pi**(3/2))
```

3.241.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.241.8 Giac [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{2\,x^2+2}}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

3.242 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx$

3.242.1 Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

output
```
1/2*EllipticF(x,1/2*I*2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

input
```
Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

3.242.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{x^2+2}}dx$$

$$\Big\downarrow \ 323$$

$$\frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+2}}dx}{\sqrt{x^2-1}}$$

$$\Big\downarrow \ 321$$

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

input
```
Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

3.242.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

3.242.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{iF\left(\frac{ix\sqrt{2}}{2},i\sqrt{2}\right)\sqrt{-x^{2}+1}}{\sqrt{x^{2}-1}}$	34
elliptic	$-\frac{i\sqrt{\left(x^{2}-1\right)\left(x^{2}+2\right)}\,\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{-x^{2}+1}\,F\left(\frac{ix\sqrt{2}}{2},i\sqrt{2}\right)}{2\sqrt{x^{2}-1}\,\sqrt{x^{2}+2}\,\sqrt{x^{4}+x^{2}-2}}$	76

input
```
int(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*EllipticF(1/2*I*x*2^(1/2),I*2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = -\frac{1}{2}\,\sqrt{-2}F(\arcsin{(x)}\mid -\frac{1}{2})$$

input
```
integrate(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-2)*elliptic_f(arcsin(x), -1/2)
```

3.242.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{(x-1)\,(x+1)}\sqrt{x^2+2}}\,dx$$

input
```
integrate(1/(x**2-1)**(1/2)/(x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(x**2 + 2)), x)
```

3.242.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.242.8 Giac [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{x^2+2}}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(x^2 + 2)^(1/2)),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(x^2 + 2)^(1/2)), x)
```

3.243 $\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx$

3.243.1 Optimal result

Integrand size = 21, antiderivative size = 12

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = -\operatorname{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{2}}\right),2\right)$$

output
```
-(x^2)^(1/2)/x*EllipticF(1/2*(-2*x^2+4)^(1/2),2^(1/2))
```

3.243.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(12) = 24.

Time = 10.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.92

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\sqrt{1-\frac{x^2}{2}}\operatorname{EllipticF}\left(\arcsin(x),\frac{1}{2}\right)}{\sqrt{-2+3x^2-x^4}}$$

input
```
Integrate[1/(Sqrt[2 - x^2]*Sqrt[-1 + x^2]),x]
```

output
```
(Sqrt[1 - x^2]*Sqrt[1 - x^2/2]*EllipticF[ArcSin[x], 1/2])/Sqrt[-2 + 3*x^2
- x^4]
```

3.243.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {322}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2-1}}\,dx$$

$$\Big\downarrow \text{ 322}$$

$$-\text{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{2}}\right),2\right)$$

input
```
Int[1/(Sqrt[2 - x^2]*Sqrt[-1 + x^2]),x]
```

output
```
-EllipticF[ArcCos[x/Sqrt[2]], 2]
```

3.243.3.1 Defintions of rubi rules used

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

3.243.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

method	result	size
default	$\frac{F\left(\frac{\sqrt{2}\,x}{2},\sqrt{2}\right)\sqrt{-x^2+1}}{\sqrt{x^2-1}}$	28
elliptic	$\frac{\sqrt{-(x^2-1)(x^2-2)}\,\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{-x^2+1}\,F\left(\frac{\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{-x^2+2}\,\sqrt{x^2-1}\,\sqrt{-x^4+3x^2-2}}$	78

input
```
int(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticF(1/2*2^(1/2)*x,2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin{(x)}\mid\frac{1}{2})$$

input
```
integrate(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-2)*elliptic_f(arcsin(x), 1/2)
```

3.243.6 Sympy [F]

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{2-x^2}}\,dx$$

input
```
integrate(1/(-x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(2 - x**2)), x)
```

3.243.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-x^2+2}}\,dx$$

input
```
integrate(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)), x)
```

3.243.8 Giac [F]

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-x^2+2}}\,dx$$

input
```
integrate(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-x^2}}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(2 - x^2)^(1/2)),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(2 - x^2)^(1/2)), x)
```

3.244 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx$

3.244.1 Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = -\frac{\sqrt{-1+x^2}\operatorname{arctanh}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

output
```
-1/2*arctanh(x)*(x^2-1)^(1/2)*2^(1/2)/(-x^2+1)^(1/2)
```

3.244.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = -\frac{(-1+x^2)\operatorname{arctanh}(x)}{\sqrt{2}\sqrt{-(-1+x^2)^2}}$$

input
```
Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]
```

output
```
-(((-1 + x^2)*ArcTanh[x])/(Sqrt[2]*Sqrt[-(-1 + x^2)^2]))
```

3.244.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {283, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{x^2-1}}dx$$

$$\Big\downarrow \; 283$$

$$\frac{\sqrt{x^2-1}\int \frac{1}{x^2-1}dx}{\sqrt{2}\sqrt{1-x^2}}$$

$$\Big\downarrow \; 220$$

$$-\frac{\sqrt{x^2-1}\text{arctanh}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

input
```
Int[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]
```

output
```
-((Sqrt[-1 + x^2]*ArcTanh[x])/(Sqrt[2]*Sqrt[1 - x^2]))
```

3.244.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

3.244.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-x^{2}+1}\,\operatorname{arctanh}(x)}{2\sqrt{x^{2}-1}}$	24
meijerg	$\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}(x^{2}-1)}\,\operatorname{arctanh}(x)}{2\sqrt{\operatorname{signum}(x^{2}-1)}}$	26
risch	$\frac{\sqrt{\frac{-2x^{2}+2}{x^{2}-1}}\,\sqrt{x^{2}-1}\left(-\frac{\sqrt{-2}\,\ln(-1+x)}{4}+\frac{\sqrt{-2}\,\ln(1+x)}{4}\right)}{\sqrt{-2x^{2}+2}}$	54

input
```
int(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)^(1/2)*arctanh(x)
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{-2\,x^2+2}x}{x^4-1}\right)$$

input
```
integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(sqrt(2)*sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)*x/(x^4 - 1))
```

3.244.6 Sympy [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{2}\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2-1}}\,dx}{2}$$

input
```
integrate(1/(-2*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

output
```
sqrt(2)*Integral(1/(sqrt(1 - x**2)*sqrt(x**2 - 1)), x)/2
```

3.244.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

input
```
integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

3.244.8 Giac [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

input
```
integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-2\,x^2}}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

3.245 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx$

3.245.1 Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

output
```
1/2*EllipticF(x,1/2*6^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

3.245.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{3}\sqrt{-1+x^2}}$$

input
```
Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 + x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/(Sqrt[3]*Sqrt[-1 + x^2
])
```

3.245.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{x^2-1}}\,dx$$

$$\Big\downarrow \; 323$$

$$\frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}dx}{\sqrt{x^2-1}}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

input
```
Int[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 + x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], 3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

3.245.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.245.4 Maple [A] (verified)

Time = 3.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{F\left(x,\frac{\sqrt{6}}{2}\right)\sqrt{-x^2+1}\,\sqrt{2}}{2\sqrt{x^2-1}}$	29
elliptic	$\frac{\sqrt{-(3x^2-2)(x^2-1)}\,\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,F\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{-3x^2+2}\,\sqrt{x^2-1}\,\sqrt{-3x^4+5x^2-2}}$	74

input
```
int(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,1/2*6^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = -\frac{1}{2}\,\sqrt{-2}F(\arcsin{(x)}\mid\frac{3}{2})$$

input
```
integrate(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-2)*elliptic_f(arcsin(x), 3/2)
```

3.245.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \begin{cases} -\frac{\sqrt{3}iF\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{2}{3}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

input
```
integrate(1/(-3*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

output
```
Piecewise((-sqrt(3)*I*elliptic_f(asin(sqrt(6)*x/2), 2/3)/3, (x > -sqrt(6)/
3) & (x < sqrt(6)/3)))
```

3.245.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

3.245.8 Giac [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-3\,x^2}}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.246 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx$

3.246.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),2)}{\sqrt{2}\sqrt{-1+x^2}}$$

output
```
1/2*EllipticF(x,2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

3.246.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),\frac{1}{2}\right)}{2\sqrt{-1+x^2}}$$

input
```
Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 + x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2]*x], 1/2])/(2*Sqrt[-1 + x^2])
```

3.246.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {323, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{x^2-1}}dx$$
$$\Big\downarrow \text{ 323}$$
$$\frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2}\sqrt{1-2x^2}\sqrt{1-x^2}}dx}{\sqrt{x^2-1}}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-2x^2}\sqrt{1-x^2}}dx}{\sqrt{2}\sqrt{x^2-1}}$$
$$\Big\downarrow \text{ 321}$$
$$\frac{\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),2)}{\sqrt{2}\sqrt{x^2-1}}$$

input
```
Int[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 + x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

3.246.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.246.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{F\left(x,\sqrt{2}\right)\sqrt{-x^2+1}\,\sqrt{2}}{2\sqrt{x^2-1}}$	27
elliptic	$\frac{\sqrt{-(2x^2-1)(x^2-1)}\,\sqrt{-x^2+1}\,F\left(x,\sqrt{2}\right)}{\sqrt{x^2-1}\,\sqrt{-4x^4+6x^2-2}}$	53

input
```
int(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = -\frac{1}{2}\,\sqrt{-2}F(\arcsin{(x)}\;|2)$$

input
```
integrate(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-2)*elliptic_f(arcsin(x), 2)
```

3.246.6 Sympy [A] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{2}\left(\begin{cases} -\frac{\sqrt{2}iF\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|\frac{1}{2}\right)}{2} & \text{for } x > -\frac{\sqrt{2}}{2} \wedge x < \frac{\sqrt{2}}{2} \end{cases}\right)}{2}$$

input
```
integrate(1/(-4*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

output
```
sqrt(2)*Piecewise((-sqrt(2)*I*elliptic_f(asin(sqrt(2)*x), 1/2)/2, (x > -sq
rt(2)/2) & (x < sqrt(2)/2)))/2
```

3.246.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

input
```
integrate(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

3.246.8 Giac [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

input
```
integrate(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-4\,x^2}}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(2 - 4*x^2)^(1/2)),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(2 - 4*x^2)^(1/2)), x)
```

3.247 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx$

3.247.1 Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),\frac{5}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

output
```
1/2*EllipticF(x,1/2*10^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

3.247.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),\frac{2}{5}\right)}{\sqrt{5}\sqrt{-1+x^2}}$$

input
```
Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 + x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], 2/5])/(Sqrt[5]*Sqrt[-1 + x^2
])
```

3.247.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{x^2-1}}\,dx$$

$$\Big\downarrow \; 323$$

$$\frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}dx}{\sqrt{x^2-1}}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

input
```
Int[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 + x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], 5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

3.247.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.247.4 Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{F\left(x,\frac{\sqrt{10}}{2}\right)\sqrt{-x^2+1}\,\sqrt{2}}{2\sqrt{x^2-1}}$	29
elliptic	$\frac{\sqrt{-(5x^2-2)(x^2-1)}\,\sqrt{-x^2+1}\,\sqrt{-10x^2+4}\,F\left(x,\frac{\sqrt{10}}{2}\right)}{2\sqrt{-5x^2+2}\,\sqrt{x^2-1}\,\sqrt{-5x^4+7x^2-2}}$	74

input
```
int(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(x,1/2*10^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin(x) \mid \frac{5}{2})$$

input
```
integrate(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-2)*elliptic_f(arcsin(x), 5/2)
```

3.247.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \begin{cases} -\frac{\sqrt{5}iF\left(\operatorname{asin}\left(\frac{\sqrt{10}x}{2}\right)\middle|\frac{2}{5}\right)}{5} & \text{for } x > -\frac{\sqrt{10}}{5} \wedge x < \frac{\sqrt{10}}{5} \end{cases}$$

input
```
integrate(1/(-5*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

output
```
Piecewise((-sqrt(5)*I*elliptic_f(asin(sqrt(10)*x/2), 2/5)/5, (x > -sqrt(10
)/5) & (x < sqrt(10)/5)))
```

3.247.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

input
```
integrate(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

3.247.8 Giac [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

input
```
integrate(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-5\,x^2}}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(2 - 5*x^2)^(1/2)),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(2 - 5*x^2)^(1/2)), x)
```

3.248 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}} dx$

3.248.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}} dx = \frac{\sqrt{2+5x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+5x^2}{1+x^2}}}$$

output
```
1/2*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*6^(1/2
))*(5*x^2+2)^(1/2)*2^(1/2)/(-x^2-1)^(1/2)/((5*x^2+2)/(x^2+1))^(1/2)
```

3.248.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}} dx = -\frac{i\sqrt{1+x^2}\,\text{EllipticF}\left(i\text{arcsinh}(x),\frac{5}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}}$$

input
```
Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

output
```
((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 5/2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

3.248.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{5x^2+2}}dx$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{5x^2+2}\,\text{EllipticF}\left(\arctan(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{5x^2+2}{x^2+1}}}$$

input
```
Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

output
```
(Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[
(2 + 5*x^2)/(1 + x^2)])
```

3.248.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.248.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{i F\left(\frac{i x \sqrt{10}}{2}, \frac{\sqrt{10}}{5}\right) \sqrt{5}\, \sqrt{-x^{2}-1}}{5 \sqrt{x^{2}+1}}$	36
elliptic	$-\frac{i \sqrt{-\left(x^{2}+1\right)\left(5 x^{2}+2\right)}\, \sqrt{10}\, \sqrt{10 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i x \sqrt{10}}{2}, \frac{\sqrt{10}}{5}\right)}{10 \sqrt{-x^{2}-1}\, \sqrt{5 x^{2}+2}\, \sqrt{-5 x^{4}-7 x^{2}-2}}$	84

input
```
int(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*I*EllipticF(1/2*I*x*10^(1/2),1/5*10^(1/2))/(x^2+1)^(1/2)*5^(1/2)*(-x^2
-1)^(1/2)
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx = \frac{1}{2}i\sqrt{-2}F\left(\arcsin\left(ix\right) \mid \frac{5}{2}\right)$$

input
```
integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*I*sqrt(-2)*elliptic_f(arcsin(I*x), 5/2)
```

3.248.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{5x^2+2}}\,dx$$

input
```
integrate(1/(-x**2-1)**(1/2)/(5*x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-x**2 - 1)*sqrt(5*x**2 + 2)), x)
```

3.248.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.248.8 Giac [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{5\,x^2+2}}\,dx$$

input
```
int(1/((- x^2 - 1)^(1/2)*(5*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((- x^2 - 1)^(1/2)*(5*x^2 + 2)^(1/2)), x)
```

3.249 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx$

3.249.1 Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{1+2x^2}\,\text{EllipticF}(\arctan(x),-1)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{1+2x^2}{1+x^2}}}$$

output
```
1/2*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),I)*(2*x^2+1)
^(1/2)*2^(1/2)/(-x^2-1)^(1/2)/((2*x^2+1)/(x^2+1))^(1/2)
```

3.249.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = -\frac{i\sqrt{1+x^2}\,\text{EllipticF}(i\text{arcsinh}(x),2)}{\sqrt{2}\sqrt{-1-x^2}}$$

input
```
Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

output
```
((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

3.249.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{4x^2+2}}dx$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{2x^2+1}\,\text{EllipticF}(\arctan(x),-1)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

input
```
Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

output
```
(Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(1
 + 2*x^2)/(1 + x^2)])
```

3.249.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.249.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{iF\left(ix\sqrt{2},\frac{\sqrt{2}}{2}\right)\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	33
elliptic	$-\frac{i\sqrt{-(x^2+1)(2x^2+1)}\,\sqrt{2}\,\sqrt{x^2+1}\,F\left(ix\sqrt{2},\frac{\sqrt{2}}{2}\right)}{2\sqrt{-x^2-1}\,\sqrt{-4x^4-6x^2-2}}$	66

input
```
int(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*EllipticF(I*2^(1/2)*x,1/2*2^(1/2))/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \frac{1}{2}i\sqrt{-2}F(\arcsin{(i\,x)}\,|2)$$

input
```
integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*I*sqrt(-2)*elliptic_f(arcsin(I*x), 2)
```

3.249.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{2}\int \frac{1}{\sqrt{-x^2-1}\sqrt{2x^2+1}}\,dx}{2}$$

input
```
integrate(1/(-x**2-1)**(1/2)/(4*x**2+2)**(1/2),x)
```

output
```
sqrt(2)*Integral(1/(sqrt(-x**2 - 1)*sqrt(2*x**2 + 1)), x)/2
```

3.249.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.249.8 Giac [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{4\,x^2+2}}\,dx$$

input
```
int(1/((- x^2 - 1)^(1/2)*(4*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((- x^2 - 1)^(1/2)*(4*x^2 + 2)^(1/2)), x)
```

3.250 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx$

3.250.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = \frac{\sqrt{2+3x^2}\,\mathrm{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

output
```
1/2*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*2^(1/2
))*(3*x^2+2)^(1/2)*2^(1/2)/(-x^2-1)^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

3.250.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = -\frac{i\sqrt{1+x^2}\,\mathrm{EllipticF}\left(i\,\mathrm{arcsinh}(x),\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}}$$

input
```
Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 3/2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

3.250.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

input
```
Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
(Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[
(2 + 3*x^2)/(1 + x^2)])
```

3.250.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.250.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{iF\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)\sqrt{3}\,\sqrt{-x^2-1}}{3\sqrt{x^2+1}}$	36
elliptic	$-\frac{i\sqrt{-(3x^2+2)(x^2+1)}\,\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{6\sqrt{-x^2-1}\,\sqrt{3x^2+2}\,\sqrt{-3x^4-5x^2-2}}$	84

input
```
int(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*I*EllipticF(1/2*I*x*6^(1/2),1/3*6^(1/2))/(x^2+1)^(1/2)*3^(1/2)*(-x^2-1
)^(1/2)
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = \frac{1}{2}i\,\sqrt{-2}F(\arcsin{(i\,x)} \mid \frac{3}{2})$$

input
```
integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*I*sqrt(-2)*elliptic_f(arcsin(I*x), 3/2)
```

3.250.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{3x^2+2}}\,dx$$

input
```
integrate(1/(-x**2-1)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-x**2 - 1)*sqrt(3*x**2 + 2)), x)
```

3.250.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.250.8 Giac [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{3\,x^2+2}}\,dx$$

input
```
int(1/((- x^2 - 1)^(1/2)*(3*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((- x^2 - 1)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.251 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx$

3.251.1 Optimal result

Integrand size = 23, antiderivative size = 28

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \frac{\sqrt{1+x^2}\arctan(x)}{\sqrt{2}\sqrt{-1-x^2}}$$

output
```
1/2*arctan(x)*(x^2+1)^(1/2)*2^(1/2)/(-x^2-1)^(1/2)
```

3.251.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \frac{(1+x^2)\arctan(x)}{\sqrt{2}\sqrt{-(1+x^2)^2}}$$

input
```
Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

output
```
((1 + x^2)*ArcTan[x])/(Sqrt[2]*Sqrt[-(1 + x^2)^2])
```

3.251.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {283, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{2x^2+2}}dx$$

$$\Big\downarrow \text{283}$$

$$\frac{\sqrt{-x^2-1}\int \frac{1}{-x^2-1}dx}{\sqrt{2}\sqrt{x^2+1}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\sqrt{-x^2-1}\arctan(x)}{\sqrt{2}\sqrt{x^2+1}}$$

input
```
Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

output
```
-((Sqrt[-1 - x^2]*ArcTan[x])/(Sqrt[2]*Sqrt[1 + x^2]))
```

3.251.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

3.251.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.49 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.32

method	result	size
meijerg	$-\frac{i\sqrt{2}\,\arctan(x)}{2}$	9
default	$-\frac{\sqrt{-x^2-1}\,\sqrt{2}\,\arctan(x)}{2\sqrt{x^2+1}}$	24
risch	$\frac{\sqrt{\frac{(-x^2-1)(2x^2+2)}{(x^2+1)^2}}\,(x^2+1)\left(-\frac{i\sqrt{-2}\,\ln(x+i)}{4}+\frac{i\sqrt{-2}\,\ln(x-i)}{4}\right)}{\sqrt{-x^2-1}\,\sqrt{2x^2+2}}$	72

input
```
int(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*2^(1/2)*arctan(x)
```

3.251.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(23) = 46.

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.71

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2\,x^2+2}\sqrt{-x^2-1}x+\sqrt{2}(x^4-1)\right)}{x^4+2\,x^2+1}\right) - \frac{1}{8}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2\,x^2+2}\sqrt{-x^2-1}x-\sqrt{2}(x^4-1)\right)}{x^4+2\,x^2+1}\right)$$

input
```
integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(2*(2*sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)*x + sqrt(2)*(x^4 - 1))
/(x^4 + 2*x^2 + 1)) - 1/8*sqrt(2)*log(2*(2*sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)*
x - sqrt(2)*(x^4 - 1))/(x^4 + 2*x^2 + 1))
```

3.251.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \frac{\sqrt{2}\int \frac{1}{\sqrt{-x^2-1}\sqrt{x^2+1}}\,dx}{2}$$

input
```
integrate(1/(-x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
```

output
```
sqrt(2)*Integral(1/(sqrt(-x**2 - 1)*sqrt(x**2 + 1)), x)/2
```

3.251.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.251.8 Giac [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2\,x^2+2}}\,dx$$

input
```
int(1/((- x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((- x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

3.252 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx$

3.252.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \frac{\sqrt{2+x^2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

output
```
1/2*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))
*(x^2+2)^(1/2)*2^(1/2)/(-x^2-1)^(1/2)/((x^2+2)/(x^2+1))^(1/2)
```

3.252.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(i\text{arcsinh}(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{-((1+x^2)(2+x^2))}}$$

input
```
Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + x^2]),x]
```

output
```
((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x], 1/2])/(Sqrt[2]*S
qrt[-((1 + x^2)*(2 + x^2))])
```

3.252.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{x^2+2}}\,dx$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

input
```
Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + x^2]),x]
```

output
```
(Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(2
+ x^2)/(1 + x^2)])
```

3.252.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.252.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.46 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{i F\left(i x, \frac{\sqrt{2}}{2}\right) \sqrt{2}\, \sqrt{-x^{2}-1}}{2 \sqrt{x^{2}+1}}$	33
elliptic	$-\frac{i \sqrt{-\left(x^{2}+1\right) \left(x^{2}+2\right)}\, \sqrt{x^{2}+1}\, \sqrt{2 x^{2}+4}\, F\left(i x, \frac{\sqrt{2}}{2}\right)}{2 \sqrt{-x^{2}-1}\, \sqrt{x^{2}+2}\, \sqrt{-x^{4}-3 x^{2}-2}}$	74

input
```
int(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*EllipticF(I*x,1/2*2^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

3.252.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \frac{1}{2}i\sqrt{2}\sqrt{-2}F\left(\arcsin\left(\frac{1}{2}i\sqrt{2}x\right)\mid 2\right)$$

input
```
integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*I*sqrt(2)*sqrt(-2)*elliptic_f(arcsin(1/2*I*sqrt(2)*x), 2)
```

3.252.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{x^2+2}}\,dx$$

input
```
integrate(1/(-x**2-1)**(1/2)/(x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-x**2 - 1)*sqrt(x**2 + 2)), x)
```

3.252.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.252.8 Giac [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{x^2+2}}\,dx$$

input
```
int(1/((- x^2 - 1)^(1/2)*(x^2 + 2)^(1/2)),x)
```

output
```
int(1/((- x^2 - 1)^(1/2)*(x^2 + 2)^(1/2)), x)
```

3.253 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx$

3.253.1 Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx=\frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{\sqrt{-1-x^2}}$$

output
```
EllipticF(1/2*x*2^(1/2),I*2^(1/2))*(x^2+1)^(1/2)/(-x^2-1)^(1/2)
```

3.253.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx=-\frac{i\sqrt{1+x^2}\,\text{EllipticF}\left(i\text{arcsinh}(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}}$$

input
```
Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 - x^2]),x]
```

output
```
((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], -1/2])/(Sqrt[2]*Sqrt[-1 - x^2]
)
```

3.253.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{2-x^2}}dx$$
$$\Big\downarrow \text{ 323}$$
$$\frac{\sqrt{x^2+1}\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx}{\sqrt{-x^2-1}}$$
$$\Big\downarrow \text{ 321}$$
$$\frac{\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{\sqrt{-x^2-1}}$$

input
```
Int[1/(Sqrt[-1 - x^2]*Sqrt[2 - x^2]),x]
```

output
```
(Sqrt[1 + x^2]*EllipticF[ArcSin[x/Sqrt[2]], -2])/Sqrt[-1 - x^2]
```

3.253.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.253.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{iF\left(ix,\frac{i\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	34
elliptic	$-\frac{i\sqrt{(x^2-2)(x^2+1)}\,\sqrt{x^2+1}\,\sqrt{-2x^2+4}\,F\left(ix,\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-x^2-1}\,\sqrt{-x^2+2}\,\sqrt{x^4-x^2-2}}$	74

input
```
int(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*EllipticF(I*x,1/2*I*2^(1/2))/(x^2+1)^(1/2)*2^(1/2)*(-x^2-1)^(1/2)
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx = -\frac{1}{2}\,\sqrt{2}\sqrt{-2}F\left(\arcsin\left(\frac{1}{2}\,\sqrt{2}x\right)\,|\,-2\right)$$

input
```
integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt(-2)*elliptic_f(arcsin(1/2*sqrt(2)*x), -2)
```

3.253.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{2-x^2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x**2-1)**(1/2)/(-x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(2 - x**2)*sqrt(-x**2 - 1)), x)
```

3.253.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.253.8 Giac [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-x^2}}\,dx$$

input
```
int(1/((- x^2 - 1)^(1/2)*(2 - x^2)^(1/2)),x)
```

output
```
int(1/((- x^2 - 1)^(1/2)*(2 - x^2)^(1/2)), x)
```

3.254 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx$

3.254.1 Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = -\frac{\sqrt{1-\frac{1}{x^4}}\,x^2\,\mathrm{EllipticF}\left(\csc^{-1}(x),-1\right)}{\sqrt{2-2x^2}\sqrt{-1-x^2}}$$

output
```
-x^2*EllipticF(1/x,I)*(1-1/x^4)^(1/2)/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2)
```

3.254.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = \frac{x\sqrt{1-x^4}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)}{\sqrt{2-2x^2}\sqrt{-1-x^2}}$$

input
```
Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 - x^2]),x]
```

output
```
(x*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4])/(Sqrt[2 - 2*x^2]*S
qrt[-1 - x^2])
```

3.254.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {288, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-x^2-1}}dx$$

$$\Big\downarrow \; 288$$

$$\frac{\sqrt{x^2+1}\int \frac{1}{\sqrt{2-2x^4}}dx}{\sqrt{-x^2-1}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{x^2+1}\,\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}\sqrt{-x^2-1}}$$

input
```
Int[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 - x^2]),x]
```

output
```
(Sqrt[1 + x^2]*EllipticF[ArcSin[x], -1])/(Sqrt[2]*Sqrt[-1 - x^2])
```

3.254.3.1 Defintions of rubi rules used

rule 288
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(c + d*x^2)^FracPart[p]/((-1)^IntPart[p]*(-c - d*x^2)^FracPart[p])   Int[
((-a)*c - b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d,
 0] && GtQ[a, 0] && LtQ[c, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

3.254.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{iF(ix,i)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	30
elliptic	$-\frac{i\sqrt{x^4-1}\,\sqrt{x^2+1}\,F(ix,i)}{\sqrt{-x^2-1}\,\sqrt{2x^4-2}}$	43

input
```
int(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*EllipticF(I*x,I)*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin{(x)}\,|-1)$$

input
```
integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-2)*elliptic_f(arcsin(x), -1)
```

3.254.6 Sympy [A] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{2}G_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4} \\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0 \end{matrix}\middle|\,\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}} - \frac{\sqrt{2}G_{6,6}^{3,5}\left(\begin{matrix} -\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1 \\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4} \end{matrix}\middle|\,\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

input
```
integrate(1/(-2*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

output
```
sqrt(2)*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4),
(0,)), x**(-4))/(16*pi**(3/2)) - sqrt(2)*meijerg(((-1/4, 0, 1/4, 1/2, 3/4)
, (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), exp_polar(-2*I*pi)/x**4)/(16*pi*
*(3/2))
```

3.254.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

input
```
integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

3.254.8 Giac [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

input
```
integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-2\,x^2}}\,dx$$

input
```
int(1/((- x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)),x)
```

output
```
int(1/((- x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

3.255 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx$

3.255.1 Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx=\frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-1-x^2}}$$

output
```
1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*(x^2+1)^(1/2)*3^(1/2)/(-x^2-1)^
(1/2)
```

3.255.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx=\frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-1-x^2}}$$

input
```
Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 - x^2]),x]
```

output
```
(Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/(Sqrt[3]*Sqrt[-1 - x^
2])
```

3.255.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-x^2-1}}\,dx$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\sqrt{x^2+1}\int \frac{1}{\sqrt{2-3x^2}\sqrt{x^2+1}}dx}{\sqrt{-x^2-1}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}}$$

input
```
Int[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 - x^2]),x]
```

output
```
(Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/(Sqrt[3]*Sqrt[-1 - x^
2])
```

3.255.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

3.255.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{iF\left(ix, \frac{i\sqrt{6}}{2}\right)\sqrt{-x^2-1}\,\sqrt{2}}{2\sqrt{x^2+1}}$	34
elliptic	$-\frac{i\sqrt{(3x^2-2)(x^2+1)}\,\sqrt{x^2+1}\,\sqrt{-6x^2+4}\,F\left(ix, \frac{i\sqrt{6}}{2}\right)}{2\sqrt{-3x^2+2}\,\sqrt{-x^2-1}\,\sqrt{3x^4+x^2-2}}$	76

input
```
int(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*EllipticF(I*x,1/2*I*6^(1/2))/(x^2+1)^(1/2)*(-x^2-1)^(1/2)*2^(1/2)
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = -\frac{1}{6}\sqrt{3}\sqrt{2}\sqrt{-2}F(\arcsin\left(\frac{1}{2}\sqrt{3}\sqrt{2}x\right)|-\frac{2}{3})$$

input
```
integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*sqrt(2)*sqrt(-2)*elliptic_f(arcsin(1/2*sqrt(3)*sqrt(2)*x), -2
/3)
```

3.255.6 Sympy [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{2-3x^2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-3*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

output
```
Integral(1/(sqrt(2 - 3*x**2)*sqrt(-x**2 - 1)), x)
```

3.255.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

3.255.8 Giac [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-3\,x^2}}\,dx$$

input
```
int(1/((- x^2 - 1)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

output
```
int(1/((- x^2 - 1)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.256 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx$

3.256.1 Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{2}\right)}{2\sqrt{-1-x^2}}$$

output
```
1/2*EllipticF(x*2^(1/2),1/2*I*2^(1/2))*(x^2+1)^(1/2)/(-x^2-1)^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{2}\right)}{2\sqrt{-1-x^2}}$$

input
```
Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 - x^2]),x]
```

output
```
(Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/2])/(2*Sqrt[-1 - x^2])
```

3.256.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {323, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-x^2-1}}dx$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{x^2+1}\int \frac{1}{\sqrt{2}\sqrt{1-2x^2}\sqrt{x^2+1}}dx}{\sqrt{-x^2-1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x^2+1}\int \frac{1}{\sqrt{1-2x^2}\sqrt{x^2+1}}dx}{\sqrt{2}\sqrt{-x^2-1}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{2}\right)}{2\sqrt{-x^2-1}}$$

input
```
Int[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 - x^2]),x]
```

output
```
(Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/2])/(2*Sqrt[-1 - x^2])
```

3.256.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.256.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{iF\left(ix,i\sqrt{2}\right)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	34
elliptic	$-\frac{i\sqrt{(2x^2-1)(x^2+1)}\,\sqrt{x^2+1}\,F\left(ix,i\sqrt{2}\right)}{\sqrt{-x^2-1}\,\sqrt{4x^4+2x^2-2}}$	60

input
```
int(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*EllipticF(I*x,I*2^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = -\frac{1}{4}\sqrt{2}\sqrt{-2}F(\arcsin\left(\sqrt{2}x\right) \mid -\frac{1}{2})$$

input
```
integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*sqrt(-2)*elliptic_f(arcsin(sqrt(2)*x), -1/2)
```

3.256.6 Sympy [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{2}\left(\begin{cases} -\frac{\sqrt{2}iF\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|-\frac{1}{2}\right)}{2} & \text{for } x > -\frac{\sqrt{2}}{2} \wedge x < \frac{\sqrt{2}}{2} \end{cases}\right)}{2}$$

input
```
integrate(1/(-4*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

output
```
sqrt(2)*Piecewise((-sqrt(2)*I*elliptic_f(asin(sqrt(2)*x), -1/2)/2, (x > -s
qrt(2)/2) & (x < sqrt(2)/2)))/2
```

3.256.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

input
```
integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

3.256.8 Giac [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

input
```
integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-4\,x^2}}\,dx$$

input
```
int(1/((- x^2 - 1)^(1/2)*(2 - 4*x^2)^(1/2)),x)
```

output
```
int(1/((- x^2 - 1)^(1/2)*(2 - 4*x^2)^(1/2)), x)
```

3.257 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx$

3.257.1 Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-1-x^2}}$$

output
```
1/5*EllipticF(1/2*x*10^(1/2),1/5*I*10^(1/2))*(x^2+1)^(1/2)*5^(1/2)/(-x^2-1
)^(1/2)
```

3.257.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-1-x^2}}$$

input
```
Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 - x^2]),x]
```

output
```
(Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], -2/5])/(Sqrt[5]*Sqrt[-1 - x^
2])
```

3.257.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-x^2-1}}dx$$
$$\Big\downarrow \; 323$$
$$\frac{\sqrt{x^2+1}\int \frac{1}{\sqrt{2-5x^2}\sqrt{x^2+1}}dx}{\sqrt{-x^2-1}}$$
$$\Big\downarrow \; 321$$
$$\frac{\sqrt{x^2+1}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-x^2-1}}$$

input
```
Int[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 - x^2]),x]
```

output
```
(Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], -2/5])/(Sqrt[5]*Sqrt[-1 - x^
2])
```

3.257.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

3.257.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{iF\left(ix,\frac{i\sqrt{10}}{2}\right)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	34
elliptic	$-\frac{i\sqrt{(5x^2-2)(x^2+1)}\,\sqrt{x^2+1}\,\sqrt{-10x^2+4}\,F\left(ix,\frac{i\sqrt{10}}{2}\right)}{2\sqrt{-5x^2+2}\,\sqrt{-x^2-1}\,\sqrt{5x^4+3x^2-2}}$	78

input
```
int(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*EllipticF(I*x,1/2*I*10^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = -\frac{1}{10}\sqrt{5}\sqrt{2}\sqrt{-2}F(\arcsin\left(\frac{1}{2}\sqrt{5}\sqrt{2}x\right)|-\frac{2}{5})$$

input
```
integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/10*sqrt(5)*sqrt(2)*sqrt(-2)*elliptic_f(arcsin(1/2*sqrt(5)*sqrt(2)*x), -
2/5)
```

3.257.6 Sympy [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{2-5x^2}\sqrt{-x^2-1}}\,dx$$

input
```
integrate(1/(-5*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

output
```
Integral(1/(sqrt(2 - 5*x**2)*sqrt(-x**2 - 1)), x)
```

3.257.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

input
```
integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

3.257.8 Giac [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

input
```
integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-5\,x^2}}\,dx$$

input
```
int(1/((- x^2 - 1)^(1/2)*(2 - 5*x^2)^(1/2)),x)
```

output
```
int(1/((- x^2 - 1)^(1/2)*(2 - 5*x^2)^(1/2)), x)
```

3.258 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx$

3.258.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

output
```
EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)*(1-d
*x^2/c)^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)
```

3.258.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/Sqrt[c - d*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c
)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2])
```

3.258.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \text{ 331}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{ 330}$$

$$\frac{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

input
```
Int[Sqrt[a + b*x^2]/Sqrt[c - d*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
```

3.258.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.258.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}\,a\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,E\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)}{\left(-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{\frac{d}{c}}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}}-\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\cdots}\right)\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}}\right)}{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}}$

input
```
int((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)*a*((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2
)*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)
/(d/c)^(1/2)
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx = -\frac{\sqrt{-bd}bc^2x\sqrt{\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}\right)+\sqrt{bx^2+a}\sqrt{-dx^2+c}bcd-(bc^2+ad^2)\sqrt{-bd}x\sqrt{\frac{c}{d}}F\left(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}\right)}{bcd^2x}$$

input
```
integrate((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-b*d)*b*c^2*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c))
 + sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*b*c*d - (b*c^2 + a*d^2)*sqrt(-b*d)*x*s
qrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)))/(b*c*d^2*x)
```

3.258.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/sqrt(c - d*x**2), x)
```

3.258.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

3.258.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}}{\sqrt{c-d\,x^2}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c - d*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(c - d*x^2)^(1/2), x)
```

3.259 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}} \, dx$

3.259.1 Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}} \, dx = \frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

output
```
EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2-a)^(1/2)*(1-
d*x^2/c)^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}} \, dx = \frac{\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}}$$

input
```
Integrate[Sqrt[-a - b*x^2]/Sqrt[c - d*x^2],x]
```

output
```
(Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*
c)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2])
```

3.259.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \; 331$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-bx^2-a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}$$

$$\Big\downarrow \; 330$$

$$\frac{\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

input
```
Int[Sqrt[-a - b*x^2]/Sqrt[c - d*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)
/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
```

3.259.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.259.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.

Time = 2.48 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.87

method	result
default	$\frac{\sqrt{-b\,x^{2}-a}\,\sqrt{-d\,x^{2}+c}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\left(aF\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d+bcF\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)-bcE\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\right)}{\left(-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\sqrt{-(b\,x^{2}+a)(-d\,x^{2}+c)}\,\Bigg(-\frac{a\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}-cb\,x^{2}-ac}}-\frac{bc\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\Big(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\Big(x\sqrt{-\frac{b}{a}},}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}-cb\,x^{2}-ac}\,d}$ / $\sqrt{-b\,x^{2}-a}\,\sqrt{-d\,x^{2}+c}$

input
```
int((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*x^2-a)^(1/2)*(-d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)
*(a*EllipticF(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(-b/a)^(1
/2),(-a*d/b/c)^(1/2))-b*c*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2)))/(-b*
d*x^4-a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx =$$
$$-\frac{\sqrt{bd}bc^2x\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}) + \sqrt{-bx^2-a}\sqrt{-dx^2+c}bcd - (bc^2+ad^2)\sqrt{bd}x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right.}{bcd^2x}$$

input
```
integrate((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*b*c^2*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c))
+ sqrt(-b*x^2 - a)*sqrt(-d*x^2 + c)*b*c*d - (b*c^2 + a*d^2)*sqrt(b*d)*x*sq
rt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)))/(b*c*d^2*x)
```

3.259.6 Sympy [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx$$

input
```
integrate((-b*x**2-a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-a - b*x**2)/sqrt(c - d*x**2), x)
```

3.259.7 Maxima [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

3.259.8 Giac [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{-b\,x^2-a}}{\sqrt{c-d\,x^2}}\,dx$$

input
```
int((- a - b*x^2)^(1/2)/(c - d*x^2)^(1/2),x)
```

output
```
int((- a - b*x^2)^(1/2)/(c - d*x^2)^(1/2), x)
```

3.260 $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx$

3.260.1 Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

output
```
EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)*(1-d
*x^2/c)^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)
```

3.260.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c+dx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/Sqrt[-c + d*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c
)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2])
```

3.260.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{dx^2-c}}dx$$

$$\Big\downarrow \ 331$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{dx^2-c}}$$

$$\Big\downarrow \ 330$$

$$\frac{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

input
```
Int[Sqrt[a + b*x^2]/Sqrt[-c + d*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
```

3.260.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.260.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(73) = 146$.

Time = 2.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.90

method	result
default	$\frac{\left(-aF\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d-bcF\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)+bcE\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\right)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2-c}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}}{(-bd\,x^4-ad\,x^2+cb\,x^2+ac)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{-(b\,x^2+a)(-d\,x^2+c)}\left(\frac{a\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1-\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2-cb\,x^2-ac}}+\frac{bc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1-\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2-cb\,x^2-ac}\,d}\cdots}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2-c}}$

input
```
int((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*EllipticF(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^(1
/2),(-a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2)))*(b*x
^2+a)^(1/2)*(d*x^2-c)^(1/2)*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)/(-b*d
*x^4-a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{bd}bc^2x\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}) + \sqrt{bx^2+a}\sqrt{dx^2-c}bcd - (bc^2+ad^2)\sqrt{bd}x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc})}{bcd^2x}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(b*d)*b*c^2*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) +
 sqrt(b*x^2 + a)*sqrt(d*x^2 - c)*b*c*d - (b*c^2 + a*d^2)*sqrt(b*d)*x*sqrt(
c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)))/(b*c*d^2*x)
```

3.260.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2-c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/sqrt(-c + d*x**2), x)
```

3.260.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2-c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c), x)
```

3.260.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2-c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c), x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2-c}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(d*x^2 - c)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(d*x^2 - c)^(1/2), x)
```

3.261 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}} \, dx$

3.261.1 Optimal result

Integrand size = 28, antiderivative size = 91

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}} \, dx = \frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

output
```
EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2-a)^(1/2)*(1-
d*x^2/c)^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}} \, dx = \frac{\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c+dx^2}}$$

input
```
Integrate[Sqrt[-a - b*x^2]/Sqrt[-c + d*x^2],x]
```

output
```
(Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*
c)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2])
```

3.261.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{dx^2-c}}dx$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-bx^2-a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{dx^2-c}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

input
```
Int[Sqrt[-a - b*x^2]/Sqrt[-c + d*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)
/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
```

3.261.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.261.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{-b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}\,a\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,E\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)}{\left(bd\,x^{4}+ad\,x^{2}-cb\,x^{2}-ac\right)\sqrt{\frac{d}{c}}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(-\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}}+\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}}\right.}{\sqrt{-b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}}$

input
```
int((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*d*x^4+a*d*x^2-b*c*x^2-a*c)/(d/c)^(1/2)*(-b*x^2-a)^(1/2)*(d*x^2-c)^(1/
2)*a*((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(-b*
c/a/d)^(1/2))
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{-bd}bc^2x\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right) | -\frac{ad}{bc}) + \sqrt{-bx^2-a}\sqrt{dx^2-c}bcd - (bc^2+ad^2)\sqrt{-bd}x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)}{bcd^2x}$$

input
```
integrate((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b*d)*b*c^2*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c))
+ sqrt(-b*x^2 - a)*sqrt(d*x^2 - c)*b*c*d - (b*c^2 + a*d^2)*sqrt(-b*d)*x*sq
rt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)))/(b*c*d^2*x)
```

3.261.6 Sympy [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx$$

input
```
integrate((-b*x**2-a)**(1/2)/(d*x**2-c)**(1/2),x)
```

output
```
Integral(sqrt(-a - b*x**2)/sqrt(-c + d*x**2), x)
```

3.261.7 Maxima [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

input
```
integrate((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c), x)
```

3.261.8 Giac [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx=\int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

input
```
integrate((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c), x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx=\int \frac{\sqrt{-b\,x^2-a}}{\sqrt{d\,x^2-c}}\,dx$$

input
```
int((- a - b*x^2)^(1/2)/(d*x^2 - c)^(1/2),x)
```

output
```
int((- a - b*x^2)^(1/2)/(d*x^2 - c)^(1/2), x)
```

3.262 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx$

3.262.1 Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

output
```
EllipticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2+a)^(1/2)*(1-d
*x^2/c)^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}}$$

input
```
Integrate[Sqrt[a - b*x^2]/Sqrt[c - d*x^2],x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/
(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2])
```

3.262.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \text{ 331}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{ 330}$$

$$\frac{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

input
```
Int[Sqrt[a - b*x^2]/Sqrt[c - d*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2])
```

3.262.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.262.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{-b x^2+a}\,\sqrt{-d x^2+c}\,a\sqrt{\frac{-d x^2+c}{c}}\,\sqrt{\frac{-b x^2+a}{a}}\,E\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)}{\left(bd x^4-ad x^2-cb x^2+ac\right)\sqrt{\frac{d}{c}}}$
elliptic	$\frac{\sqrt{(-b x^2+a)(-d x^2+c)}\left(\frac{a\sqrt{1-\frac{d x^2}{c}}\,\sqrt{1-\frac{b x^2}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd x^4-ad x^2-cb x^2+ac}}-\frac{a\sqrt{1-\frac{d x^2}{c}}\,\sqrt{1-\frac{b x^2}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\cdots}\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{bd x^4-ad x^2-cb x^2+ac}}\right.}{\sqrt{-b x^2+a}\,\sqrt{-d x^2+c}}$

input
```
int((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)*a*((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1
/2)*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2))/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)
/(d/c)^(1/2)
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx =$$
$$-\frac{\sqrt{bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) - \sqrt{bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{-dx^2+c}b}{bdx}$$

input
```
integrate((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) - sqr
t(b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)) + sq
rt(-b*x^2 + a)*sqrt(-d*x^2 + c)*b)/(b*d*x)
```

3.262.6 Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)/sqrt(c - d*x**2), x)
```

3.262.7 Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

3.262.8 Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{a-b\,x^2}}{\sqrt{c-d\,x^2}}\,dx$$

input
```
int((a - b*x^2)^(1/2)/(c - d*x^2)^(1/2),x)
```

output
```
int((a - b*x^2)^(1/2)/(c - d*x^2)^(1/2), x)
```

3.263 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx$

3.263.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx=\frac{\sqrt{c}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

output
```
EllipticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(b*x^2-a)^(1/2)*(1-d*
x^2/c)^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)
```

3.263.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx=\frac{\sqrt{-a+bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}}$$

input
```
Integrate[Sqrt[-a + b*x^2]/Sqrt[c - d*x^2],x]
```

output
```
(Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)
/(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2])
```

3.263.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2-a}}{\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \text{ 331}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{bx^2-a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{ 330}$$

$$\frac{\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{c}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

input
```
Int[Sqrt[-a + b*x^2]/Sqrt[c - d*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)
/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2])
```

3.263.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.263.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(74) = 148$.

Time = 2.49 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.81

method	result
default	$\frac{\sqrt{b\,x^{2}-a}\,\sqrt{-d\,x^{2}+c}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\left(aF\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bcF\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bcE\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)}{\left(bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\,\left(-\frac{a\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac}}+\frac{bc\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac}\,d}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{-d\,x^{2}+c}}$

input
```
int((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2-a)^(1/2)*(-d*x^2+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)
*(a*EllipticF(x*(b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(b/a)^(1/2)
,(a*d/b/c)^(1/2))+b*c*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2)))/(b*d*x^4-a
*d*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx = -\frac{\sqrt{-bd}ax\sqrt{\frac{a}{b}}E\left(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}\right) - \sqrt{-bd}(a-b)x\sqrt{\frac{a}{b}}F\left(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}\right) + \sqrt{bx^2-a}\sqrt{-dx^2+c}b}{bdx}$$

input
```
integrate((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) - sq
rt(-b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)) +
sqrt(b*x^2 - a)*sqrt(-d*x^2 + c)*b)/(b*d*x)
```

3.263.6 Sympy [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx$$

input
```
integrate((b*x**2-a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-a + b*x**2)/sqrt(c - d*x**2), x)
```

3.263.7 Maxima [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

3.263.8 Giac [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{bx^2-a}}{\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{b\,x^2-a}}{\sqrt{c-d\,x^2}}\,dx$$

input
```
int((b*x^2 - a)^(1/2)/(c - d*x^2)^(1/2),x)
```

output
```
int((b*x^2 - a)^(1/2)/(c - d*x^2)^(1/2), x)
```

3.264 $\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx$

3.264.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx=\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

output
```
EllipticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2+a)^(1/2)*(1-d
*x^2/c)^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)
```

3.264.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c+dx^2}}$$

input
```
Integrate[Sqrt[a - b*x^2]/Sqrt[-c + d*x^2],x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/
(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2])
```

3.264.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2-c}}dx$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{dx^2-c}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

input
```
Int[Sqrt[a - b*x^2]/Sqrt[-c + d*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2])
```

3.264.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.264.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(74) = 148$.

Time = 2.56 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.82

method	result
default	$\frac{\left(-aF\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+bcF\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-bcE\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)\sqrt{-b\,x^{2}+a}\,\sqrt{d\,x^{2}-c}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}}{\left(bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(\frac{a\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac}}-\frac{bc\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac}\,d}}{\sqrt{-b\,x^{2}+a}\,\sqrt{d\,x^{2}-c}}$

input
```
int((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*EllipticF(x*(b/a)^(1/2),(a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(b/a)^(1/2)
,(a*d/b/c)^(1/2))-b*c*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2)))*(-b*x^2+a)
^(1/2)*(d*x^2-c)^(1/2)*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)/(b*d*x^4-
a*d*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{-bd}ax\sqrt{\frac{a}{b}}E\left(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}\right) - \sqrt{-bd}(a-b)x\sqrt{\frac{a}{b}}F\left(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}\right) + \sqrt{-bx^2+a}\sqrt{dx^2-c}b}{bdx}$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) - sqr
t(-b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)) + s
qrt(-b*x^2 + a)*sqrt(d*x^2 - c)*b)/(b*d*x)
```

3.264.6 Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)/(d*x**2-c)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)/sqrt(-c + d*x**2), x)
```

3.264.7 Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{dx^2-c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c), x)
```

3.264.8 Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx=\int \frac{\sqrt{-bx^2+a}}{\sqrt{dx^2-c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c), x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx=\int \frac{\sqrt{a-b\,x^2}}{\sqrt{d\,x^2-c}}\,dx$$

input
```
int((a - b*x^2)^(1/2)/(d*x^2 - c)^(1/2),x)
```

output
```
int((a - b*x^2)^(1/2)/(d*x^2 - c)^(1/2), x)
```

3.265 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx$

3.265.1 Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

output
```
EllipticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(b*x^2-a)^(1/2)*(1-d*
x^2/c)^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)
```

3.265.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{-a+bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c+dx^2}}$$

input
```
Integrate[Sqrt[-a + b*x^2]/Sqrt[-c + d*x^2],x]
```

output
```
(Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)
/(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2])
```

3.265.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2-c}}dx$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{bx^2-a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{dx^2-c}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{c}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

input
```
Int[Sqrt[-a + b*x^2]/Sqrt[-c + d*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)
/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2])
```

3.265.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.265.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}\,a\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,E\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)}{\left(-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac\right)\sqrt{\frac{d}{c}}}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\,\left(-\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}}+\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-\cdots}\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}}\cdots\right)}{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}}$

input
```
int((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-b*d*x^4+a*d*x^2+b*c*x^2-a*c)/(d/c)^(1/2)*(b*x^2-a)^(1/2)*(d*x^2-c)^(1/
2)*a*((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(b*
c/a/d)^(1/2))
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) - \sqrt{bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) + \sqrt{bx^2-a}\sqrt{dx^2-c}b}{bdx}$$

input
```
integrate((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) - sqrt
(b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)) + sqr
t(b*x^2 - a)*sqrt(d*x^2 - c)*b)/(b*d*x)
```

3.265.6 Sympy [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx$$

input
```
integrate((b*x**2-a)**(1/2)/(d*x**2-c)**(1/2),x)
```

output
```
Integral(sqrt(-a + b*x**2)/sqrt(-c + d*x**2), x)
```

3.265.7 Maxima [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

input
```
integrate((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c), x)
```

3.265.8 Giac [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

input
```
integrate((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c), x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2-a}}{\sqrt{d\,x^2-c}}\,dx$$

input
```
int((b*x^2 - a)^(1/2)/(d*x^2 - c)^(1/2),x)
```

output
```
int((b*x^2 - a)^(1/2)/(d*x^2 - c)^(1/2), x)
```

3.266 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.266.1 Optimal result

Integrand size = 23, antiderivative size = 194

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(
b*x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.266.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*
c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2])
```

3.266.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \ 324$$

$$a\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 320$$

$$b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \ 388$$

$$b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \ 313$$

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)$$

input
```
Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

output
```
b*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

3.266.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.266.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

method	result
default	$\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\left(aF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bcF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bcE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)}{(bd x^4+ad x^2+cb x^2+ac)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{a\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+cb x^2+ac}}-\frac{bc\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+cb x^2+ac}\,d}}{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(a
*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx =$$
$$-\frac{\sqrt{bd}bc^2x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bx^2+a}\sqrt{dx^2+c}bcd - (bc^2+ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right.}{bcd^2x}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*b*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
 - sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*c*d - (b*c^2 + a*d^2)*sqrt(b*d)*x*sqr
t(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)
```

3.266.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

3.266.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

3.266.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c + d*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.267 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx$

3.267.1 Optimal result

Integrand size = 26, antiderivative size = 203

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
x*(-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*
(-b*x^2-a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(
1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2-a)^(1/2)/d^(1/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.267.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx=\frac{\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[-a - b*x^2]/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b
*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2])
```

3.267.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 324$$

$$-a\int \frac{1}{\sqrt{-bx^2-a}\sqrt{dx^2+c}}dx-b\int \frac{x^2}{\sqrt{-bx^2-a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 320$$

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-b\int \frac{x^2}{\sqrt{-bx^2-a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 388$$

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-b\left(\frac{c\int \frac{\sqrt{-bx^2-a}}{(dx^2+c)^{3/2}}dx}{b}-\frac{x\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \ 313$$

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-b\left(\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}\right)$$

input
```
Int[Sqrt[-a - b*x^2]/Sqrt[c + d*x^2],x]
```

output
```
-(b*(-((x*Sqrt[-a - b*x^2])/(b*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[-a - b*x^
2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (Sqrt[c]*Sqrt[-a -
 b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

3.267.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.267.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.51

method	result
default	$\frac{\sqrt{-b x^2-a}\,\sqrt{d x^2+c}\, a\sqrt{\frac{d x^2+c}{c}}\,\sqrt{\frac{b x^2+a}{a}}\, E\left(x\sqrt{-\frac{d}{c}}, \sqrt{\frac{bc}{ad}}\right)}{(bd x^4+ad x^2+cb x^2+ac)\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{-(b x^2+a)(d x^2+c)}\left(-\frac{a\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{b x^2}{a}}\, F\left(x\sqrt{-\frac{d}{c}}, \sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd x^4-ad x^2-cb x^2-ac}}+\frac{a\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{b x^2}{a}}\left(F\left(x\sqrt{-\frac{d}{c}}, \sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}},\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd x^4-ad x^2-cb x^2-ac}}\right.}{\sqrt{-b x^2-a}\,\sqrt{d x^2+c}}$

input
```
int((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*x^2-a)^(1/2)*(d*x^2+c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)
*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(
-d/c)^(1/2)
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx =$$
$$-\frac{\sqrt{-bd}bc^2x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{-bx^2-a}\sqrt{dx^2+c}bcd - (bc^2+ad^2)\sqrt{-bd}x\sqrt{-\frac{c}{d}}F(\arcsin}{bcd^2x}$$

input
```
integrate((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-b*d)*b*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)
) - sqrt(-b*x^2 - a)*sqrt(d*x^2 + c)*b*c*d - (b*c^2 + a*d^2)*sqrt(-b*d)*x*
sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)
```

3.267.6 Sympy [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((-b*x**2-a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-a - b*x**2)/sqrt(c + d*x**2), x)
```

3.267.7 Maxima [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c), x)
```

3.267.8 Giac [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c), x)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{-b\,x^2-a}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((- a - b*x^2)^(1/2)/(c + d*x^2)^(1/2),x)
```

output
```
int((- a - b*x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.268 $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx$

3.268.1 Optimal result

Integrand size = 26, antiderivative size = 203

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
x*(b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*
(b*x^2+a)^(1/2)/d^(1/2)/(-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+(
1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(1/2)/(-d*x^2-c)^(
1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.268.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)\mid\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/Sqrt[-c - d*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*
c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2])
```

3.268.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}dx$$

$$\Big\downarrow \; 324$$

$$a\int \frac{1}{\sqrt{bx^2+a}\sqrt{-dx^2-c}}dx+b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2-c}}dx$$

$$\Big\downarrow \; 320$$

$$b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2-c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \; 388$$

$$b\left(\frac{c\int \frac{\sqrt{bx^2+a}}{(-dx^2-c)^{3/2}}dx}{b}+\frac{x\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \; 313$$

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+ b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)$$

input
```
Int[Sqrt[a + b*x^2]/Sqrt[-c - d*x^2],x]
```

output
```
b*((x*Sqrt[a + b*x^2])/(b*Sqrt[-c - d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[-c -
 d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])) + (Sqrt[c]*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c
 - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

3.268.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.268. $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx$

3.268.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.53

method	result
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}-c}\,a\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)}{\left(-bd\,x^{4}-ad\,x^{2}-cb\,x^{2}-ac\right)\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{-\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}-cb\,x^{2}-ac}}-\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}},\sqrt{-\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}-cb\,x^{2}-ac}}}{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}-c}}$

input
```
int((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(-d*x^2-c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)
*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))/(-b*d*x^4-a*d*x^2-b*c*x^2-a*c)/
(-d/c)^(1/2)
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx$$
$$=\frac{\sqrt{-bd}bc^2x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{bx^2+a}\sqrt{-dx^2-c}bcd-(bc^2+ad^2)\sqrt{-bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{}}{}\right.}{bcd^2x}$$

input
```
integrate((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b*d)*b*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
 - sqrt(b*x^2 + a)*sqrt(-d*x^2 - c)*b*c*d - (b*c^2 + a*d^2)*sqrt(-b*d)*x*s
qrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)
```

3.268.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/sqrt(-c - d*x**2), x)
```

3.268.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-dx^2-c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

3.268.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-dx^2-c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{-d\,x^2-c}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(- c - d*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(- c - d*x^2)^(1/2), x)
```

3.269 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx$

3.269.1 Optimal result

Integrand size = 29, antiderivative size = 212

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}} - \frac{\sqrt{c}\sqrt{-a-bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
x*(-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2
)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)
*(-b*x^2-a)^(1/2)/d^(1/2)/(-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
+(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2-a)^(1/2)/d^(1/2)/(-d*x^2-c
)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.269.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}}$$

input
```
Integrate[Sqrt[-a - b*x^2]/Sqrt[-c - d*x^2],x]
```

output
```
(Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b
*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2])
```

3.269.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx$$

$$\Big\downarrow \; 324$$

$$-a\int \frac{1}{\sqrt{-bx^2-a}\sqrt{-dx^2-c}}dx - b\int \frac{x^2}{\sqrt{-bx^2-a}\sqrt{-dx^2-c}}dx$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - b\int \frac{x^2}{\sqrt{-bx^2-a}\sqrt{-dx^2-c}}dx$$

$$\Big\downarrow \; 388$$

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - b\left(-\frac{c\int \frac{\sqrt{-bx^2-a}}{(-dx^2-c)^{3/2}}dx}{b} - \frac{x\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}\right)$$

$$\Big\downarrow \; 313$$

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-$$
$$b\left(\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}\right)$$

input
```
Int[Sqrt[-a - b*x^2]/Sqrt[-c - d*x^2],x]
```

output
```
-(b*(-((x*Sqrt[-a - b*x^2])/(b*Sqrt[-c - d*x^2])) + (Sqrt[c]*Sqrt[-a - b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sq
rt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]))) + (Sqrt[c]*Sqrt[-a
 - b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d
]*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

3.269.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

3.269.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.78

method	result
default	$\frac{\left(-aF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+bcF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-bcE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)\sqrt{-b\,x^{2}-a}\,\sqrt{-d\,x^{2}-c}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}}{\left(bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}\left(-\frac{a\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}+\frac{bc\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}\,d}}{\sqrt{-b\,x^{2}-a}\,\sqrt{-d\,x^{2}-c}}$

input
```
int((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))-b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))*(-b*x^2
-a)^(1/2)*(-d*x^2-c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx$$
$$= \frac{\sqrt{bd}bc^2x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{-bx^2-a}\sqrt{-dx^2-c}bcd - (bc^2+ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-}}{x}\right.}{bcd^2x}$$

input
```
integrate((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(b*d)*b*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
- sqrt(-b*x^2 - a)*sqrt(-d*x^2 - c)*b*c*d - (b*c^2 + a*d^2)*sqrt(b*d)*x*sq
rt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)
```

3.269.6 Sympy [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx=\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx$$

input
```
integrate((-b*x**2-a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

output
```
Integral(sqrt(-a - b*x**2)/sqrt(-c - d*x**2), x)
```

3.269.7 Maxima [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx=\int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2-c}}\,dx$$

input
```
integrate((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

3.269.8 Giac [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx=\int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2-c}}\,dx$$

input
```
integrate((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx=\int \frac{\sqrt{-b\,x^2-a}}{\sqrt{-d\,x^2-c}}\,dx$$

input
```
int((- a - b*x^2)^(1/2)/(- c - d*x^2)^(1/2),x)
```

output
```
int((- a - b*x^2)^(1/2)/(- c - d*x^2)^(1/2), x)
```

3.270 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx$

3.270.1 Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)
^(1/2)*(d*x^2+c)^(1/2)/d/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+(a*d+b*c)*Elli
pticF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/d/b^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.270.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}\,E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[a - b*x^2]/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((
b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2])
```

3.270.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{326}$$

$$\frac{(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

input
```
Int[Sqrt[a - b*x^2]/Sqrt[c + d*x^2],x]
```

output
```
-((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c
])) + (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2
]*Sqrt[c + d*x^2])
```

3.270.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.270.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.85

method	result
default	$\frac{\sqrt{-b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\left(aF\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d+bcF\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)-bcE\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\right)}{\left(-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(\frac{a\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}}+\frac{bc\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-}{c}}\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}\,d}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$

input
```
int((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
(a*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(b/a)^(1/2)
,(-a*d/b/c)^(1/2))-b*c*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2)))/(-b*d*x^
4+a*d*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{-bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)|-\frac{bc}{ad}) - \sqrt{-bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)|-\frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{dx^2+c}b}{bdx}$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - sq
rt(-b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)) +
 sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*b)/(b*d*x)
```

3.270.6 Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)/sqrt(c + d*x**2), x)
```

3.270.7 Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c), x)
```

3.270.8 Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c), x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a-b\,x^2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a - b*x^2)^(1/2)/(c + d*x^2)^(1/2),x)
```

output
```
int((a - b*x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.271 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.271.1 Optimal result

Integrand size = 25, antiderivative size = 191

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{-a+bx^2}\sqrt{c+dx^2}}$$

output
```
EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^
(1/2)*(d*x^2+c)^(1/2)/d/(b*x^2-a)^(1/2)/(1+d*x^2/c)^(1/2)-(a*d+b*c)*Ellipt
icF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/d/b^(1/2)/(b*x^2-a)^(1/2)/(d*x^2+c)^(1/2)
```

3.271.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{-a+bx^2}\sqrt{\frac{c+dx^2}{c}}\,E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[-a + b*x^2]/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -(
(b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2])
```

3.271.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2-a}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{326}$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2-a}}dx}{d}-\frac{(ad+bc)\int \frac{1}{\sqrt{bx^2-a}\sqrt{dx^2+c}}dx}{d}$$

$$\Big\downarrow \text{323}$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2-a}}dx}{d}-\frac{\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2-a}}dx}{d}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{bx^2-a}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2-a}}dx}{d}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{331}$$

$$\frac{b\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{bx^2-a}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{c+dx^2}}$$

input
```
Int[Sqrt[-a + b*x^2]/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqr
t[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c]
) - (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]
*Sqrt[c + d*x^2])
```

3.271.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.271.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.56

method	result
default	$\frac{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}+c}\,a\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,E\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)}{\left(-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(-\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac}}+\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}}\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac}}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}+c}}$

input
```
int((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2-a)^(1/2)*(d*x^2+c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)
*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
/(-d/c)^(1/2)
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{bd}ax\sqrt{\frac{a}{b}}E\left(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) | -\frac{bc}{ad}\right) - \sqrt{bd}(a-b)x\sqrt{\frac{a}{b}}F\left(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) | -\frac{bc}{ad}\right) + \sqrt{bx^2-a}\sqrt{dx^2+c}b}{bdx}$$

input
```
integrate((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - sqr
t(b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)) + s
qrt(b*x^2 - a)*sqrt(d*x^2 + c)*b)/(b*d*x)
```

3.271.6 Sympy [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2-a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-a + b*x**2)/sqrt(c + d*x**2), x)
```

3.271.7 Maxima [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c), x)
```

3.271.8 Giac [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c), x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2-a}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((b*x^2 - a)^(1/2)/(c + d*x^2)^(1/2),x)
```

output
```
int((b*x^2 - a)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.272 $\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx$

3.272.1 Optimal result

Integrand size = 27, antiderivative size = 194

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}$$

output
```
EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^
(1/2)*(-d*x^2-c)^(1/2)/d/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+(a*d+b*c)*Elli
pticF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/d/b^(1/2)/(-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2)
```

3.272.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}\,E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}}$$

input
```
Integrate[Sqrt[a - b*x^2]/Sqrt[-c - d*x^2],x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((
b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2])
```

3.272.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}dx$$

$$\Big\downarrow \text{326}$$

$$\frac{(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{-dx^2-c}}dx}{d}+\frac{b\int \frac{\sqrt{-dx^2-c}}{\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{323}$$

$$\frac{b\int \frac{\sqrt{-dx^2-c}}{\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{-c-dx^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{b\int \frac{\sqrt{-dx^2-c}}{\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{-c-dx^2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{b\int \frac{\sqrt{-dx^2-c}}{\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}$$

$$\Big\downarrow \text{331}$$

$$\frac{b\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-dx^2-c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}+\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

input
```
Int[Sqrt[a - b*x^2]/Sqrt[-c - d*x^2],x]
```

output
```
(Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]
) + (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*
Sqrt[-c - d*x^2])
```

3.272.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.272.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.56

method	result
default	$\frac{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}-c}\,a\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,E\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)}{\left(bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac\right)\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac}}-\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}},\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac}}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}-c}}$

input
```
int((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*x^2+a)^(1/2)*(-d*x^2-c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/
2)*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))/(b*d*x^4-a*d*x^2+b*c*x^2-a*c
)/(-d/c)^(1/2)
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = -\frac{\sqrt{bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) | -\frac{bc}{ad}) - \sqrt{bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) | -\frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{-dx^2-cb}}{bdx}$$

input
```
integrate((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - sq
rt(b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)) +
sqrt(-b*x^2 + a)*sqrt(-d*x^2 - c)*b)/(b*d*x)
```

3.272.6 Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)/sqrt(-c - d*x**2), x)
```

3.272.7 Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2-c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

3.272.8 Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2-c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{a-b\,x^2}}{\sqrt{-d\,x^2-c}}\,dx$$

input
```
int((a - b*x^2)^(1/2)/(- c - d*x^2)^(1/2),x)
```

output
```
int((a - b*x^2)^(1/2)/(- c - d*x^2)^(1/2), x)
```

3.273 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx$

3.273.1 Optimal result

Integrand size = 28, antiderivative size = 198

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = -\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{-a+bx^2}\sqrt{-c-dx^2}}$$

output
```
-EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)
^(1/2)*(-d*x^2-c)^(1/2)/d/(b*x^2-a)^(1/2)/(1+d*x^2/c)^(1/2)-(a*d+b*c)*Elli
pticF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/d/b^(1/2)/(b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2)
```

3.273.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{\sqrt{-a+bx^2}\sqrt{\frac{c+dx^2}{c}}\,E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}}$$

input
```
Integrate[Sqrt[-a + b*x^2]/Sqrt[-c - d*x^2],x]
```

output
```
(Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -(
(b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2])
```

3.273.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2-a}}{\sqrt{-c-dx^2}}dx$$

$$\Big\downarrow \ 326$$

$$-\frac{(ad+bc)\int \frac{1}{\sqrt{bx^2-a}\sqrt{-dx^2-c}}dx}{d}-\frac{b\int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}}dx}{d}$$

$$\Big\downarrow \ 323$$

$$-\frac{b\int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}}dx}{d}-\frac{\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{-c-dx^2}}$$

$$\Big\downarrow \ 323$$

$$-\frac{b\int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}}dx}{d}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{bx^2-a}\sqrt{-c-dx^2}}$$

$$\Big\downarrow \ 321$$

$$-\frac{b\int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}}dx}{d}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{-c-dx^2}}$$

$$\Big\downarrow \ 331$$

$$-\frac{b\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-dx^2-c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{bx^2-a}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{-c-dx^2}}$$

$$\Big\downarrow \ 330$$

$$-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{-c-dx^2}}$$

$$\Big\downarrow \; 327$$

$$-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{-c-dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

input
```
Int[Sqrt[-a + b*x^2]/Sqrt[-c - d*x^2],x]
```

output
```
-((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(
Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)
/c])) - (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*
x^2]*Sqrt[-c - d*x^2])
```

3.273.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

3.273. $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx$

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.273.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.84

method	result
default	$\frac{\left(-aF\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d-bcF\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)+bcE\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\right)\sqrt{b\,x^{2}-a}\,\sqrt{-d\,x^{2}-c}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}}{\left(-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(d\,x^{2}+c)}\left(-\frac{a\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}}-\frac{bc\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{a}{}}\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}\,d}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{-d\,x^{2}-c}}$

input
```
int((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(b/a)^(1/2
),(-a*d/b/c)^(1/2))+b*c*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2)))*(b*x^2-
a)^(1/2)*(-d*x^2-c)^(1/2)*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)/(-b*d*x
^4+a*d*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = -\frac{\sqrt{-bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)|-\frac{bc}{ad}) - \sqrt{-bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)|-\frac{bc}{ad}) + \sqrt{bx^2-a}\sqrt{-dx^2-}}{bdx}$$

input
```
integrate((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - s
qrt(-b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d))
+ sqrt(b*x^2 - a)*sqrt(-d*x^2 - c)*b)/(b*d*x)
```

3.273.6 Sympy [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx$$

input
```
integrate((b*x**2-a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

output
```
Integral(sqrt(-a + b*x**2)/sqrt(-c - d*x**2), x)
```

3.273.7 Maxima [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{-dx^2-c}}\,dx$$

input
```
integrate((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

3.273.8 Giac [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{-dx^2-c}}\,dx$$

input
```
integrate((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{b\,x^2-a}}{\sqrt{-d\,x^2-c}}\,dx$$

input
```
int((b*x^2 - a)^(1/2)/(- c - d*x^2)^(1/2),x)
```

output
```
int((b*x^2 - a)^(1/2)/(- c - d*x^2)^(1/2), x)
```

3.274 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx$

3.274.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

output
```
EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(d
*x^2+c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)
```

3.274.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

input
```
Integrate[Sqrt[c + d*x^2]/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d
)/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

3.274.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{ 331}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{ 330}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^2]/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
```

3.274.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.274.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{d\,x^{2}+c}\,\sqrt{-b\,x^{2}+a}\,c\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,E\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)}{\left(-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{b}{a}}}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}}-\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-}{cb}}\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$

input
```
int((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)*c*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
/(b/a)^(1/2)
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx =$$
$$-\frac{\sqrt{-bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)|-\frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{dx^2+c}cabd - (b^2c+a^2d)\sqrt{-bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\right.}{ab^2dx}$$

input
```
integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d))
 + sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*a*b*d - (b^2*c + a^2*d)*sqrt(-b*d)*x*s
qrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)))/(a*b^2*d*x)
```

3.274.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/sqrt(a - b*x**2), x)
```

3.274.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

3.274.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(a - b*x^2)^(1/2),x)
```

output
```
int((c + d*x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.275 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx$

3.275.1 Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

output
```
EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(-
d*x^2-c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)
```

3.275.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

input
```
Integrate[Sqrt[-c - d*x^2]/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*
d)/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

3.275.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-dx^2-c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

input
```
Int[Sqrt[-c - d*x^2]/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)
/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
```

3.275.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.275.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.

Time = 2.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.87

method	result
default	$\frac{\sqrt{-d\,x^2-c}\,\sqrt{-b\,x^2+a}\,\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{-b\,x^2+a}{a}}\left(adF\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)+cF\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b-adE\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\right)}{(-bd\,x^4+ad\,x^2-cb\,x^2+ac)\sqrt{-\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{-(-b\,x^2+a)(d\,x^2+c)}\left(-\frac{c\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2+cb\,x^2-ac}}-\frac{da\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{-\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2+cb\,x^2-ac}\,b}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{-d\,x^2-c}}$

input
```
int((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-d*x^2-c)^(1/2)*(-b*x^2+a)^(1/2)*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)
*(a*d*EllipticF(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))+c*EllipticF(x*(-d/c)^(1/2
),(-b*c/a/d)^(1/2))*b-a*d*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2)))/(-b*
d*x^4+a*d*x^2-b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid -\frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{-dx^2-c}\,cabd - (b^2c+a^2d)\sqrt{bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right.}{ab^2dx}$$

input
```
integrate((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d))
+ sqrt(-b*x^2 + a)*sqrt(-d*x^2 - c)*a*b*d - (b^2*c + a^2*d)*sqrt(b*d)*x*sq
rt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)))/(a*b^2*d*x)
```

3.275.6 Sympy [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((-d*x**2-c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(-c - d*x**2)/sqrt(a - b*x**2), x)
```

3.275.7 Maxima [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

3.275.8 Giac [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-d\,x^2-c}}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int((- c - d*x^2)^(1/2)/(a - b*x^2)^(1/2),x)
```

output
```
int((- c - d*x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.276 $\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx$

3.276.1 Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

output
```
EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(d
*x^2+c)^(1/2)/b^(1/2)/(b*x^2-a)^(1/2)/(1+d*x^2/c)^(1/2)
```

3.276.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{-a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

input
```
Integrate[Sqrt[c + d*x^2]/Sqrt[-a + b*x^2],x]
```

output
```
(Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d
)/(b*c))])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

3.276.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{bx^2-a}}dx$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^2]/Sqrt[-a + b*x^2],x]
```

output
```
(Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
```

3.276.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.276.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(73) = 146$.

Time = 2.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.90

method	result
default	$\frac{\left(-adF\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)-cF\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b+adE\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\right)\sqrt{d\,x^2+c}\,\sqrt{b\,x^2-a}\,\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{-b\,x^2+a}{a}}}{\left(-bd\,x^4+ad\,x^2-cb\,x^2+ac\right)\sqrt{-\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{-(-b\,x^2+a)(d\,x^2+c)}\left(\frac{c\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2+cb\,x^2-ac}}+\frac{da\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}},\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2+cb\,x^2-ac}\,b}\right.}{\sqrt{b\,x^2-a}\,\sqrt{d\,x^2+c}}$

input
```
int((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*d*EllipticF(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))-c*EllipticF(x*(-d/c)^(1/2
),(-b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2)))*(d*x
^2+c)^(1/2)*(b*x^2-a)^(1/2)*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)/(-b*d
*x^4+a*d*x^2-b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) | -\frac{bc}{ad}) + \sqrt{bx^2-a}\sqrt{dx^2+c}abd - (b^2c+a^2d)\sqrt{bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) | -}{ab^2dx}$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) +
 sqrt(b*x^2 - a)*sqrt(d*x^2 + c)*a*b*d - (b^2*c + a^2*d)*sqrt(b*d)*x*sqrt(
a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)))/(a*b^2*d*x)
```

3.276.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2-a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/sqrt(-a + b*x**2), x)
```

3.276.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2-a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a), x)
```

3.276.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx=\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2-a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a), x)
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx=\int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2-a}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(b*x^2 - a)^(1/2),x)
```

output
```
int((c + d*x^2)^(1/2)/(b*x^2 - a)^(1/2), x)
```

3.277 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx$

3.277.1 Optimal result

Integrand size = 28, antiderivative size = 91

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

output
```
EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(-
d*x^2-c)^(1/2)/b^(1/2)/(b*x^2-a)^(1/2)/(1+d*x^2/c)^(1/2)
```

3.277.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{-a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

input
```
Integrate[Sqrt[-c - d*x^2]/Sqrt[-a + b*x^2],x]
```

output
```
(Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*
d)/(b*c))])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

3.277.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{bx^2-a}}\,dx$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-dx^2-c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

input
```
Int[Sqrt[-c - d*x^2]/Sqrt[-a + b*x^2],x]
```

output
```
(Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)
/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
```

3.277.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.277.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{-d\,x^{2}-c}\,\sqrt{b\,x^{2}-a}\,c\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,E\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)}{\left(bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac\right)\sqrt{\frac{b}{a}}}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(-\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}}+\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad}{}}\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{-d\,x^{2}-c}}$

input
```
int((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*d*x^4-a*d*x^2+b*c*x^2-a*c)/(b/a)^(1/2)*(-d*x^2-c)^(1/2)*(b*x^2-a)^(1/
2)*c*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*
d/b/c)^(1/2))
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{-bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)|-\frac{bc}{ad}) + \sqrt{bx^2-a}\sqrt{-dx^2-c}abd - (b^2c+a^2d)\sqrt{-bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)}{ab^2dx}$$

input
```
integrate((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d))
+ sqrt(b*x^2 - a)*sqrt(-d*x^2 - c)*a*b*d - (b^2*c + a^2*d)*sqrt(-b*d)*x*sq
rt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)))/(a*b^2*d*x)
```

3.277.6 Sympy [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx$$

input
```
integrate((-d*x**2-c)**(1/2)/(b*x**2-a)**(1/2),x)
```

output
```
Integral(sqrt(-c - d*x**2)/sqrt(-a + b*x**2), x)
```

3.277.7 Maxima [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

input
```
integrate((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a), x)
```

3.277.8 Giac [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

input
```
integrate((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a), x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-d\,x^2-c}}{\sqrt{b\,x^2-a}}\,dx$$

input
```
int((- c - d*x^2)^(1/2)/(b*x^2 - a)^(1/2),x)
```

output
```
int((- c - d*x^2)^(1/2)/(b*x^2 - a)^(1/2), x)
```

3.278 $\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx$

3.278.1 Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

output
```
EllipticE(x*b^(1/2)/a^(1/2),(a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(-d
*x^2+c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(1-d*x^2/c)^(1/2)
```

3.278.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

input
```
Integrate[Sqrt[c - d*x^2]/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/
(b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

3.278.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \ 331$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \ 330$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

input
```
Int[Sqrt[c - d*x^2]/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c])
```

3.278.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.278.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(73) = 146$.

Time = 2.44 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.82

method	result
default	$\frac{\left(-adF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)+cF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b+adE\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\right)\sqrt{-d\,x^{2}+c}\,\sqrt{-b\,x^{2}+a}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}}{\left(bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(\frac{c\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}}-\frac{da\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-1\cdots\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}\,b}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}}$

input
```
int((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*d*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))+c*EllipticF(x*(d/c)^(1/2),(
b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2)))*(-d*x^2+c)
^(1/2)*(-b*x^2+a)^(1/2)*((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)/(b*d*x^4
-a*d*x^2-b*c*x^2+a*c)/(d/c)^(1/2)/b
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx =$$
$$-\frac{\sqrt{bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{-dx^2+c}abd + (b^2c-a^2d)\sqrt{bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)}{ab^2dx}$$

input
```
integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) +
 sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*a*b*d + (b^2*c - a^2*d)*sqrt(b*d)*x*sqr
t(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)))/(a*b^2*d*x)
```

3.278.6 Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(c - d*x**2)/sqrt(a - b*x**2), x)
```

3.278.7 Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

3.278.8 Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{c-d\,x^2}}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int((c - d*x^2)^(1/2)/(a - b*x^2)^(1/2),x)
```

output
```
int((c - d*x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.279 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx$

3.279.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

output
```
EllipticE(x*b^(1/2)/a^(1/2),(a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(d*
x^2-c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(1-d*x^2/c)^(1/2)
```

3.279.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c+dx^2}\,E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

input
```
Integrate[Sqrt[-c + d*x^2]/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)
/(b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

3.279.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2-c}}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2-c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

input
```
Int[Sqrt[-c + d*x^2]/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)
/Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c])
```

3.279.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.279.4 Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{d\,x^2-c}\,\sqrt{-b\,x^2+a}\,c\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}\,E\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{\left(bd\,x^4-ad\,x^2-cb\,x^2+ac\right)\sqrt{\frac{b}{a}}}$
elliptic	$\frac{\sqrt{-(-b\,x^2+a)(-d\,x^2+c)}\left(-\frac{c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1-\frac{d\,x^2}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2+cb\,x^2-ac}}+\frac{c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1-\frac{d\,x^2}{c}}\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\cdots}\right)\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2+cb\,x^2-ac}}\cdots\right)}{\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2-c}}$

input
```
int((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x^2-c)^(1/2)*(-b*x^2+a)^(1/2)*c*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/
2)*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)/
(b/a)^(1/2)
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx =$$
$$-\frac{\sqrt{-bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid\frac{bc}{ad})+\sqrt{-bx^2+a}\sqrt{dx^2-c}abd+(b^2c-a^2d)\sqrt{-bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right.}{ab^2dx}$$

input
```
integrate((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d))
+ sqrt(-b*x^2 + a)*sqrt(d*x^2 - c)*a*b*d + (b^2*c - a^2*d)*sqrt(-b*d)*x*sq
rt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)))/(a*b^2*d*x)
```

3.279.6 Sympy [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((d*x**2-c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(-c + d*x**2)/sqrt(a - b*x**2), x)
```

3.279.7 Maxima [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

3.279.8 Giac [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{d\,x^2-c}}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int((d*x^2 - c)^(1/2)/(a - b*x^2)^(1/2),x)
```

output
```
int((d*x^2 - c)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.280 $\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx$

3.280.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

output
```
EllipticE(x*b^(1/2)/a^(1/2),(a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(-d
*x^2+c)^(1/2)/b^(1/2)/(b*x^2-a)^(1/2)/(1-d*x^2/c)^(1/2)
```

3.280.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{-a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

input
```
Integrate[Sqrt[c - d*x^2]/Sqrt[-a + b*x^2],x]
```

output
```
(Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/
(b*c)])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

3.280.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{bx^2-a}}dx$$

$$\Big\downarrow \ 331$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \ 330$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

input
```
Int[Sqrt[c - d*x^2]/Sqrt[-a + b*x^2],x]
```

output
```
(Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c])
```

3.280.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.280.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{-d\,x^{2}+c}\,\sqrt{b\,x^{2}-a}\,c\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,E\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{\left(-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac\right)\sqrt{\frac{b}{a}}}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\,\left(\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac}}-\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\right.\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac}}}{\sqrt{b\,x^{2}-a}\,\sqrt{-d\,x^{2}+c}}$

input
```
int((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-d*x^2+c)^(1/2)*(b*x^2-a)^(1/2)*c*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/
2)*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2))/(-b*d*x^4+a*d*x^2+b*c*x^2-a*c)
/(b/a)^(1/2)
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{-bd}a^2dx\sqrt{\frac{a}{b}}E\left(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}\right) + \sqrt{bx^2-a}\sqrt{-dx^2+c}abd + (b^2c-a^2d)\sqrt{-bd}x\sqrt{\frac{a}{b}}F\left(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\right.}{ab^2dx}$$

input
```
integrate((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) +
 sqrt(b*x^2 - a)*sqrt(-d*x^2 + c)*a*b*d + (b^2*c - a^2*d)*sqrt(-b*d)*x*sqr
t(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)))/(a*b^2*d*x)
```

3.280.6 Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)/(b*x**2-a)**(1/2),x)
```

output
```
Integral(sqrt(c - d*x**2)/sqrt(-a + b*x**2), x)
```

3.280.7 Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{bx^2-a}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a), x)
```

3.280.8 Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{bx^2-a}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a), x)
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{c-d\,x^2}}{\sqrt{b\,x^2-a}}\,dx$$

input
```
int((c - d*x^2)^(1/2)/(b*x^2 - a)^(1/2),x)
```

output
```
int((c - d*x^2)^(1/2)/(b*x^2 - a)^(1/2), x)
```

3.281 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}} \, dx$

3.281.1 Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}} \, dx = \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

output
```
EllipticE(x*b^(1/2)/a^(1/2),(a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(d*
x^2-c)^(1/2)/b^(1/2)/(b*x^2-a)^(1/2)/(1-d*x^2/c)^(1/2)
```

3.281.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}} \, dx = \frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c+dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{-a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

input
```
Integrate[Sqrt[-c + d*x^2]/Sqrt[-a + b*x^2],x]
```

output
```
(Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)
/(b*c)])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

3.281.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2-c}}{\sqrt{bx^2-a}}dx$$

$$\Big\downarrow \text{ 331}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2-c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{ 330}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

input
```
Int[Sqrt[-c + d*x^2]/Sqrt[-a + b*x^2],x]
```

output
```
(Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)
/Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c])
```

3.281.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.281.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(75) = 150$.

Time = 2.43 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.81

method	result
default	$\frac{\sqrt{d\,x^{2}-c}\,\sqrt{b\,x^{2}-a}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\left(adF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)-cF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b-adE\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\right)}{\left(bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(-d\,x^{2}+c)}\,\left(-\frac{c\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}}+\frac{da\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-\cdots}\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}\,b}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}}$

input
```
int((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x^2-c)^(1/2)*(b*x^2-a)^(1/2)*((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*
(a*d*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))-c*EllipticF(x*(d/c)^(1/2),(b
*c/a/d)^(1/2))*b-a*d*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2)))/(b*d*x^4-a*
d*x^2-b*c*x^2+a*c)/(d/c)^(1/2)/b
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx$$
$$= \frac{\sqrt{bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) + \sqrt{bx^2-a}\sqrt{dx^2-c}abd + (b^2c-a^2d)\sqrt{bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad})}{ab^2dx}$$

input
```
integrate((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) +
sqrt(b*x^2 - a)*sqrt(d*x^2 - c)*a*b*d + (b^2*c - a^2*d)*sqrt(b*d)*x*sqrt(a
/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)))/(a*b^2*d*x)
```

3.281.6 Sympy [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx$$

input
```
integrate((d*x**2-c)**(1/2)/(b*x**2-a)**(1/2),x)
```

output
```
Integral(sqrt(-c + d*x**2)/sqrt(-a + b*x**2), x)
```

3.281.7 Maxima [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

input
```
integrate((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a), x)
```

3.281.8 Giac [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

input
```
integrate((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a), x)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{d\,x^2-c}}{\sqrt{b\,x^2-a}}\,dx$$

input
```
int((d*x^2 - c)^(1/2)/(b*x^2 - a)^(1/2),x)
```

output
```
int((d*x^2 - c)^(1/2)/(b*x^2 - a)^(1/2), x)
```

3.282 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$

3.282.1 Optimal result

Integrand size = 23, antiderivative size = 204

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
d*x*(b*x^2+a)^(1/2)/b/(d*x^2+c)^(1/2)+c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x
^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/
2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+
d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/b/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.282.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

input
```
Integrate[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*
d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

3.282.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \ 324$$

$$c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx + d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 320$$

$$d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx + \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \ 388$$

$$d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right) + \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \ 313$$

$$\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+$$
$$d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)$$

input
```
Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

output
```
d*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

3.282.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.282.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

method	result
default	$\frac{\sqrt{d x^2+c}\,\sqrt{b x^2+a}\,c\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{\left(bd\,x^4+ad\,x^2+cb\,x^2+ac\right)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}\left(\frac{c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ac}{}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}\right.}{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-
b/a)^(1/2)
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{bd}cx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bd}(c+d)x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bx^2+a}\sqrt{dx^2+c}d}{bdx}$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - s
qrt(b*d)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c))
- sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d)/(b*d*x)
```

3.282.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/sqrt(a + b*x**2), x)
```

3.282.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

3.282.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx=\int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^(1/2),x)
```

output
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.283 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx$

3.283.1 Optimal result

Integrand size = 26, antiderivative size = 214

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = -\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
-d*x*(b*x^2+a)^(1/2)/b/(-d*x^2-c)^(1/2)-c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d
*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1
/2))*(b*x^2+a)^(1/2)/a/d^(1/2)/(-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)+(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/b/(-d
*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.283.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

input
```
Integrate[Sqrt[-c - d*x^2]/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a
*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

3.283.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \; 324$$

$$-c\int \frac{1}{\sqrt{bx^2+a}\sqrt{-dx^2-c}}dx - d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2-c}}dx$$

$$\Big\downarrow \; 320$$

$$-d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2-c}}dx - \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \; 388$$

$$-d\left(\frac{c\int \frac{\sqrt{bx^2+a}}{(-dx^2-c)^{3/2}}dx}{b} + \frac{x\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}\right) - \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \; 313$$

$$-\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)$$

input
```
Int[Sqrt[-c - d*x^2]/Sqrt[a + b*x^2],x]
```

output
```
-(d*((x*Sqrt[a + b*x^2])/(b*Sqrt[-c - d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[-c
 - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]))) - (c^(3/2)*Sqrt[a + b*x
^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sq
rt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

3.283.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.283.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.75

method	result
default	$\frac{\left(-adF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)+cF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b+adE\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\right)\sqrt{-d\,x^{2}-c}\,\sqrt{b\,x^{2}+a}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}}{\left(bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{-\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{-\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(-\frac{c\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}-cb\,x^{2}-ac}}+\frac{da\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}},\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}-cb\,x^{2}-ac}\,b}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}-c}}$

input
```
int((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*d*EllipticF(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))+c*EllipticF(x*(-d/c)^(1/2)
,(b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2)))*(-d*x^2
-c)^(1/2)*(b*x^2+a)^(1/2)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx =$$

$$-\frac{\sqrt{-bd}cx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}) - \sqrt{-bd}(c+d)x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}) - \sqrt{bx^2+a}\sqrt{-dx^2}}{bdx}$$

input
```
integrate((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-b*d)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
sqrt(-b*d)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)
) - sqrt(b*x^2 + a)*sqrt(-d*x^2 - c)*d)/(b*d*x)
```

3.283.6 Sympy [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((-d*x**2-c)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(-c - d*x**2)/sqrt(a + b*x**2), x)
```

3.283.7 Maxima [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a), x)
```

3.283.8 Giac [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a), x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-d\,x^2-c}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((- c - d*x^2)^(1/2)/(a + b*x^2)^(1/2),x)
```

output
```
int((- c - d*x^2)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.284 $\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx$

3.284.1 Optimal result

Integrand size = 26, antiderivative size = 214

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = -\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-d*x*(-b*x^2-a)^(1/2)/b/(d*x^2+c)^(1/2)-c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d
*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1
/2))*(-b*x^2-a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^
(1/2)+(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(-b*x^2-a)^(1/2)/b/(c
*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.284.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx=\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

input
```
Integrate[Sqrt[c + d*x^2]/Sqrt[-a - b*x^2],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*
d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

3.284.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx$$

$$\Big\downarrow \text{324}$$

$$c\int \frac{1}{\sqrt{-bx^2-a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{-bx^2-a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{320}$$

$$d\int \frac{x^2}{\sqrt{-bx^2-a}\sqrt{dx^2+c}}dx-\frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{388}$$

$$d\left(\frac{c\int \frac{\sqrt{-bx^2-a}}{(dx^2+c)^{3/2}}dx}{b}-\frac{x\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}\right)-\frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{313}$$

$$d\left(\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}\right)-\frac{c^{3/2}\sqrt{-a-bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[Sqrt[c + d*x^2]/Sqrt[-a - b*x^2],x]
```

output
```
d*(-((x*Sqrt[-a - b*x^2])/(b*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[-a - b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) - (c^(3/2)*Sqrt[-a - b*
x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

3.284.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.284.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.76

method	result
default	$\frac{\sqrt{d\,x^{2}+c}\,\sqrt{-b\,x^{2}-a}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,\left(adF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)-cF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b-adE\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\right)}{\left(bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{-\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{-\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(\frac{c\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}-cb\,x^{2}-ac}}-\frac{da\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}},\sqrt{\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}-cb\,x^{2}-ac}\,b}\right.}{\sqrt{-b\,x^{2}-a}\,\sqrt{d\,x^{2}+c}}$

input
```
int((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x^2+c)^(1/2)*(-b*x^2-a)^(1/2)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*(
a*d*EllipticF(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))-c*EllipticF(x*(-d/c)^(1/2),(
b*c/a/d)^(1/2))*b-a*d*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2)))/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx$$
$$=\frac{\sqrt{-bd}cx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{-bd}(c+d)x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{-bx^2-a}\sqrt{dx^2+}}{bdx}$$

input
```
integrate((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b*d)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - s
qrt(-b*d)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c))
 - sqrt(-b*x^2 - a)*sqrt(d*x^2 + c)*d)/(b*d*x)
```

3.284.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/sqrt(-a - b*x**2), x)
```

3.284.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

3.284.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\sqrt{-b\,x^2-a}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(- a - b*x^2)^(1/2),x)
```

output
```
int((c + d*x^2)^(1/2)/(- a - b*x^2)^(1/2), x)
```

3.285 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx$

3.285.1 Optimal result

Integrand size = 29, antiderivative size = 222

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
d*x*(-b*x^2-a)^(1/2)/b/(-d*x^2-c)^(1/2)+c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d
*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1
/2))*(-b*x^2-a)^(1/2)/a/d^(1/2)/(-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(-b*x^2-a)^(1/2)/b/(
-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.285.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx=\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

input
```
Integrate[Sqrt[-c - d*x^2]/Sqrt[-a - b*x^2],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a
*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

3.285.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}dx$$

$$\Big\downarrow \text{324}$$

$$-c\int \frac{1}{\sqrt{-bx^2-a}\sqrt{-dx^2-c}}dx-d\int \frac{x^2}{\sqrt{-bx^2-a}\sqrt{-dx^2-c}}dx$$

$$\Big\downarrow \text{320}$$

$$\frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d\int \frac{x^2}{\sqrt{-bx^2-a}\sqrt{-dx^2-c}}dx$$

$$\Big\downarrow \text{388}$$

$$\frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d\left(-\frac{c\int \frac{\sqrt{-bx^2-a}}{(-dx^2-c)^{3/2}}dx}{b}-\frac{x\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}\right)$$

$$\Big\downarrow \text{313}$$

$$\frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d\left(\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}\right)$$

input
```
Int[Sqrt[-c - d*x^2]/Sqrt[-a - b*x^2],x]
```

output
```
-(d*(-((x*Sqrt[-a - b*x^2])/(b*Sqrt[-c - d*x^2])) + (Sqrt[c]*Sqrt[-a - b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sq
rt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]))) + (c^(3/2)*Sqrt[-a
 - b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt
[d]*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

3.285.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

3.285.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.50

method	result
default	$\frac{\sqrt{-d\,x^2-c}\,\sqrt{-b\,x^2-a}\,c\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{\left(-bd\,x^4-ad\,x^2-cb\,x^2-ac\right)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}+\frac{c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}\right.}{\sqrt{-b\,x^2-a}\,\sqrt{-d\,x^2-c}}$

input
```
int((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-b*d*x^4-a*d*x^2-b*c*x^2-a*c)/(-b/a)^(1/2)*(-d*x^2-c)^(1/2)*(-b*x^2-a)^
(1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{\sqrt{bd}cx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bd}(c+d)x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{-bx^2-a}\sqrt{-dx^2-c}\cdots}{bdx}$$

input
```
integrate((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(b*d)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sq
rt(b*d)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
 sqrt(-b*x^2 - a)*sqrt(-d*x^2 - c)*d)/(b*d*x)
```

3.285.6 Sympy [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx$$

input
```
integrate((-d*x**2-c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

output
```
Integral(sqrt(-c - d*x**2)/sqrt(-a - b*x**2), x)
```

3.285.7 Maxima [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2-a}}\,dx$$

input
```
integrate((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

3.285.8 Giac [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2-a}}\,dx$$

input
```
integrate((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{-d\,x^2-c}}{\sqrt{-b\,x^2-a}}\,dx$$

input
```
int((- c - d*x^2)^(1/2)/(- a - b*x^2)^(1/2),x)
```

output
```
int((- c - d*x^2)^(1/2)/(- a - b*x^2)^(1/2), x)
```

3.286 $\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx$

3.286.1 Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

output
```
-EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(
1/2)*(1-d*x^2/c)^(1/2)/b/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+(a*d+b*c)*Elli
pticF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x
^2/c)^(1/2)/b/d^(1/2)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

3.286.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

input
```
Integrate[Sqrt[c - d*x^2]/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((
a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

3.286.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{326}$$

$$\frac{(ad+bc)\int \frac{1}{\sqrt{bx^2+a}\sqrt{c-dx^2}}dx}{b}-\frac{d\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{b}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}(ad+bc)\int \frac{1}{\sqrt{bx^2+a}\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{c-dx^2}}-\frac{d\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{b}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{d\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{b}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{d\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{b}$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{d\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{d\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\int\frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

$$\Big\downarrow 327$$

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

input
```
Int[Sqrt[c - d*x^2]/Sqrt[a + b*x^2],x]
```

output
```
-((Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2
])) + (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Ellipti
cF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2
]*Sqrt[c - d*x^2])
```

3.286.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.286.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.85

method	result
default	$\frac{\sqrt{-d\,x^{2}+c}\,\sqrt{b\,x^{2}+a}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,\left(adF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)+cF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b-adE\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\right)}{\left(-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(-d\,x^{2}+c)}\left(\frac{c\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}}+\frac{da\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}\,b}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}}$

input
```
int((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*
(a*d*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))+c*EllipticF(x*(d/c)^(1/2),(
-b*c/a/d)^(1/2))*b-a*d*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2)))/(-b*d*x^
4-a*d*x^2+b*c*x^2+a*c)/(d/c)^(1/2)/b
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{-bd}cx\sqrt{\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}\right)-\sqrt{-bd}(c-d)x\sqrt{\frac{c}{d}}F\left(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}\right)+\sqrt{bx^2+a}\sqrt{-dx^2+c}d}{bdx}$$

input
```
integrate((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b*d)*c*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - sq
rt(-b*d)*(c - d)*x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)) +
 sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*d)/(b*d*x)
```

3.286.6 Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(c - d*x**2)/sqrt(a + b*x**2), x)
```

3.286.7 Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a), x)
```

3.286.8 Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a), x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c-d\,x^2}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c - d*x^2)^(1/2)/(a + b*x^2)^(1/2),x)
```

output
```
int((c - d*x^2)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.287 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx$

3.287.1 Optimal result

Integrand size = 25, antiderivative size = 191

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}} - \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{-c+dx^2}}$$

output
```
EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1
/2)*(1-d*x^2/c)^(1/2)/b/(1+b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)-(a*d+b*c)*Ellipt
icF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2
/c)^(1/2)/b/d^(1/2)/(b*x^2+a)^(1/2)/(d*x^2-c)^(1/2)
```

3.287.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c+dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

input
```
Integrate[Sqrt[-c + d*x^2]/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -(
(a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

3.287.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2-c}}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 326}$$

$$\frac{d\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2-c}}dx}{b}-\frac{(ad+bc)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2-c}}dx}{b}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{d\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2-c}}dx}{b}-\frac{\sqrt{1-\frac{dx^2}{c}}(ad+bc)\int \frac{1}{\sqrt{bx^2+a}\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{dx^2-c}}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{d\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2-c}}dx}{b}-\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{a+bx^2}\sqrt{dx^2-c}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{d\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2-c}}dx}{b}-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{dx^2-c}}$$

$$\Big\downarrow \text{ 331}$$

$$\frac{d\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{dx^2-c}}$$

$$\Big\downarrow \text{ 330}$$

$$\frac{d\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{dx^2-c}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{dx^2-c}}$$

input
```
Int[Sqrt[-c + d*x^2]/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqr
t[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2]
) - (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF
[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*
Sqrt[-c + d*x^2])
```

3.287.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

3.287. $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx$

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.287.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.56

method	result
default	$\frac{\sqrt{d\,x^{2}-c}\,\sqrt{b\,x^{2}+a}\,c\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)}{\left(-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{-(b\,x^{2}+a)(-d\,x^{2}+c)}\,\Big(-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}-cb\,x^{2}-ac}}+\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\Big(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\Big(x\sqrt{-\frac{b}{a}},\sqrt{\cdots}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}-cb\,x^{2}-ac}}}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}-c}}$

input
```
int((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x^2-c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)
*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)
/(-b/a)^(1/2)
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{bd}cx\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}) - \sqrt{bd}(c-d)x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}) + \sqrt{bx^2+a}\sqrt{dx^2-c}d}{bdx}$$

input
```
integrate((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(b*d)*c*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - sqr
t(b*d)*(c - d)*x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)) + s
qrt(b*x^2 + a)*sqrt(d*x^2 - c)*d)/(b*d*x)
```

3.287.6 Sympy [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2-c)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(-c + d*x**2)/sqrt(a + b*x**2), x)
```

3.287.7 Maxima [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a), x)
```

3.287.8 Giac [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a), x)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{d\,x^2-c}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((d*x^2 - c)^(1/2)/(a + b*x^2)^(1/2),x)
```

output
```
int((d*x^2 - c)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.288 $\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}} dx$

3.288.1 Optimal result

Integrand size = 27, antiderivative size = 194

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}} dx = \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}$$

output
```
EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(-b*x^2-a)^(
1/2)*(1-d*x^2/c)^(1/2)/b/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+(a*d+b*c)*Elli
pticF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x
^2/c)^(1/2)/b/d^(1/2)/(-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2)
```

3.288.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}} dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

input
```
Integrate[Sqrt[c - d*x^2]/Sqrt[-a - b*x^2],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((
a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

3.288.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}dx$$

$$\Big\downarrow \text{326}$$

$$\frac{(ad+bc)\int \frac{1}{\sqrt{-bx^2-a}\sqrt{c-dx^2}}dx}{b}+\frac{d\int \frac{\sqrt{-bx^2-a}}{\sqrt{c-dx^2}}dx}{b}$$

$$\Big\downarrow \text{323}$$

$$\frac{d\int \frac{\sqrt{-bx^2-a}}{\sqrt{c-dx^2}}dx}{b}+\frac{\sqrt{1-\frac{dx^2}{c}}(ad+bc)\int \frac{1}{\sqrt{-bx^2-a}\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{d\int \frac{\sqrt{-bx^2-a}}{\sqrt{c-dx^2}}dx}{b}+\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{-a-bx^2}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{d\int \frac{\sqrt{-bx^2-a}}{\sqrt{c-dx^2}}dx}{b}+\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{331}$$

$$\frac{d\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-bx^2-a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{c-dx^2}}+\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{d\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}+\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}$$

$$\Big\downarrow 327$$

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

input
```
Int[Sqrt[c - d*x^2]/Sqrt[-a - b*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2]
) + (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF
[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]
*Sqrt[c - d*x^2])
```

3.288.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.288.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.56

method	result
default	$\frac{\sqrt{-d\,x^{2}+c}\,\sqrt{-b\,x^{2}-a}\,c\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)}{\left(bd\,x^{4}+ad\,x^{2}-cb\,x^{2}-ac\right)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{-\left(b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\,\left(\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}-cb\,x^{2}-ac}}-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1 \cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}-cb\,x^{2}-ac}}\right.}{\sqrt{-b\,x^{2}-a}\,\sqrt{-d\,x^{2}+c}}$

input
```
int((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-d*x^2+c)^(1/2)*(-b*x^2-a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/
2)*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))/(b*d*x^4+a*d*x^2-b*c*x^2-a*c
)/(-b/a)^(1/2)
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx = -\frac{\sqrt{bd}cx\sqrt{\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}\right)-\sqrt{bd}(c-d)x\sqrt{\frac{c}{d}}F\left(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}\right)+\sqrt{-bx^2-a}\sqrt{-dx^2+cd}}{bdx}$$

input
```
integrate((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*c*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - sq
rt(b*d)*(c - d)*x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)) +
sqrt(-b*x^2 - a)*sqrt(-d*x^2 + c)*d)/(b*d*x)
```

3.288.6 Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

output
```
Integral(sqrt(c - d*x**2)/sqrt(-a - b*x**2), x)
```

3.288.7 Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

3.288.8 Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{c-d\,x^2}}{\sqrt{-b\,x^2-a}}\,dx$$

input
```
int((c - d*x^2)^(1/2)/(- a - b*x^2)^(1/2),x)
```

output
```
int((c - d*x^2)^(1/2)/(- a - b*x^2)^(1/2), x)
```

3.289 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx$

3.289.1 Optimal result

Integrand size = 28, antiderivative size = 198

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = -\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}} - \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{-c+dx^2}}$$

output
```
-EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(-b*x^2-a)^
(1/2)*(1-d*x^2/c)^(1/2)/b/(1+b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)-(a*d+b*c)*Elli
pticF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x
^2/c)^(1/2)/b/d^(1/2)/(-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2)
```

3.289.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c+dx^2}\,E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

input
```
Integrate[Sqrt[-c + d*x^2]/Sqrt[-a - b*x^2],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -(
(a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

3.289.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2-c}}{\sqrt{-a-bx^2}}dx$$

$$\Big\downarrow \; 326$$

$$-\frac{(ad+bc)\int \frac{1}{\sqrt{-bx^2-a}\sqrt{dx^2-c}}dx}{b}-\frac{d\int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2-c}}dx}{b}$$

$$\Big\downarrow \; 323$$

$$-\frac{d\int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2-c}}dx}{b}-\frac{\sqrt{1-\frac{dx^2}{c}}(ad+bc)\int \frac{1}{\sqrt{-bx^2-a}\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{dx^2-c}}$$

$$\Big\downarrow \; 323$$

$$-\frac{d\int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2-c}}dx}{b}-\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{-a-bx^2}\sqrt{dx^2-c}}$$

$$\Big\downarrow \; 321$$

$$-\frac{d\int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2-c}}dx}{b}-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{dx^2-c}}$$

$$\Big\downarrow \; 331$$

$$-\frac{d\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-bx^2-a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{dx^2-c}}$$

$$\Big\downarrow \; 330$$

$$-\frac{d\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\int\frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{dx^2-c}}$$

$$\Big\downarrow \ 327$$

$$-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

input
```
Int[Sqrt[-c + d*x^2]/Sqrt[-a - b*x^2],x]
```

output
```
-((Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(
Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x
^2])) - (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Ellip
ticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*
x^2]*Sqrt[-c + d*x^2])
```

3.289.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

3.289.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.84

method	result
default	$\frac{\left(-adF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)-cF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b+adE\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\right)\sqrt{d\,x^{2}-c}\,\sqrt{-b\,x^{2}-a}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}}{\left(-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(-\frac{c\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}}-\frac{da\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-1 \cdots}\right)\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}\,b}\cdots\right)}{\sqrt{-b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}}$

input
```
int((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*d*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))-c*EllipticF(x*(d/c)^(1/2),
(-b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2)))*(d*x^2-
c)^(1/2)*(-b*x^2-a)^(1/2)*((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)/(-b*d*x
^4-a*d*x^2+b*c*x^2+a*c)/(d/c)^(1/2)/b
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx =$$
$$-\frac{\sqrt{-bd}cx\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}) - \sqrt{-bd}(c-d)x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}) + \sqrt{-bx^2-a}\sqrt{dx^2-}}{bdx}$$

input
```
integrate((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-b*d)*c*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - s
qrt(-b*d)*(c - d)*x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c))
+ sqrt(-b*x^2 - a)*sqrt(d*x^2 - c)*d)/(b*d*x)
```

3.289.6 Sympy [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx$$

input
```
integrate((d*x**2-c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

output
```
Integral(sqrt(-c + d*x**2)/sqrt(-a - b*x**2), x)
```

3.289.7 Maxima [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2-a}}\,dx$$

input
```
integrate((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

3.289.8 Giac [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx=\int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2-a}}\,dx$$

input
```
integrate((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx=\int \frac{\sqrt{d\,x^2-c}}{\sqrt{-b\,x^2-a}}\,dx$$

input
```
int((d*x^2 - c)^(1/2)/(- a - b*x^2)^(1/2),x)
```

output
```
int((d*x^2 - c)^(1/2)/(- a - b*x^2)^(1/2), x)
```

3.290 $\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx$

3.290.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx=\frac{\sqrt{2+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

output
```
1/2*(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticF(x*d^(1/2)*3^(1/2)/(3
*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/d^(1/2)/((b*x
^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)
```

3.290.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-b}x}{\sqrt{2}}\right),\frac{2d}{3b}\right)}{\sqrt{3}\sqrt{-b}}$$

input
```
Integrate[1/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

output
```
EllipticF[ArcSin[(Sqrt[-b]*x)/Sqrt[2]], (2*d)/(3*b)]/(Sqrt[3]*Sqrt[-b])
```

3.290.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

$$\Big\downarrow \ 320$$

$$\frac{\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

input
```
Int[1/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

output
```
(Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/
(Sqrt[2]*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

3.290.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

3.290.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{F\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)\sqrt{2}}{2\sqrt{-d}}$	38
elliptic	$\frac{\sqrt{(b\,x^2+2)(d\,x^2+3)}\,\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{2\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\,\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}$	112

input
```
int(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))*2^
(1/2)/(-d)^(1/2)
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = -\frac{\sqrt{6}\sqrt{2}\sqrt{-b}F\left(\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{-b}x\right)\mid\frac{2\,d}{3\,b}\right)}{6\,b}$$

input
```
integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(6)*sqrt(2)*sqrt(-b)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(-b)*x), 2
/3*d/b)/b
```

3.290.6 Sympy [F]

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

input
```
integrate(1/(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

output
```
Integral(1/(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3)), x)
```

3.290.7 Maxima [F]

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

input
```
integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

3.290.8 Giac [F]

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

input
```
integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}\,dx$$

input
```
int(1/((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2)),x)
```

output
```
int(1/((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2)), x)
```

3.291 $\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx$

3.291.1 Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

output
```
EllipticF(1/2*x,2*(-d/c)^(1/2))*(1+d*x^2/c)^(1/2)/(d*x^2+c)^(1/2)
```

3.291.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

input
```
Integrate[1/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/Sqrt[c + d*x^2]
```

3.291.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \; 323$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{4-x^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

input
```
Int[1/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/Sqrt[c + d*x^2]
```

3.291.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

3.291.4 Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{\frac{d\,x^{2}+c}{c}}\, F\left(\frac{x}{2}, 2\sqrt{-\frac{d}{c}}\right)}{\sqrt{d\,x^{2}+c}}$	38
elliptic	$\frac{\sqrt{-\left(d\,x^{2}+c\right)\left(x^{2}-4\right)}\, \sqrt{1+\frac{d\,x^{2}}{c}}\, F\left(\frac{x}{2}, \sqrt{-1-\frac{-c+4d}{c}}\right)}{\sqrt{d\,x^{2}+c}\, \sqrt{-d\,x^{4}-c\,x^{2}+4d\,x^{2}+4c}}$	83

input
```
int(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(1/2*x,2*(-d/c)^(1/2))
```

3.291.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \frac{F\left(\arcsin\left(\frac{1}{2}\,x\right) \mid -\frac{4\,d}{c}\right)}{\sqrt{c}}$$

input
```
integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
elliptic_f(arcsin(1/2*x), -4*d/c)/sqrt(c)
```

3.291.6 Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \begin{cases} \frac{F\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle| -\frac{4d}{c}\right)}{\sqrt{c}} & \text{for}\: x > -2 \wedge x < 2 \end{cases}$$

input
```
integrate(1/(-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((elliptic_f(asin(x/2), -4*d/c)/sqrt(c), (x > -2) & (x < 2)))
```

3.291.7 Maxima [F]

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

input
```
integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

3.291.8 Giac [F]

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

input
```
integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{4-x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/((4 - x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/((4 - x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.292 $\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx$

3.292.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}$$

output
```
(1/(x^2+4))^(1/2)*EllipticF(x/(x^2+4)^(1/2),(1-4*d/c)^(1/2))*(d*x^2+c)^(1/
2)/c/((d*x^2+c)/c/(x^2+4))^(1/2)
```

3.292.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = -\frac{i\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{2}\right),\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

input
```
Integrate[1/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

output
```
((-I)*Sqrt[(c + d*x^2)/c]*EllipticF[I*ArcSinh[x/2], (4*d)/c])/Sqrt[c + d*x
^2]
```

3.292.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+4}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

input
```
Int[1/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*d)/c])/(c*Sqrt[4 + x^2]*Sqr
t[(c + d*x^2)/(c*(4 + x^2))])
```

3.292.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.292.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\sqrt{\frac{d\,x^{2}+c}{c}}\,F\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{\frac{c}{d}}}{2}\right)}{2\sqrt{d\,x^{2}+c}\,\sqrt{-\frac{d}{c}}}$	53
elliptic	$\frac{\sqrt{(d\,x^{2}+c)(x^{2}+4)}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right)}{2\sqrt{d\,x^{2}+c}\,\sqrt{-\frac{d}{c}}\,\sqrt{d\,x^{4}+c\,x^{2}+4d\,x^{2}+4c}}$	95

input
```
int(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-d/c)^(1/2),1/2*(c/d)
^(1/2))/(-d/c)^(1/2)
```

3.292.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.26

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = -\frac{i\,F\left(\arcsin\left(\frac{1}{2}i\,x\right)\mid\frac{4d}{c}\right)}{\sqrt{c}}$$

input
```
integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-I*elliptic_f(arcsin(1/2*I*x), 4*d/c)/sqrt(c)
```

3.292.6 Sympy [F]

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{c+dx^2}\sqrt{x^2+4}}\,dx$$

input
```
integrate(1/(x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(c + d*x**2)*sqrt(x**2 + 4)), x)
```

3.292.7 Maxima [F]

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{x^2+4}}\,dx$$

input
```
integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

3.292.8 Giac [F]

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{x^2+4}}\,dx$$

input
```
integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{x^2+4}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/((x^2 + 4)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/((x^2 + 4)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.293 $\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx$

3.293.1 Optimal result

Integrand size = 23, antiderivative size = 6

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = -\operatorname{EllipticF}(\arccos(x),2)$$

output
```
-(x^2)^(1/2)/x*EllipticF((-x^2+1)^(1/2),2^(1/2))
```

3.293.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(6) = 12.

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 4.50

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{1-2x^2}\operatorname{EllipticF}(\arcsin(x),2)}{\sqrt{-1+2x^2}}$$

input
```
Integrate[1/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]
```

output
```
(Sqrt[1 - 2*x^2]*EllipticF[ArcSin[x], 2])/Sqrt[-1 + 2*x^2]
```

3.293.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {322}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2x^2-1}}\,dx$$
$$\Big\downarrow \text{ 322}$$
$$-\text{EllipticF}(\arccos(x),2)$$

input
```
Int[1/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]
```

output
```
-EllipticF[ArcCos[x], 2]
```

3.293.3.1 Defintions of rubi rules used

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

3.293.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 4.17

method	result	size
default	$\frac{F\left(x,\sqrt{2}\right)\sqrt{-2x^2+1}}{\sqrt{2x^2-1}}$	25
elliptic	$\frac{\sqrt{-(2x^2-1)(x^2-1)}\,\sqrt{-2x^2+1}\,F\left(x,\sqrt{2}\right)}{\sqrt{2x^2-1}\,\sqrt{-2x^4+3x^2-1}}$	55

input
```
int(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticF(x,2^(1/2))*(-2*x^2+1)^(1/2)/(2*x^2-1)^(1/2)
```

3.293.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = -i\,F(\arcsin{(x)}\,|2)$$

input
```
integrate(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-I*elliptic_f(arcsin(x), 2)
```

3.293.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{2x^2-1}}\,dx$$

input
```
integrate(1/(-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(2*x**2 - 1)), x)
```

3.293.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

3.293.8 Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2\,x^2-1}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(2*x^2 - 1)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(2*x^2 - 1)^(1/2)), x)
```

3.294 $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$

3.294.1 Optimal result

Integrand size = 28, antiderivative size = 23

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = -\frac{E(\arcsin(cx)|-1)}{c} + \frac{2\,\text{EllipticF}(\arcsin(cx),-1)}{c}$$

output
```
-EllipticE(c*x,I)/c+2*EllipticF(c*x,I)/c
```

3.294.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{-c^2}x\right)\middle|-1\right)}{\sqrt{-c^2}}$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

output
```
EllipticE[ArcSin[Sqrt[-c^2]*x], -1]/Sqrt[-c^2]
```

3.294.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \ 326$$

$$2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}dx-\int \frac{\sqrt{c^2x^2+1}}{\sqrt{1-c^2x^2}}dx$$

$$\Big\downarrow \ 284$$

$$2\int \frac{1}{\sqrt{1-c^4x^4}}dx-\int \frac{\sqrt{c^2x^2+1}}{\sqrt{1-c^2x^2}}dx$$

$$\Big\downarrow \ 327$$

$$2\int \frac{1}{\sqrt{1-c^4x^4}}dx-\frac{E(\arcsin(cx)|-1)}{c}$$

$$\Big\downarrow \ 762$$

$$\frac{2\,\text{EllipticF}(\arcsin(cx),-1)}{c}-\frac{E(\arcsin(cx)|-1)}{c}$$

input
```
Int[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

output
```
-(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

3.294.3.1 Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

3.294.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{(2F(x\ \mathrm{csgn}(c)c,i)-E(x\ \mathrm{csgn}(c)c,i))\,\mathrm{csgn}(c)}{c}$	28
elliptic	$\frac{\sqrt{-c^4x^4+1}\left(\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,F\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}+\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(F\left(x\sqrt{c^2},i\right)-E\left(x\sqrt{c^2},i\right)\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}\right)}{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}}$	153

input
```
int((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2*EllipticF(x*csgn(c)*c,I)-EllipticE(x*csgn(c)*c,I))*csgn(c)/c
```

3.294.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.17

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}c^3+\sqrt{-c^4}\left((c^2-1)xF\left(\arcsin\left(\frac{1}{cx}\right)|-1\right)+xE\left(\arcsin\left(\frac{1}{cx}\right)|-1\right)\right)}{c^5x}$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)*c^3 + sqrt(-c^4)*((c^2 - 1)*x*ellipt
ic_f(arcsin(1/(c*x)), -1) + x*elliptic_e(arcsin(1/(c*x)), -1)))/(c^5*x)
```

3.294.6 Sympy [F]

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/sqrt(c**2*x**2 + 1), x)
```

3.294.7 Maxima [F]

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

3.294.8 Giac [F]

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx=\int \frac{\sqrt{1-c^2\,x^2}}{\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2), x)
```

3.295 $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}} \, dx$

3.295.1 Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}} \, dx = \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} - \frac{\sqrt{2}\sqrt{2+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} + \frac{\sqrt{2}\sqrt{2+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

output
```
x*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2)-(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*
EllipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)
*(b*x^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)+(1/(3
*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticF(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)
^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*
x^2+3))^(1/2)/(d*x^2+3)^(1/2)
```

3.295.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx=\frac{\sqrt{2}E\left(\arcsin\left(\frac{\sqrt{-d}x}{\sqrt{3}}\right)|\frac{3b}{2d}\right)}{\sqrt{-d}}$$

input
```
Integrate[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

output
```
(Sqrt[2]*EllipticE[ArcSin[(Sqrt[-d]*x)/Sqrt[3]], (3*b)/(2*d)])/Sqrt[-d]
```

3.295.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}dx$$

$$\Big\downarrow \text{ 324}$$

$$2\int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx+b\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx$$

$$\Big\downarrow \text{ 320}$$

$$b\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx+\frac{\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

$$\Big\downarrow \text{ 388}$$

$$b\left(\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{3\int \frac{\sqrt{bx^2+2}}{(dx^2+3)^{3/2}}dx}{b}\right)+\frac{\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+ b\left(\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}\right)$$

input
```
Int[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

output
```
b*((x*Sqrt[2 + b*x^2])/(b*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(b*Sqrt[d]*Sqrt[(2 +
b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])) + (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF
[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/
(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

3.295.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

3.295. $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

3.295.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.20

method	result
default	$\frac{E\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)\sqrt{2}}{\sqrt{-d}}$
elliptic	$\frac{\sqrt{(b\,x^2+2)(d\,x^2+3)}\left(\frac{\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}-\frac{\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\left(F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)-E\left(\frac{x\sqrt{-3d}}{3},\sqrt{-\cdots}\right)\right)}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}\cdots\right)}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}$

input
```
int((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))*2^(1/2
)/(-d)^(1/2)
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = -\frac{9\sqrt{3}\sqrt{bd}bx\sqrt{-\frac{1}{d}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right) \mid \frac{2\,d}{3\,b}) - \sqrt{3}\sqrt{bd}(2\,d^2+9\,b)x\sqrt{-\frac{1}{d}}F(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right) \mid \frac{2\,d}{3\,b}) - 3\,\sqrt{\cdots}}{3\,bd^2x}$$

input
```
integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(9*sqrt(3)*sqrt(b*d)*b*x*sqrt(-1/d)*elliptic_e(arcsin(sqrt(3)*sqrt(-1
/d)/x), 2/3*d/b) - sqrt(3)*sqrt(b*d)*(2*d^2 + 9*b)*x*sqrt(-1/d)*elliptic_f
(arcsin(sqrt(3)*sqrt(-1/d)/x), 2/3*d/b) - 3*sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3
)*b*d)/(b*d^2*x)
```

3.295.6 Sympy [F]

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

input
```
integrate((b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

output
```
Integral(sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

3.295.7 Maxima [F]

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

input
```
integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

3.295.8 Giac [F]

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

input
```
integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+2}}{\sqrt{d\,x^2+3}}\,dx$$

input
```
int((b*x^2 + 2)^(1/2)/(d*x^2 + 3)^(1/2),x)
```

output
```
int((b*x^2 + 2)^(1/2)/(d*x^2 + 3)^(1/2), x)
```

3.296 $\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

3.296.1 Optimal result

Integrand size = 23, antiderivative size = 19

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{E\left(\arccos\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

output
```
-1/3*(x^2)^(1/2)/x*EllipticE(1/2*(-6*x^2+4)^(1/2),2^(1/2))*3^(1/2)
```

3.296.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \frac{\sqrt{-1+3x^2}E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3-9x^2}}$$

input
```
Integrate[Sqrt[-1 + 3*x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
(Sqrt[-1 + 3*x^2]*EllipticE[ArcSin[Sqrt[3/2]*x], 2])/Sqrt[3 - 9*x^2]
```

3.296.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

$$\Big\downarrow \; 328$$

$$-\frac{E\left(\arccos\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

input
```
Int[Sqrt[-1 + 3*x^2]/Sqrt[2 - 3*x^2],x]
```

output
```
-(EllipticE[ArcCos[Sqrt[3/2]*x], 2]/Sqrt[3])
```

3.296.3.1 Defintions of rubi rules used

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

3.296.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

method	result	size
default	$-\frac{E\left(\frac{x\sqrt{2}\,\sqrt{3}}{2},\sqrt{2}\right)\sqrt{-3x^2+1}\,\sqrt{3}}{3\sqrt{3x^2-1}}$	37
elliptic	$\frac{\sqrt{-(3x^2-2)(3x^2-1)}\left(-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,F\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)}{6\sqrt{-9x^4+9x^2-2}}+\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,\left(F\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)-E\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)\right)}{6\sqrt{-9x^4+9x^2-2}}\right)}{\sqrt{3x^2-1}\,\sqrt{-3x^2+2}}$	146

input
```
int((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*EllipticE(1/2*x*2^(1/2)*3^(1/2),2^(1/2))*(-3*x^2+1)^(1/2)*3^(1/2)/(3*
x^2-1)^(1/2)
```

3.296.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.79

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$$
$$= \frac{-4i\sqrt{3}\sqrt{2}xE\left(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)\mid\frac{1}{2}\right)+i\sqrt{3}\sqrt{2}xF\left(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)\mid\frac{1}{2}\right)-6\sqrt{3x^2-1}\sqrt{-3x^2+2}}{18x}$$

input
```
integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/18*(-4*I*sqrt(3)*sqrt(2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/x), 1/2
) + I*sqrt(3)*sqrt(2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(2)/x), 1/2) - 6
*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2))/x
```

3.296.6 Sympy [F]

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

input
```
integrate((3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

3.296.7 Maxima [F]

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{3\,x^2-1}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2), x)
```

3.296.8 Giac [F]

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{3\,x^2-1}}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2), x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{3\,x^2-1}}{\sqrt{2-3\,x^2}}\,dx$$

input
```
int((3*x^2 - 1)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

output
```
int((3*x^2 - 1)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.297 $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.297.1 Optimal result

Integrand size = 59, antiderivative size = 95

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\frac{\sqrt{b+\sqrt{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

output
```
1/2*EllipticE(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2),((-b-(-4*a*c+
b^2)^(1/2))/(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2)/c^(1/2)
```

3.297.2 Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\frac{\sqrt{b+\sqrt{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

input
```
Integrate[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])],x]
```

output
```
(Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))])
/(Sqrt[2]*Sqrt[c])
```

3.297.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.017$, Rules used = {327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}}dx$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

input
```
Int[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])],x]
```

output

```
(Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))])
/(Sqrt[2]*Sqrt[c])
```


3.297.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.297.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(80) = 160$.

Time = 2.89 (sec) , antiderivative size = 809, normalized size of antiderivative = 8.52

method	result
elliptic	$\sqrt{\frac{-2c\,x^2+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}}\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{-\frac{\left(-2c\,x^2+\sqrt{-4ac+b^2}-b\right)\left(-2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{ac}}\left(\frac{\sqrt{2}\sqrt{1-\frac{2c\,x^2}{-b+\sqrt{-4ac+b^2}}}\sqrt{1-\frac{2c\,x^2}{b+\sqrt{-4ac}}}}{2\sqrt{\frac{c}{-b+\sqrt{-4ac+b}}}}\right.$

input
```
int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((-2*c
*x^2+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-b+(-4*a*c+b^2)^
(1/2))*(-(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a
/c)^(1/2)/(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(1/2*2^(1/2)/(c/(-b+(-4*a*c+b^2)
^(1/2)))^(1/2)*(1-2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2)*(1-2*c*x^2/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a
*c+b^2)^(1/2))-4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1
/2)*EllipticF(x*2^(1/2)*(c/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4+2*(-2*c/
(b+(-4*a*c+b^2)^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2)))/c*(b-(-4*a*c+b^2)^(1/2)
))^(1/2))+2*c/(-b+(-4*a*c+b^2)^(1/2))*2^(1/2)/(c/(-b+(-4*a*c+b^2)^(1/2)))^
(1/2)*(1-2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2)*(1-2*c*x^2/(b+(-4*a*c+b^2)
^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2)^(
1/2))-4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)/(-2*c
/(b+(-4*a*c+b^2)^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2))-b/a)*(EllipticF(x*2^(1/
2)*(c/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4+2*(-2*c/(b+(-4*a*c+b^2)^(1/2)
)+2*c/(b-(-4*a*c+b^2)^(1/2)))/c*(b-(-4*a*c+b^2)^(1/2)))^(1/2))-EllipticE(x
*2^(1/2)*(c/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4+2*(-2*c/(b+(-4*a*c+b^2)
^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2)))/c*(b-(-4*a*c+b^2)^(1/2)))^(1/2))))
```

3.297. $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.297.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(77) = 154$.

Time = 0.14 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.35

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = -\frac{2\sqrt{\frac{1}{2}}\left(\sqrt{b^2-4\,ac}bx+(b^2-2\,ac)x\right)\sqrt{\frac{b+\sqrt{b^2-4\,ac}}{c}}\sqrt{-\frac{c}{a}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{b+\sqrt{b^2-4\,ac}}{c}}}{x}\right)\mid-\frac{b^2-2\,ac-\sqrt{b^2-4\,ac}b}{2\,ac})-}{}$$

input
```
integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(1/2)*(sqrt(b^2 - 4*a*c)*b*x + (b^2 - 2*a*c)*x)*sqrt((b + sqrt
(b^2 - 4*a*c))/c)*sqrt(-c/a)*elliptic_e(arcsin(sqrt(1/2)*sqrt((b + sqrt(b^
2 - 4*a*c))/c)/x), -1/2*(b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*b)/(a*c)) - 2*sqr
t(1/2)*(sqrt(b^2 - 4*a*c)*(b - c)*x + (b^2 - (2*a - b)*c)*x)*sqrt((b + sqr
t(b^2 - 4*a*c))/c)*sqrt(-c/a)*elliptic_f(arcsin(sqrt(1/2)*sqrt((b + sqrt(b
^2 - 4*a*c))/c)/x), -1/2*(b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*b)/(a*c)) + (b*c
 + sqrt(b^2 - 4*a*c)*c)*sqrt((b*x^2 + sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a)*sqrt
(-(b*x^2 - sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a))/(c^2*x)
```

3.297.6 Sympy [F]

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{-\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

input
```
integrate((1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1-2*c*x**2/(b+(-4*
a*c+b**2)**(1/2)))**(1/2),x)
```

output
```
Integral(sqrt((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**2
)))/sqrt(-(-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2)))
, x)
```

3.297.7 Maxima [F]

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

input
```
integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqr
t(b^2 - 4*a*c)) + 1), x)
```

3.297.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{\sqrt{\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}+1}}{\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,dx$$

input
```
int(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)/(1 - (2*c*x^2)/(b + (b
^2 - 4*a*c)^(1/2)))^(1/2),x)
```

output
```
int(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)/(1 - (2*c*x^2)/(b + (b
^2 - 4*a*c)^(1/2)))^(1/2), x)
```

3.297. $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.298 $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.298.1 Optimal result

Integrand size = 59, antiderivative size = 94

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

output
```
1/2*EllipticE(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2),((b+(-4*a*c+b
^2)^(1/2))/(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2)/c^(1/2)
```

3.298.2 Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

input
```
Integrate[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])],x]
```

output
```
(Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c]))]
)/(Sqrt[2]*Sqrt[c])
```

3.298.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.017$, Rules used = {327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}}dx$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

input
```
Int[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])],x]
```

output
```
(Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(S
qrt[2]*Sqrt[c])
```

3.298.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.298.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. $2(76) = 152$.

Time = 2.72 (sec) , antiderivative size = 1388, normalized size of antiderivative = 14.77

method	result	size
elliptic	Expression too large to display	1388

input
```
int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((-2*c*
x^2+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-b+(-4*a*c+b^2)^(
1/2))*(-(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c
)^(1/2)/(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(1/2/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*
c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*
c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2
)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))-2*c*x^2/(b-(-4*a*c+b
^2)^(1/2))+4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)*
EllipticF(1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b
+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(-2*c/(b+
(-4*a*c+b^2)^(1/2))-2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*
c+b^2)^(1/2)*b^2-4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1
/2)))^(1/2))-2*c/(-b+(-4*a*c+b^2)^(1/2))/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b
^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b
^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(
-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))-2*c*x^2/(b-(-4*a*c+b...
```

3.298.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(76) = 152$.

Time = 0.10 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.36

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(b^2x+\sqrt{b^2-4\,ac}bx+\left(bcx+\sqrt{b^2-4\,ac}cx\right)\sqrt{\frac{b^2-4\,ac}{c^2}}\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}+b}{c}}\sqrt{\frac{c}{a}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}+b}{c}}}{x}\right.}{}$$

input
```
integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(1/2)*(b^2*x + sqrt(b^2 - 4*a*c)*b*x + (b*c*x + sqrt(b^2 - 4*a*c
)*c*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/c)*sq
rt(c/a)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/c
)/x), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^2 + 2*a*c)/(a*c)) - sqrt(1/2)*
(sqrt(b^2 - 4*a*c)*b*x + (b^2 - 2*b*c)*x + (sqrt(b^2 - 4*a*c)*c*x + (b*c +
 2*c^2)*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/c
)*sqrt(c/a)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) +
b)/c)/x), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^2 + 2*a*c)/(a*c)) + (b*c +
 sqrt(b^2 - 4*a*c)*c)*sqrt(-(b*x^2 + sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a)*sqrt(
-(b*x^2 - sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a))/(c^2*x)
```

3.298.6 Sympy [F]

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{-\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{-\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

input
```
integrate((1-2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1-2*c*x**2/(b+(-4*
a*c+b**2)**(1/2)))**(1/2),x)
```

output
```
Integral(sqrt(-(-b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b*
*2)))/sqrt(-(-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2)
)), x)
```

3.298.7 Maxima [F]

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

input
```
integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sq
rt(b^2 - 4*a*c)) + 1), x)
```

3.298.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}}{\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,dx$$

input
```
int((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)))^(1/2)/(1 - (2*c*x^2)/(b + (b
^2 - 4*a*c)^(1/2)))^(1/2),x)
```

output
```
int((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)))^(1/2)/(1 - (2*c*x^2)/(b + (b
^2 - 4*a*c)^(1/2)))^(1/2), x)
```

3.299 $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.299.1 Optimal result

Integrand size = 59, antiderivative size = 478

$$\begin{aligned}
&\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx\\
&=\frac{x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&-\frac{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}E\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&+\frac{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right),-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}
\end{aligned}$$

output
```
x*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)-1/2*(1/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)*EllipticE(x*2^(
1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)
))^(1/2),(-2*(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/
(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)
/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(
1/2)+1/2*(1/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)*EllipticF(x*2^(1/2)*
c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1
/2),(-2*(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/((1+
2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

3.299.2 Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\frac{\sqrt{-b-\sqrt{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

input
```
Integrate[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])],x]
```

output
```
(Sqrt[-b - Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b
 - Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/
(Sqrt[2]*Sqrt[c])
```

3.299.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}dx$$

$$\Big\downarrow \text{ 324}$$

$$\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx+\frac{2c\int \frac{x^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{b-\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{2c\int \frac{x^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{b-\sqrt{b^2-4ac}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right),-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{2c\left(\frac{x\left(b-\sqrt{b^2-4ac}\right)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{2c\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\left(b-\sqrt{b^2-4ac}\right)\int \frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{2c}\right)}{b-\sqrt{b^2-4ac}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right),-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{2c\left(\frac{x\left(b-\sqrt{b^2-4ac}\right)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{2c\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}E\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\right)}{b-\sqrt{b^2-4ac}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right),-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

3.299. $\displaystyle\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}dx$

input
```
Int[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])],x]
```

output
```
(2*c*(((b - Sqrt[b^2 - 4*a*c])*x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]
)])/(2*c*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) - ((b - Sqrt[b^2 - 4
*a*c])*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-
2*Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[(1
+ (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])))/(b - Sqrt[b^2 - 4*a*c])
 + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*EllipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

3.299.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.299. $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.299.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. $2(457) = 914$.

Time = 2.69 (sec) , antiderivative size = 1388, normalized size of antiderivative = 2.90

method	result	size
elliptic	Expression too large to display	1388

input
```
int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*
x^2+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-b+(-4*a*c+b^2)^(
1/2))*(-(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c
)^(1/2)/(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(1/2/(-2*((-4*a*c+b^2)^(3/2)-(-4*a
*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a
*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/
2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+
b^2)^(1/2))+4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)
*EllipticF(1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(
b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(b+
(-4*a*c+b^2)^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*
c+b^2)^(1/2)*b^2+4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1
/2)))^(1/2))+2*c/(-b+(-4*a*c+b^2)^(1/2))/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b
^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b
^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(
-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b...
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = -\frac{\sqrt{\frac{1}{2}}\left(b^2x+\sqrt{b^2-4\,ac}bx-\left(bcx+\sqrt{b^2-4\,ac}cx\right)\sqrt{\frac{b^2-4\,ac}{c^2}}\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}\sqrt{\frac{c}{a}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right.\right.}{}$$

input
```
integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(1/2)*(b^2*x + sqrt(b^2 - 4*a*c)*b*x - (b*c*x + sqrt(b^2 - 4*a*c
)*c*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*sq
rt(c/a)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c
)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*(
sqrt(b^2 - 4*a*c)*b*x + (b^2 + 2*b*c)*x - (sqrt(b^2 - 4*a*c)*c*x + (b*c -
2*c^2)*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)
*sqrt(c/a)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b
)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - (b*c + s
qrt(b^2 - 4*a*c)*c)*sqrt((b*x^2 + sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a)*sqrt((b*
x^2 - sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a))/(c^2*x)
```

3.299.6 Sympy [F]

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

input
```
integrate((1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*
a*c+b**2)**(1/2)))**(1/2),x)
```

output
```
Integral(sqrt((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**2
)))/sqrt((b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2))),
x)
```

3.299. $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.299.7 Maxima [F]

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

input
```
integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt
(b^2 - 4*a*c)) + 1), x)
```

3.299.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}+1}}{\sqrt{\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}+1}}\,dx$$

input
```
int(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)/((2*c*x^2)/(b + (b^2 -
 4*a*c)^(1/2)) + 1)^(1/2),x)
```

output
```
int(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)/((2*c*x^2)/(b + (b^2 -
 4*a*c)^(1/2)) + 1)^(1/2), x)
```

3.299. $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.300 $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.300.1 Optimal result

Integrand size = 59, antiderivative size = 215

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = -\frac{\left(b+\sqrt{b^2-4ac}\right)E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\middle|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}b\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right),-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}$$

output
```
b*EllipticF(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2),((-b+(-4*a*c+b^
2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)-1/2*EllipticE(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2),(
(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b+(-4*a*c+b^2)^(1/
2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
```

3.300.2 Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\frac{\sqrt{-b-\sqrt{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

input
```
Integrate[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])],x]
```

output
```
(Sqrt[-b - Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b
 - Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])
/(Sqrt[2]*Sqrt[c])
```

3.300.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {326, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}dx$$

$$\Big\downarrow \text{326}$$

$$\frac{2b\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{b-\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}+b\right)\int \frac{\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}dx}{b-\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{2}b\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right),-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\sqrt{b^2-4ac}+b\right)\int \frac{\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}dx}{b-\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{2}b\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right),-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\sqrt{b^2-4ac}+b\right)E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}$$

input
```
Int[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])],x]
```

output
```
-(((b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]))])/(
Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) + (Sqrt[2]*b*EllipticF[ArcSi
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*
c])/(b + Sqrt[b^2 - 4*a*c]))])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

3.300.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.300. $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.300.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(173) = 346$.

Time = 2.63 (sec) , antiderivative size = 872, normalized size of antiderivative = 4.06

method	result
elliptic	$\sqrt{\frac{2c\,x^2+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}}\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{-\frac{\left(2c\,x^2+\sqrt{-4ac+b^2}-b\right)\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{ac}}\left(\frac{\sqrt{1+\frac{2c\,x^2}{b+\sqrt{-4ac+b^2}}}\sqrt{1+\frac{2c\,x^2}{-b+\sqrt{-4ac+b^2}}}\,F}{\sqrt{-\frac{2c}{b+\sqrt{-4ac+b^2}}}}\right.$

input
```
int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x
^2+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-b+(-4*a*c+b^2)^(1
/2))*(-(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^
(1/2)/(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(1/(-2*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2
)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c/(-b+(-4*a*c+b^2)^(1/2))*
x^2)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
)-4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)*EllipticF
(x*(-2*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4-2*(2*c/(b+(-4*a*c+b^2)^(1/2
))-2*c/(b-(-4*a*c+b^2)^(1/2)))/c/(-b+(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(
1/2))*(b+(-4*a*c+b^2)^(1/2)))^(1/2))-4*c/(-b+(-4*a*c+b^2)^(1/2))/(-2*c/(b+
(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c
/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))-2*c*
x^2/(b-(-4*a*c+b^2)^(1/2))-4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1
/2))*x^4)^(1/2)/(2*c/(b+(-4*a*c+b^2)^(1/2))-2*c/(b-(-4*a*c+b^2)^(1/2))-b/a
)*(EllipticF(x*(-2*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4-2*(2*c/(b+(-4*a
*c+b^2)^(1/2))-2*c/(b-(-4*a*c+b^2)^(1/2)))/c/(-b+(-4*a*c+b^2)^(1/2))*(b-(-
4*a*c+b^2)^(1/2))*(b+(-4*a*c+b^2)^(1/2)))^(1/2))-EllipticE(x*(-2*c/(b+(-4*
a*c+b^2)^(1/2)))^(1/2),1/2*(-4-2*(2*c/(b+(-4*a*c+b^2)^(1/2))-2*c/(b-(-4*a*
c+b^2)^(1/2)))/c/(-b+(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2))*(b+(-4*a*c
+b^2)^(1/2)))^(1/2))))
```

3.300. $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.300.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$$
$$= \frac{4\sqrt{\frac{1}{2}}ax\sqrt{\frac{b-\sqrt{b^2-4\,ac}}{c}}\sqrt{-\frac{c}{a}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{b-\sqrt{b^2-4\,ac}}{c}}}{x}\right)|-\frac{b^2-2\,ac+\sqrt{b^2-4\,ac}b}{2\,ac}) - 2\sqrt{\frac{1}{2}}((2\,a-b)x-\sqrt{b^2-4\,ac}}{}$$

input
```
integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(4*sqrt(1/2)*a*x*sqrt((b - sqrt(b^2 - 4*a*c))/c)*sqrt(-c/a)*elliptic_e
(arcsin(sqrt(1/2)*sqrt((b - sqrt(b^2 - 4*a*c))/c)/x), -1/2*(b^2 - 2*a*c +
sqrt(b^2 - 4*a*c)*b)/(a*c)) - 2*sqrt(1/2)*((2*a - b)*x - sqrt(b^2 - 4*a*c)
*x)*sqrt((b - sqrt(b^2 - 4*a*c))/c)*sqrt(-c/a)*elliptic_f(arcsin(sqrt(1/2)
*sqrt((b - sqrt(b^2 - 4*a*c))/c)/x), -1/2*(b^2 - 2*a*c + sqrt(b^2 - 4*a*c)
*b)/(a*c)) + (b + sqrt(b^2 - 4*a*c))*sqrt(-(b*x^2 + sqrt(b^2 - 4*a*c)*x^2
- 2*a)/a)*sqrt((b*x^2 - sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a))/(c*x)
```

3.300.6 Sympy [F]

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{-\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

input
```
integrate((1-2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*
a*c+b**2)**(1/2)))**(1/2),x)
```

output
```
Integral(sqrt(-(-b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b*
*2)))/sqrt((b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2)))
, x)
```

3.300.7 Maxima [F]

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

input
```
integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqr
t(b^2 - 4*a*c)) + 1), x)
```

3.300.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}}{\sqrt{\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}+1}}\,dx$$

input
```
int((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)))^(1/2)/((2*c*x^2)/(b + (b^2 -
4*a*c)^(1/2)) + 1)^(1/2),x)
```

output
```
int((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)))^(1/2)/((2*c*x^2)/(b + (b^2 -
4*a*c)^(1/2)) + 1)^(1/2), x)
```

3.300. $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.301 $\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx$

3.301.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = -\frac{2^{-2-m}\sqrt{x^2}(2-4x^2)^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},(1-2x^2)^2\right)}{(1+m)x}$$

output
```
-2^(-2-m)*(-4*x^2+2)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],(-2*x^2+
1)^2)*(x^2)^(1/2)/(1+m)/x
```

3.301.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.97

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx$$

$$= \frac{3x(1-2x^2)^m\,\text{AppellF1}\left(\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},2x^2,x^2\right)}{\sqrt{1-x^2}\left(3\,\text{AppellF1}\left(\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},2x^2,x^2\right)+x^2\left(-4m\,\text{AppellF1}\left(\frac{3}{2},1-m,\frac{1}{2},\frac{5}{2},2x^2,x^2\right)+\text{AppellF1}\left(\frac{3}{2},\right.\right.\right.}$$

input
```
Integrate[(1 - 2*x^2)^m/Sqrt[1 - x^2],x]
```

output
```
(3*x*(1 - 2*x^2)^m*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2])/(Sqrt[1 - x^2]
*(3*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2] + x^2*(-4*m*AppellF1[3/2, 1 -
m, 1/2, 5/2, 2*x^2, x^2] + AppellF1[3/2, -m, 3/2, 5/2, 2*x^2, x^2])))
```

3.301.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.37, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}} \, dx$$
$$\Big\downarrow \; 333$$
$$x \operatorname{AppellF1}\left(\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 2x^2, x^2\right)$$

input
```
Int[(1 - 2*x^2)^m/Sqrt[1 - x^2],x]
```

output
```
x*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2]
```

3.301.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

3.301.4 Maple [F]

$$\int \frac{(-2x^2+1)^m}{\sqrt{-x^2+1}} dx$$

input
```
int((-2*x^2+1)^m/(-x^2+1)^(1/2),x)
```

output
```
int((-2*x^2+1)^m/(-x^2+1)^(1/2),x)
```

3.301.5 Fricas [F]

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(-2\,x^2+1)^m}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((-2*x^2+1)^m/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 + 1)*(-2*x^2 + 1)^m/(x^2 - 1), x)
```

3.301.6 Sympy [F]

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(1-2x^2)^m}{\sqrt{-(x-1)\,(x+1)}}\,dx$$

input
```
integrate((-2*x**2+1)**m/(-x**2+1)**(1/2),x)
```

output
```
Integral((1 - 2*x**2)**m/sqrt(-(x - 1)*(x + 1)), x)
```

3.301.7 Maxima [F]

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(-2\,x^2+1)^m}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((-2*x^2+1)^m/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1), x)
```

3.301.8 Giac [F]

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(-2\,x^2+1)^m}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((-2*x^2+1)^m/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1), x)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(1-2\,x^2)^m}{\sqrt{1-x^2}}\,dx$$

input
```
int((1 - 2*x^2)^m/(1 - x^2)^(1/2),x)
```

output
```
int((1 - 2*x^2)^m/(1 - x^2)^(1/2), x)
```

3.302 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx$

3.302.1 Optimal result

Integrand size = 26, antiderivative size = 46

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{-1+x^2}}$$

output
```
EllipticF(x,I*3^(1/2)+2*I)*(-x^2+1)^(1/2)/(x^2-1)^(1/2)/(2-3^(1/2))
```

3.302.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),\frac{1}{-7+4\sqrt{3}}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{-1+x^2}}$$

input
```
Integrate[1/(Sqrt[-1 + x^2]*Sqrt[7 - 4*Sqrt[3] + x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], (-7 + 4*Sqrt[3])^(-1)])/(Sqrt[7 - 4*Sq
rt[3]]*Sqrt[-1 + x^2])
```

3.302.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{x^2-4\sqrt{3}+7}}dx$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2-4\sqrt{3}+7}}dx}{\sqrt{x^2-1}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{x^2-1}}$$

input
```
Int[1/(Sqrt[-1 + x^2]*Sqrt[7 - 4*Sqrt[3] + x^2]),x]
```

output
```
(Sqrt[1 - x^2]*EllipticF[ArcSin[x], -7 - 4*Sqrt[3]])/(Sqrt[7 - 4*Sqrt[3]]*
Sqrt[-1 + x^2])
```

3.302.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

3.302.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(37) = 74$.

Time = 2.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.54

method	result	size
default	$-\frac{iF\left(\frac{ix}{-2+\sqrt{3}},2i-i\sqrt{3}\right)\sqrt{-x^2+1}\,\sqrt{-\left(-x^2+4\sqrt{3}-7\right)\left(-4\sqrt{3}+7\right)}\,\left(-2+\sqrt{3}\right)\sqrt{x^2-1}\,\sqrt{7+x^2-4\sqrt{3}}}{\left(4\sqrt{3}-7\right)\left(-x^4+4x^2\sqrt{3}-6x^2-4\sqrt{3}+7\right)}$	117
elliptic	$-\frac{i\sqrt{-(x^2-1)\left(-x^2+4\sqrt{3}-7\right)}\,\sqrt{-4\sqrt{3}+7}\,\sqrt{1-\frac{x^2}{4\sqrt{3}-7}}\,\sqrt{-x^2+1}\,F\left(\frac{ix}{\sqrt{-4\sqrt{3}+7}},2i-i\sqrt{3}\right)}{\sqrt{x^2-1}\,\sqrt{7+x^2-4\sqrt{3}}\,\sqrt{6x^2-7+x^4-4x^2\sqrt{3}+4\sqrt{3}}}$	128

input
```
int(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*EllipticF(I*x/(-2+3^(1/2)),2*I-I*3^(1/2))*(-x^2+1)^(1/2)*(-(-x^2+4*3^(1
/2)-7)*(-4*3^(1/2)+7))^(1/2)/(4*3^(1/2)-7)*(-2+3^(1/2))*(x^2-1)^(1/2)*(7+x
^2-4*3^(1/2))^(1/2)/(-x^4+4*x^2*3^(1/2)-6*x^2-4*3^(1/2)+7)
```

3.302.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(34) = 68$.

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.57

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx =$$
$$-\frac{1}{2}\left(\left(2\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{4\sqrt{3}-7}+2\sqrt{2}\sqrt{4\sqrt{3}+7}\sqrt{4\sqrt{3}-7}\right)\sqrt{-4\sqrt{3}+4\sqrt{4\sqrt{3}+7}-6}F(\text{arcsin}$$
$$-4\sqrt{4\sqrt{3}+7}\left(2\sqrt{3}-3\right)-7)$$

input
```
integrate(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(4*sqrt(3) - 7) + 2*sqrt(2)*sqrt
(4*sqrt(3) + 7)*sqrt(4*sqrt(3) - 7))*sqrt(-4*sqrt(3) + 4*sqrt(4*sqrt(3) +
7) - 6)*elliptic_f(arcsin(1/2*sqrt(2)*x*sqrt(-4*sqrt(3) + 4*sqrt(4*sqrt(3)
 + 7) - 6)), -4*sqrt(4*sqrt(3) + 7)*(2*sqrt(3) - 3) - 7)
```

3.302.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{x^2-4\sqrt{3}+7}}\,dx$$

input
```
integrate(1/(x**2-1)**(1/2)/(7+x**2-4*3**(1/2))**(1/2),x)
```

output
```
Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(x**2 - 4*sqrt(3) + 7)), x)
```

3.302.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-4\sqrt{3}+7}\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)), x)
```

3.302.8 Giac [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-4\sqrt{3}+7}\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)), x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{x^2-4\,\sqrt{3}+7}}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(x^2 - 4*3^(1/2) + 7)^(1/2)),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(x^2 - 4*3^(1/2) + 7)^(1/2)), x)
```

3.303 $\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx$

3.303.1 Optimal result

Integrand size = 41, antiderivative size = 47

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx = -\frac{1}{6}\sqrt{3+\sqrt{3}}\,\text{EllipticF}\left(\arccos\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

output
```
-1/6*(x^2*(9-3*3^(1/2)))^(1/2)/x/(9-3*3^(1/2))^(1/2)*EllipticF(1/3*(9-x^2*
(9-3*3^(1/2)))^(1/2),1/2*(2+2*3^(1/2))^(1/2))*(3+3^(1/2))^(1/2)
```

3.303.2 Mathematica [A] (warning: unable to verify)

Time = 1.98 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx = \frac{\sqrt{3+\sqrt{3}-2x^2}\sqrt{-3+3\sqrt{3}-2\sqrt{3}x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{1-\frac{1}{\sqrt{3}}}x\right),2+\sqrt{3}\right)}{6\sqrt{\left(-2+\sqrt{3}\right)\left(3-6x^2+2x^4\right)}}$$

input
```
Integrate[1/(Sqrt[3 - 3*Sqrt[3] + 2*Sqrt[3]*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x
^2]),x]
```

output
```
(Sqrt[3 + Sqrt[3] - 2*x^2]*Sqrt[-3 + 3*Sqrt[3] - 2*Sqrt[3]*x^2]*EllipticF[
ArcSin[Sqrt[1 - 1/Sqrt[3]]*x], 2 + Sqrt[3]])/(6*Sqrt[(-2 + Sqrt[3])*(3 - 6
*x^2 + 2*x^4)])
```

3.303.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {322}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2\sqrt{3}x^2-3\sqrt{3}+3}\sqrt{\left(\sqrt{3}-3\right)x^2+3}}dx$$

$$\Big\downarrow \text{322}$$

$$-\frac{1}{6}\sqrt{3+\sqrt{3}}\,\text{EllipticF}\left(\arccos\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

input
```
Int[1/(Sqrt[3 - 3*Sqrt[3] + 2*Sqrt[3]*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]),x
]
```

output
```
-1/6*(Sqrt[3 + Sqrt[3]]*EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sq
rt[3])/2])
```

3.303.3.1 Defintions of rubi rules used

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

3.303.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(64) = 128$.

Time = 2.61 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.19

method	result
default	$\frac{\sqrt{x^{2}\sqrt{3}-3x^{2}+3}\,\sqrt{3-3\sqrt{3}+2x^{2}\sqrt{3}}\,\sqrt{2}\,\sqrt{-12x^{2}\sqrt{3}+18x^{2}+9\sqrt{3}-9}\,\sqrt{3}\,\sqrt{-\left(3-3\sqrt{3}+2x^{2}\sqrt{3}\right)\left(\sqrt{3}-1\right)}\,F\left(\frac{x\sqrt{2}\,\sqrt{3}\,\sqrt{\left(2\sqrt{3}-3\right)\left(\sqrt{3}-1\right)}}{3\sqrt{3}-3}\right)}{54\left(\sqrt{3}-1\right)^{2}\sqrt{2\sqrt{3}-3}\,\left(2x^{4}\sqrt{3}-2x^{4}-6x^{2}\sqrt{3}+6x^{2}+3\sqrt{3}-3\right)}$
elliptic	$\frac{\sqrt{\left(x^{2}\sqrt{3}-3x^{2}+3\right)\left(3-3\sqrt{3}+2x^{2}\sqrt{3}\right)}\,\sqrt{6}\,\sqrt{9-\frac{6\left(2\sqrt{3}-3\right)x^{2}}{\sqrt{3}-1}}\,\sqrt{9-\frac{6\sqrt{3}\,x^{2}}{\sqrt{3}-1}}\,F\left(\frac{x\sqrt{6}\,\sqrt{\frac{2\sqrt{3}-3}{\sqrt{3}-1}}}{3},\frac{\sqrt{-9-\frac{6\left(18\sqrt{3}-18\right)\sqrt{3}}{\left(\sqrt{3}-1\right)\left(6-6\sqrt{3}\right)}}}{3}\right)}{18\sqrt{x^{2}\sqrt{3}-3x^{2}+3}\,\sqrt{3-3\sqrt{3}+2x^{2}\sqrt{3}}\,\sqrt{\frac{2\sqrt{3}-3}{\sqrt{3}-1}}\,\sqrt{18x^{2}\sqrt{3}-18x^{2}+6x^{4}-6x^{4}\sqrt{3}+9-9\sqrt{3}}}$

input
```
int(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*x^2*3^(1/2))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/54*(x^2*3^(1/2)-3*x^2+3)^(1/2)*(3-3*3^(1/2)+2*x^2*3^(1/2))^(1/2)*2^(1/2)
*(-12*x^2*3^(1/2)+18*x^2+9*3^(1/2)-9)^(1/2)/(3^(1/2)-1)^2*3^(1/2)*(-(3-3*3
^(1/2)+2*x^2*3^(1/2))*(3^(1/2)-1))^(1/2)*EllipticF(1/3*x*2^(1/2)*3^(1/2)/(
3^(1/2)-1)*((2*3^(1/2)-3)*(3^(1/2)-1))^(1/2),1/(3^(1/2)-1)*((3^(1/2)-1)*(1
+3^(1/2)))^(1/2))*(-3+3^(1/2))/(2*3^(1/2)-3)^(1/2)/(2*x^4*3^(1/2)-2*x^4-6*
x^2*3^(1/2)+6*x^2+3*3^(1/2)-3)
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx$$
$$= -\frac{1}{18}\sqrt{\sqrt{3}+3}\sqrt{-9\sqrt{3}+9}F(\arcsin\left(\frac{1}{3}\sqrt{3}x\sqrt{\sqrt{3}+3}\right) | -\sqrt{3}+2)$$

input
```
integrate(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*x^2*3^(1/2))^(1/2),x
, algorithm="fricas")
```

output
```
-1/18*sqrt(sqrt(3) + 3)*sqrt(-9*sqrt(3) + 9)*elliptic_f(arcsin(1/3*sqrt(3)
*x*sqrt(sqrt(3) + 3)), -sqrt(3) + 2)
```

3.303.6 Sympy [F]

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx$$
$$=\int \frac{1}{\sqrt{-3x^2+\sqrt{3}x^2+3}\sqrt{2\sqrt{3}x^2-3\sqrt{3}+3}}\,dx$$

input
```
integrate(1/(3+x**2*(-3+3**(1/2)))**(1/2)/(3-3*3**(1/2)+2*x**2*3**(1/2))**
(1/2),x)
```

output
```
Integral(1/(sqrt(-3*x**2 + sqrt(3)*x**2 + 3)*sqrt(2*sqrt(3)*x**2 - 3*sqrt(
3) + 3)), x)
```

3.303.7 Maxima [F]

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx$$
$$=\int \frac{1}{\sqrt{x^2\left(\sqrt{3}-3\right)+3}\sqrt{2\sqrt{3}x^2-3\sqrt{3}+3}}\,dx$$

input
```
integrate(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*x^2*3^(1/2))^(1/2),x
, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*sqrt(3) +
3)), x)
```

3.303.8 Giac [F]

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx = \int \frac{1}{\sqrt{x^2\left(\sqrt{3}-3\right)+3}\sqrt{2\sqrt{3}x^2-3\sqrt{3}+3}}\,dx$$

input
```
integrate(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*x^2*3^(1/2))^(1/2),x
, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*sqrt(3) +
3)), x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx = \int \frac{1}{\sqrt{\left(\sqrt{3}-3\right)\,x^2+3}\,\sqrt{2\,\sqrt{3}\,x^2-3\,\sqrt{3}+3}}\,dx$$

input
```
int(1/((x^2*(3^(1/2) - 3) + 3)^(1/2)*(2*3^(1/2)*x^2 - 3*3^(1/2) + 3)^(1/2)
),x)
```

output
```
int(1/((x^2*(3^(1/2) - 3) + 3)^(1/2)*(2*3^(1/2)*x^2 - 3*3^(1/2) + 3)^(1/2)
), x)
```

3.304 $\int \frac{1}{\sqrt[4]{2+3x^2}(4+3x^2)}\,dx$

3.304.1 Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx = -\frac{\arctan\left(\frac{2\,2^{3/4}+2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{2\,2^{3/4}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

output
```
-1/12*arctan(1/6*(2*2^(3/4)+2*2^(1/4)*(3*x^2+2)^(1/2))/x/(3*x^2+2)^(1/4)*3
^(1/2))*2^(1/4)*3^(1/2)-1/12*arctanh(1/6*(2*2^(3/4)-2*2^(1/4)*(3*x^2+2)^(1
/2))/x/(3*x^2+2)^(1/4)*3^(1/2))*2^(1/4)*3^(1/2)
```

3.304.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx = \frac{\arctan\left(\frac{3\sqrt{2}x^2-4\sqrt{2+3x^2}}{2\,2^{3/4}\sqrt{3}x\sqrt[4]{2+3x^2}}\right)+\operatorname{arctanh}\left(\frac{2\,2^{3/4}\sqrt{3}x\sqrt[4]{2+3x^2}}{3\sqrt{2}x^2+4\sqrt{2+3x^2}}\right)}{4\,2^{3/4}\sqrt{3}}$$

input
```
Integrate[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```

output
```
(ArcTan[(3*Sqrt[2]*x^2 - 4*Sqrt[2 + 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2 + 3*x^
2)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[3]*x*(2 + 3*x^2)^(1/4))/(3*Sqrt[2]*x^
2 + 4*Sqrt[2 + 3*x^2])])/(4*2^(3/4)*Sqrt[3])
```

3.304.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{3x^2+2}\,(3x^2+4)}\,dx$$

$$\Big\downarrow \ 308$$

$$-\frac{\arctan\left(\frac{2\sqrt[4]{2}\sqrt{3x^2+2}+2\ 2^{3/4}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\ 2^{3/4}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{3x^2+2}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

input
```
Int[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```

output
```
-1/2*ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^
2)^(1/4))]/(2^(3/4)*Sqrt[3]) - ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + 3*x
^2])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

3.304.3.1 Defintions of rubi rules used

rule 308
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

3.304.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.45 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.45

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^4+72\right) \ln\left(-\frac{6\left(3x^2+2\right)^{\frac{3}{4}} \operatorname{RootOf}\left(\_Z^4+72\right)+\left(3x^2+2\right)^{\frac{1}{4}} \operatorname{RootOf}\left(\_Z^4+72\right)^3-18\sqrt{3x^2+2}\,x-3\operatorname{RootOf}\left(\_Z^4+72\right)^2 x}{3x^2+4}\right)}{24}$

input
```
int(1/(3*x^2+2)^(1/4)/(3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/24*RootOf(_Z^4+72)*ln(-(6*(3*x^2+2)^(3/4)*RootOf(_Z^4+72)+(3*x^2+2)^(1/
4)*RootOf(_Z^4+72)^3-18*(3*x^2+2)^(1/2)*x-3*RootOf(_Z^4+72)^2*x)/(3*x^2+4)
)+1/24*RootOf(_Z^2+RootOf(_Z^4+72)^2)*ln((6*(3*x^2+2)^(3/4)*RootOf(_Z^2+Ro
otOf(_Z^4+72)^2)-(3*x^2+2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+72)^2)*RootOf(_Z^
4+72)^2+18*(3*x^2+2)^(1/2)*x-3*RootOf(_Z^4+72)^2*x)/(3*x^2+4))
```

3.304.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.24

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx = \left(\frac{1}{288}i+\frac{1}{288}\right) \cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i+1)\cdot 18^{\frac{3}{4}}\sqrt{2}x-(3i-3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{3\,x^2+2}x+12i\,\sqrt{2}(3\,x^2+2)^{\frac{1}{4}}+12\,(3\,x^2+2)^{\frac{3}{4}}}{3\,x^2+4}\right) - \left(\frac{1}{288}i-\frac{1}{288}\right) \cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i-1)\cdot 18^{\frac{3}{4}}\sqrt{2}x+(3i+3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{3\,x^2+2}x-12i\,\sqrt{2}(3\,x^2+2)^{\frac{1}{4}}+12\,(3\,x^2+2)^{\frac{3}{4}}}{3\,x^2+4}\right) + \left(\frac{1}{288}i-\frac{1}{288}\right) \cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i-1)\cdot 18^{\frac{3}{4}}\sqrt{2}x-(3i+3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{3\,x^2+2}x-12i\,\sqrt{2}(3\,x^2+2)^{\frac{1}{4}}+12\,(3\,x^2+2)^{\frac{3}{4}}}{3\,x^2+4}\right) - \left(\frac{1}{288}i+\frac{1}{288}\right) \cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i+1)\cdot 18^{\frac{3}{4}}\sqrt{2}x+(3i-3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{3\,x^2+2}x+12i\,\sqrt{2}(3\,x^2+2)^{\frac{1}{4}}+12\,(3\,x^2+2)^{\frac{3}{4}}}{3\,x^2+4}\right)$$

input
```
integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="fricas")
```

output
```
(1/288*I + 1/288)*18^(3/4)*sqrt(2)*log(((I + 1)*18^(3/4)*sqrt(2)*x - (3*I
- 3)*18^(1/4)*sqrt(2)*sqrt(3*x^2 + 2)*x + 12*I*sqrt(2)*(3*x^2 + 2)^(1/4) +
 12*(3*x^2 + 2)^(3/4))/(3*x^2 + 4)) - (1/288*I - 1/288)*18^(3/4)*sqrt(2)*l
og((-(I - 1)*18^(3/4)*sqrt(2)*x + (3*I + 3)*18^(1/4)*sqrt(2)*sqrt(3*x^2 +
2)*x - 12*I*sqrt(2)*(3*x^2 + 2)^(1/4) + 12*(3*x^2 + 2)^(3/4))/(3*x^2 + 4))
 + (1/288*I - 1/288)*18^(3/4)*sqrt(2)*log(((I - 1)*18^(3/4)*sqrt(2)*x - (3
*I + 3)*18^(1/4)*sqrt(2)*sqrt(3*x^2 + 2)*x - 12*I*sqrt(2)*(3*x^2 + 2)^(1/4
) + 12*(3*x^2 + 2)^(3/4))/(3*x^2 + 4)) - (1/288*I + 1/288)*18^(3/4)*sqrt(2
)*log((-(I + 1)*18^(3/4)*sqrt(2)*x + (3*I - 3)*18^(1/4)*sqrt(2)*sqrt(3*x^2
 + 2)*x + 12*I*sqrt(2)*(3*x^2 + 2)^(1/4) + 12*(3*x^2 + 2)^(3/4))/(3*x^2 +
4))
```

3.304. $\int \frac{1}{\sqrt[4]{2+3x^2}(4+3x^2)}\,dx$

3.304.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx=\int \frac{1}{\sqrt[4]{3x^2+2}\cdot(3x^2+4)}\,dx$$

input
```
integrate(1/(3*x**2+2)**(1/4)/(3*x**2+4),x)
```

output
```
Integral(1/((3*x**2 + 2)**(1/4)*(3*x**2 + 4)), x)
```

3.304.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx=\int \frac{1}{(3\,x^2+4)(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)), x)
```

3.304.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx=\int \frac{1}{(3\,x^2+4)(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)), x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx = \int \frac{1}{(3\,x^2+2)^{1/4}\,(3\,x^2+4)}\,dx$$

input
```
int(1/((3*x^2 + 2)^(1/4)*(3*x^2 + 4)),x)
```

output
```
int(1/((3*x^2 + 2)^(1/4)*(3*x^2 + 4)), x)
```

3.305 $\int \frac{1}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

3.305.1 Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx=\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

output
```
1/12*arctan(1/6*(2-2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)/x/(-3*x^2+2)^(1/4)*3^
(1/2))*2^(1/4)*3^(1/2)+1/12*arctanh(1/6*(2+2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/
4)/x/(-3*x^2+2)^(1/4)*3^(1/2))*2^(1/4)*3^(1/2)
```

3.305.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx=\frac{\arctan\left(\frac{3\sqrt{2}x^2-4\sqrt{2-3x^2}}{2\ 2^{3/4}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)+\operatorname{arctanh}\left(\frac{2\ 2^{3/4}\sqrt{3}x\sqrt[4]{2-3x^2}}{3\sqrt{2}x^2+4\sqrt{2-3x^2}}\right)}{4\ 2^{3/4}\sqrt{3}}$$

input
```
Integrate[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(ArcTan[(3*Sqrt[2]*x^2 - 4*Sqrt[2 - 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2 - 3*x^
2)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))/(3*Sqrt[2]*x^
2 + 4*Sqrt[2 - 3*x^2])])/(4*2^(3/4)*Sqrt[3])
```

3.305.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$

$$\Big\downarrow \ 308$$

$$\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

input
```
Int[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))
]/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqr
t[3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

3.305.3.1 Defintions of rubi rules used

rule 308
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

3.305.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.47 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.57

method	result
trager	$-\frac{\text{RootOf}\left(\_Z^2+\text{RootOf}\left(\_Z^4+72\right)^2\right)\ln\left(-\frac{6\left(-3x^2+2\right)^{\frac{3}{4}}\text{RootOf}\left(\_Z^2+\text{RootOf}\left(\_Z^4+72\right)^2\right)+\left(-3x^2+2\right)^{\frac{1}{4}}\text{RootOf}\left(\_Z^2+\text{Roo}\right.}{3x^2-4}\right.}{24}$

input
```
int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/24*RootOf(_Z^2+RootOf(_Z^4+72)^2)*ln(-(6*(-3*x^2+2)^(3/4)*RootOf(_Z^2+R
ootOf(_Z^4+72)^2)+(-3*x^2+2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+72)^2)*RootOf(_
Z^4+72)^2-18*(-3*x^2+2)^(1/2)*x-3*RootOf(_Z^4+72)^2*x)/(3*x^2-4))-1/24*Roo
tOf(_Z^4+72)*ln(-(6*(-3*x^2+2)^(3/4)*RootOf(_Z^4+72)-(-3*x^2+2)^(1/4)*Root
Of(_Z^4+72)^3-18*(-3*x^2+2)^(1/2)*x+3*RootOf(_Z^4+72)^2*x)/(3*x^2-4))
```

3.305.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\left(\frac{1}{288}i+\frac{1}{288}\right)\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i+1)\cdot 18^{\frac{3}{4}}\sqrt{2}x+(3i-3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{-3\,x^2+2}x-12i\,\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+12\,(-3\,x^2+2)^{\frac{3}{4}}}{3\,x^2-4}\right)$$
$$+\left(\frac{1}{288}i-\frac{1}{288}\right)\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i-1)\cdot 18^{\frac{3}{4}}\sqrt{2}x-(3i+3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{-3\,x^2+2}x+12i\,\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+12\,(-3\,x^2+2)^{\frac{3}{4}}}{3\,x^2-4}\right)$$
$$-\left(\frac{1}{288}i-\frac{1}{288}\right)\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i-1)\cdot 18^{\frac{3}{4}}\sqrt{2}x+(3i+3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{-3\,x^2+2}x+12i\,\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+12\,(-3\,x^2+2)^{\frac{3}{4}}}{3\,x^2-4}\right)$$
$$+\left(\frac{1}{288}i+\frac{1}{288}\right)\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i+1)\cdot 18^{\frac{3}{4}}\sqrt{2}x-(3i-3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{-3\,x^2+2}x-12i\,\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+12\,(-3\,x^2+2)^{\frac{3}{4}}}{3\,x^2-4}\right)$$

input
```
integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
-(1/288*I + 1/288)*18^(3/4)*sqrt(2)*log(((I + 1)*18^(3/4)*sqrt(2)*x + (3*I
 - 3)*18^(1/4)*sqrt(2)*sqrt(-3*x^2 + 2)*x - 12*I*sqrt(2)*(-3*x^2 + 2)^(1/4
) + 12*(-3*x^2 + 2)^(3/4))/(3*x^2 - 4)) + (1/288*I - 1/288)*18^(3/4)*sqrt(
2)*log((-(I - 1)*18^(3/4)*sqrt(2)*x - (3*I + 3)*18^(1/4)*sqrt(2)*sqrt(-3*x
^2 + 2)*x + 12*I*sqrt(2)*(-3*x^2 + 2)^(1/4) + 12*(-3*x^2 + 2)^(3/4))/(3*x^
2 - 4)) - (1/288*I - 1/288)*18^(3/4)*sqrt(2)*log(((I - 1)*18^(3/4)*sqrt(2)
*x + (3*I + 3)*18^(1/4)*sqrt(2)*sqrt(-3*x^2 + 2)*x + 12*I*sqrt(2)*(-3*x^2
+ 2)^(1/4) + 12*(-3*x^2 + 2)^(3/4))/(3*x^2 - 4)) + (1/288*I + 1/288)*18^(3
/4)*sqrt(2)*log((-(I + 1)*18^(3/4)*sqrt(2)*x - (3*I - 3)*18^(1/4)*sqrt(2)*
sqrt(-3*x^2 + 2)*x - 12*I*sqrt(2)*(-3*x^2 + 2)^(1/4) + 12*(-3*x^2 + 2)^(3/
4))/(3*x^2 - 4))
```

3.305. $\int \frac{1}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

3.305.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{1}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

input
```
integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

output
```
-Integral(1/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.305.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.305.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{1}{(2-3\,x^2)^{1/4}\,(3\,x^2-4)}\,dx$$

input
```
int(-1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

output
```
-int(1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.306 $\int \frac{1}{\sqrt[4]{2+bx^2}(4+bx^2)}\,dx$

3.306.1 Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx=-\frac{\arctan\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}x\sqrt[4]{2+bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}x\sqrt[4]{2+bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

output
```
-1/4*arctan(1/2*(2*2^(3/4)+2*2^(1/4)*(b*x^2+2)^(1/2))/x/(b*x^2+2)^(1/4)/b^
(1/2))*2^(1/4)/b^(1/2)-1/4*arctanh(1/2*(2*2^(3/4)-2*2^(1/4)*(b*x^2+2)^(1/2
))/x/(b*x^2+2)^(1/4)/b^(1/2))*2^(1/4)/b^(1/2)
```

3.306.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx=\frac{\arctan\left(\frac{2^{3/4}bx^2-4\sqrt[4]{2}\sqrt{2+bx^2}}{4\sqrt{b}x\sqrt[4]{2+bx^2}}\right)+\operatorname{arctanh}\left(\frac{2\ 2^{3/4}\sqrt{b}x\sqrt[4]{2+bx^2}}{\sqrt{2}bx^2+4\sqrt{2+bx^2}}\right)}{4\ 2^{3/4}\sqrt{b}}$$

input
```
Integrate[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
```

output
```
(ArcTan[(2^(3/4)*b*x^2 - 4*2^(1/4)*Sqrt[2 + b*x^2])/(4*Sqrt[b]*x*(2 + b*x^
2)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[b]*x*(2 + b*x^2)^(1/4))/(Sqrt[2]*b*x^
2 + 4*Sqrt[2 + b*x^2])])/(4*2^(3/4)*Sqrt[b])
```

3.306.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{bx^2+2}\,(bx^2+4)}\,dx$$

$$\Big\downarrow \text{ 308}$$

$$-\frac{\arctan\left(\frac{2\sqrt[4]{2}\sqrt{bx^2+2}+2\ 2^{3/4}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{2\ 2^{3/4}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{bx^2+2}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

input
```
Int[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
```

output
```
-1/2*ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^
2)^(1/4))]/(2^(3/4)*Sqrt[b]) - ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + b*x
^2])/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

3.306.3.1 Defintions of rubi rules used

rule 308
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

3.306.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+2\right)^{\frac{1}{4}} \left(b\,x^{2}+4\right)}dx$$

input
```
int(1/(b*x^2+2)^(1/4)/(b*x^2+4),x)
```

output
```
int(1/(b*x^2+2)^(1/4)/(b*x^2+4),x)
```

3.306.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.28 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.93

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx =$$

$$-\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}-\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}+2\sqrt{\frac{1}{2}}(bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}+(bx^2+}{bx^2+4}\right.$$

$$+\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}-\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}-2\sqrt{\frac{1}{2}}(bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}-(bx^2}{bx^2+4}\right.$$

$$+\frac{1}{8}i\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(\frac{i\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}+i\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}-2\sqrt{\frac{1}{2}}(bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}+(bx}{bx^2+4}\right.$$

$$-\frac{1}{8}i\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}-i\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}-2\sqrt{\frac{1}{2}}(bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}+(}{bx^2+4}\right.$$

input
```
integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="fricas")
```

output
```
-1/8*(1/2)^(1/4)*(-1/b^2)^(1/4)*log(((1/2)^(3/4)*sqrt(b*x^2 + 2)*b^2*x*(-1
/b^2)^(3/4) - (1/2)^(1/4)*b*x*(-1/b^2)^(1/4) + 2*sqrt(1/2)*(b*x^2 + 2)^(1/
4)*b*sqrt(-1/b^2) + (b*x^2 + 2)^(3/4))/(b*x^2 + 4)) + 1/8*(1/2)^(1/4)*(-1/
b^2)^(1/4)*log(-((1/2)^(3/4)*sqrt(b*x^2 + 2)*b^2*x*(-1/b^2)^(3/4) - (1/2)^
(1/4)*b*x*(-1/b^2)^(1/4) - 2*sqrt(1/2)*(b*x^2 + 2)^(1/4)*b*sqrt(-1/b^2) -
(b*x^2 + 2)^(3/4))/(b*x^2 + 4)) + 1/8*I*(1/2)^(1/4)*(-1/b^2)^(1/4)*log((I*
(1/2)^(3/4)*sqrt(b*x^2 + 2)*b^2*x*(-1/b^2)^(3/4) + I*(1/2)^(1/4)*b*x*(-1/b
^2)^(1/4) - 2*sqrt(1/2)*(b*x^2 + 2)^(1/4)*b*sqrt(-1/b^2) + (b*x^2 + 2)^(3/
4))/(b*x^2 + 4)) - 1/8*I*(1/2)^(1/4)*(-1/b^2)^(1/4)*log((-I*(1/2)^(3/4)*sq
rt(b*x^2 + 2)*b^2*x*(-1/b^2)^(3/4) - I*(1/2)^(1/4)*b*x*(-1/b^2)^(1/4) - 2*
sqrt(1/2)*(b*x^2 + 2)^(1/4)*b*sqrt(-1/b^2) + (b*x^2 + 2)^(3/4))/(b*x^2 + 4
))
```

3.306.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx = \int \frac{1}{\sqrt[4]{bx^2+2}\,(bx^2+4)}\,dx$$

input
```
integrate(1/(b*x**2+2)**(1/4)/(b*x**2+4),x)
```

output
```
Integral(1/((b*x**2 + 2)**(1/4)*(b*x**2 + 4)), x)
```

3.306.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx = \int \frac{1}{(bx^2+4)(bx^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)), x)
```

3.306.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx = \int \frac{1}{(bx^2+4)(bx^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)), x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx = \int \frac{1}{(b\,x^2+2)^{1/4}\,(b\,x^2+4)}\,dx$$

input
```
int(1/((b*x^2 + 2)^(1/4)*(b*x^2 + 4)),x)
```

output
```
int(1/((b*x^2 + 2)^(1/4)*(b*x^2 + 4)), x)
```

3.307 $\int \frac{1}{\sqrt[4]{2-bx^2}(4-bx^2)}\,dx$

3.307.1 Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx=\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{2+\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

output
```
1/4*arctan(1/2*(2-2^(1/2)*(-b*x^2+2)^(1/2))*2^(3/4)/x/(-b*x^2+2)^(1/4)/b^(
1/2))*2^(1/4)/b^(1/2)+1/4*arctanh(1/2*(2+2^(1/2)*(-b*x^2+2)^(1/2))*2^(3/4)
/x/(-b*x^2+2)^(1/4)/b^(1/2))*2^(1/4)/b^(1/2)
```

3.307.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx=\frac{\arctan\left(\frac{2^{3/4}bx^2-4\sqrt[4]{2}\sqrt{2-bx^2}}{4\sqrt{b}x\sqrt[4]{2-bx^2}}\right)+\operatorname{arctanh}\left(\frac{2\ 2^{3/4}\sqrt{b}x\sqrt[4]{2-bx^2}}{\sqrt{2}bx^2+4\sqrt{2-bx^2}}\right)}{4\ 2^{3/4}\sqrt{b}}$$

input
```
Integrate[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
```

output
```
(ArcTan[(2^(3/4)*b*x^2 - 4*2^(1/4)*Sqrt[2 - b*x^2])/(4*Sqrt[b]*x*(2 - b*x^
2)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))/(Sqrt[2]*b*x^
2 + 4*Sqrt[2 - b*x^2])])/(4*2^(3/4)*Sqrt[b])
```

3.307.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx$$

$$\Big\downarrow \; 308$$

$$\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\,\sqrt[4]{2-bx^2}}\right)}{2\;2^{3/4}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}x\,\sqrt[4]{2-bx^2}}\right)}{2\;2^{3/4}\sqrt{b}}$$

input
```
Int[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
```

output
```
ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))
]/(2*2^(3/4)*Sqrt[b]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqr
t[b]*x*(2 - b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

3.307.3.1 Defintions of rubi rules used

rule 308
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

3.307.4 Maple [F]

$$\int \frac{1}{(-b\,x^2+2)^{\frac{1}{4}}\,(-b\,x^2+4)}dx$$

input
```
int(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x)
```

output
```
int(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x)
```

3.307.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.25 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.13

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx$$
$$= \frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{-bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}+\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}+2\sqrt{\frac{1}{2}}(-bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}-(-}{bx^2-4}\right.$$
$$-\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{-bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}+\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}-2\sqrt{\frac{1}{2}}(-bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}+(-}{bx^2-4}\right.$$
$$+\frac{1}{8}i\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(\frac{i\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{-bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}-i\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}+2\sqrt{\frac{1}{2}}(-bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}+}{bx^2-4}\right.$$
$$-\frac{1}{8}i\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{-bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}+i\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}+2\sqrt{\frac{1}{2}}(-bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}}{bx^2-4}\right.$$

input
```
integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="fricas")
```

output
```
1/8*(1/2)^(1/4)*(-1/b^2)^(1/4)*log(-((1/2)^(3/4)*sqrt(-b*x^2 + 2)*b^2*x*(-
1/b^2)^(3/4) + (1/2)^(1/4)*b*x*(-1/b^2)^(1/4) + 2*sqrt(1/2)*(-b*x^2 + 2)^(
1/4)*b*sqrt(-1/b^2) - (-b*x^2 + 2)^(3/4))/(b*x^2 - 4)) - 1/8*(1/2)^(1/4)*(
-1/b^2)^(1/4)*log(((1/2)^(3/4)*sqrt(-b*x^2 + 2)*b^2*x*(-1/b^2)^(3/4) + (1/
2)^(1/4)*b*x*(-1/b^2)^(1/4) - 2*sqrt(1/2)*(-b*x^2 + 2)^(1/4)*b*sqrt(-1/b^2
) + (-b*x^2 + 2)^(3/4))/(b*x^2 - 4)) + 1/8*I*(1/2)^(1/4)*(-1/b^2)^(1/4)*lo
g((I*(1/2)^(3/4)*sqrt(-b*x^2 + 2)*b^2*x*(-1/b^2)^(3/4) - I*(1/2)^(1/4)*b*x
*(-1/b^2)^(1/4) + 2*sqrt(1/2)*(-b*x^2 + 2)^(1/4)*b*sqrt(-1/b^2) + (-b*x^2
+ 2)^(3/4))/(b*x^2 - 4)) - 1/8*I*(1/2)^(1/4)*(-1/b^2)^(1/4)*log((-I*(1/2)^
(3/4)*sqrt(-b*x^2 + 2)*b^2*x*(-1/b^2)^(3/4) + I*(1/2)^(1/4)*b*x*(-1/b^2)^(
1/4) + 2*sqrt(1/2)*(-b*x^2 + 2)^(1/4)*b*sqrt(-1/b^2) + (-b*x^2 + 2)^(3/4))
/(b*x^2 - 4))
```

3.307.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx = -\int \frac{1}{bx^2\sqrt[4]{-bx^2+2}-4\sqrt[4]{-bx^2+2}}\,dx$$

input
```
integrate(1/(-b*x**2+2)**(1/4)/(-b*x**2+4),x)
```

output
```
-Integral(1/(b*x**2*(-b*x**2 + 2)**(1/4) - 4*(-b*x**2 + 2)**(1/4)), x)
```

3.307.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx = \int -\frac{1}{(bx^2-4)(-bx^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)), x)
```

3.307.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx = \int -\frac{1}{(bx^2-4)(-bx^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)), x)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx = -\int \frac{1}{(2-b\,x^2)^{1/4}\,(b\,x^2-4)}\,dx$$

input
```
int(-1/((2 - b*x^2)^(1/4)*(b*x^2 - 4)),x)
```

output
```
-int(1/((2 - b*x^2)^(1/4)*(b*x^2 - 4)), x)
```

3.308 $\int \frac{1}{\sqrt[4]{a+3x^2}(2a+3x^2)}\,dx$

3.308.1 Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx=-\frac{\arctan\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}-\frac{\text{arctanh}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

output
```
-1/6*arctan(1/3*a^(3/4)*(1+(3*x^2+a)^(1/2)/a^(1/2))/x/(3*x^2+a)^(1/4)*3^(1
/2))/a^(3/4)*3^(1/2)-1/6*arctanh(1/3*a^(3/4)*(1-(3*x^2+a)^(1/2)/a^(1/2))/x
/(3*x^2+a)^(1/4)*3^(1/2))/a^(3/4)*3^(1/2)
```

3.308.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx=\frac{-\arctan\left(\frac{-3x^2+2\sqrt{a}\sqrt{a+3x^2}}{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a+3x^2}}\right)+\text{arctanh}\left(\frac{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a+3x^2}}{3x^2+2\sqrt{a}\sqrt{a+3x^2}}\right)}{4\sqrt{3}a^{3/4}}$$

input
```
Integrate[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
```

output
```
(-ArcTan[(-3*x^2 + 2*Sqrt[a]*Sqrt[a + 3*x^2])/(2*Sqrt[3]*a^(1/4)*x*(a + 3*
x^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(a + 3*x^2)^(1/4))/(3*x^2 + 2*
Sqrt[a]*Sqrt[a + 3*x^2])])/(4*Sqrt[3]*a^(3/4))
```

3.308.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx$$

$$\Big\downarrow \; 308$$

$$-\frac{\arctan\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

input
```
Int[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
```

output
```
-1/2*ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)
^(1/4))]/(Sqrt[3]*a^(3/4)) - ArcTanh[(a^(3/4)*(1 - Sqrt[a + 3*x^2]/Sqrt[a]
))/(Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4))
```

3.308.3.1 Defintions of rubi rules used

rule 308
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

3.308.4 Maple [F]

$$\int \frac{1}{(3x^2+a)^{\frac{1}{4}}(3x^2+2a)}dx$$

input
```
int(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x)
```

output
```
int(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x)
```

3.308.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.03 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt[4]{a+3x^2}(2a+3x^2)}dx =$$

$$-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+(3x^2+a)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}+\ldots}{3x^2+2a}\right.$$

$$+\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-(3x^2+a)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}-\ldots}{3x^2+2a}\right.$$

$$+\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-(3x^2+a)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}+3i\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}-\ldots}{3x^2+2a}\right.$$

$$-\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{-18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-(3x^2+a)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}-3i\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)\ldots}{3x^2+2a}\right.$$

input
```
integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="fricas")
```

output
```
-1/4*(1/36)^(1/4)*(-1/a^3)^(1/4)*log((18*(1/36)^(3/4)*sqrt(3*x^2 + a)*a^2*
x*(-1/a^3)^(3/4) + (3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) - 3*(1/36)^(1/4)*a*x
*(-1/a^3)^(1/4) + (3*x^2 + a)^(3/4))/(3*x^2 + 2*a)) + 1/4*(1/36)^(1/4)*(-1
/a^3)^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (
3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) - 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) - (3
*x^2 + a)^(3/4))/(3*x^2 + 2*a)) + 1/4*I*(1/36)^(1/4)*(-1/a^3)^(1/4)*log((1
8*I*(1/36)^(3/4)*sqrt(3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (3*x^2 + a)^(1/4)*
a^2*sqrt(-1/a^3) + 3*I*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) + (3*x^2 + a)^(3/4)
)/(3*x^2 + 2*a)) - 1/4*I*(1/36)^(1/4)*(-1/a^3)^(1/4)*log((-18*I*(1/36)^(3/
4)*sqrt(3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^
3) - 3*I*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) + (3*x^2 + a)^(3/4))/(3*x^2 + 2*a
))
```

3.308.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx = \int \frac{1}{\sqrt[4]{a+3x^2}\cdot(2a+3x^2)}\,dx$$

input
```
integrate(1/(3*x**2+a)**(1/4)/(3*x**2+2*a),x)
```

output
```
Integral(1/((a + 3*x**2)**(1/4)*(2*a + 3*x**2)), x)
```

3.308.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx = \int \frac{1}{(3\,x^2+2\,a)(3\,x^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)), x)
```

3.308.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx = \int \frac{1}{(3\,x^2+2\,a)(3\,x^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)), x)
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx = \int \frac{1}{(3\,x^2+2\,a)\,\left(3\,x^2+a\right)^{1/4}}\,dx$$

input
```
int(1/((2*a + 3*x^2)*(a + 3*x^2)^(1/4)),x)
```

output
```
int(1/((2*a + 3*x^2)*(a + 3*x^2)^(1/4)), x)
```

3.309 $\int \frac{1}{\sqrt[4]{a-3x^2}(2a-3x^2)}\,dx$

3.309.1 Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx=\frac{\arctan\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

output
```
1/6*arctan(1/3*a^(3/4)*(1-(-3*x^2+a)^(1/2)/a^(1/2))/x/(-3*x^2+a)^(1/4)*3^(
1/2))/a^(3/4)*3^(1/2)+1/6*arctanh(1/3*a^(3/4)*(1+(-3*x^2+a)^(1/2)/a^(1/2))
/x/(-3*x^2+a)^(1/4)*3^(1/2))/a^(3/4)*3^(1/2)
```

3.309.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx=\frac{-\arctan\left(\frac{-3x^2+2\sqrt{a}\sqrt{a-3x^2}}{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a-3x^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a-3x^2}}{3x^2+2\sqrt{a}\sqrt{a-3x^2}}\right)}{4\sqrt{3}a^{3/4}}$$

input
```
Integrate[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

output
```
(-ArcTan[(-3*x^2 + 2*Sqrt[a]*Sqrt[a - 3*x^2])/(2*Sqrt[3]*a^(1/4)*x*(a - 3*
x^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(a - 3*x^2)^(1/4))/(3*x^2 + 2*
Sqrt[a]*Sqrt[a - 3*x^2])])/(4*Sqrt[3]*a^(3/4))
```

3.309.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx$$

$$\Big\downarrow \ 308$$

$$\frac{\arctan\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}+\frac{\text{arctanh}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

input
```
Int[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

output
```
ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4
))]/(2*Sqrt[3]*a^(3/4)) + ArcTanh[(a^(3/4)*(1 + Sqrt[a - 3*x^2]/Sqrt[a]))/
(Sqrt[3]*x*(a - 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4))
```

3.309.3.1 Defintions of rubi rules used

rule 308
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

3.309.4 Maple [F]

$$\int \frac{1}{(-3x^2+a)^{\frac{1}{4}}(-3x^2+2a)}dx$$

input
```
int(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x)
```

output
```
int(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x)
```

3.309.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.05 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.18

$$\int \frac{1}{\sqrt[4]{a-3x^2}(2a-3x^2)}\,dx$$
$$= \frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+(-3x^2+a)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}}{3x^2-2a}\right.$$
$$-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-(-3x^2+a)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}}{3x^2-2a}\right.$$
$$+\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+(-3x^2+a)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}-3i\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right.}{3x^2-2a}\right.$$
$$-\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{-18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+(-3x^2+a)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}+3i\left(\frac{1}{36}\right)^{\frac{1}{4}}ax(-}{3x^2-2a}\right.$$

input
```
integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="fricas")
```

output
```
1/4*(1/36)^(1/4)*(-1/a^3)^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(-3*x^2 + a)*a^2
*x*(-1/a^3)^(3/4) + (-3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) + 3*(1/36)^(1/4)*a
*x*(-1/a^3)^(1/4) - (-3*x^2 + a)^(3/4))/(3*x^2 - 2*a)) - 1/4*(1/36)^(1/4)*
(-1/a^3)^(1/4)*log((18*(1/36)^(3/4)*sqrt(-3*x^2 + a)*a^2*x*(-1/a^3)^(3/4)
- (-3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) + 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4)
+ (-3*x^2 + a)^(3/4))/(3*x^2 - 2*a)) + 1/4*I*(1/36)^(1/4)*(-1/a^3)^(1/4)*l
og((18*I*(1/36)^(3/4)*sqrt(-3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) + (-3*x^2 + a)
^(1/4)*a^2*sqrt(-1/a^3) - 3*I*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) + (-3*x^2 +
a)^(3/4))/(3*x^2 - 2*a)) - 1/4*I*(1/36)^(1/4)*(-1/a^3)^(1/4)*log((-18*I*(1
/36)^(3/4)*sqrt(-3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) + (-3*x^2 + a)^(1/4)*a^2*
sqrt(-1/a^3) + 3*I*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) + (-3*x^2 + a)^(3/4))/(
3*x^2 - 2*a))
```

3.309.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx = -\int \frac{1}{-2a\sqrt[4]{a-3x^2}+3x^2\sqrt[4]{a-3x^2}}\,dx$$

input
```
integrate(1/(-3*x**2+a)**(1/4)/(-3*x**2+2*a),x)
```

output
```
-Integral(1/(-2*a*(a - 3*x**2)**(1/4) + 3*x**2*(a - 3*x**2)**(1/4)), x)
```

3.309.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-2\,a)(-3\,x^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)), x)
```

3.309.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-2\,a)(-3\,x^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="giac")
```

output
```
integrate(-1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)), x)
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx = \int \frac{1}{(2\,a-3\,x^2)\,\left(a-3\,x^2\right)^{1/4}}\,dx$$

input
```
int(1/((2*a - 3*x^2)*(a - 3*x^2)^(1/4)),x)
```

output
```
int(1/((2*a - 3*x^2)*(a - 3*x^2)^(1/4)), x)
```

3.310 $\int \frac{1}{\sqrt[4]{a+bx^2}(2a+bx^2)}\,dx$

3.310.1 Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx=-\frac{\arctan\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

output
```
-1/2*arctan(a^(3/4)*(1+(b*x^2+a)^(1/2)/a^(1/2))/x/(b*x^2+a)^(1/4)/b^(1/2))
/a^(3/4)/b^(1/2)-1/2*arctanh(a^(3/4)*(1-(b*x^2+a)^(1/2)/a^(1/2))/x/(b*x^2+
a)^(1/4)/b^(1/2))/a^(3/4)/b^(1/2)
```

3.310.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx=\frac{\arctan\left(\frac{bx^2-2\sqrt{a}\sqrt{a+bx^2}}{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a+bx^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a+bx^2}}{bx^2+2\sqrt{a}\sqrt{a+bx^2}}\right)}{4a^{3/4}\sqrt{b}}$$

input
```
Integrate[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
```

output
```
(ArcTan[(b*x^2 - 2*Sqrt[a]*Sqrt[a + b*x^2])/(2*a^(1/4)*Sqrt[b]*x*(a + b*x^
2)^(1/4))] + ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(a + b*x^2)^(1/4))/(b*x^2 + 2*Sq
rt[a]*Sqrt[a + b*x^2])])/(4*a^(3/4)*Sqrt[b])
```

3.310.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx$$

$$\Big\downarrow \; 308$$

$$-\frac{\arctan\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

input
```
Int[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
```

output
```
-1/2*ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)
^(1/4))]/(a^(3/4)*Sqrt[b]) - ArcTanh[(a^(3/4)*(1 - Sqrt[a + b*x^2]/Sqrt[a]
))/(Sqrt[b]*x*(a + b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b])
```

3.310.3.1 Defintions of rubi rules used

rule 308
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

3.310.4 Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,(b\,x^2+2a)}dx$$

input
```
int(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x)
```

output
```
int(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x)
```

3.310.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 24.40 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.76

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx =$$

$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2+a}a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}}{bx^2+2\,a}\right.$$

$$+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2+a}a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right.}{bx^2+2\,a}\right.$$

$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2+a}a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}+i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right.}{bx^2+2\,a}\right.$$

$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{-2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2+a}a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}-i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{}{a}\right.}{bx^2+2\,a}\right.$$

input
```
integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="fricas")
```

output
```
-1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt(b*x^2 + a)*a
^2*b^2*x*(-1/(a^3*b^2))^(3/4) + (b*x^2 + a)^(1/4)*a^2*b*sqrt(-1/(a^3*b^2))
 - (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (b*x^2 + a)^(3/4))/(b*x^2 + 2*
a)) + 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(b*x^2
+ a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) - (b*x^2 + a)^(1/4)*a^2*b*sqrt(-1/(a^3
*b^2)) - (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) - (b*x^2 + a)^(3/4))/(b*x^
2 + 2*a)) + 1/4*I*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((2*I*(1/4)^(3/4)*sq
rt(b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) - (b*x^2 + a)^(1/4)*a^2*b*sqr
t(-1/(a^3*b^2)) + I*(1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (b*x^2 + a)^(
3/4))/(b*x^2 + 2*a)) - 1/4*I*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((-2*I*(1
/4)^(3/4)*sqrt(b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) - (b*x^2 + a)^(1/
4)*a^2*b*sqrt(-1/(a^3*b^2)) - I*(1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (
b*x^2 + a)^(3/4))/(b*x^2 + 2*a))
```

3.310.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx = \int \frac{1}{\sqrt[4]{a+bx^2}\cdot(2a+bx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/4)/(b*x**2+2*a),x)
```

output
```
Integral(1/((a + b*x**2)**(1/4)*(2*a + b*x**2)), x)
```

3.310.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx = \int \frac{1}{(bx^2+2\,a)(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)), x)
```

3.310.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx = \int \frac{1}{(bx^2+2\,a)(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)), x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{1/4}\,\left(b\,x^2+2\,a\right)}\,dx$$

input
```
int(1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(1/4)*(2*a + b*x^2)), x)
```

3.311 $\int \frac{1}{\sqrt[4]{a-bx^2}(2a-bx^2)}\,dx$

3.311.1 Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx=\frac{\arctan\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

output
```
1/2*arctan(a^(3/4)*(1-(-b*x^2+a)^(1/2)/a^(1/2))/x/(-b*x^2+a)^(1/4)/b^(1/2)
)/a^(3/4)/b^(1/2)+1/2*arctanh(a^(3/4)*(1+(-b*x^2+a)^(1/2)/a^(1/2))/x/(-b*x
^2+a)^(1/4)/b^(1/2))/a^(3/4)/b^(1/2)
```

3.311.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx=\frac{\arctan\left(\frac{bx^2-2\sqrt{a}\sqrt{a-bx^2}}{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a-bx^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a-bx^2}}{bx^2+2\sqrt{a}\sqrt{a-bx^2}}\right)}{4a^{3/4}\sqrt{b}}$$

input
```
Integrate[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
```

output
```
(ArcTan[(b*x^2 - 2*Sqrt[a]*Sqrt[a - b*x^2])/(2*a^(1/4)*Sqrt[b]*x*(a - b*x^
2)^(1/4))] + ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(a - b*x^2)^(1/4))/(b*x^2 + 2*Sq
rt[a]*Sqrt[a - b*x^2])])/(4*a^(3/4)*Sqrt[b])
```

3.311.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx$$

$$\Big\downarrow \text{ 308}$$

$$\frac{\arctan\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

input
```
Int[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
```

output
```
ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4
))]/(2*a^(3/4)*Sqrt[b]) + ArcTanh[(a^(3/4)*(1 + Sqrt[a - b*x^2]/Sqrt[a]))/
(Sqrt[b]*x*(a - b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b])
```

3.311.3.1 Defintions of rubi rules used

rule 308
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

3.311.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{4}} \left(-b\,x^{2}+2a\right)}dx$$

input
```
int(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x)
```

output
```
int(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x)
```

3.311.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.30 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.70

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx$$
$$= \frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2+a}a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(-bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b}\right.}{bx^2-2\,a}\right.$$
$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2+a}a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(-bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3}\right.}{bx^2-2\,a}\right.$$
$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2+a}a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(-bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}-i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\right.}{bx^2-2\,a}\right.$$
$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{-2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2+a}a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(-bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}+i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx}{bx^2-2\,a}\right.$$

input
```
integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="fricas")
```

output
```
1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(-b*x^2 + a)*
a^2*b^2*x*(-1/(a^3*b^2))^(3/4) + (-b*x^2 + a)^(1/4)*a^2*b*sqrt(-1/(a^3*b^2
)) + (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) - (-b*x^2 + a)^(3/4))/(b*x^2 -
 2*a)) - 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt(-b*x
^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) - (-b*x^2 + a)^(1/4)*a^2*b*sqrt(-1/
(a^3*b^2)) + (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (-b*x^2 + a)^(3/4))/
(b*x^2 - 2*a)) + 1/4*I*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((2*I*(1/4)^(3/
4)*sqrt(-b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) + (-b*x^2 + a)^(1/4)*a^
2*b*sqrt(-1/(a^3*b^2)) - I*(1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (-b*x^
2 + a)^(3/4))/(b*x^2 - 2*a)) - 1/4*I*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log(
(-2*I*(1/4)^(3/4)*sqrt(-b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) + (-b*x^
2 + a)^(1/4)*a^2*b*sqrt(-1/(a^3*b^2)) + I*(1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))
^(1/4) + (-b*x^2 + a)^(3/4))/(b*x^2 - 2*a))
```

3.311.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx = -\int \frac{1}{-2a\sqrt[4]{a-bx^2}+bx^2\sqrt[4]{a-bx^2}}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(1/4)/(-b*x**2+2*a),x)
```

output
```
-Integral(1/(-2*a*(a - b*x**2)**(1/4) + b*x**2*(a - b*x**2)**(1/4)), x)
```

3.311.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx = \int -\frac{1}{(bx^2-2\,a)(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)), x)
```

3.311.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx = \int -\frac{1}{(bx^2-2\,a)(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)), x)
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx = \int \frac{1}{(a-b\,x^2)^{1/4}\,(2\,a-b\,x^2)}\,dx$$

input
```
int(1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x)
```

output
```
int(1/((a - b*x^2)^(1/4)*(2*a - b*x^2)), x)
```

3.312 $\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.312.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

output
```
-1/12*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/12*arctanh(1/2*x*6^(
1/2)/(3*x^2-1)^(1/4))*6^(1/2)
```

3.312.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{-1+3x^2}}{x}\right) - \operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

input
```
Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(ArcTan[(Sqrt[2/3]*(-1 + 3*x^2)^(1/4))/x] - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*
x^2)^(1/4)])/(2*Sqrt[6])
```

3.312.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {309}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2-2)\sqrt[4]{3x^2-1}}dx$$

$$\Big\downarrow \ 309$$

$$-\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

input
```
Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
-1/2*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/Sqrt[6] - ArcTanh[(Sqrt[3/2]
*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

3.312.3.1 Defintions of rubi rules used

rule 309
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

3.312.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.26

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\left(3x^2-1\right)^{\frac{3}{4}}+3\sqrt{3x^2-1}\,x-\operatorname{RootOf}\left(\_Z^2+6\right)\left(3x^2-1\right)^{\frac{1}{4}}-3x}{3x^2-2}\right)}{12}+\frac{\operatorname{RootOf}\left(\_Z^2-6\right)\ln\left(-\right.}{}$

input
```
int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/12*RootOf(_Z^2+6)*ln((RootOf(_Z^2+6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*
x-RootOf(_Z^2+6)*(3*x^2-1)^(1/4)-3*x)/(3*x^2-2))+1/12*RootOf(_Z^2-6)*ln(-(
RootOf(_Z^2-6)*(3*x^2-1)^(3/4)-3*(3*x^2-1)^(1/2)*x+RootOf(_Z^2-6)*(3*x^2-1
)^(1/4)-3*x)/(3*x^2-2))
```

3.312.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(43) = 86$.

Time = 2.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.70

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}(3x^2-1)^{\frac{1}{4}}}{3x}\right) + \frac{1}{24}\sqrt{6}\log\left(-\frac{9x^4-6\sqrt{6}(3x^2-1)^{\frac{1}{4}}x^3+12\sqrt{3x^2-1}x^2-4\sqrt{6}(3x^2-1)^{\frac{3}{4}}x+12x^2-4}{9x^4-12x^2+4}\right)$$

input
```
integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
1/12*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/24*sqrt(6)*log(-(
9*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(
6)*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

3.312.6 Sympy [F]

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

input
```
integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

output
```
Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

3.312.7 Maxima [F]

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.312.8 Giac [F]

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{(3\,x^2-1)^{1/4}\,(3\,x^2-2)}\,dx$$

input
```
int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

output
```
int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.313 $\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx$

3.313.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}$$

output
```
-1/12*arctan(1/2*x*6^(1/2)/(-3*x^2-1)^(1/4))*6^(1/2)-1/12*arctanh(1/2*x*6^
(1/2)/(-3*x^2-1)^(1/4))*6^(1/2)
```

3.313.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = -\frac{-\arctan\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{-1-3x^2}}{x}\right) + \operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}$$

input
```
Integrate[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
```

output
```
-1/2*(-ArcTan[(Sqrt[2/3]*(-1 - 3*x^2)^(1/4))/x] + ArcTanh[(Sqrt[3/2]*x)/(-
1 - 3*x^2)^(1/4)])/Sqrt[6]
```

3.313.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {309}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^2-2)\sqrt[4]{-3x^2-1}}\,dx$$
$$\Big\downarrow \; 309$$
$$-\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}-\frac{\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}$$

input
```
Int[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
```

output
```
-1/2*ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/Sqrt[6] - ArcTanh[(Sqrt[3/2]
*x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6])
```

3.313.3.1 Defintions of rubi rules used

rule 309
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

3.313.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.26

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\left(-3x^2-1\right)^{\frac{3}{4}}-3\sqrt{-3x^2-1}\,x+\operatorname{RootOf}\left(\_Z^2+6\right)\left(-3x^2-1\right)^{\frac{1}{4}}-3x}{3x^2+2}\right)}{12}-\frac{\operatorname{RootOf}\left(\_Z^2-6\right)\ln}{}$

input
```
int(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/12*RootOf(_Z^2+6)*ln(-(RootOf(_Z^2+6)*(-3*x^2-1)^(3/4)-3*(-3*x^2-1)^(1/2
)*x+RootOf(_Z^2+6)*(-3*x^2-1)^(1/4)-3*x)/(3*x^2+2))-1/12*RootOf(_Z^2-6)*ln
((RootOf(_Z^2-6)*(-3*x^2-1)^(3/4)+3*(-3*x^2-1)^(1/2)*x-RootOf(_Z^2-6)*(-3*
x^2-1)^(1/4)-3*x)/(3*x^2+2))
```

3.313.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.98

$$\begin{aligned}\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx &= -\frac{1}{24}\sqrt{6}\log\left(\frac{\sqrt{6}\sqrt{-3\,x^2-1}x-\sqrt{6}x+2\,(-3\,x^2-1)^{\frac{3}{4}}-2\,(-3\,x^2-1)^{\frac{1}{4}}}{3\,(3\,x^2+2)}\right)\\&\quad+\frac{1}{24}\sqrt{6}\log\left(-\frac{\sqrt{6}\sqrt{-3\,x^2-1}x-\sqrt{6}x-2\,(-3\,x^2-1)^{\frac{3}{4}}+2\,(-3\,x^2-1)^{\frac{1}{4}}}{3\,(3\,x^2+2)}\right)\\&\quad+\frac{1}{24}i\sqrt{6}\log\left(\frac{i\sqrt{6}\sqrt{-3\,x^2-1}x+i\sqrt{6}x+2\,(-3\,x^2-1)^{\frac{3}{4}}+2\,(-3\,x^2-1)^{\frac{1}{4}}}{3\,(3\,x^2+2)}\right)\\&\quad-\frac{1}{24}i\sqrt{6}\log\left(\frac{-i\sqrt{6}\sqrt{-3\,x^2-1}x-i\sqrt{6}x+2\,(-3\,x^2-1)^{\frac{3}{4}}+2\,(-3\,x^2-1)^{\frac{1}{4}}}{3\,(3\,x^2+2)}\right)\end{aligned}$$

input
```
integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
-1/24*sqrt(6)*log(1/3*(sqrt(6)*sqrt(-3*x^2 - 1)*x - sqrt(6)*x + 2*(-3*x^2
- 1)^(3/4) - 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2)) + 1/24*sqrt(6)*log(-1/3*(s
qrt(6)*sqrt(-3*x^2 - 1)*x - sqrt(6)*x - 2*(-3*x^2 - 1)^(3/4) + 2*(-3*x^2 -
 1)^(1/4))/(3*x^2 + 2)) + 1/24*I*sqrt(6)*log(1/3*(I*sqrt(6)*sqrt(-3*x^2 -
1)*x + I*sqrt(6)*x + 2*(-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)^(1/4))/(3*x^2 +
 2)) - 1/24*I*sqrt(6)*log(1/3*(-I*sqrt(6)*sqrt(-3*x^2 - 1)*x - I*sqrt(6)*x
 + 2*(-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2))
```

3.313.6 Sympy [F]

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = -\int \frac{1}{3x^2\sqrt[4]{-3x^2-1}+2\sqrt[4]{-3x^2-1}}\,dx$$

input
```
integrate(1/(-3*x**2-2)/(-3*x**2-1)**(1/4),x)
```

output
```
-Integral(1/(3*x**2*(-3*x**2 - 1)**(1/4) + 2*(-3*x**2 - 1)**(1/4)), x)
```

3.313.7 Maxima [F]

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = \int -\frac{1}{(3\,x^2+2)(-3\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)), x)
```

3.313.8 Giac [F]

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = \int -\frac{1}{(3\,x^2+2)(-3\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(-1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)), x)
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = -\int \frac{1}{\left(-3\,x^2-1\right)^{1/4}\,\left(3\,x^2+2\right)}\,dx$$

input
```
int(-1/((- 3*x^2 - 1)^(1/4)*(3*x^2 + 2)),x)
```

output
```
-int(1/((- 3*x^2 - 1)^(1/4)*(3*x^2 + 2)), x)
```

3.314 $\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx$

3.314.1 Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

output
```
-1/4*arctan(1/2*x*b^(1/2)/(b*x^2-1)^(1/4)*2^(1/2))*2^(1/2)/b^(1/2)-1/4*arc
tanh(1/2*x*b^(1/2)/(b*x^2-1)^(1/4)*2^(1/2))*2^(1/2)/b^(1/2)
```

3.314.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{-1+bx^2}}{\sqrt{b}x}\right)-\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

input
```
Integrate[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
```

output
```
(ArcTan[(Sqrt[2]*(-1 + b*x^2)^(1/4))/(Sqrt[b]*x)] - ArcTanh[(Sqrt[b]*x)/(S
qrt[2]*(-1 + b*x^2)^(1/4))])/(2*Sqrt[2]*Sqrt[b])
```

3.314.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {309}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-2)\sqrt[4]{bx^2-1}}dx$$

$$\Big\downarrow \; 309$$

$$-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

input
```
Int[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
```

output
```
-1/2*ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*Sqrt[b]) - 
ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

3.314.3.1 Defintions of rubi rules used

rule 309
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

3.314.4 Maple [F]

$$\int \frac{1}{(b\,x^2-2)\,(b\,x^2-1)^{\frac{1}{4}}}dx$$

input
```
int(1/(b*x^2-2)/(b*x^2-1)^(1/4),x)
```

output
```
int(1/(b*x^2-2)/(b*x^2-1)^(1/4),x)
```

3.314.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(55) = 110.

Time = 5.88 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.56

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx = \left[\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}(bx^2-1)^{\frac{1}{4}}}{\sqrt{b}x}\right)+\sqrt{2}\sqrt{b}\log\left(-\frac{b^2x^4-2\sqrt{2}(bx^2-1)^{\frac{1}{4}}b^{\frac{3}{2}}x^3+4\sqrt{bx^2-1}bx^2+4bx^2-4\sqrt{2}(bx^2-1)^{\frac{3}{4}}\sqrt{b}x-4}{b^2x^4-4bx^2+4}\right)}{8b}, -\right.$$

input
```
integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(b*x^2 - 1)^(1/4)/(sqrt(b)*x)) + sq
rt(2)*sqrt(b)*log(-(b^2*x^4 - 2*sqrt(2)*(b*x^2 - 1)^(1/4)*b^(3/2)*x^3 + 4*
sqrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(2)*(b*x^2 - 1)^(3/4)*sqrt(b)*x -
4)/(b^2*x^4 - 4*b*x^2 + 4)))/b, 1/8*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(b*
x^2 - 1)^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 + 2*sqrt(2
)*(b*x^2 - 1)^(1/4)*sqrt(-b)*b*x^3 - 4*sqrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4
*sqrt(2)*(b*x^2 - 1)^(3/4)*sqrt(-b)*x - 4)/(b^2*x^4 - 4*b*x^2 + 4)))/b]
```

3.314.6 Sympy [F]

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx = \int \frac{1}{(bx^2-2)\sqrt[4]{bx^2-1}}\,dx$$

input
```
integrate(1/(b*x**2-2)/(b*x**2-1)**(1/4),x)
```

output
```
Integral(1/((b*x**2 - 2)*(b*x**2 - 1)**(1/4)), x)
```

3.314.7 Maxima [F]

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx = \int \frac{1}{(bx^2-1)^{\frac{1}{4}}(bx^2-2)}\,dx$$

input
```
integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

3.314.8 Giac [F]

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx = \int \frac{1}{(bx^2-1)^{\frac{1}{4}}(bx^2-2)}\,dx$$

input
```
integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx = \int \frac{1}{(b\,x^2-1)^{1/4}\,(b\,x^2-2)}\,dx$$

input
```
int(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)),x)
```

output
```
int(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

3.315 $\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx$

3.315.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

output
```
-1/4*arctan(1/2*x*b^(1/2)/(-b*x^2-1)^(1/4)*2^(1/2))*2^(1/2)/b^(1/2)-1/4*ar
ctanh(1/2*x*b^(1/2)/(-b*x^2-1)^(1/4)*2^(1/2))*2^(1/2)/b^(1/2)
```

3.315.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx=-\frac{-\arctan\left(\frac{\sqrt{2}\sqrt[4]{-1-bx^2}}{\sqrt{b}x}\right)+\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

input
```
Integrate[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
```

output
```
-1/2*(-ArcTan[(Sqrt[2]*(-1 - b*x^2)^(1/4))/(Sqrt[b]*x)] + ArcTanh[(Sqrt[b]
*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))])/(Sqrt[2]*Sqrt[b])
```

3.315.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {309}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-bx^2-2)\sqrt[4]{-bx^2-1}}dx$$

$$\Big\downarrow \text{309}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

input
```
Int[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
```

output
```
-1/2*ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*Sqrt[b]) -
ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

3.315.3.1 Defintions of rubi rules used

rule 309
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

3.315.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}-2\right)\left(-b\,x^{2}-1\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x)
```

output
```
int(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x)
```

3.315.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(57) = 114.

Time = 5.87 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.46

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx$$
$$= \left[\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}(-bx^2-1)^{\frac{1}{4}}}{\sqrt{b}x}\right)+\sqrt{2}\sqrt{b}\log\left(-\frac{b^2x^4+4\sqrt{-bx^2-1}bx^2-4bx^2-2\sqrt{2}\left((-bx^2-1)^{\frac{1}{4}}bx^3+2(-bx^2-1)^{\frac{3}{4}}x\right)\sqrt{b}-4}{b^2x^4+4bx^2+4}\right.}{8b}\right.$$

input
```
integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1/4)/(sqrt(b)*x)) + s
qrt(2)*sqrt(b)*log(-(b^2*x^4 + 4*sqrt(-b*x^2 - 1)*b*x^2 - 4*b*x^2 - 2*sqrt
(2)*((-b*x^2 - 1)^(1/4)*b*x^3 + 2*(-b*x^2 - 1)^(3/4)*x)*sqrt(b) - 4)/(b^2*
x^4 + 4*b*x^2 + 4)))/b, 1/8*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(-b*x^2 - 1
)^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 4*sqrt(-b*x^2 -
 1)*b*x^2 - 4*b*x^2 + 2*sqrt(2)*((-b*x^2 - 1)^(1/4)*b*x^3 - 2*(-b*x^2 - 1)
^(3/4)*x)*sqrt(-b) - 4)/(b^2*x^4 + 4*b*x^2 + 4)))/b]
```

3.315.6 Sympy [F]

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx = -\int \frac{1}{bx^2\sqrt[4]{-bx^2-1}+2\sqrt[4]{-bx^2-1}}\,dx$$

input
```
integrate(1/(-b*x**2-2)/(-b*x**2-1)**(1/4),x)
```

output
```
-Integral(1/(b*x**2*(-b*x**2 - 1)**(1/4) + 2*(-b*x**2 - 1)**(1/4)), x)
```

3.315.7 Maxima [F]

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx = \int -\frac{1}{(bx^2+2)(-bx^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)), x)
```

3.315.8 Giac [F]

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx = \int -\frac{1}{(bx^2+2)(-bx^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)), x)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx = -\int \frac{1}{(-b\,x^2-1)^{1/4}\,(b\,x^2+2)}\,dx$$

input
```
int(-1/((- b*x^2 - 1)^(1/4)*(b*x^2 + 2)),x)
```

output
```
-int(1/((- b*x^2 - 1)^(1/4)*(b*x^2 + 2)), x)
```

3.316 $\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx$

3.316.1 Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}a^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

output
```
-1/12*arctan(1/2*x*6^(1/2)/a^(1/4)/(3*x^2-a)^(1/4))/a^(3/4)*6^(1/2)-1/12*a
rctanh(1/2*x*6^(1/2)/a^(1/4)/(3*x^2-a)^(1/4))/a^(3/4)*6^(1/2)
```

3.316.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a+3x^2}}{x}\right) - \operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a+3x^2}}{x}\right)}{2\sqrt{6}a^{3/4}}$$

input
```
Integrate[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
```

output
```
(ArcTan[(Sqrt[2/3]*a^(1/4)*(-a + 3*x^2)^(1/4))/x] - ArcTanh[(Sqrt[2/3]*a^(
1/4)*(-a + 3*x^2)^(1/4))/x])/(2*Sqrt[6]*a^(3/4))
```

3.316.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {309}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2-2a)\sqrt[4]{3x^2-a}}\,dx$$

$$\Big\downarrow \text{309}$$

$$-\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}$$

input
```
Int[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
```

output
```
-1/2*ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(Sqrt[6]*a^(3/4))
- ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4))
```

3.316.3.1 Defintions of rubi rules used

rule 309
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

3.316.4 Maple [F]

$$\int \frac{1}{(3x^2-2a)(3x^2-a)^{\frac{1}{4}}}dx$$

input
```
int(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x)
```

output
```
int(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x)
```

3.316.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.09 (sec) , antiderivative size = 375, normalized size of antiderivative = 4.41

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx =$$

$$-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2-a}a^2\frac{1}{a^3}^{\frac{3}{4}}x+(3x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+(3x^2-a)^{\frac{3}{4}}}{3x^2-2a}\right)$$
$$+\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2-a}a^2\frac{1}{a^3}^{\frac{3}{4}}x-(3x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x-(3x^2-a)^{\frac{3}{4}}}{3x^2-2a}\right)$$
$$+\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2-a}a^2\frac{1}{a^3}^{\frac{3}{4}}x-(3x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}-3i\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+(3x^2-a)^{\frac{3}{4}}}{3x^2-2a}\right)$$
$$-\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{-18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2-a}a^2\frac{1}{a^3}^{\frac{3}{4}}x-(3x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}+3i\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+(3x^2-a)^{\frac{3}{4}}}{3x^2-2a}\right.$$

input
```
integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="fricas")
```

output
```
-1/4*(1/36)^(1/4)*(a^(-3))^(1/4)*log((18*(1/36)^(3/4)*sqrt(3*x^2 - a)*a^2*
(a^(-3))^(3/4)*x + (3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) + 3*(1/36)^(1/4)*a*(
a^(-3))^(1/4)*x + (3*x^2 - a)^(3/4))/(3*x^2 - 2*a)) + 1/4*(1/36)^(1/4)*(a^
(-3))^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(3*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (
3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) + 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x - (3
*x^2 - a)^(3/4))/(3*x^2 - 2*a)) + 1/4*I*(1/36)^(1/4)*(a^(-3))^(1/4)*log((1
8*I*(1/36)^(3/4)*sqrt(3*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (3*x^2 - a)^(1/4)*
a^2*sqrt(a^(-3)) - 3*I*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x + (3*x^2 - a)^(3/4)
)/(3*x^2 - 2*a)) - 1/4*I*(1/36)^(1/4)*(a^(-3))^(1/4)*log((-18*I*(1/36)^(3/
4)*sqrt(3*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3
)) + 3*I*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x + (3*x^2 - a)^(3/4))/(3*x^2 - 2*a
))
```

3.316.6 Sympy [F]

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx = \int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx$$

input
```
integrate(1/(3*x**2-2*a)/(3*x**2-a)**(1/4),x)
```

output
```
Integral(1/((-2*a + 3*x**2)*(-a + 3*x**2)**(1/4)), x)
```

3.316.7 Maxima [F]

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx = \int \frac{1}{(3\,x^2-a)^{\frac{1}{4}}(3\,x^2-2\,a)}\,dx$$

input
```
integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)), x)
```

3.316.8 Giac [F]

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx = \int \frac{1}{(3\,x^2-a)^{\frac{1}{4}}(3\,x^2-2\,a)}\,dx$$

input
```
integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)), x)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx = -\int \frac{1}{(2\,a-3\,x^2)\,(3\,x^2-a)^{1/4}}\,dx$$

input
```
int(-1/((2*a - 3*x^2)*(3*x^2 - a)^(1/4)),x)
```

output
```
-int(1/((2*a - 3*x^2)*(3*x^2 - a)^(1/4)), x)
```

3.317 $\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx$

3.317.1 Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

output
```
-1/12*arctan(1/2*x*6^(1/2)/a^(1/4)/(-3*x^2-a)^(1/4))/a^(3/4)*6^(1/2)-1/12*
arctanh(1/2*x*6^(1/2)/a^(1/4)/(-3*x^2-a)^(1/4))/a^(3/4)*6^(1/2)
```

3.317.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = -\frac{-\arctan\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a-3x^2}}{x}\right)+\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a-3x^2}}{x}\right)}{2\sqrt{6}a^{3/4}}$$

input
```
Integrate[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
```

output
```
-1/2*(-ArcTan[(Sqrt[2/3]*a^(1/4)*(-a - 3*x^2)^(1/4))/x] + ArcTanh[(Sqrt[2/
3]*a^(1/4)*(-a - 3*x^2)^(1/4))/x])/(Sqrt[6]*a^(3/4))
```

3.317.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {309}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx$$

$$\Big\downarrow \ 309$$

$$-\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

input
```
Int[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
```

output
```
-1/2*ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(Sqrt[6]*a^(3/4))
- ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4))
```

3.317.3.1 Defintions of rubi rules used

rule 309
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

3.317.4 Maple [F]

$$\int \frac{1}{(-3x^2-2a)\,(-3x^2-a)^{\frac{1}{4}}}\,dx$$

input
```
int(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x)
```

output
```
int(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x)
```

3.317.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.89 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.39

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx$$

$$= \frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2-a}\,a^2\frac{1}{a^3}^{\frac{3}{4}}x+(-3x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x-(-3x^2-a)^{\frac{3}{4}}}{3x^2+2a}\right.$$

$$-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2-a}\,a^2\frac{1}{a^3}^{\frac{3}{4}}x-(-3x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+(-3x^2-a)\right.}{3x^2+2a}$$

$$+\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2-a}\,a^2\frac{1}{a^3}^{\frac{3}{4}}x+(-3x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}+3i\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+(-3x^2-}{3x^2+2a}\right.$$

$$-\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{-18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2-a}\,a^2\frac{1}{a^3}^{\frac{3}{4}}x+(-3x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}-3i\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+(-3x^2}{3x^2+2a}\right.$$

input
```
integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="fricas")
```

output
```
1/4*(1/36)^(1/4)*(a^(-3))^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(-3*x^2 - a)*a^2
*(a^(-3))^(3/4)*x + (-3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) - 3*(1/36)^(1/4)*a
*(a^(-3))^(1/4)*x - (-3*x^2 - a)^(3/4))/(3*x^2 + 2*a)) - 1/4*(1/36)^(1/4)*
(a^(-3))^(1/4)*log((18*(1/36)^(3/4)*sqrt(-3*x^2 - a)*a^2*(a^(-3))^(3/4)*x
- (-3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) - 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x
+ (-3*x^2 - a)^(3/4))/(3*x^2 + 2*a)) + 1/4*I*(1/36)^(1/4)*(a^(-3))^(1/4)*l
og((18*I*(1/36)^(3/4)*sqrt(-3*x^2 - a)*a^2*(a^(-3))^(3/4)*x + (-3*x^2 - a)
^(1/4)*a^2*sqrt(a^(-3)) + 3*I*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x + (-3*x^2 -
a)^(3/4))/(3*x^2 + 2*a)) - 1/4*I*(1/36)^(1/4)*(a^(-3))^(1/4)*log((-18*I*(1
/36)^(3/4)*sqrt(-3*x^2 - a)*a^2*(a^(-3))^(3/4)*x + (-3*x^2 - a)^(1/4)*a^2*
sqrt(a^(-3)) - 3*I*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x + (-3*x^2 - a)^(3/4))/(
3*x^2 + 2*a))
```

3.317.6 Sympy [F]

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = -\int \frac{1}{2a\sqrt[4]{-a-3x^2}+3x^2\sqrt[4]{-a-3x^2}}\,dx$$

input
```
integrate(1/(-3*x**2-2*a)/(-3*x**2-a)**(1/4),x)
```

output
```
-Integral(1/(2*a*(-a - 3*x**2)**(1/4) + 3*x**2*(-a - 3*x**2)**(1/4)), x)
```

3.317.7 Maxima [F]

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = \int -\frac{1}{(3\,x^2+2\,a)(-3\,x^2-a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)), x)
```

3.317.8 Giac [F]

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = \int -\frac{1}{(3\,x^2+2\,a)(-3\,x^2-a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="giac")
```

output
```
integrate(-1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)), x)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = -\int \frac{1}{(3\,x^2+2\,a)\,(-3\,x^2-a)^{1/4}}\,dx$$

input
```
int(-1/((2*a + 3*x^2)*(- a - 3*x^2)^(1/4)),x)
```

output
```
-int(1/((2*a + 3*x^2)*(- a - 3*x^2)^(1/4)), x)
```

3.318 $\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx$

3.318.1 Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

output
```
-1/4*arctan(1/2*x*b^(1/2)/a^(1/4)/(b*x^2-a)^(1/4)*2^(1/2))/a^(3/4)*2^(1/2)
/b^(1/2)-1/4*arctanh(1/2*x*b^(1/2)/a^(1/4)/(b*x^2-a)^(1/4)*2^(1/2))/a^(3/4
)*2^(1/2)/b^(1/2)
```

3.318.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}{\sqrt{b}x}\right) - \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}{\sqrt{b}x}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

input
```
Integrate[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
```

output
```
(ArcTan[(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))/(Sqrt[b]*x)] - ArcTanh[(Sqrt[
2]*a^(1/4)*(-a + b*x^2)^(1/4))/(Sqrt[b]*x)])/(2*Sqrt[2]*a^(3/4)*Sqrt[b])
```

3.318.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {309}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-2a)\sqrt[4]{bx^2-a}}dx$$

$$\Big\downarrow \text{309}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

input
```
Int[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
```

output
```
-1/2*ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2]*a^(
3/4)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/
(2*Sqrt[2]*a^(3/4)*Sqrt[b])
```

3.318.3.1 Defintions of rubi rules used

rule 309
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

3.318.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}-2a\right)\left(b\,x^{2}-a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x)
```

output
```
int(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x)
```

3.318.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.58 (sec) , antiderivative size = 457, normalized size of antiderivative = 4.52

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx =$$

$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+(bx^2}{bx^2-2a}\right.$$

$$+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}-(b}{bx^2-2a}\right.$$

$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}-i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+(}{bx^2-2a}\right.$$

$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{-2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}+i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+}{bx^2-2a}\right.$$

input
```
integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="fricas")
```

output
```
-1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt(b*x^2 - a)*a^
2*b^2*x*(1/(a^3*b^2))^(3/4) + (b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^2)) +
(1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (b*x^2 - a)^(3/4))/(b*x^2 - 2*a))
+ 1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(b*x^2 - a)*
a^2*b^2*x*(1/(a^3*b^2))^(3/4) - (b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^2))
+ (1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) - (b*x^2 - a)^(3/4))/(b*x^2 - 2*a)
) + 1/4*I*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((2*I*(1/4)^(3/4)*sqrt(b*x^2
- a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) - (b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b
^2)) - I*(1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (b*x^2 - a)^(3/4))/(b*x^2
 - 2*a)) - 1/4*I*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((-2*I*(1/4)^(3/4)*sqr
t(b*x^2 - a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) - (b*x^2 - a)^(1/4)*a^2*b*sqrt(
1/(a^3*b^2)) + I*(1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (b*x^2 - a)^(3/4)
)/(b*x^2 - 2*a))
```

3.318.6 Sympy [F]

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = \int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx$$

input
```
integrate(1/(b*x**2-2*a)/(b*x**2-a)**(1/4),x)
```

output
```
Integral(1/((-2*a + b*x**2)*(-a + b*x**2)**(1/4)), x)
```

3.318.7 Maxima [F]

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{4}}(bx^2-2\,a)}\,dx$$

input
```
integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)), x)
```

3.318.8 Giac [F]

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{4}}(bx^2-2\,a)}\,dx$$

input
```
integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)), x)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = -\int \frac{1}{(b\,x^2-a)^{1/4}\,(2\,a-b\,x^2)}\,dx$$

input
```
int(-1/((b*x^2 - a)^(1/4)*(2*a - b*x^2)),x)
```

output
```
-int(1/((b*x^2 - a)^(1/4)*(2*a - b*x^2)), x)
```

3.319 $\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx$

3.319.1 Optimal result

Integrand size = 27, antiderivative size = 103

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

output
```
-1/4*arctan(1/2*x*b^(1/2)/a^(1/4)/(-b*x^2-a)^(1/4)*2^(1/2))/a^(3/4)*2^(1/2
)/b^(1/2)-1/4*arctanh(1/2*x*b^(1/2)/a^(1/4)/(-b*x^2-a)^(1/4)*2^(1/2))/a^(3
/4)*2^(1/2)/b^(1/2)
```

3.319.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx = -\frac{-\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}{\sqrt{b}x}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}{\sqrt{b}x}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

input
```
Integrate[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
```

output
```
-1/2*(-ArcTan[(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))/(Sqrt[b]*x)] + ArcTanh[
(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))/(Sqrt[b]*x)])/(Sqrt[2]*a^(3/4)*Sqrt[b
])
```

3.319.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {309}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}dx$$

$$\Big\downarrow \text{309}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

input
```
Int[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
```

output
```
-1/2*ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2]*a^(
3/4)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/
(2*Sqrt[2]*a^(3/4)*Sqrt[b])
```

3.319.3.1 Defintions of rubi rules used

rule 309
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

3.319.4 Maple [F]

$$\int \frac{1}{(-b\,x^2-2a)\,(-b\,x^2-a)^{\frac{1}{4}}}dx$$

input
```
int(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x)
```

output
```
int(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x)
```

3.319.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.42 (sec) , antiderivative size = 469, normalized size of antiderivative = 4.55

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx$$
$$= \frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(-bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}-(}{bx^2+2a}\right.$$
$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(-bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+(}{bx^2+2a}\right.$$
$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(-bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}+i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}}{bx^2+2a}\right.$$
$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{-2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(-bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}-i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)}{bx^2+2a}\right.$$

input
```
integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="fricas")
```

output
```
1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(-b*x^2 - a)*a
^2*b^2*x*(1/(a^3*b^2))^(3/4) + (-b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^2))
- (1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) - (-b*x^2 - a)^(3/4))/(b*x^2 + 2*a
)) - 1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt(-b*x^2 -
a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) - (-b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^
2)) - (1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (-b*x^2 - a)^(3/4))/(b*x^2 +
 2*a)) + 1/4*I*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((2*I*(1/4)^(3/4)*sqrt(-
b*x^2 - a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) + (-b*x^2 - a)^(1/4)*a^2*b*sqrt(1
/(a^3*b^2)) + I*(1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (-b*x^2 - a)^(3/4)
)/(b*x^2 + 2*a)) - 1/4*I*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((-2*I*(1/4)^(
3/4)*sqrt(-b*x^2 - a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) + (-b*x^2 - a)^(1/4)*a
^2*b*sqrt(1/(a^3*b^2)) - I*(1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (-b*x^2
 - a)^(3/4))/(b*x^2 + 2*a))
```

3.319.6 Sympy [F]

$$\int \frac{1}{(-2a - bx^2)\sqrt[4]{-a - bx^2}}\,dx = -\int \frac{1}{2a\sqrt[4]{-a - bx^2} + bx^2\sqrt[4]{-a - bx^2}}\,dx$$

input
```
integrate(1/(-b*x**2-2*a)/(-b*x**2-a)**(1/4),x)
```

output
```
-Integral(1/(2*a*(-a - b*x**2)**(1/4) + b*x**2*(-a - b*x**2)**(1/4)), x)
```

3.319.7 Maxima [F]

$$\int \frac{1}{(-2a - bx^2)\sqrt[4]{-a - bx^2}}\,dx = \int -\frac{1}{(bx^2 + 2\,a)(-bx^2 - a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)), x)
```

3.319.8 Giac [F]

$$\int \frac{1}{(-2a - bx^2)\sqrt[4]{-a - bx^2}}\,dx = \int -\frac{1}{(bx^2 + 2\,a)(-bx^2 - a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)), x)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2a - bx^2)\sqrt[4]{-a - bx^2}}\,dx = -\int \frac{1}{(-b\,x^2 - a)^{1/4}\,(b\,x^2 + 2\,a)}\,dx$$

input
```
int(-1/((- a - b*x^2)^(1/4)*(2*a + b*x^2)),x)
```

output
```
-int(1/((- a - b*x^2)^(1/4)*(2*a + b*x^2)), x)
```

3.320 $\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$

3.320.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}$$

output
```
1/4*arctan(1/2*x/(x^2-1)^(1/4)*2^(1/2))*2^(1/2)+1/4*arctanh(1/2*x/(x^2-1)^
(1/4)*2^(1/2))*2^(1/2)
```

3.320.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \frac{-\arctan\left(\frac{\sqrt{2}\sqrt[4]{-1+x^2}}{x}\right) + \operatorname{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}$$

input
```
Integrate[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

output
```
(-ArcTan[(Sqrt[2]*(-1 + x^2)^(1/4))/x] + ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/
4))])/(2*Sqrt[2])
```

3.320.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {309}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-x^2)\sqrt[4]{x^2-1}}\,dx$$

$$\Big\downarrow \text{ 309}$$

$$\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}$$

input
```
Int[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

output
```
ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))]/(2*Sqrt[2]) + ArcTanh[x/(Sqrt[2]*(-1
+ x^2)^(1/4))]/(2*Sqrt[2])
```

3.320.3.1 Defintions of rubi rules used

rule 309
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

3.320.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.21

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\left(x^2-1\right)^{\frac{3}{4}}+x\sqrt{x^2-1}+\operatorname{RootOf}\left(\_Z^2-2\right)\left(x^2-1\right)^{\frac{1}{4}}+x}{x^2-2}\right)}{4}-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\ldots\right.}{\ldots}\right.}{\ldots}$

input
```
int(1/(-x^2+2)/(x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/4*RootOf(_Z^2-2)*ln((RootOf(_Z^2-2)*(x^2-1)^(3/4)+x*(x^2-1)^(1/2)+RootOf
(_Z^2-2)*(x^2-1)^(1/4)+x)/(x^2-2))-1/4*RootOf(_Z^2+2)*ln(-(RootOf(_Z^2+2)*
(x^2-1)^(3/4)-x*(x^2-1)^(1/2)-RootOf(_Z^2+2)*(x^2-1)^(1/4)+x)/(x^2-2))
```

3.320.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(39) = 78$.

Time = 1.81 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.72

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2-1)^{\frac{1}{4}}}{x}\right) + \frac{1}{8}\sqrt{2}\log\left(-\frac{x^4+2\sqrt{2}(x^2-1)^{\frac{1}{4}}x^3+4\sqrt{x^2-1}x^2+4\sqrt{2}(x^2-1)^{\frac{3}{4}}x+4x^2-4}{x^4-4x^2+4}\right)$$

input
```
integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(sqrt(2)*(x^2 - 1)^(1/4)/x) + 1/8*sqrt(2)*log(-(x^4 + 2
*sqrt(2)*(x^2 - 1)^(1/4)*x^3 + 4*sqrt(x^2 - 1)*x^2 + 4*sqrt(2)*(x^2 - 1)^(
3/4)*x + 4*x^2 - 4)/(x^4 - 4*x^2 + 4))
```

3.320.6 Sympy [F]

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = -\int \frac{1}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx$$

input
```
integrate(1/(-x**2+2)/(x**2-1)**(1/4),x)
```

output
```
-Integral(1/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

3.320.7 Maxima [F]

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \int -\frac{1}{(x^2-1)^{\frac{1}{4}}(x^2-2)}\,dx$$

input
```
integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
-integrate(1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.320.8 Giac [F]

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \int -\frac{1}{(x^2-1)^{\frac{1}{4}}(x^2-2)}\,dx$$

input
```
integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(-1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = -\int \frac{1}{(x^2-1)^{1/4}\,(x^2-2)}\,dx$$

input
```
int(-1/((x^2 - 1)^(1/4)*(x^2 - 2)),x)
```

output
```
-int(1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.321 $\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx$

3.321.1 Optimal result

Integrand size = 21, antiderivative size = 362

$$\begin{aligned}\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx &= \frac{6abx}{5d\sqrt[4]{a+bx^2}}-\frac{2b(bc-ad)x}{d^2\sqrt[4]{a+bx^2}} \\ &+\frac{2bx(a+bx^2)^{3/4}}{5d}-\frac{6a^{3/2}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5d\sqrt[4]{a+bx^2}} \\ &+\frac{2\sqrt{a}\sqrt{b}(bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\sqrt[4]{a+bx^2}} \\ &+\frac{\sqrt[4]{a}(-bc+ad)^{3/2}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{d^{5/2}x} \\ &-\frac{\sqrt[4]{a}(-bc+ad)^{3/2}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{d^{5/2}x}\end{aligned}$$

output
```
6/5*a*b*x/d/(b*x^2+a)^(1/4)-2*b*(-a*d+b*c)*x/d^2/(b*x^2+a)^(1/4)+2/5*b*x*(
b*x^2+a)^(3/4)/d-6/5*a^(3/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arc
tan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/d/(b*x^2+a)^(1/4)+2*(-a*d+b*c)*(1
+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcta
n(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2)
)*a^(1/2)*b^(1/2)/d^2/(b*x^2+a)^(1/4)+a^(1/4)*(a*d-b*c)^(3/2)*EllipticPi((
b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2
)/d^(5/2)/x-a^(1/4)*(a*d-b*c)^(3/2)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(
1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d^(5/2)/x
```

3.321.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 9.12 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \frac{x\left(\frac{b(-5bc+8ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}+\frac{6\left(-3ac(5a^2d+2abdx^2+2b^2x^2(c+dx^2))\,\text{Appe}\ldots}{(c+dx^2)\left(-6ac\,\text{AppellF}\ldots}\right.}{1\ldots}$$

input
```
Integrate[(a + b*x^2)^(7/4)/(c + d*x^2),x]
```

output
```
(x*((b*(-5*b*c + 8*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/
2, -((b*x^2)/a), -((d*x^2)/c)])/c + (6*(-3*a*c*(5*a^2*d + 2*a*b*d*x^2 + 2*
b^2*x^2*(c + d*x^2))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)
] + b*x^2*(a + b*x^2)*(c + d*x^2)*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x
^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*
x^2)/c)])))/((c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a),
-((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x
^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/
(15*d*(a + b*x^2)^(1/4))
```

3.321.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {301, 211, 227, 225, 212, 301, 227, 225, 212, 310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx$$

$$\Big\downarrow \text{301}$$

$$\frac{b\int (bx^2+a)^{3/4}dx}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^{3/4}}{dx^2+c}dx}{d}$$

$$\Big\downarrow \text{211}$$

$$\frac{b\left(\frac{3}{5}a\int \frac{1}{\sqrt[4]{bx^2+a}}dx+\frac{2}{5}x(a+bx^2)^{3/4}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^{3/4}}{dx^2+c}dx}{d}$$

$$\Big\downarrow \text{227}$$

$$\frac{b\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x(a+bx^2)^{3/4}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^{3/4}}{dx^2+c}dx}{d}$$

$$\Big\downarrow \text{225}$$

$$\frac{b\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x(a+bx^2)^{3/4}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^{3/4}}{dx^2+c}dx}{d}$$

$$\Big\downarrow \text{212}$$

$$\frac{b\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}\right)}{d}-\frac{(bc-ad)\int\frac{\left(bx^2+a\right)^{3/4}}{dx^2+c}dx}{d}$$

$\downarrow$ 301

$$\frac{b\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}\right)}{d}-\frac{(bc-ad)\left(\frac{b\int\frac{1}{\sqrt[4]{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{d}$$

$\downarrow$ 227

$$\frac{b\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}\right)}{d}-\frac{(bc-ad)\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{d\sqrt[4]{a+bx^2}}-\frac{(bc-ad)\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{d}$$

$\downarrow$ 225

3.321. $\int\frac{\left(a+bx^2\right)^{7/4}}{c+dx^2}dx$

$$\frac{b\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}\right)}{d}-\frac{(bc-ad)\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{d\sqrt[4]{a+bx^2}}-\frac{(bc-ad)\int\frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}\right)}{d}$$

$$\Big\downarrow \; 212$$

$$\frac{b\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}\right)}{d}-\frac{(bc-ad)\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}-\frac{(bc-ad)\int\frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}\right)}{d}$$

$$\Big\downarrow \; 310$$

3.321. $\int \frac{\left(a+bx^2\right)^{7/4}}{c+dx^2}\,dx$

$$\frac{b\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}\right)}{d}-\frac{(bc-ad)\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\int\frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{bx^2+a}{a}}\left(bc-ad+d\left(bx^2+a\right)\right)}d\sqrt[4]{bx^2+a}}{dx}\right)}{d}$$

$$\Big\downarrow \; 993$$

$$\frac{b\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}\right)}{d}-\frac{(bc-ad)\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\left(\frac{\int\frac{1}{\left(\sqrt{ad-bc}+\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+}}{2\sqrt{d}}\right.}{dx}\right)}{d}$$

$$\Big\downarrow \; 1542$$

3.321. $\int \frac{\left(a+bx^2\right)^{7/4}}{c+dx^2}\,dx$

$$\frac{b\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}\right)}{d}-$$

$$\frac{(bc-ad)\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\left(\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right.\right.}{2\sqrt{d}\sqrt{ad-bc}}\right.}{dx}\right.}{d}$$

input
```
Int[(a + b*x^2)^(7/4)/(c + d*x^2),x]
```

output
```
(b*((2*x*(a + b*x^2)^(3/4))/5 + (3*a*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*
x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sq
rt[b]))/(5*(a + b*x^2)^(1/4))))/d - ((b*c - a*d)*((b*(1 + (b*x^2)/a)^(1/4)
*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sq
rt[a]]/2, 2])/Sqrt[b]))/(d*(a + b*x^2)^(1/4)) - (2*(b*c - a*d)*Sqrt[-((b*x
^2)/a)]*((a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcS
in[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d]) - (a^(1
/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1
/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d])))/(d*x)))/d
```

3.321.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

3.321. $\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx$

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.321. $\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx$

3.321.4 Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{7}{4}}}{d\,x^2+c}dx$$

input
```
int((b*x^2+a)^(7/4)/(d*x^2+c),x)
```

output
```
int((b*x^2+a)^(7/4)/(d*x^2+c),x)
```

3.321.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.321.6 Sympy [F]

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \int \frac{(a+bx^2)^{\frac{7}{4}}}{c+dx^2}\,dx$$

input
```
integrate((b*x**2+a)**(7/4)/(d*x**2+c),x)
```

output
```
Integral((a + b*x**2)**(7/4)/(c + d*x**2), x)
```

3.321.7 Maxima [F]

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{4}}}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/4)/(d*x^2 + c), x)
```

3.321.8 Giac [F]

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{4}}}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/4)/(d*x^2 + c), x)
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \int \frac{(b\,x^2+a)^{7/4}}{d\,x^2+c}\,dx$$

input
```
int((a + b*x^2)^(7/4)/(c + d*x^2),x)
```

output
```
int((a + b*x^2)^(7/4)/(c + d*x^2), x)
```

3.322 $\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx$

3.322.1 Optimal result

Integrand size = 21, antiderivative size = 302

$$
\begin{aligned}
\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx &= \frac{2bx\sqrt[4]{a+bx^2}}{3d} \\
&+ \frac{2a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3d\,(a+bx^2)^{3/4}} \\
&- \frac{2\sqrt{a}\sqrt{b}(bc-ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d^2\,(a+bx^2)^{3/4}} \\
&+ \frac{\sqrt[4]{a}(bc-ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{d^2x} \\
&+ \frac{\sqrt[4]{a}(bc-ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{d^2x}
\end{aligned}
$$

output
```
2/3*b*x*(b*x^2+a)^(1/4)/d+2/3*a^(3/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*
b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(si
n(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/d/(b*x^2+a)^(3/4)-2*(-a*
d+b*c)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(
1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))
),2^(1/2))*a^(1/2)*b^(1/2)/d^2/(b*x^2+a)^(3/4)+a^(1/4)*(-a*d+b*c)*Elliptic
Pi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^
(1/2)/d^2/x+a^(1/4)*(-a*d+b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*
d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d^2/x
```

3.322.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \frac{x\left(\frac{b(-3bc+4ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}+\frac{6\left(-3ac\left(3a^2d+2abdx^2+2b^2x^2(c+dx^2)\right)\text{Appell}\right.}{(c+dx^2)\left(-6ac\,\text{AppellF1}\right.}\right.}{9d}$$

input
```
Integrate[(a + b*x^2)^(5/4)/(c + d*x^2),x]
```

output
```
(x*((b*(-3*b*c + 4*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/
2, -((b*x^2)/a), -((d*x^2)/c)])/c + (6*(-3*a*c*(3*a^2*d + 2*a*b*d*x^2 + 2*
b^2*x^2*(c + d*x^2))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)
] + b*x^2*(a + b*x^2)*(c + d*x^2)*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x
^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((
d*x^2)/c)])))/((c + d*x^2)*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a)
, -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d
*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))
)))/(9*d*(a + b*x^2)^(3/4))
```

3.322.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {301, 211, 231, 229, 301, 231, 229, 312, 118, 25, 925, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/4}}{c+dx^2}dx$$

$$\Big\downarrow \text{301}$$

$$\frac{b\int \sqrt[4]{bx^2+a}dx}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^2+a}}{dx^2+c}dx}{d}$$

$$\Big\downarrow \text{211}$$

$$\frac{b\left(\frac{1}{3}a\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^2+a}}{dx^2+c}dx}{d}$$

$$\Big\downarrow \text{231}$$

$$\frac{b\left(\frac{a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^2+a}}{dx^2+c}dx}{d}$$

$$\Big\downarrow \text{229}$$

$$\frac{b\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^2+a}}{dx^2+c}dx}{d}$$

$$\Big\downarrow \text{301}$$

$$\frac{b\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{d}-\frac{(bc-ad)\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{231}$$

$$\frac{b\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}(a+bx^2)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{d(a+bx^2)^{3/4}}-\frac{(bc-ad)\int\frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx}{d}\right)}{d}$$

$$\downarrow 229$$

$$\frac{b\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}(a+bx^2)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{d}-\frac{(bc-ad)\left(\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d(a+bx^2)^{3/4}}-\frac{(bc-ad)\int\frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx}{d}\right)}{d}$$

$$\downarrow 312$$

$$\frac{b\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}(a+bx^2)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{d}-\frac{(bc-ad)\left(\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d(a+bx^2)^{3/4}}-\frac{\sqrt{-\frac{bx^2}{a}}(bc-ad)\int\frac{1}{\sqrt{-\frac{bx^2}{a}}(bx^2+a)^{3/4}(dx^2+c)}dx^2}{2dx}\right)}{d}$$

$$\downarrow 118$$

$$\frac{b\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}(a+bx^2)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{d}-\frac{(bc-ad)\left(\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\int-\frac{1}{\sqrt{1-\frac{x^8}{a}}(dx^8+bc-ad)}d\sqrt[4]{bx^2+a}}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d(a+bx^2)^{3/4}}\right)}{d}$$

$$\downarrow 25$$

3.322. $\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx$

$$\frac{b\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}(a+bx^2)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{d}-\frac{(bc-ad)\left(\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d(a+bx^2)^{3/4}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\int\frac{1}{\sqrt{1-\frac{x^8}{a}}(dx^8+bc-ad)}d\sqrt[4]{bx^2+a}}{dx}\right)}{d}$$

$$\Big\downarrow \text{925}$$

$$\frac{b\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}(a+bx^2)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{d}-\frac{(bc-ad)\left(\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\left(-\frac{\int\frac{1}{\left(1-\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}-\frac{\int\frac{1}{\left(\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}+1\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}\right)}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{Ellip}}{d(a+bx}\right.}{d}$$

$$\Big\downarrow \text{1542}$$

$$\frac{b\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}(a+bx^2)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{d}-\frac{(bc-ad)\left(\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\left(-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}\right)}{dx}\right.}{d}$$

input
```
Int[(a + b*x^2)^(5/4)/(c + d*x^2),x]
```

3.322. $\int\frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx$

output
```
(b*((2*x*(a + b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF
[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a + b*x^2)^(3/4))))/d - ((
b*c - a*d)*((2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqr
t[b]*x)/Sqrt[a]]/2, 2])/(d*(a + b*x^2)^(3/4)) + (2*(b*c - a*d)*Sqrt[-((b*x
^2)/a)]*(-1/2*(a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]),
 ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(b*c - a*d) - (a^(1/4)*EllipticPi
[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/(2*(b*c - a*d))))/(d*x)))/d
```

3.322.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.322.4 Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{5}{4}}}{d\,x^{2}+c}dx$$

input
```
int((b*x^2+a)^(5/4)/(d*x^2+c),x)
```

output
```
int((b*x^2+a)^(5/4)/(d*x^2+c),x)
```

3.322.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.322.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{4}}}{c+dx^2}\,dx$$

input
```
integrate((b*x**2+a)**(5/4)/(d*x**2+c),x)
```

output
```
Integral((a + b*x**2)**(5/4)/(c + d*x**2), x)
```

3.322.7 Maxima [F]

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/4)/(d*x^2 + c), x)
```

3.322.8 Giac [F]

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/4)/(d*x^2 + c), x)
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \int \frac{(b\,x^2+a)^{5/4}}{d\,x^2+c}\,dx$$

input
```
int((a + b*x^2)^(5/4)/(c + d*x^2),x)
```

output
```
int((a + b*x^2)^(5/4)/(c + d*x^2), x)
```

3.323 $\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx$

3.323.1 Optimal result

Integrand size = 21, antiderivative size = 244

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \frac{2bx}{d\sqrt[4]{a+bx^2}} - \frac{2\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{-bc+ad}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{d^{3/2}x} - \frac{\sqrt[4]{a}\sqrt{-bc+ad}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{d^{3/2}x}$$

output
```
2*b*x/d/(b*x^2+a)^(1/4)-2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1
/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan
(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/d/(b*x^2+a)^(1/4)+a^(1/4)*El
lipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(a*d-
b*c)^(1/2)*(-b*x^2/a)^(1/2)/d^(3/2)/x-a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a
^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(a*d-b*c)^(1/2)*(-b*x^2/a)^(1/2)
/d^(3/2)/x
```

3.323.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 9.24 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \frac{6acx(a+bx^2)^{3/4}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{3}{4},1,\frac{3}{2},-\frac{bx^2}{a}\right.}{(c+dx^2)\left(6ac\operatorname{AppellF1}\left(\frac{1}{2},-\frac{3}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(-4ad\operatorname{AppellF1}\left(\frac{3}{2},-\frac{3}{4},2,\frac{5}{2},-\right.\right.\right.}$$

input
```
Integrate[(a + b*x^2)^(3/4)/(c + d*x^2),x]
```

output
```
(6*a*c*x*(a + b*x^2)^(3/4)*AppellF1[1/2, -3/4, 1, 3/2, -((b*x^2)/a), -((d*
x^2)/c)])/((c + d*x^2)*(6*a*c*AppellF1[1/2, -3/4, 1, 3/2, -((b*x^2)/a), -(
(d*x^2)/c)] + x^2*(-4*a*d*AppellF1[3/2, -3/4, 2, 5/2, -((b*x^2)/a), -((d*x
^2)/c)] + 3*b*c*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

3.323.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {301, 227, 225, 212, 310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx$$

$$\Big\downarrow \ 301$$

$$\frac{b\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}$$

$$\Big\downarrow \ 227$$

$$\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{d\sqrt[4]{a+bx^2}}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}$$

$$\Big\downarrow \ 225$$

$$\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{d\sqrt[4]{a+bx^2}}-\frac{(bc-ad)\int\frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}$$

$$\Big\downarrow \text{212}$$

$$\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}-\frac{(bc-ad)\int\frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}$$

$$\Big\downarrow \text{310}$$

$$\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\int\frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{bx^2+a}{a}}(bc-ad+d(bx^2+a))}d\sqrt[4]{bx^2+a}}{dx}$$

$$\Big\downarrow \text{993}$$

$$\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\left(\frac{\int\frac{1}{\left(\sqrt{ad-bc}+\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}-\frac{\int\frac{1}{\left(\sqrt{ad-bc}-\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}\right)}{dx}$$

$$\Big\downarrow \text{1542}$$

3.323. $\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx$

$$\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\left(\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}\right)}{dx}$$

input
```
Int[(a + b*x^2)^(3/4)/(c + d*x^2),x]
```

output
```
(b*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*Ellipti
cE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(d*(a + b*x^2)^(1/4)) - (2
*(b*c - a*d)*Sqrt[-((b*x^2)/a)]*((a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/S
qrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqr
t[-(b*c) + a*d]) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d
], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d]))
)/(d*x)
```

3.323.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.323.4 Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{d\,x^2+c}dx$$

input
```
int((b*x^2+a)^(3/4)/(d*x^2+c),x)
```

output
```
int((b*x^2+a)^(3/4)/(d*x^2+c),x)
```

3.323.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.323.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{4}}}{c+dx^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/4)/(d*x**2+c),x)
```

output
```
Integral((a + b*x**2)**(3/4)/(c + d*x**2), x)
```

3.323.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/4)/(d*x^2 + c), x)
```

3.323.8 Giac [F]

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/4)/(d*x^2 + c), x)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \int \frac{(b\,x^2+a)^{3/4}}{d\,x^2+c}\,dx$$

input
```
int((a + b*x^2)^(3/4)/(c + d*x^2),x)
```

output
```
int((a + b*x^2)^(3/4)/(c + d*x^2), x)
```

3.324 $\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx$

3.324.1 Optimal result

Integrand size = 21, antiderivative size = 199

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx = \frac{2\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d\left(a+bx^2\right)^{3/4}} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{dx} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{dx}$$

output
```
2*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*a
rctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(
1/2))*a^(1/2)*b^(1/2)/d/(b*x^2+a)^(3/4)-a^(1/4)*EllipticPi((b*x^2+a)^(1/4)
/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d/x-a^(1/4)*
EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*
x^2/a)^(1/2)/d/x
```

3.324.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx$$
$$= \frac{6acx\sqrt[4]{a+bx^2}\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)\left(6ac\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(-4ad\,\text{AppellF1}\left(\frac{3}{2},-\frac{1}{4},2,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bc\,\text{App}\right.\right.}$$

input
```
Integrate[(a + b*x^2)^(1/4)/(c + d*x^2),x]
```

output
```
(6*a*c*x*(a + b*x^2)^(1/4)*AppellF1[1/2, -1/4, 1, 3/2, -((b*x^2)/a), -((d*
x^2)/c)])/((c + d*x^2)*(6*a*c*AppellF1[1/2, -1/4, 1, 3/2, -((b*x^2)/a), -(
(d*x^2)/c)] + x^2*(-4*a*d*AppellF1[3/2, -1/4, 2, 5/2, -((b*x^2)/a), -((d*x
^2)/c)] + b*c*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

3.324.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {301, 231, 229, 312, 118, 25, 925, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx$$
$$\Big\downarrow \text{301}$$
$$\frac{b\int \frac{1}{(bx^2+a)^{3/4}}dx}{d}-\frac{(bc-ad)\int \frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx}{d}$$
$$\Big\downarrow \text{231}$$
$$\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{d\,(a+bx^2)^{3/4}}-\frac{(bc-ad)\int \frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx}{d}$$
$$\Big\downarrow \text{229}$$

$$\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d\left(a+bx^2\right)^{3/4}}-\frac{(bc-ad)\int\frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx}{d}$$

$$\Big\downarrow \text{312}$$

$$\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d\left(a+bx^2\right)^{3/4}}-\frac{\sqrt{-\frac{bx^2}{a}}(bc-ad)\int\frac{1}{\sqrt{-\frac{bx^2}{a}}(bx^2+a)^{3/4}(dx^2+c)}dx^2}{2dx}$$

$$\Big\downarrow \text{118}$$

$$\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\int-\frac{1}{\sqrt{1-\frac{x^8}{a}}(dx^8+bc-ad)}d\sqrt[4]{bx^2+a}}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\int\frac{1}{\sqrt{1-\frac{x^8}{a}}(dx^8+bc-ad)}d\sqrt[4]{bx^2+a}}{dx}$$

$$\Big\downarrow \text{925}$$

$$\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\left(-\frac{\int\frac{1}{\left(1-\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}-\frac{\int\frac{1}{\left(\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}+1\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}\right)}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{1542}$$

$$\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)\left(-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}\right)}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d\left(a+bx^2\right)^{3/4}}$$

3.324. $\int\frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx$

input
```
Int[(a + b*x^2)^(1/4)/(c + d*x^2),x]
```

output
```
(2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt
[a]]/2, 2])/(d*(a + b*x^2)^(3/4)) + (2*(b*c - a*d)*Sqrt[-((b*x^2)/a)]*(-1/
2*(a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a +
 b*x^2)^(1/4)/a^(1/4)], -1])/(b*c - a*d) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sq
rt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*(b*c
 - a*d))))/(d*x)
```

3.324.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.324.4 Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}{d\,x^{2}+c}dx$$

input
```
int((b*x^2+a)^(1/4)/(d*x^2+c),x)
```

output
```
int((b*x^2+a)^(1/4)/(d*x^2+c),x)
```

3.324.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.324.6 Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx = \int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/4)/(d*x**2+c),x)
```

output
```
Integral((a + b*x**2)**(1/4)/(c + d*x**2), x)
```

3.324.7 Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/(d*x^2 + c), x)
```

3.324.8 Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/(d*x^2 + c), x)
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{1/4}}{d\,x^2+c}\,dx$$

input
```
int((a + b*x^2)^(1/4)/(c + d*x^2),x)
```

output
```
int((a + b*x^2)^(1/4)/(c + d*x^2), x)
```

3.325 $\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx$

3.325.1 Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{d}\sqrt{-bc+ad}\,x} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{d}\sqrt{-bc+ad}\,x}$$

output
```
a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2
),I)*(-b*x^2/a)^(1/2)/x/d^(1/2)/(a*d-b*c)^(1/2)-a^(1/4)*EllipticPi((b*x^2+
a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/x/d^(
1/2)/(a*d-b*c)^(1/2)
```

3.325.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = -\frac{6acx\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\sqrt[4]{a+bx^2}\,(c+dx^2)\left(-6ac\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4ad\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right.\right.}$$

input
```
Integrate[1/((a + b*x^2)^(1/4)*(c + d*x^2)),x]
```

output
```
(-6*a*c*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((a + b*
x^2)^(1/4)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -(
(d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2
)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

3.325.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}dx$$

$$\Big\downarrow \; 310$$

$$\frac{2\sqrt{-\frac{bx^2}{a}}\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{bx^2+a}{a}}\,(bc-ad+d(bx^2+a))}d\sqrt[4]{bx^2+a}}{x}$$

$$\Big\downarrow \; 993$$

$$\frac{2\sqrt{-\frac{bx^2}{a}}\left(\frac{\int \frac{1}{\left(\sqrt{ad-bc}+\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}-\frac{\int \frac{1}{\left(\sqrt{ad-bc}-\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}\right)}{x}$$

$$\Big\downarrow \; 1542$$

$$\frac{2\sqrt{-\frac{bx^2}{a}}\left(\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}\right)}{x}$$

input
```
Int[1/((a + b*x^2)^(1/4)*(c + d*x^2)),x]
```

output
```
(2*Sqrt[-((b*x^2)/a)]*((a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c)
 + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) +
 a*d]) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[
(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d])))/x
```

3.325.3.1 Defintions of rubi rules used

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.325.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(d\,x^{2}+c\right)}dx$$

input
```
int(1/(b*x^2+a)^(1/4)/(d*x^2+c),x)
```

output
```
int(1/(b*x^2+a)^(1/4)/(d*x^2+c),x)
```

3.325.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.325.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/4)/(d*x**2+c),x)
```

output
```
Integral(1/((a + b*x**2)**(1/4)*(c + d*x**2)), x)
```

3.325.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(dx^2+c)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)), x)
```

3.325.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(dx^2+c)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)), x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{1/4}\,(d\,x^2+c)}\,dx$$

input
```
int(1/((a + b*x^2)^(1/4)*(c + d*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(1/4)*(c + d*x^2)), x)
```

3.326 $\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)} dx$

3.326.1 Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)} dx = \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}}, \arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right), -1\right)}{(bc-ad)x} + \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}}, \arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right), -1\right)}{(bc-ad)x}$$

output
```
a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2
),I)*(-b*x^2/a)^(1/2)/(-a*d+b*c)/x+a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1
/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(-a*d+b*c)/x
```

3.326.2 Mathematica [A] (verified)

Time = 8.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)} dx = \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}}, \arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right), -1\right) + \text{EllipticPi}\left(\right.\right.}{(bc-ad)x}$$

input
```
Integrate[1/((a + b*x^2)^(3/4)*(c + d*x^2)),x]
```

output
```
(a^(1/4)*Sqrt[-((b*x^2)/a)]*(EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) +
a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1] + EllipticPi[(Sqrt[a]*Sqrt[d
])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1]))/((b*c - a*
d)*x)
```

3.326.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {312, 118, 25, 925, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}dx$$

$$\Big\downarrow \text{312}$$

$$\frac{\sqrt{-\frac{bx^2}{a}}\int \frac{1}{\sqrt{-\frac{bx^2}{a}}(bx^2+a)^{3/4}(dx^2+c)}dx^2}{2x}$$

$$\Big\downarrow \text{118}$$

$$-\frac{2\sqrt{-\frac{bx^2}{a}}\int -\frac{1}{\sqrt{1-\frac{x^8}{a}}(dx^8+bc-ad)}d\sqrt[4]{bx^2+a}}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sqrt{-\frac{bx^2}{a}}\int \frac{1}{\sqrt{1-\frac{x^8}{a}}(dx^8+bc-ad)}d\sqrt[4]{bx^2+a}}{x}$$

$$\Big\downarrow \text{925}$$

$$-\frac{2\sqrt{-\frac{bx^2}{a}}\left(-\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}-\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}+1\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}\right)}{x}$$

$$\Big\downarrow \text{1542}$$

$$-\frac{2\sqrt{-\frac{bx^2}{a}}\left(-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}\right)}{x}$$

input
```
Int[1/((a + b*x^2)^(3/4)*(c + d*x^2)),x]
```

output
```
(-2*Sqrt[-((b*x^2)/a)]*(-1/2*(a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[
-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(b*c - a*d) - (a^(
1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(
1/4)/a^(1/4)], -1])/(2*(b*c - a*d))))/x
```

3.326.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.326.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{3}{4}} \left(d\,x^{2}+c\right)}d x$$

input
```
int(1/(b*x^2+a)^(3/4)/(d*x^2+c),x)
```

output
```
int(1/(b*x^2+a)^(3/4)/(d*x^2+c),x)
```

3.326.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.326.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx = \int \frac{1}{\left(a+bx^{2}\right)^{\frac{3}{4}}\left(c+dx^{2}\right)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/4)/(d*x**2+c),x)
```

output
```
Integral(1/((a + b*x**2)**(3/4)*(c + d*x**2)), x)
```

3.326.7 Maxima [F]

$$\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)), x)
```

3.326.8 Giac [F]

$$\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)), x)
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{3/4}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int(1/((a + b*x^2)^(3/4)*(c + d*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(3/4)*(c + d*x^2)), x)
```

3.327 $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx$

3.327.1 Optimal result

Integrand size = 21, antiderivative size = 233

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \frac{2\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}(bc-ad)\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(-bc+ad)^{3/2}x} - \frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(-bc+ad)^{3/2}x}$$

output
```
2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*a
rctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(
1/2))*b^(1/2)/(-a*d+b*c)/(b*x^2+a)^(1/4)/a^(1/2)+a^(1/4)*EllipticPi((b*x^2
+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*d^(1/2)*(-b*x^2/a)^(
1/2)/(a*d-b*c)^(3/2)/x-a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*
d^(1/2)/(a*d-b*c)^(1/2),I)*d^(1/2)*(-b*x^2/a)^(1/2)/(a*d-b*c)^(3/2)/x
```

3.327.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.49 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}\,dx = \frac{x\left(\frac{bdx^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}+\frac{6\left(3ac\left(ad-b\left(c+2dx^2\right)\right)\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2}\right.}{\left(c+dx^2\right)\left(6ac\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2}\right.\right.}\right.}{3a(-bc+}$$

input
```
Integrate[1/((a + b*x^2)^(5/4)*(c + d*x^2)),x]
```

output
```
(x*((b*d*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a)
, -((d*x^2)/c)])/c + (6*(3*a*c*(a*d - b*(c + 2*d*x^2))*AppellF1[1/2, 1/4,
1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^2*(c + d*x^2)*(4*a*d*AppellF1[3/
2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/
2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1[1/2, 1/4, 1
, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2,
 -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a)
, -((d*x^2)/c)])))))/(3*a*(-(b*c) + a*d)*(a + b*x^2)^(1/4))
```

3.327.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {302, 213, 212, 310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}dx$$

$$\Big\downarrow \ 302$$

$$\frac{b\int \frac{1}{\left(bx^2+a\right)^{5/4}}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{bc-ad}$$

$$\Big\downarrow \ 213$$

$$\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{d\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{bc-ad}$$

$$\Big\downarrow \text{212}$$

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}(bc-ad)}-\frac{d\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{bc-ad}$$

$$\Big\downarrow \text{310}$$

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}(bc-ad)}-\frac{2d\sqrt{-\frac{bx^2}{a}}\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{bx^2+a}{a}}(bc-ad+d(bx^2+a))}d\sqrt[4]{bx^2+a}}{x(bc-ad)}$$

$$\Big\downarrow \text{993}$$

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}(bc-ad)}-\frac{2d\sqrt{-\frac{bx^2}{a}}\left(\frac{\int \frac{1}{\left(\sqrt{ad-bc}+\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}-\frac{\int \frac{1}{\left(\sqrt{ad-bc}-\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}\right)}{x(bc-ad)}$$

$$\Big\downarrow \text{1542}$$

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}(bc-ad)}-\frac{2d\sqrt{-\frac{bx^2}{a}}\left(\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}\right)}{x(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(5/4)*(c + d*x^2)),x]
```

3.327. $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx$

output
```
(2*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^2)^(1/4)) - (2*d*Sqrt[-((b*x^2)/a)]*((a^
(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^
2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d]) - (a^(1/4)*Elliptic
Pi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)]
, -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d])))/((b*c - a*d)*x)
```

3.327.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 302
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/(
b*c - a*d)   Int[(a + b*x^2)^p, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^
2)^(p + 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && EqQ[Denominator[p], 4] && (EqQ[p, -5/4] || EqQ[p, -7/4]
)
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

3.327. $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.327.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{5}{4}} \left(d\,x^{2}+c\right)}d x$$

input
```
int(1/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

output
```
int(1/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

3.327.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.327.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{4}}\,(c+dx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/4)/(d*x**2+c),x)
```

output
```
Integral(1/((a + b*x**2)**(5/4)*(c + d*x**2)), x)
```

3.327.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)), x)
```

3.327.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)), x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{5/4}\,(d\,x^2+c)}\,dx$$

input
```
int(1/((a + b*x^2)^(5/4)*(c + d*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(5/4)*(c + d*x^2)), x)
```

3.328 $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx$

3.328.1 Optimal result

Integrand size = 21, antiderivative size = 254

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx = \frac{2bx}{3a(bc-ad)(a+bx^2)^{3/4}} + \frac{2\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}(bc-ad)(a+bx^2)^{3/4}} - \frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(bc-ad)^2x} - \frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(bc-ad)^2x}$$

output
```
2/3*b*x/a/(-a*d+b*c)/(b*x^2+a)^(3/4)+2/3*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan
(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF
(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(-a*d+b*c)/(b*x^2+a)^
(3/4)/a^(1/2)-a^(1/4)*d*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2
)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(-a*d+b*c)^2/x-a^(1/4)*d*EllipticPi(
(b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2
)/(-a*d+b*c)^2/x
```

3.328.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx = \frac{x\left(-\frac{bdx^2\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}+\frac{6\left(3ac(-3bc+3ad-2bdx^2)\operatorname{AppellF1}\left(\frac{1}{2},\frac{3}{4},\right.\right.}{(c+dx^2)\left(6ac\operatorname{AppellF1}\left(\frac{1}{2},\frac{3}{4},1,\right.\right.}\right.}{9a(-bc+}$$

input
```
Integrate[1/((a + b*x^2)^(7/4)*(c + d*x^2)),x]
```

output
```
(x*(-((b*d*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/
a), -((d*x^2)/c)])/c) + (6*(3*a*c*(-3*b*c + 3*a*d - 2*b*d*x^2)*AppellF1[1/
2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^2*(c + d*x^2)*(4*a*d*App
ellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2,
7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1[1
/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 3/
4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2,
-((b*x^2)/a), -((d*x^2)/c)])))))/(9*a*(-(b*c) + a*d)*(a + b*x^2)^(3/4))
```

3.328.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {302, 215, 231, 229, 312, 118, 25, 925, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}dx$$

$$\Big\downarrow \; 302$$

$$\frac{b\int \frac{1}{(bx^2+a)^{7/4}}dx}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx}{bc-ad}$$

$$\Big\downarrow \; 215$$

$$\frac{b\left(\frac{\int \frac{1}{(bx^2+a)^{3/4}}dx}{3a}+\frac{2x}{3a(a+bx^2)^{3/4}}\right)}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx}{bc-ad}$$

$$\Big\downarrow \; 231$$

$$\frac{b\left(\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3a(a+bx^2)^{3/4}}+\frac{2x}{3a(a+bx^2)^{3/4}}\right)}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx}{bc-ad}$$

$$\Big\downarrow \; 229$$

$$\frac{b\left(\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}(a+bx^2)^{3/4}}+\frac{2x}{3a(a+bx^2)^{3/4}}\right)}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx}{bc-ad}$$

$$\Big\downarrow \; 312$$

$$\frac{b\left(\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}(a+bx^2)^{3/4}}+\frac{2x}{3a(a+bx^2)^{3/4}}\right)}{bc-ad}-\frac{d\sqrt{-\frac{bx^2}{a}}\int \frac{1}{\sqrt{-\frac{bx^2}{a}}(bx^2+a)^{3/4}(dx^2+c)}dx^2}{2x(bc-ad)}$$

$$\Big\downarrow \; 118$$

$$\frac{2d\sqrt{-\frac{bx^2}{a}}\int -\frac{1}{\sqrt{1-\frac{x^8}{a}}(dx^8+bc-ad)}d\sqrt[4]{bx^2+a}}{x(bc-ad)}+\frac{b\left(\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}(a+bx^2)^{3/4}}+\frac{2x}{3a(a+bx^2)^{3/4}}\right)}{bc-ad}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}(a+bx^2)^{3/4}}+\frac{2x}{3a(a+bx^2)^{3/4}}\right)}{bc-ad}-\frac{2d\sqrt{-\frac{bx^2}{a}}\int \frac{1}{\sqrt{1-\frac{x^8}{a}}(dx^8+bc-ad)}d\sqrt[4]{bx^2+a}}{x(bc-ad)}$$

$$\Big\downarrow \; 925$$

$$\frac{2d\sqrt{-\frac{bx^2}{a}}\left(-\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}-\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}+1\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}\right)}{x(bc-ad)}+\frac{b\left(\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}(a+bx^2)^{3/4}}+\frac{2x}{3a(a+bx^2)^{3/4}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1542$$

3.328. $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx$

$$\frac{2d\sqrt{-\frac{bx^2}{a}}\left(-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}\right)}{x(bc-ad)}+\frac{b\left(\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}(a+bx^2)^{3/4}}+\frac{2x}{3a(a+bx^2)^{3/4}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^2)^(7/4)*(c + d*x^2)),x]
```

output
```
(b*((2*x)/(3*a*(a + b*x^2)^(3/4)) + (2*(1 + (b*x^2)/a)^(3/4)*EllipticF[Arc
Tan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a + b*x^2)^(3/4))))/(b
*c - a*d) + (2*d*Sqrt[-((b*x^2)/a)]*(-1/2*(a^(1/4)*EllipticPi[-((Sqrt[a]*S
qrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(b*c
- a*d) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[
(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*(b*c - a*d))))/((b*c - a*d)*x)
```

3.328.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

3.328. $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx$

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 302
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/(
b*c - a*d)   Int[(a + b*x^2)^p, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^
2)^(p + 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && EqQ[Denominator[p], 4] && (EqQ[p, -5/4] || EqQ[p, -7/4]
)
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.328.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{7}{4}}\left(d\,x^{2}+c\right)}d x$$

input
```
int(1/(b*x^2+a)^(7/4)/(d*x^2+c),x)
```

output
```
int(1/(b*x^2+a)^(7/4)/(d*x^2+c),x)
```

3.328.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.328.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{7}{4}}(c+dx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(7/4)/(d*x**2+c),x)
```

output
```
Integral(1/((a + b*x**2)**(7/4)*(c + d*x**2)), x)
```

3.328.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}(dx^2+c)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)), x)
```

3.328.8 Giac [F]

$$\int \frac{1}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}\left(dx^2+c\right)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)), x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{7/4}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int(1/((a + b*x^2)^(7/4)*(c + d*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(7/4)*(c + d*x^2)), x)
```

3.329 $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx$

3.329.1 Optimal result

Integrand size = 21, antiderivative size = 274

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \frac{2bx}{5a(bc-ad)(a+bx^2)^{5/4}} + \frac{2\sqrt{b}(3bc-8ad)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}(bc-ad)^2\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(-bc+ad)^{5/2}x} - \frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(-bc+ad)^{5/2}x}$$

output
```
2/5*b*x/a/(-a*d+b*c)/(b*x^2+a)^(5/4)+2/5*(-8*a*d+3*b*c)*(1+b*x^2/a)^(1/4)*
(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1
/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2
)/(-a*d+b*c)^2/(b*x^2+a)^(1/4)+a^(1/4)*d^(3/2)*EllipticPi((b*x^2+a)^(1/4)/
a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(a*d-b*c)^(5/
2)/x-a^(1/4)*d^(3/2)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a
*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(a*d-b*c)^(5/2)/x
```

3.329.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.70 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.53

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \frac{x\left(\frac{bd(-3bc+8ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c} - \frac{6\left(3ac(5a^3d^2+3b^3cx^2(c+2dx^2\right.}{}\right.}{}$$

input
```
Integrate[1/((a + b*x^2)^(9/4)*(c + d*x^2)),x]
```

output
```
(x*((b*d*(-3*b*c + 8*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1,
5/2, -((b*x^2)/a), -((d*x^2)/c)])/c - (6*(3*a*c*(5*a^3*d^2 + 3*b^3*c*x^2*(
c + 2*d*x^2) - a^2*b*d*(10*c + 13*d*x^2) + a*b^2*(5*c^2 - 16*d^2*x^4))*App
ellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^2*(c + d*x^2)*(9
*a^2*d - 3*b^2*c*x^2 - 4*a*b*(c - 2*d*x^2))*(4*a*d*AppellF1[3/2, 1/4, 2, 5
/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)
/a), -((d*x^2)/c)])))/((a + b*x^2)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4,
1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2
, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a
), -((d*x^2)/c)])))))/(15*a^2*(b*c - a*d)^2*(a + b*x^2)^(1/4))
```

3.329.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {316, 27, 402, 27, 405, 227, 225, 212, 310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}dx$$

$$\Big\downarrow \; 316$$

$$\frac{2bx}{5a(a+bx^2)^{5/4}(bc-ad)} - \frac{2\int -\frac{3bdx^2+3bc-5ad}{2(bx^2+a)^{5/4}(dx^2+c)}dx}{5a(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3bdx^2+3bc-5ad}{(bx^2+a)^{5/4}(dx^2+c)}dx}{5a(bc-ad)}+\frac{2bx}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{2bx(3bc-8ad)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{2\int \frac{3b^2c^2-8abdc-5a^2d^2+bd(3bc-8ad)x^2}{2\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{2bx}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2bx(3bc-8ad)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\int \frac{3b^2c^2-8abdc-5a^2d^2+bd(3bc-8ad)x^2}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{2bx}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

$$\Big\downarrow \ 405$$

$$\frac{\frac{2bx(3bc-8ad)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{b(3bc-8ad)\int \frac{1}{\sqrt[4]{bx^2+a}}dx-5a^2d^2\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{2bx}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

$$\Big\downarrow \ 227$$

$$\frac{\frac{2bx(3bc-8ad)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(3bc-8ad)\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}-5a^2d^2\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{2bx}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

$$\Big\downarrow \ 225$$

$$\frac{\frac{2bx(3bc-8ad)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(3bc-8ad)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{\sqrt[4]{a+bx^2}}-5a^2d^2\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{2bx}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

3.329. $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}dx$

$\Big\downarrow$ 212

$$\frac{\frac{2bx(3bc-8ad)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(3bc-8ad)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}-5a^2d^2\int\frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{2bx}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

$\Big\downarrow$ 310

$$\frac{\frac{2bx(3bc-8ad)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(3bc-8ad)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}-\frac{10a^2d^2\sqrt{-\frac{bx^2}{a}}\int\frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{bx^2+a}{a}}\left(bc-ad+d\left(bx^2+a\right)\right)}d}{x}}{a(bc-ad)}}{5a(bc-ad)}$$

$$\frac{2bx}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

$\Big\downarrow$ 993

$$\frac{\frac{2bx(3bc-8ad)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(3bc-8ad)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}-10a^2d^2\sqrt{-\frac{bx^2}{a}}\left(\frac{\int\frac{1}{\left(\sqrt{ad-bc}+\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2}{a}}}}{2\sqrt{d}}\right.}{a(bc-ad)}}{5a(bc-ad)}$$

$$\frac{2bx}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

$\Big\downarrow$ 1542

3.329. $\displaystyle\int\frac{1}{\left(a+bx^2\right)^{9/4}\left(c+dx^2\right)}\,dx$

$$\frac{\frac{2bx(3bc-8ad)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(3bc-8ad)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}-\frac{10a^2d^2\sqrt{-\frac{bx^2}{a}}\left(\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\ldots\right.\right.}{2\sqrt{d}\sqrt{ad-bc}\ldots}\right.}{\ldots}}{a(bc-ad)}}{5a(bc-ad)}$$

$$\frac{2bx}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(9/4)*(c + d*x^2)),x]
```

output
```
(2*b*x)/(5*a*(b*c - a*d)*(a + b*x^2)^(5/4)) + ((2*b*(3*b*c - 8*a*d)*x)/(a*
(b*c - a*d)*(a + b*x^2)^(1/4)) - ((b*(3*b*c - 8*a*d)*(1 + (b*x^2)/a)^(1/4)
*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sq
rt[a]]/2, 2])/Sqrt[b]))/(a + b*x^2)^(1/4) - (10*a^2*d^2*Sqrt[-((b*x^2)/a)]
*((a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a +
 b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d]) - (a^(1/4)*Ell
ipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(
1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d])))/x)/(a*(b*c - a*d)))/(5*a*(b*c
 - a*d))
```

3.329.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

3.329. $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.329.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{9}{4}} \left(d\,x^{2}+c\right)}dx$$

input
```
int(1/(b*x^2+a)^(9/4)/(d*x^2+c),x)
```

output
```
int(1/(b*x^2+a)^(9/4)/(d*x^2+c),x)
```

3.329.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.329.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{9}{4}}(c+dx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(9/4)/(d*x**2+c),x)
```

output
```
Integral(1/((a + b*x**2)**(9/4)*(c + d*x**2)), x)
```

3.329.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)), x)
```

3.329.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)), x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{9/4}\,(d\,x^2+c)}\,dx$$

input
```
int(1/((a + b*x^2)^(9/4)*(c + d*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(9/4)*(c + d*x^2)), x)
```

3.330 $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx$

3.330.1 Optimal result

Integrand size = 21, antiderivative size = 304

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx &= \frac{2bx}{7a(bc-ad)(a+bx^2)^{7/4}} \\ &+ \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2(a+bx^2)^{3/4}} \\ &+ \frac{2\sqrt{b}(5bc-12ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{3/2}(bc-ad)^2(a+bx^2)^{3/4}} \\ &+ \frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(bc-ad)^3x} \\ &+ \frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(bc-ad)^3x}\end{aligned}$$

output
```
2/7*b*x/a/(-a*d+b*c)/(b*x^2+a)^(7/4)+2/21*b*(-12*a*d+5*b*c)*x/a^2/(-a*d+b*
c)^2/(b*x^2+a)^(3/4)+2/21*(-12*a*d+5*b*c)*(1+b*x^2/a)^(3/4)*(cos(1/2*arcta
n(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*Elliptic
F(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2)/(-a*d+b*c)^2
/(b*x^2+a)^(3/4)+a^(1/4)*d^2*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d
^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(-a*d+b*c)^3/x+a^(1/4)*d^2*Elli
pticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/
a)^(1/2)/(-a*d+b*c)^3/x
```

3.330.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.83 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(a+bx^2\right)^{11/4}\left(c+dx^2\right)}\,dx =$$

$$-\frac{x\left(\frac{bd(-5bc+12ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}+\frac{6\left(3ac\left(21a^3d^2+5b^3cx^2(3c+2dx^2)-3a^2bd(14c+3dx^2)+ab^2(21c^2-20cd\ldots\right.\right.}{(a+bx^2)(c+dx^2)\left(\ldots\right.}\right.}{6\ldots}$$

input
```
Integrate[1/((a + b*x^2)^(11/4)*(c + d*x^2)),x]
```

output
```
-1/63*(x*((b*d*(-5*b*c + 12*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3
/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/c + (6*(3*a*c*(21*a^3*d^2 + 5*b^3
*c*x^2*(3*c + 2*d*x^2) - 3*a^2*b*d*(14*c + 3*d*x^2) + a*b^2*(21*c^2 - 20*c
*d*x^2 - 24*d^2*x^4))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c
)] + b*x^2*(c + d*x^2)*(15*a^2*d - 5*b^2*c*x^2 + a*b*(-8*c + 12*d*x^2))*(4
*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*Appell
F1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((a + b*x^2)*(c + d*x^
2)*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4
*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*Appell
F1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(a^2*(b*c - a*d)^2*(
a + b*x^2)^(3/4))
```

3.330.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {316, 27, 402, 27, 405, 231, 229, 312, 118, 25, 925, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^2\right)^{11/4}\left(c+dx^2\right)}\,dx$$

$$\Big\downarrow \; 316$$

$$\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}-\frac{2\int -\frac{5bdx^2+5bc-7ad}{2(bx^2+a)^{7/4}(dx^2+c)}dx}{7a(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{5bdx^2+5bc-7ad}{(bx^2+a)^{7/4}(dx^2+c)}dx}{7a(bc-ad)}+\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{2bx(5bc-12ad)}{3a(a+bx^2)^{3/4}(bc-ad)}-\frac{2\int -\frac{5b^2c^2-12abdc+21a^2d^2+bd(5bc-12ad)x^2}{2\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{3a(bc-ad)}}{7a(bc-ad)}+\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{5b^2c^2-12abdc+21a^2d^2+bd(5bc-12ad)x^2}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{3a(bc-ad)}+\frac{2bx(5bc-12ad)}{3a(a+bx^2)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 405$$

$$\frac{\frac{21a^2d^2\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx+b(5bc-12ad)\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx}{3a(bc-ad)}+\frac{2bx(5bc-12ad)}{3a(a+bx^2)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 231$$

$$\frac{\frac{21a^2d^2\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx+\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{\left(a+bx^2\right)^{3/4}}}{3a(bc-ad)}+\frac{2bx(5bc-12ad)}{3a(a+bx^2)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 229$$

$$\frac{\frac{21a^2d^2\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}}{3a(bc-ad)}+\frac{2bx(5bc-12ad)}{3a(a+bx^2)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 312$$

3.330. $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}dx$

$$\frac{\frac{\frac{21a^2d^2\sqrt{-\frac{bx^2}{a}}\int \frac{1}{\sqrt{-\frac{bx^2}{a}}\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx^2}{2x}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}}{3a(bc-ad)}+\frac{2bx(5bc-12ad)}{3a(a+bx^2)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 118$$

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}-\frac{42a^2d^2\sqrt{-\frac{bx^2}{a}}\int -\frac{1}{\sqrt{1-\frac{x^8}{a}}\left(dx^8+bc-ad\right)}d\sqrt[4]{bx^2+a}}{x}}{3a(bc-ad)}+\frac{2bx(5bc-12ad)}{3a(a+bx^2)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{42a^2d^2\sqrt{-\frac{bx^2}{a}}\int \frac{1}{\sqrt{1-\frac{x^8}{a}}\left(dx^8+bc-ad\right)}d\sqrt[4]{bx^2+a}}{x}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}}{3a(bc-ad)}+\frac{2bx(5bc-12ad)}{3a(a+bx^2)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 925$$

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}-\frac{42a^2d^2\sqrt{-\frac{bx^2}{a}}\left(-\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}-\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}+1\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{b\cdots}}{2(bc-ad)}\cdots\right)}{x}}{3a(bc-ad)}\cdots}{7a(bc-ad)}$$

$$\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 1542$$

3.330. $\int \frac{1}{\left(a+bx^2\right)^{11/4}\left(c+dx^2\right)}\,dx$

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{(a+bx^2)^{3/4}}-\frac{42a^2d^2\sqrt{-\frac{bx^2}{a}}\left(-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\cdots\right.}{\cdots}\right.}{x}}{3a(bc-ad)}}{7a(bc-ad)}$$

$$\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(11/4)*(c + d*x^2)),x]
```

output
```
(2*b*x)/(7*a*(b*c - a*d)*(a + b*x^2)^(7/4)) + ((2*b*(5*b*c - 12*a*d)*x)/(3
*a*(b*c - a*d)*(a + b*x^2)^(3/4)) + ((2*Sqrt[a]*Sqrt[b]*(5*b*c - 12*a*d)*(
1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a + b*x
^2)^(3/4) - (42*a^2*d^2*Sqrt[-((b*x^2)/a)]*(-1/2*(a^(1/4)*EllipticPi[-((Sq
rt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1]
)/(b*c - a*d) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d],
ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*(b*c - a*d))))/x)/(3*a*(b*c - a
*d)))/(7*a*(b*c - a*d))
```

3.330.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.330. $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx$

3.330.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{11}{4}} \left(d\,x^{2}+c\right)}dx$$

input
```
int(1/(b*x^2+a)^(11/4)/(d*x^2+c),x)
```

output
```
int(1/(b*x^2+a)^(11/4)/(d*x^2+c),x)
```

3.330.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.330.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{11}{4}}(c+dx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(11/4)/(d*x**2+c),x)
```

output
```
Integral(1/((a + b*x**2)**(11/4)*(c + d*x**2)), x)
```

3.330.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{4}}(dx^2+c)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)), x)
```

3.330.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{4}}(dx^2+c)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)), x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{11/4}\,(d\,x^2+c)}\,dx$$

input
```
int(1/((a + b*x^2)^(11/4)*(c + d*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(11/4)*(c + d*x^2)), x)
```

3.331 $\int \frac{\left(a+bx^2\right)^{7/4}}{\left(c+dx^2\right)^2} dx$

3.331.1 Optimal result

Integrand size = 21, antiderivative size = 340

$$\int \frac{\left(a+bx^2\right)^{7/4}}{\left(c+dx^2\right)^2} dx = \frac{b(5bc-ad)x}{2cd^2\sqrt[4]{a+bx^2}} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/4}}{2cd\left(c+dx^2\right)} - \frac{\sqrt{a}\sqrt{b}(5bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{-bc+ad}(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd^{5/2}x} - \frac{\sqrt[4]{a}\sqrt{-bc+ad}(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd^{5/2}x}$$

output
```
1/2*b*(-a*d+5*b*c)*x/c/d^2/(b*x^2+a)^(1/4)-1/2*(-a*d+b*c)*x*(b*x^2+a)^(3/4
)/c/d/(d*x^2+c)-1/2*(-a*d+5*b*c)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2
*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/c/d^2/(b*x^2+a)^(1/4)
+1/4*a^(1/4)*(2*a*d+5*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(
1/2)/(a*d-b*c)^(1/2),I)*(a*d-b*c)^(1/2)*(-b*x^2/a)^(1/2)/c/d^(5/2)/x-1/4*a
^(1/4)*(2*a*d+5*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a
*d-b*c)^(1/2),I)*(a*d-b*c)^(1/2)*(-b*x^2/a)^(1/2)/c/d^(5/2)/x
```

3.331.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \frac{x\left(-b(-5bc+ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{6c\left(-6ac\left(2a^2d-b^2cx^2+a\right.\right.}{(c+dx^2)}\right.}{}$$

input
```
Integrate[(a + b*x^2)^(7/4)/(c + d*x^2)^2,x]
```

output
```
(x*(-(b*(-5*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)]) + (6*c*(-6*a*c*(2*a^2*d - b^2*c*x^2 + a*b*d
*x^2)*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + (-(b*c) + a
*d)*x^2*(a + b*x^2)*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x
^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((
c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)]
+ x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*
AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(12*c^2*d*(a +
 b*x^2)^(1/4))
```

3.331.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {315, 27, 405, 227, 225, 212, 310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{315}$$

$$\frac{\int \frac{b(5bc-ad)x^2+2a(bc+ad)}{2\sqrt[4]{bx^2+a}\,(dx^2+c)}dx}{2cd}-\frac{x(a+bx^2)^{3/4}(bc-ad)}{2cd\,(c+dx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b(5bc-ad)x^2+2a(bc+ad)}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{4cd}-\frac{x\left(a+bx^2\right)^{3/4}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 405$$

$$\frac{\frac{b(5bc-ad)\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{d}-\frac{(bc-ad)(2ad+5bc)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^2\right)^{3/4}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 227$$

$$\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(5bc-ad)\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{d\sqrt[4]{a+bx^2}}-\frac{(bc-ad)(2ad+5bc)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^2\right)^{3/4}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 225$$

$$\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(5bc-ad)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{d\sqrt[4]{a+bx^2}}-\frac{(bc-ad)(2ad+5bc)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^2\right)^{3/4}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 212$$

$$\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(5bc-ad)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}-\frac{(bc-ad)(2ad+5bc)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^2\right)^{3/4}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 310$$

3.331. $\int \frac{\left(a+bx^2\right)^{7/4}}{\left(c+dx^2\right)^2}dx$

$$\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(5bc-ad)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)(2ad+5bc)\int\frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{bx^2+a}{a}}\left(bc-ad+d\left(bx^2+a\right)\right)}d\sqrt[4]{bx^2-}}{dx}}{4cd}$$

$$\frac{x\left(a+bx^2\right)^{3/4}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$\downarrow$ 993

$$\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(5bc-ad)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)(2ad+5bc)\left(\frac{\int\frac{1}{\left(\sqrt{ad-bc}+\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{b}}{2\sqrt{d}}\right.}{dx}}{4cd}$$

$$\frac{x\left(a+bx^2\right)^{3/4}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$\downarrow$ 1542

$$\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(5bc-ad)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)(2ad+5bc)\left(\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{a}}\right.\right.}{2\sqrt{d}\sqrt{ad-bc}}\right.}{dx}}{4cd}$$

$$\frac{x\left(a+bx^2\right)^{3/4}(bc-ad)}{2cd\left(c+dx^2\right)}$$

input
```
Int[(a + b*x^2)^(7/4)/(c + d*x^2)^2,x]
```

output
```
-1/2*((b*c - a*d)*x*(a + b*x^2)^(3/4))/(c*d*(c + d*x^2)) + ((b*(5*b*c - a*
d)*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*Ellipti
cE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(d*(a + b*x^2)^(1/4)) - (2
*(b*c - a*d)*(5*b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*((a^(1/4)*EllipticPi[-((Sq
rt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1]
)/(2*Sqrt[d]*Sqrt[-(b*c) + a*d]) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/S
qrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt
[-(b*c) + a*d])))/(d*x))/(4*c*d)
```

3.331. $\int \frac{\left(a+bx^2\right)^{7/4}}{\left(c+dx^2\right)^2}\,dx$

3.331.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

3.331. $\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx$

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.331.4 Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{7}{4}}}{\left(d\,x^{2}+c\right)^{2}}dx$$

input
```
int((b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

output
```
int((b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

3.331.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.331.6 Sympy [F]

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{7}{4}}}{(c+dx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(7/4)/(d*x**2+c)**2,x)
```

output
```
Integral((a + b*x**2)**(7/4)/(c + d*x**2)**2, x)
```

3.331.7 Maxima [F]

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{4}}}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/4)/(d*x^2 + c)^2, x)
```

3.331.8 Giac [F]

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{4}}}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/4)/(d*x^2 + c)^2, x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{7/4}}{(d\,x^2+c)^2}\,dx$$

input
```
int((a + b*x^2)^(7/4)/(c + d*x^2)^2,x)
```

output
```
int((a + b*x^2)^(7/4)/(c + d*x^2)^2, x)
```

3.332 $\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2} dx$

3.332.1 Optimal result

Integrand size = 21, antiderivative size = 279

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2} dx = -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd(c+dx^2)} + \frac{\sqrt{a}\sqrt{b}(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2cd^2(a+bx^2)^{3/4}} - \frac{\sqrt[4]{a}(3bc+2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd^2x} - \frac{\sqrt[4]{a}(3bc+2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd^2x}$$

output

```
-1/2*(-a*d+b*c)*x*(b*x^2+a)^(1/4)/c/d/(d*x^2+c)+1/2*(a*d+3*b*c)*(1+b*x^2/a
)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1
/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2
)*b^(1/2)/c/d^2/(b*x^2+a)^(3/4)-1/4*a^(1/4)*(2*a*d+3*b*c)*EllipticPi((b*x^
2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/
d^2/x-1/4*a^(1/4)*(2*a*d+3*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)
*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d^2/x
```

3.332.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.36 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = \frac{x\left(b(3bc+ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{6c\left(-6ac(2a^2d-b^2cx^2+abd\cdots\right.}{(c+dx^2)\left(\cdots\right.}\right.}{\cdots}$$

input
```
Integrate[(a + b*x^2)^(5/4)/(c + d*x^2)^2,x]
```

output
```
(x*(b*(3*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -
((b*x^2)/a), -((d*x^2)/c)] + (6*c*(-6*a*c*(2*a^2*d - b^2*c*x^2 + a*b*d*x^2
)*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + (-(b*c) + a*d)*
x^2*(a + b*x^2)*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/
c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c
+ d*x^2)*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] +
x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*
AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))))/(12*c^2*d*(a +
 b*x^2)^(3/4))
```

3.332.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {315, 27, 405, 231, 229, 312, 118, 25, 925, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx$$

$$\Big\downarrow \; 315$$

$$\frac{\int \frac{b(3bc+ad)x^2+2a(bc+ad)}{2(bx^2+a)^{3/4}(dx^2+c)}dx}{2cd}-\frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\,(c+dx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{b(3bc+ad)x^2+2a(bc+ad)}{(bx^2+a)^{3/4}(dx^2+c)}dx}{4cd} - \frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 405$$

$$\frac{\frac{b(ad+3bc)\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx}{d} - \frac{(bc-ad)(2ad+3bc)\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{d}}{4cd} - \frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 231$$

$$\frac{\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+3bc)\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{d(a+bx^2)^{3/4}} - \frac{(bc-ad)(2ad+3bc)\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{d}}{4cd} - \frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 229$$

$$\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+3bc)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d(a+bx^2)^{3/4}} - \frac{(bc-ad)(2ad+3bc)\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{d}}{4cd} - \frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 312$$

$$\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+3bc)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d(a+bx^2)^{3/4}} - \frac{\sqrt{-\frac{bx^2}{a}}(bc-ad)(2ad+3bc)\int \frac{1}{\sqrt{-\frac{bx^2}{a}}\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx^2}{2dx}}{4cd} - \frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 118$$

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)(2ad+3bc)\int -\frac{1}{\sqrt{1-\frac{x^8}{a}}\left(dx^8+bc-ad\right)}d\sqrt[4]{bx^2+a}}{dx} + \frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+3bc)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d(a+bx^2)^{3/4}}}{4cd} - \frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 25$$

3.332. $\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx$

$$\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+3bc)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d(a+bx^2)^{3/4}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)(2ad+3bc)\int\frac{1}{\sqrt{1-\frac{x^8}{a}}\left(dx^8+bc-ad\right)}d\sqrt[4]{bx^2+a}}{dx}}{4cd}-\frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 925$$

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)(2ad+3bc)\left(-\frac{\int\frac{1}{\left(1-\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}-\frac{\int\frac{1}{\left(\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}+1\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}\right)}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+3bc)}{d(a+b}}{4cd}$$

$$\frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

$$\Big\downarrow \; 1542$$

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(bc-ad)(2ad+3bc)\left(-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}\right)}{dx}+\frac{2}{}}{4cd}$$

$$\frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

input
```
Int[(a + b*x^2)^(5/4)/(c + d*x^2)^2,x]
```

output
```
-1/2*((b*c - a*d)*x*(a + b*x^2)^(1/4))/(c*d*(c + d*x^2)) + ((2*Sqrt[a]*Sqr
t[b]*(3*b*c + a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt
[a]]/2, 2])/(d*(a + b*x^2)^(3/4)) + (2*(b*c - a*d)*(3*b*c + 2*a*d)*Sqrt[-(
(b*x^2)/a)]*(-1/2*(a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*
d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(b*c - a*d) - (a^(1/4)*Ellipt
icPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4
)], -1])/(2*(b*c - a*d))))/(d*x))/(4*c*d)
```

3.332. $\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx$

3.332.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

3.332. $\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx$

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.332.4 Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{4}}}{(d\,x^2+c)^2}dx$$

input
```
int((b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

output
```
int((b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

3.332.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.332.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{4}}}{(c+dx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
```

output
```
Integral((a + b*x**2)**(5/4)/(c + d*x**2)**2, x)
```

3.332.7 Maxima [F]

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2, x)
```

3.332.8 Giac [F]

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2, x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{5/4}}{(d\,x^2+c)^2}\,dx$$

input
```
int((a + b*x^2)^(5/4)/(c + d*x^2)^2,x)
```

output
```
int((a + b*x^2)^(5/4)/(c + d*x^2)^2, x)
```

3.333 $\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2} dx$

3.333.1 Optimal result

Integrand size = 21, antiderivative size = 309

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2} dx = -\frac{bx}{2cd\sqrt[4]{a+bx^2}} + \frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)} + \frac{\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}(bc+2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}}, \arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right), -1\right)}{4cd^{3/2}\sqrt{-bc+ad}\,x} - \frac{\sqrt[4]{a}(bc+2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}}, \arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right), -1\right)}{4cd^{3/2}\sqrt{-bc+ad}\,x}$$

output
```
-1/2*b*x/c/d/(b*x^2+a)^(1/4)+1/2*x*(b*x^2+a)^(3/4)/c/(d*x^2+c)+1/2*(1+b*x^
2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b
^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(
1/2)*b^(1/2)/c/d/(b*x^2+a)^(1/4)+1/4*a^(1/4)*(2*a*d+b*c)*EllipticPi((b*x^2
+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d
^(3/2)/x/(a*d-b*c)^(1/2)-1/4*a^(1/4)*(2*a*d+b*c)*EllipticPi((b*x^2+a)^(1/4
)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d^(3/2)/x/
(a*d-b*c)^(1/2)
```

3.333.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+bx^2\right)^{3/4}}{\left(c+dx^2\right)^2}\,dx = \frac{x\left(-\frac{bx^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}+\frac{6\left(\frac{a+bx^2}{c}-\frac{6a^2\ \cdots}{-6ac\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4\cdots\right.}\right.}{\cdots}\right.}{12\sqrt[4]{a+bx^2}\ \cdots}$$

input
```
Integrate[(a + b*x^2)^(3/4)/(c + d*x^2)^2,x]
```

output
```
(x*(-((b*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a)
, -((d*x^2)/c)])/c^2) + (6*((a + b*x^2)/c - (6*a^2*AppellF1[1/2, 1/4, 1, 3
/2, -((b*x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x
^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a)
, -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c
)])))))/(c + d*x^2)))/(12*(a + b*x^2)^(1/4))
```

3.333.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {314, 27, 405, 227, 225, 212, 310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/4}}{\left(c+dx^2\right)^2}dx$$

$$\Big\downarrow \ 314$$

$$\frac{x\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)}-\frac{\int -\frac{2a-bx^2}{2\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{2c}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2a-bx^2}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{4c}+\frac{x\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)}$$

$$\Big\downarrow \; 405$$

$$\frac{\frac{(2ad+bc)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}-\frac{b\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{d}}{4c}+\frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)}$$

$$\Big\downarrow \; 227$$

$$\frac{\frac{(2ad+bc)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}-\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{d\sqrt[4]{a+bx^2}}}{4c}+\frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)}$$

$$\Big\downarrow \; 225$$

$$\frac{\frac{(2ad+bc)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}-\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{d\sqrt[4]{a+bx^2}}}{4c}+\frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)}$$

$$\Big\downarrow \; 212$$

$$\frac{\frac{(2ad+bc)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{d}-\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}}{4c}+\frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)}$$

$$\Big\downarrow \; 310$$

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(2ad+bc)\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{bx^2+a}{a}}\left(bc-ad+d\left(bx^2+a\right)\right)}d\sqrt[4]{bx^2+a}}{dx}-\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{d\sqrt[4]{a+bx^2}}}{4c}+\frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)}$$

$$\Big\downarrow \; 993$$

3.333. $\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx$

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(2ad+bc)\left(\frac{\int \frac{1}{\left(\sqrt{ad-bc}+\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}-\frac{\int \frac{1}{\left(\sqrt{ad-bc}-\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}\right)}{dx}-\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\Bigg(}{}}{4c}$$

$$\frac{x\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)}$$

$\Big\downarrow$ 1542

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(2ad+bc)\left(\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}\right)}{dx}-\frac{b\sqrt[4]{\frac{bx^2}{a}+}}{}}{4c}$$

$$\frac{x\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)}$$

input
```
Int[(a + b*x^2)^(3/4)/(c + d*x^2)^2,x]
```

output
```
(x*(a + b*x^2)^(3/4))/(2*c*(c + d*x^2)) + (-((b*(1 + (b*x^2)/a)^(1/4)*((2*
x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]
]/2, 2])/Sqrt[b]))/(d*(a + b*x^2)^(1/4))) + (2*(b*c + 2*a*d)*Sqrt[-((b*x^2
)/a)]*((a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin
[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d]) - (a^(1/4
)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4
)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d])))/(d*x))/(4*c)
```

3.333.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

3.333. $\int \frac{\left(a+bx^2\right)^{3/4}}{\left(c+dx^2\right)^2}\,dx$

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.333. $\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx$

3.333.4 Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{(d\,x^2+c)^2}dx$$

input
```
int((b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

output
```
int((b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

3.333.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.333.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{4}}}{(c+dx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/4)/(d*x**2+c)**2,x)
```

output
```
Integral((a + b*x**2)**(3/4)/(c + d*x**2)**2, x)
```

3.333.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/4)/(d*x^2 + c)^2, x)
```

3.333.8 Giac [F]

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/4)/(d*x^2 + c)^2, x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/4}}{(d\,x^2+c)^2}\,dx$$

input
```
int((a + b*x^2)^(3/4)/(c + d*x^2)^2,x)
```

output
```
int((a + b*x^2)^(3/4)/(c + d*x^2)^2, x)
```

3.334 $\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx$

3.334.1 Optimal result

Integrand size = 21, antiderivative size = 278

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx = \frac{x\sqrt[4]{a+bx^2}}{2c(c+dx^2)} + \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2cd(a+bx^2)^{3/4}} - \frac{\sqrt[4]{a}(bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd(bc-ad)x} - \frac{\sqrt[4]{a}(bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd(bc-ad)x}$$

output
```
1/2*x*(b*x^2+a)^(1/4)/c/(d*x^2+c)+1/2*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*
b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(si
n(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/c/d/(b*x^2+a)^(3
/4)-1/4*a^(1/4)*(-2*a*d+b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d
^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d/(-a*d+b*c)/x-1/4*a^(1/4)*(-
2*a*d+b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1
/2),I)*(-b*x^2/a)^(1/2)/c/d/(-a*d+b*c)/x
```

3.334.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx$$
$$= \frac{x\left(\frac{bx^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}+\frac{6\left(\frac{a+bx^2}{c}-\frac{6a^2\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{-6ac\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4ad\,\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},2,\frac{5}{2},-\frac{bx^2}{a},-\frac{d}{}\right.\right.}\right.}{c+dx^2}\right.}{12\,(a+bx^2)^{3/4}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/(c + d*x^2)^2,x]
```

output
```
(x*((b*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a),
-((d*x^2)/c)])/c^2 + (6*((a + b*x^2)/c - (6*a^2*AppellF1[1/2, 3/4, 1, 3/2,
 -((b*x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)
/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -
((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)
])))))/(c + d*x^2)))/(12*(a + b*x^2)^(3/4))
```

3.334.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {314, 27, 405, 231, 229, 312, 118, 25, 925, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx$$
$$\Big\downarrow \text{314}$$
$$\frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)}-\frac{\int -\frac{bx^2+2a}{2(bx^2+a)^{3/4}(dx^2+c)}dx}{2c}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{bx^2+2a}{(bx^2+a)^{3/4}(dx^2+c)}dx}{4c}+\frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)}$$

$$\Big\downarrow \text{ 405}$$

$$\frac{\frac{b\int \frac{1}{(bx^2+a)^{3/4}}dx}{d}-\frac{(bc-2ad)\int \frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx}{d}}{4c}+\frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)}$$

$$\Big\downarrow \text{ 231}$$

$$\frac{\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{d(a+bx^2)^{3/4}}-\frac{(bc-2ad)\int \frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx}{d}}{4c}+\frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)}$$

$$\Big\downarrow \text{ 229}$$

$$\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d(a+bx^2)^{3/4}}-\frac{(bc-2ad)\int \frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx}{d}}{4c}+\frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)}$$

$$\Big\downarrow \text{ 312}$$

$$\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d(a+bx^2)^{3/4}}-\frac{\sqrt{-\frac{bx^2}{a}}(bc-2ad)\int \frac{1}{\sqrt{-\frac{bx^2}{a}}(bx^2+a)^{3/4}(dx^2+c)}dx^2}{2dx}}{4c}+\frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)}$$

$$\Big\downarrow \text{ 118}$$

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(bc-2ad)\int -\frac{1}{\sqrt{1-\frac{x^8}{a}}(dx^8+bc-ad)}d\sqrt[4]{bx^2+a}}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d(a+bx^2)^{3/4}}}{4c}+\frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d(a+bx^2)^{3/4}}-\frac{2\sqrt{-\frac{bx^2}{a}}(bc-2ad)\int \frac{1}{\sqrt{1-\frac{x^8}{a}}(dx^8+bc-ad)}d\sqrt[4]{bx^2+a}}{dx}}{4c}+\frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)}$$

$$\Big\downarrow \text{ 925}$$

3.334. $\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}dx$

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(bc-2ad)\left(-\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}-\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}+1\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}\right)}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\right.}{d(a+bx^2)^{3/4}}}{4c}$$

$$\frac{x\sqrt[4]{a+bx^2}}{2c(c+dx^2)}$$

$$\downarrow 1542$$

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(bc-2ad)\left(-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}\right)}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{b}{}\right.}{}}{4c}$$

$$\frac{x\sqrt[4]{a+bx^2}}{2c(c+dx^2)}$$

input
```
Int[(a + b*x^2)^(1/4)/(c + d*x^2)^2,x]
```

output
```
(x*(a + b*x^2)^(1/4))/(2*c*(c + d*x^2)) + ((2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)
/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(d*(a + b*x^2)^(3/4
)) + (2*(b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*(-1/2*(a^(1/4)*EllipticPi[-((Sqrt
[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/
(b*c - a*d) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], Ar
cSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*(b*c - a*d))))/(d*x))/(4*c)
```

3.334.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.334. $\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx$

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

3.334. $\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.334.4 Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{(d\,x^2+c)^2}dx$$

input
```
int((b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

output
```
int((b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

3.334.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.334.6 Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx = \int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/4)/(d*x**2+c)**2,x)
```

output
```
Integral((a + b*x**2)**(1/4)/(c + d*x**2)**2, x)
```

3.334.7 Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2, x)
```

3.334.8 Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2, x)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{1/4}}{(d\,x^2+c)^2}\,dx$$

input
```
int((a + b*x^2)^(1/4)/(c + d*x^2)^2,x)
```

output
```
int((a + b*x^2)^(1/4)/(c + d*x^2)^2, x)
```

3.335 $\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)^2}\,dx$

3.335.1 Optimal result

Integrand size = 21, antiderivative size = 336

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = \frac{bx}{2c(bc-ad)\sqrt[4]{a+bx^2}} - \frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\,(c+dx^2)} - \frac{\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)\sqrt[4]{a+bx^2}} - \frac{\sqrt[4]{a}(3bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c\sqrt{d}(-bc+ad)^{3/2}x} + \frac{\sqrt[4]{a}(3bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c\sqrt{d}(-bc+ad)^{3/2}x}$$

output
```
1/2*b*x/c/(-a*d+b*c)/(b*x^2+a)^(1/4)-1/2*d*x*(b*x^2+a)^(3/4)/c/(-a*d+b*c)/
(d*x^2+c)-1/2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/
2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a
^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/c/(-a*d+b*c)/(b*x^2+a)^(1/4)-1/4*a^(1/4)
*(-2*a*d+3*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b
*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(a*d-b*c)^(3/2)/x/d^(1/2)+1/4*a^(1/4)*(-2*
a*d+3*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1
/2),I)*(-b*x^2/a)^(1/2)/c/(a*d-b*c)^(3/2)/x/d^(1/2)
```

3.335.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = \frac{-6acx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\left(-6c(-2bc+2ad+bdx^2)+bdx^2\sqrt[4]{1+\frac{bx^2}{a}}(c+dx^2)\,\text{AppellF1}\right.}{12c^2(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)\,(-6}$$

input
```
Integrate[1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x]
```

output
```
(-6*a*c*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)]*(-6*c*(-2
*b*c + 2*a*d + b*d*x^2) + b*d*x^2*(1 + (b*x^2)/a)^(1/4)*(c + d*x^2)*Appell
F1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]) - d*x^3*(6*c*(a + b*x^2)
 - b*x^2*(1 + (b*x^2)/a)^(1/4)*(c + d*x^2)*AppellF1[3/2, 1/4, 1, 5/2, -((b
*x^2)/a), -((d*x^2)/c)])*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -
((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])
)/(12*c^2*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)*(-6*a*c*AppellF1[1/2,
1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2
, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x
^2)/a), -((d*x^2)/c)])))
```

3.335.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {316, 27, 405, 227, 225, 212, 310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx$$

$$\Big\downarrow \; 316$$

$$\frac{\int \frac{bdx^2+4bc-2ad}{2\sqrt[4]{bx^2+a}(dx^2+c)}dx}{2c(bc-ad)} - \frac{dx(a+bx^2)^{3/4}}{2c(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{bdx^2+2(2bc-ad)}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{405}$$

$$\frac{(3bc-2ad)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx+b\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{227}$$

$$\frac{(3bc-2ad)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{225}$$

$$\frac{(3bc-2ad)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{\sqrt[4]{a+bx^2}}}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{212}$$

$$\frac{(3bc-2ad)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{310}$$

3.335. $\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)^2}dx$

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(3bc-2ad)\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{bx^2+a}{a}}\left(bc-ad+d\left(bx^2+a\right)\right)}d\sqrt[4]{bx^2+a}}{x}+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 993

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(3bc-2ad)\left(\frac{\int \frac{1}{\left(\sqrt{ad-bc}+\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}-\frac{\int \frac{1}{\left(\sqrt{ad-bc}-\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}\right)}{x}+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\cdots\right)}{\cdots}}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 1542

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(3bc-2ad)\left(\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}\right)}{x}+\frac{b\sqrt[4]{\frac{bx^2}{a}\cdots}}{\cdots}}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x]
```

output
```
-1/2*(d*x*(a + b*x^2)^(3/4))/(c*(b*c - a*d)*(c + d*x^2)) + ((b*(1 + (b*x^2
)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqr
t[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(a + b*x^2)^(1/4) + (2*(3*b*c - 2*a*d)*S
qrt[-((b*x^2)/a)]*((a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a
*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d
]) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a +
 b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d])))/x)/(4*c*(b*c
 - a*d))
```

3.335. $\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)^2}\,dx$

3.335.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

3.335. $\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)^2}\,dx$

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.335.4 Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,(d\,x^2+c)^2}dx$$

input
```
int(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

output
```
int(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

3.335.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.335.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/4)/(d*x**2+c)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(1/4)*(c + d*x**2)**2), x)
```

3.335.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2), x)
```

3.335.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{1/4}\,(d\,x^2+c)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(1/4)*(c + d*x^2)^2), x)
```

3.336 $\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx$

3.336.1 Optimal result

Integrand size = 21, antiderivative size = 292

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx &= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)(c+dx^2)} \\ &- \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2c(bc-ad)(a+bx^2)^{3/4}} \\ &+ \frac{\sqrt[4]{a}(5bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)^2x} \\ &+ \frac{\sqrt[4]{a}(5bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)^2x}\end{aligned}$$

output
```
-1/2*d*x*(b*x^2+a)^(1/4)/c/(-a*d+b*c)/(d*x^2+c)-1/2*(1+b*x^2/a)^(3/4)*(cos
(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2))
)*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/c/
(-a*d+b*c)/(b*x^2+a)^(3/4)+1/4*a^(1/4)*(-2*a*d+5*b*c)*EllipticPi((b*x^2+a)
^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*
d+b*c)^2/x+1/4*a^(1/4)*(-2*a*d+5*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a
^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*d+b*c)^2/x
```

3.336.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx = \frac{x\left(\frac{bdx^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{-bc+ad}+\frac{c\left(36ac(-2bc+2ad+bdx^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{4},1\cdots\right.\right.}{(bc-ad)(c+dx^2)\left(-6ac\,\text{AppellF1}\left(\frac{1}{2}\cdots\right.\right.}\right.}{12c^2\cdots}$$

input
```
Integrate[1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x]
```

output
```
(x*((b*d*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a)
, -((d*x^2)/c)])/(-(b*c) + a*d) + (c*(36*a*c*(-2*b*c + 2*a*d + b*d*x^2)*Ap
pellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - 6*d*x^2*(a + b*x^2)
*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*App
ellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((b*c - a*d)*(c + d
*x^2)*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2
*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*App
ellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(12*c^2*(a + b*x^
2)^(3/4))
```

3.336.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {316, 27, 405, 231, 229, 312, 118, 25, 925, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx$$

$$\Big\downarrow \; 316$$

$$\frac{\int \frac{-bdx^2+4bc-2ad}{2(bx^2+a)^{3/4}(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)\,(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2(2bc-ad)-bdx^2}{(bx^2+a)^{3/4}(dx^2+c)}dx}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{405}$$

$$\frac{(5bc-2ad)\int \frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx-b\int \frac{1}{(bx^2+a)^{3/4}}dx}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{231}$$

$$\frac{(5bc-2ad)\int \frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx-\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{(a+bx^2)^{3/4}}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{229}$$

$$\frac{(5bc-2ad)\int \frac{1}{(bx^2+a)^{3/4}(dx^2+c)}dx-\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{(a+bx^2)^{3/4}}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{312}$$

$$\frac{\frac{\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\int \frac{1}{\sqrt{-\frac{bx^2}{a}}\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx^2}{2x}-\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{(a+bx^2)^{3/4}}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{118}$$

$$\frac{-\frac{2\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\int -\frac{1}{\sqrt{1-\frac{x^8}{a}}\left(dx^8+bc-ad\right)}d\sqrt[4]{bx^2+a}}{x}-\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{(a+bx^2)^{3/4}}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\int \frac{1}{\sqrt{1-\frac{x^8}{a}}\left(dx^8+bc-ad\right)}d\sqrt[4]{bx^2+a}}{x}-\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{(a+bx^2)^{3/4}}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{925}$$

3.336. $\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx$

$$-\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\left(-\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}-\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}+1\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}\right)}{x}-\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\text{arc}\right.}{(a+bx^2)^{3/4}}}{4c(bc-ad)}$$

$$\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{1542}$$

$$-\frac{\frac{2\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\left(-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}\right)}{x}-\frac{2\sqrt{a}\sqrt{b}}{}}{4c(bc-ad)}$$

$$\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x]
```

output
```
-1/2*(d*x*(a + b*x^2)^(1/4))/(c*(b*c - a*d)*(c + d*x^2)) + ((-2*Sqrt[a]*Sq
rt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
a + b*x^2)^(3/4) - (2*(5*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*(-1/2*(a^(1/4)*El
lipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)
/a^(1/4)], -1])/(b*c - a*d) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-
(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*(b*c - a*d))))/x)
/(4*c*(b*c - a*d))
```

3.336.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

3.336. $\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2} \, dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.336.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{3}{4}} \left(d\,x^{2}+c\right)^{2}}dx$$

input
```
int(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

output
```
int(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

3.336.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/4}\,(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.336.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/4}\,(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{4}}\,(c+dx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/4)/(d*x**2+c)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(3/4)*(c + d*x**2)**2), x)
```

3.336.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2), x)
```

3.336.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2), x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{3/4}\,(d\,x^2+c)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(3/4)*(c + d*x^2)^2), x)
```

3.337 $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx$

3.337.1 Optimal result

Integrand size = 21, antiderivative size = 314

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = -\frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} + \frac{\sqrt{b}(4bc+ad)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}c(bc-ad)^2\sqrt[4]{a+bx^2}} - \frac{\sqrt[4]{a}\sqrt{d}(7bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(-bc+ad)^{5/2}x} + \frac{\sqrt[4]{a}\sqrt{d}(7bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(-bc+ad)^{5/2}x}$$

output
```
-1/2*d*x/c/(-a*d+b*c)/(b*x^2+a)^(1/4)/(d*x^2+c)+1/2*(a*d+4*b*c)*(1+b*x^2/a
)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1
/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2
)/c/(-a*d+b*c)^2/(b*x^2+a)^(1/4)/a^(1/2)-1/4*a^(1/4)*(-2*a*d+7*b*c)*Ellipt
icPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*d^(1/2)*(
-b*x^2/a)^(1/2)/c/(a*d-b*c)^(5/2)/x+1/4*a^(1/4)*(-2*a*d+7*b*c)*EllipticPi(
(b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*d^(1/2)*(-b*x^2
/a)^(1/2)/c/(a*d-b*c)^(5/2)/x
```

3.337.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.33 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = \frac{x\left(-bd(4bc+ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{c\left(36ac(2a^2\ldots\right.}{\ldots}\right.}{\ldots}$$

input
```
Integrate[1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x]
```

output
```
(x*(-(b*d*(4*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/
2, -((b*x^2)/a), -((d*x^2)/c)]) + (c*(36*a*c*(2*a^2*d^2 + a*b*d*(-4*c + d*
x^2) + 2*b^2*c*(c + 2*d*x^2))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((
d*x^2)/c)] - 6*x^2*(a^2*d^2 + a*b*d^2*x^2 + 4*b^2*c*(c + d*x^2))*(4*a*d*Ap
pellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5
/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1[1/
2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 1/4
, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((
b*x^2)/a), -((d*x^2)/c)])))))/(12*a*c^2*(b*c - a*d)^2*(a + b*x^2)^(1/4))
```

3.337.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {316, 27, 402, 27, 405, 227, 225, 212, 310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}dx$$

$$\Big\downarrow \; 316$$

$$\frac{\int \frac{-3bdx^2+4bc-2ad}{2(bx^2+a)^{5/4}(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx}{2c\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2(2bc-ad)-3bdx^2}{(bx^2+a)^{5/4}(dx^2+c)}dx}{4c(bc-ad)}-\frac{dx}{2c\sqrt[4]{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{2bx(ad+4bc)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{2\int \frac{4b^2c^2+8abdc-2a^2d^2+bd(4bc+ad)x^2}{2\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\sqrt[4]{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2bx(ad+4bc)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\int \frac{bd(4bc+ad)x^2+2\left(2b^2c^2+4abdc-a^2d^2\right)}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\sqrt[4]{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{ 405}$$

$$\frac{\frac{2bx(ad+4bc)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{b(ad+4bc)\int \frac{1}{\sqrt[4]{bx^2+a}}dx+ad(7bc-2ad)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\sqrt[4]{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{ 227}$$

$$\frac{\frac{2bx(ad+4bc)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(ad+4bc)\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}+ad(7bc-2ad)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\sqrt[4]{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{ 225}$$

3.337. $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx$

$$\frac{\frac{2bx(ad+4bc)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(ad+4bc)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{\sqrt[4]{a+bx^2}}+ad(7bc-2ad)\int\frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)}$$

$$\Big\downarrow \; 212$$

$$\frac{\frac{2bx(ad+4bc)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{ad(7bc-2ad)\int\frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(ad+4bc)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)}$$

$$\Big\downarrow \; 310$$

$$\frac{\frac{2bx(ad+4bc)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{2ad\sqrt{-\frac{bx^2}{a}}(7bc-2ad)\int\frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{bx^2+a}{a}}\left(bc-ad+d\left(bx^2+a\right)\right)}d\sqrt[4]{bx^2+a}}{x}+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(ad+4bc)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\right.}{}\right.}{\sqrt[4]{a+bx^2}}}{a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{2c\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)}$$

$$\Big\downarrow \; 993$$

3.337. $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx$

$$\frac{\frac{2bx(ad+4bc)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{2ad\sqrt{-\frac{bx^2}{a}}(7bc-2ad)\left(\frac{\int \frac{1}{\left(\sqrt{ad-bc}+\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}-\frac{\int \frac{1}{\left(\sqrt{ad-bc}-\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2-}}{2\sqrt{d}}\right.}{x}}{a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{2c\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)}$$

$\downarrow$ 1542

$$\frac{\frac{2bx(ad+4bc)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{2ad\sqrt{-\frac{bx^2}{a}}(7bc-2ad)\left(\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+}}{\sqrt[4]{a}}\right.\right.}{2\sqrt{d}\sqrt{ad-bc}}\right.}{x}}{a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{2c\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x]
```

output
```
-1/2*(d*x)/(c*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)) + ((2*b*(4*b*c +
a*d)*x)/(a*(b*c - a*d)*(a + b*x^2)^(1/4)) - ((b*(4*b*c + a*d)*(1 + (b*x^2)
/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt
[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(a + b*x^2)^(1/4) + (2*a*d*(7*b*c - 2*a*d
)*Sqrt[-((b*x^2)/a)]*((a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c)
+ a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) +
a*d]) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(
a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d])))/x)/(a*(b*
c - a*d)))/(4*c*(b*c - a*d))
```

3.337. $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx$

3.337.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.337. $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx$

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.337.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{5}{4}} \left(d\,x^{2}+c\right)^{2}}dx$$

input
```
int(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

output
```
int(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

3.337.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/4}\,(c+dx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.337.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{4}}(c+dx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(5/4)*(c + d*x**2)**2), x)
```

3.337.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2), x)
```

3.337.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2), x)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx=\int \frac{1}{(b\,x^2+a)^{5/4}\,(d\,x^2+c)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(5/4)*(c + d*x^2)^2), x)
```

3.338 $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2} dx$

3.338.1 Optimal result

Integrand size = 21, antiderivative size = 345

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2} dx &= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2(a+bx^2)^{3/4}} \\ &- \frac{dx}{2c(bc-ad)(a+bx^2)^{3/4}(c+dx^2)} \\ &+ \frac{\sqrt{b}(4bc+3ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\sqrt{a}c(bc-ad)^2(a+bx^2)^{3/4}} \\ &- \frac{\sqrt[4]{a}d(9bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)^3x} \\ &- \frac{\sqrt[4]{a}d(9bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)^3x}\end{aligned}$$

output
```
1/6*b*(3*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(3/4)-1/2*d*x/c/(-a*d+b*c
)/(b*x^2+a)^(3/4)/(d*x^2+c)+1/6*(3*a*d+4*b*c)*(1+b*x^2/a)^(3/4)*(cos(1/2*a
rctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*Elli
pticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/c/(-a*d+b*c)^2/(
b*x^2+a)^(3/4)/a^(1/2)-1/4*a^(1/4)*d*(-2*a*d+9*b*c)*EllipticPi((b*x^2+a)^(
1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*d+
b*c)^3/x-1/4*a^(1/4)*d*(-2*a*d+9*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a
^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*d+b*c)^3/x
```

3.338.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx = \frac{x\left(bd(4bc+3ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{c\left(36ac(6a\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x]
```

output
```
(x*(b*d*(4*b*c + 3*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/
2, -((b*x^2)/a), -((d*x^2)/c)] + (c*(36*a*c*(6*a^2*d^2 + 3*a*b*d*(-4*c + d
*x^2) + 2*b^2*c*(3*c + 2*d*x^2))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a),
-((d*x^2)/c)] - 6*x^2*(3*a^2*d^2 + 3*a*b*d^2*x^2 + 4*b^2*c*(c + d*x^2))*(4
*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*Appell
F1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*Ap
pellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1
[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4,
1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(36*a*c^2*(b*c - a*d)^2*(a + b*x^
2)^(3/4))
```

3.338.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {316, 27, 402, 27, 405, 231, 229, 312, 118, 25, 925, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx$$

$$\Big\downarrow \ 316$$

$$\frac{\int \frac{-5bdx^2+4bc-2ad}{2(bx^2+a)^{7/4}(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx}{2c\,(a+bx^2)^{3/4}\,(c+dx^2)\,(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2(2bc-ad)-5bdx^2}{(bx^2+a)^{7/4}(dx^2+c)}dx}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 402

$$\frac{\frac{2bx(3ad+4bc)}{3a(a+bx^2)^{3/4}(bc-ad)}-\frac{2\int -\frac{4b^2c^2-24abdc+6a^2d^2+bd(4bc+3ad)x^2}{2\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{3a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{bd(4bc+3ad)x^2+2\left(2b^2c^2-12abdc+3a^2d^2\right)}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{3a(bc-ad)}+\frac{2bx(3ad+4bc)}{3a(a+bx^2)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 405

$$\frac{\frac{b(3ad+4bc)\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx-3ad(9bc-2ad)\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{3a(bc-ad)}+\frac{2bx(3ad+4bc)}{3a(a+bx^2)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 231

$$\frac{\frac{\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad+4bc)\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{\left(a+bx^2\right)^{3/4}}-3ad(9bc-2ad)\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{3a(bc-ad)}+\frac{2bx(3ad+4bc)}{3a(a+bx^2)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 229

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad+4bc)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}-3ad(9bc-2ad)\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{3a(bc-ad)}+\frac{2bx(3ad+4bc)}{3a(a+bx^2)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 312

3.338. $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx$

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad+4bc)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}-\frac{3ad\sqrt{-\frac{bx^2}{a}}(9bc-2ad)\int\frac{1}{\sqrt{-\frac{bx^2}{a}}\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx^2}{2x}}{3a(bc-ad)}+\frac{2bx(3ad+4bc)}{3a(a+bx^2)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 118

$$\frac{\frac{\frac{6ad\sqrt{-\frac{bx^2}{a}}(9bc-2ad)\int-\frac{1}{\sqrt{1-\frac{x^8}{a}}\left(dx^8+bc-ad\right)}d\sqrt[4]{bx^2+a}}{x}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad+4bc)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}}{3a(bc-ad)}+\frac{2bx(3ad+4bc)}{3a(a+bx^2)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad+4bc)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}-\frac{6ad\sqrt{-\frac{bx^2}{a}}(9bc-2ad)\int\frac{1}{\sqrt{1-\frac{x^8}{a}}\left(dx^8+bc-ad\right)}d\sqrt[4]{bx^2+a}}{x}}{3a(bc-ad)}+\frac{2bx(3ad+4bc)}{3a(a+bx^2)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 925

$$\frac{\frac{\frac{6ad\sqrt{-\frac{bx^2}{a}}(9bc-2ad)\left(-\frac{\int\frac{1}{\left(1-\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}-\frac{\int\frac{1}{\left(\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}+1\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}\right)}{x}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad+4bc)\text{EllipticF}\left(\frac{1}{2}\text{ a}\right.}{\left(a+bx^2\right)^{3/4}}}{3a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 1542

3.338. $\int\frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx$

$$\frac{\frac{\frac{6ad\sqrt{-\frac{bx^2}{a}}(9bc-2ad)\left(-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2(bc-ad)}\right)}{x}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}\right.}{}}{3a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x]
```

output
```
-1/2*(d*x)/(c*(b*c - a*d)*(a + b*x^2)^(3/4)*(c + d*x^2)) + ((2*b*(4*b*c +
3*a*d)*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/4)) + ((2*Sqrt[a]*Sqrt[b]*(4*b*c
 + 3*a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(a + b*x^2)^(3/4) + (6*a*d*(9*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*(-1/2*(a^
(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^
2)^(1/4)/a^(1/4)], -1])/(b*c - a*d) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d]
)/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*(b*c - a*
d))))/x)/(3*a*(b*c - a*d)))/(4*c*(b*c - a*d))
```

3.338.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.338. $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx$

3.338.4 Maple [F]

$$\int \frac{1}{(b\,x^{2}+a)^{\frac{7}{4}}\,(d\,x^{2}+c)^{2}}dx$$

input
```
int(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

output
```
int(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

3.338.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{7/4}\,(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.338.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{7/4}\,(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{7}{4}}\,(c+dx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(7/4)/(d*x**2+c)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(7/4)*(c + d*x**2)**2), x)
```

3.338.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2), x)
```

3.338.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2), x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{7/4}\,(d\,x^2+c)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(7/4)*(c + d*x^2)^2), x)
```

3.339 $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx$

3.339.1 Optimal result

Integrand size = 21, antiderivative size = 371

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx &= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2(a+bx^2)^{5/4}} \\ &- \frac{dx}{2c(bc-ad)(a+bx^2)^{5/4}(c+dx^2)} \\ &+ \frac{\sqrt{b}(12b^2c^2-52abcd-5a^2d^2)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10a^{3/2}c(bc-ad)^3\sqrt[4]{a+bx^2}} \\ &- \frac{\sqrt[4]{a}d^{3/2}(11bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(-bc+ad)^{7/2}x} \\ &+ \frac{\sqrt[4]{a}d^{3/2}(11bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(-bc+ad)^{7/2}x}\end{aligned}$$

output
```
1/10*b*(5*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(5/4)-1/2*d*x/c/(-a*d+b*
c)/(b*x^2+a)^(5/4)/(d*x^2+c)+1/10*(-5*a^2*d^2-52*a*b*c*d+12*b^2*c^2)*(1+b*
x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x
*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b
^(1/2)/a^(3/2)/c/(-a*d+b*c)^3/(b*x^2+a)^(1/4)-1/4*a^(1/4)*d^(3/2)*(-2*a*d+
11*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2
),I)*(-b*x^2/a)^(1/2)/c/(a*d-b*c)^(7/2)/x+1/4*a^(1/4)*d^(3/2)*(-2*a*d+11*b
*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*
(-b*x^2/a)^(1/2)/c/(a*d-b*c)^(7/2)/x
```

3.339.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.70 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx = \frac{bd(-12b^2c^2+52abcd+5a^2d^2)\,x^3\sqrt[4]{1+\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right.}{}$$

input
```
Integrate[1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x]
```

output
```
(b*d*(-12*b^2*c^2 + 52*a*b*c*d + 5*a^2*d^2)*x^3*(1 + (b*x^2)/a)^(1/4)*Appe
llF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + (6*c*(-6*a*c*x*(10*a^
4*d^3 + 15*a^3*b*d^2*(-2*c + d*x^2) - 6*b^4*c^2*x^2*(c + 2*d*x^2) + a^2*b^
2*d*(30*c^2 + 26*c*d*x^2 + 5*d^2*x^4) + 2*a*b^3*c*(-5*c^2 + 5*c*d*x^2 + 26
*d^2*x^4))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^3*(5
*a^4*d^3 + 10*a^3*b*d^3*x^2 - 12*b^4*c^2*x^2*(c + d*x^2) + a^2*b^2*d*(56*c
^2 + 56*c*d*x^2 + 5*d^2*x^4) + 4*a*b^3*c*(-4*c^2 + 9*c*d*x^2 + 13*d^2*x^4)
)*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*Appe
llF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((a + b*x^2)*(c + d*
x^2)*(6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(
4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF
1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))))/(60*a^2*c^2*(b*c - a*d
)^3*(a + b*x^2)^(1/4))
```

3.339.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {316, 27, 402, 27, 402, 27, 405, 227, 225, 212, 310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx$$

$$\Big\downarrow \; 316$$

$$\frac{\int \frac{-7bdx^2+4bc-2ad}{2(bx^2+a)^{9/4}(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2(2bc-ad)-7bdx^2}{(bx^2+a)^{9/4}(dx^2+c)}dx}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{2bx(5ad+4bc)}{5a(a+bx^2)^{5/4}(bc-ad)}-\frac{2\int -\frac{12b^2c^2-40abdc+10a^2d^2+3bd(4bc+5ad)x^2}{2\left(bx^2+a\right)^{5/4}\left(dx^2+c\right)}dx}{5a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3bd(4bc+5ad)x^2+2\left(6b^2c^2-20abdc+5a^2d^2\right)}{\left(bx^2+a\right)^{5/4}\left(dx^2+c\right)}dx}{5a(bc-ad)}+\frac{2bx(5ad+4bc)}{5a(a+bx^2)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{2bx\left(-5a^2d^2-52abcd+12b^2c^2\right)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{2\int \frac{12b^3c^3-52ab^2dc^2-60a^2bd^2c+10a^3d^3+bd\left(12b^2c^2-52abdc-5a^2d^2\right)x^2}{2\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{2bx(5ad+4bc)}{5a(a+bx^2)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{2bx\left(-5a^2d^2-52abcd+12b^2c^2\right)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\int \frac{bd\left(12b^2c^2-52abdc-5a^2d^2\right)x^2+2\left(6b^3c^3-26ab^2dc^2-30a^2bd^2c+5a^3d^3\right)}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{2bx(5ad+4bc)}{5a(a+bx^2)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 405$$

3.339. $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx$

$$\frac{\frac{\frac{2bx\left(-5a^2d^2-52abcd+12b^2c^2\right)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{b\left(-5a^2d^2-52abcd+12b^2c^2\right)\int\frac{1}{\sqrt[4]{bx^2+a}}dx-5a^2d^2(11bc-2ad)\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{2bx(5ad+4bc)}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 227$$

$$\frac{\frac{\frac{2bx\left(-5a^2d^2-52abcd+12b^2c^2\right)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2d^2-52abcd+12b^2c^2\right)\int\frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}-5a^2d^2(11bc-2ad)\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{2bx(\cdots}{5a(a+bx\cdots}}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 225$$

$$\frac{\frac{\frac{2bx\left(-5a^2d^2-52abcd+12b^2c^2\right)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2d^2-52abcd+12b^2c^2\right)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{\sqrt[4]{a+bx^2}}-5a^2d^2(11bc-2ad)\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+\cdots\right.}}{a(bc-ad)}}{5a(bc-ad)}\cdots}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 212$$

$$\frac{\frac{\frac{2bx\left(-5a^2d^2-52abcd+12b^2c^2\right)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2d^2-52abcd+12b^2c^2\right)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}-5a^2d^2(11bc-2ad)\int\frac{\cdots}{\sqrt[4]{bx^2+\cdots}}}{a(bc-ad)}}{5a(bc-ad)}\cdots}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

3.339. $\int\frac{1}{\left(a+bx^2\right)^{9/4}\left(c+dx^2\right)^2}\,dx$

$$\Big\downarrow \ 310$$

$$\frac{\frac{2bx\left(-5a^2d^2-52abcd+12b^2c^2\right)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2d^2-52abcd+12b^2c^2\right)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}-\frac{10a^2d^2\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\int -\cdots}{\sqrt{\cdots}}}{a(bc-ad)}}{5a(bc-ad)}\cdots}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \ 993$$

$$\frac{\frac{2bx\left(-5a^2d^2-52abcd+12b^2c^2\right)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2d^2-52abcd+12b^2c^2\right)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}-\frac{10a^2d^2\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\left(\int \cdots\right.}{\cdots}}{a(bc-ad)}}{5a(bc-ad)}\cdots}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \ 1542$$

$$\frac{\frac{2bx\left(-5a^2d^2-52abcd+12b^2c^2\right)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2d^2-52abcd+12b^2c^2\right)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}-\frac{10a^2d^2\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\left(\sqrt[4]{\cdots}\right.}{\cdots}}{a(bc-ad)}}{5a(bc-ad)}\cdots}{4c(bc-ad\cdots}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x]
```

3.339. $\int \frac{1}{\left(a+bx^2\right)^{9/4}\left(c+dx^2\right)^2}\,dx$

output
```
-1/2*(d*x)/(c*(b*c - a*d)*(a + b*x^2)^(5/4)*(c + d*x^2)) + ((2*b*(4*b*c +
5*a*d)*x)/(5*a*(b*c - a*d)*(a + b*x^2)^(5/4)) + ((2*b*(12*b^2*c^2 - 52*a*b
*c*d - 5*a^2*d^2)*x)/(a*(b*c - a*d)*(a + b*x^2)^(1/4)) - ((b*(12*b^2*c^2 -
 52*a*b*c*d - 5*a^2*d^2)*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4
) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(a +
 b*x^2)^(1/4) - (10*a^2*d^2*(11*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*((a^(1/4)*
EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/
4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d]) - (a^(1/4)*EllipticPi[(Sq
rt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])
/(2*Sqrt[d]*Sqrt[-(b*c) + a*d])))/x)/(a*(b*c - a*d)))/(5*a*(b*c - a*d)))/(
4*c*(b*c - a*d))
```

3.339.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

3.339. $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx$

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.339.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{9}{4}} \left(d\,x^{2}+c\right)^{2}}dx$$

input
```
int(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x)
```

output
```
int(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x)
```

3.339.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.339.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{9}{4}}(c+dx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(9/4)/(d*x**2+c)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(9/4)*(c + d*x**2)**2), x)
```

3.339.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2), x)
```

3.339.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2), x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{9/4}\,(d\,x^2+c)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(9/4)*(c + d*x^2)^2), x)
```

3.340 $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx$

3.340.1 Optimal result

Integrand size = 21, antiderivative size = 419

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx &= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2(a+bx^2)^{7/4}} \\ &+ \frac{b(20b^2c^2-76abcd-21a^2d^2)\,x}{42a^2c(bc-ad)^3(a+bx^2)^{3/4}} - \frac{dx}{2c(bc-ad)(a+bx^2)^{7/4}(c+dx^2)} \\ &+ \frac{\sqrt{b}(20b^2c^2-76abcd-21a^2d^2)\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{42a^{3/2}c(bc-ad)^3(a+bx^2)^{3/4}} \\ &+ \frac{\sqrt[4]{a}d^2(13bc-2ad)\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)^4x} \\ &+ \frac{\sqrt[4]{a}d^2(13bc-2ad)\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)^4x}\end{aligned}$$

output
```
1/14*b*(7*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(7/4)+1/42*b*(-21*a^2*d^
2-76*a*b*c*d+20*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^2+a)^(3/4)-1/2*d*x/c/(-
a*d+b*c)/(b*x^2+a)^(7/4)/(d*x^2+c)+1/42*(-21*a^2*d^2-76*a*b*c*d+20*b^2*c^2
)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*a
rctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(
1/2))*b^(1/2)/a^(3/2)/c/(-a*d+b*c)^3/(b*x^2+a)^(3/4)+1/4*a^(1/4)*d^2*(-2*a
*d+13*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(
1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*d+b*c)^4/x+1/4*a^(1/4)*d^2*(-2*a*d+13*b*c)*
EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*
x^2/a)^(1/2)/c/(-a*d+b*c)^4/x
```

3.340.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.65 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx = \frac{\frac{bd(-20b^2c^2+76abcd+21a^2d^2)x^3\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(-bc+ad)^3}+\frac{6c\left(-6acx(42a^4d^3\right.}{}}{}$$

input
```
Integrate[1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x]
```

output
```
((b*d*(-20*b^2*c^2 + 76*a*b*c*d + 21*a^2*d^2)*x^3*(1 + (b*x^2)/a)^(3/4)*Ap
pellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(-(b*c) + a*d)^3 + (
6*c*(-6*a*c*x*(42*a^4*d^3 + 63*a^3*b*d^2*(-2*c + d*x^2) - 10*b^4*c^2*x^2*(
3*c + 2*d*x^2) + a^2*b^2*d*(126*c^2 - 38*c*d*x^2 + 21*d^2*x^4) + 2*a*b^3*c
*(-21*c^2 + 41*c*d*x^2 + 38*d^2*x^4))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)
/a), -((d*x^2)/c)] + x^3*(21*a^4*d^3 + 42*a^3*b*d^3*x^2 - 20*b^4*c^2*x^2*(
c + d*x^2) + 4*a*b^3*c*(-8*c^2 + 11*c*d*x^2 + 19*d^2*x^4) + a^2*b^2*d*(88*
c^2 + 88*c*d*x^2 + 21*d^2*x^4))*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2
)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*
x^2)/c)])))/((b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)*(6*a*c*AppellF1[1/2, 3/
4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 3/4, 2,
5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x
^2)/a), -((d*x^2)/c)]))))/(252*a^2*c^2*(a + b*x^2)^(3/4))
```

3.340.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {316, 27, 402, 27, 402, 27, 405, 231, 229, 312, 118, 25, 925, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}dx$$

$$\Big\downarrow \ 316$$

$$\frac{\int \frac{-9bdx^2+4bc-2ad}{2(bx^2+a)^{11/4}(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx}{2c(a+bx^2)^{7/4}(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2(2bc-ad)-9bdx^2}{(bx^2+a)^{11/4}(dx^2+c)}dx}{4c(bc-ad)}-\frac{dx}{2c(a+bx^2)^{7/4}(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{2bx(7ad+4bc)}{7a(a+bx^2)^{7/4}(bc-ad)}-\frac{2\int -\frac{20b^2c^2-56abdc+14a^2d^2+5bd(4bc+7ad)x^2}{2\left(bx^2+a\right)^{7/4}\left(dx^2+c\right)}dx}{7a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c(a+bx^2)^{7/4}(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{5bd(4bc+7ad)x^2+2\left(10b^2c^2-28abdc+7a^2d^2\right)}{\left(bx^2+a\right)^{7/4}\left(dx^2+c\right)}dx}{7a(bc-ad)}+\frac{2bx(7ad+4bc)}{7a(a+bx^2)^{7/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c(a+bx^2)^{7/4}(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\frac{2bx\left(-21a^2d^2-76abcd+20b^2c^2\right)}{3a\left(a+bx^2\right)^{3/4}(bc-ad)}-\frac{2\int -\frac{20b^3c^3-76ab^2dc^2+252a^2bd^2c-42a^3d^3+bd\left(20b^2c^2-76abdc-21a^2d^2\right)x^2}{2\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{3a(bc-ad)}}{7a(bc-ad)}+\frac{2bx(7ad+4bc)}{7a(a+bx^2)^{7/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c(a+bx^2)^{7/4}(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{bd\left(20b^2c^2-76abdc-21a^2d^2\right)x^2+2\left(10b^3c^3-38ab^2dc^2+126a^2bd^2c-21a^3d^3\right)}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{3a(bc-ad)}+\frac{2bx\left(-21a^2d^2-76abcd+20b^2c^2\right)}{3a\left(a+bx^2\right)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{2bx(7ad+4bc)}{7a(a+bx^2)^{7/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)(bc-ad)}$$

$\downarrow$ 405

$$\frac{\frac{\frac{b\left(-21a^2d^2-76abcd+20b^2c^2\right)\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx+21a^2d^2(13bc-2ad)\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{3a(bc-ad)}+\frac{2bx\left(-21a^2d^2-76abcd+20b^2c^2\right)}{3a\left(a+bx^2\right)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{2bx(7ad+4bc)}{7a(a+bx^2)^{7/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)(bc-ad)}$$

$\downarrow$ 231

$$\frac{\frac{\frac{\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-21a^2d^2-76abcd+20b^2c^2\right)\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{\left(a+bx^2\right)^{3/4}}+21a^2d^2(13bc-2ad)\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx}{3a(bc-ad)}+\frac{2bx\left(-21a^2d^2-76abcd+20b^2c^2\right)}{3a\left(a+bx^2\right)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{2bx(7ad+4bc)}{7a(a+bx^2)^{7/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)(bc-ad)}$$

$\downarrow$ 229

$$\frac{\frac{\frac{21a^2d^2(13bc-2ad)\int \frac{1}{\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-21a^2d^2-76abcd+20b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}}{3a(bc-ad)}+\frac{2bx\left(-21a^2d^2-76abcd+20b^2c^2\right)}{3a\left(a+bx^2\right)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{2bx(7ad+4bc)}{7a(a+bx^2)^{7/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{2c\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)(bc-ad)}$$

$\downarrow$ 312

3.340. $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx$

$$\frac{\frac{\frac{\frac{21a^2d^2\sqrt{-\frac{bx^2}{a}}(13bc-2ad)\int \frac{1}{\sqrt{-\frac{bx^2}{a}}\left(bx^2+a\right)^{3/4}\left(dx^2+c\right)}dx^2}{2x}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-21a^2d^2-76abcd+20b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}}{3a(bc-ad)}+\frac{2bx\left(-21a^2\ldots\right.}{3a\left(a+b\ldots\right.}}{7a(bc-ad)}\ldots}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 118$$

$$\frac{\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-21a^2d^2-76abcd+20b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}-\frac{42a^2d^2\sqrt{-\frac{bx^2}{a}}(13bc-2ad)\int -\frac{1}{\sqrt{1-\frac{x^8}{a}}\left(dx^8+bc-ad\right)}d\sqrt[4]{bx^2+a}}{x}}{3a(bc-ad)}+\frac{2bx\left(\ldots\right.}{3\ldots}}{7a(bc-ad)}\ldots}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\frac{42a^2d^2\sqrt{-\frac{bx^2}{a}}(13bc-2ad)\int \frac{1}{\sqrt{1-\frac{x^8}{a}}\left(dx^8+bc-ad\right)}d\sqrt[4]{bx^2+a}}{x}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-21a^2d^2-76abcd+20b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}}{3a(bc-ad)}+\frac{2bx\left(-2\ldots\right.}{3a\ldots}}{7a(bc-ad)}\ldots}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 925$$

$$\frac{\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-21a^2d^2-76abcd+20b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}-\frac{42a^2d^2\sqrt{-\frac{bx^2}{a}}(13bc-2ad)\left(-\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^4}{\sqrt{ad-bc}}\right)\sqrt{1-\frac{x^8}{a}}}d\sqrt[4]{bx^2+a}}{2(bc-ad)}-\frac{\int\ldots}{\left(\ldots\right.}\ldots\right.}{x}\ldots}{3a(bc-ad)}\ldots}{7a(bc-ad)}\ldots}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 1542$$

3.340. $\int \frac{1}{\left(a+bx^2\right)^{11/4}\left(c+dx^2\right)^2}\,dx$

$$\frac{\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-21a^2d^2-76abcd+20b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{(a+bx^2)^{3/4}}-\frac{42a^2d^2\sqrt{-\frac{bx^2}{a}}(13bc-2ad)\left(-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+}}{\sqrt[4]{a}}\right.\right.}{2(bc-ad)}\right.}{x}}{3a(bc-ad)}}{7a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{2c\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x]
```

output
```
-1/2*(d*x)/(c*(b*c - a*d)*(a + b*x^2)^(7/4)*(c + d*x^2)) + ((2*b*(4*b*c +
7*a*d)*x)/(7*a*(b*c - a*d)*(a + b*x^2)^(7/4)) + ((2*b*(20*b^2*c^2 - 76*a*b
*c*d - 21*a^2*d^2)*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/4)) + ((2*Sqrt[a]*Sq
rt[b]*(20*b^2*c^2 - 76*a*b*c*d - 21*a^2*d^2)*(1 + (b*x^2)/a)^(3/4)*Ellipti
cF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a + b*x^2)^(3/4) - (42*a^2*d^2*(13*
b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*(-1/2*(a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[
d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(b*c - a*
d) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a +
 b*x^2)^(1/4)/a^(1/4)], -1])/(2*(b*c - a*d))))/x)/(3*a*(b*c - a*d)))/(7*a*
(b*c - a*d)))/(4*c*(b*c - a*d))
```

3.340.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

3.340. $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2} \, dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.340.4 Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{11}{4}}\,(d\,x^2+c)^2}dx$$

input
```
int(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x)
```

output
```
int(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x)
```

3.340.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{11/4}\,(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.340.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{11/4}\,(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{11}{4}}\,(c+dx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(11/4)/(d*x**2+c)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(11/4)*(c + d*x**2)**2), x)
```

3.340.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2), x)
```

3.340.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2), x)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{11/4}\,(d\,x^2+c)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(11/4)*(c + d*x^2)^2), x)
```

3.341 $\int (a+bx^2)^p (c+dx^2)^q \, dx$

3.341.1 Optimal result

Integrand size = 19, antiderivative size = 79

$$\int (a+bx^2)^p (c+dx^2)^q \, dx = x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

output
```
x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/((1+b*
x^2/a)^p)/((1+d*x^2/c)^q)
```

3.341.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(79) = 158.

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.18

$$\int (a+bx^2)^p (c+dx^2)^q \, dx = \frac{3acx(a+bx^2)^p (c+dx^2)^q \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3ac \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right) + 2x^2 \left(bcp \text{AppellF1}\left(\frac{3}{2},1-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right) + adq \text{AppellF}\right.}$$

input
```
Integrate[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(3*a*c*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/
a), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^
2)/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q, 5/2, -((b*x^2)/a), -((d*x^2
)/c)] + a*d*q*AppellF1[3/2, -p, 1 - q, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))
```

3.341.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {334, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^p (c+dx^2)^q \, dx$$

$$\Big\downarrow \text{ 334}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \left(\frac{bx^2}{a}+1\right)^p (dx^2+c)^q dx$$

$$\Big\downarrow \text{ 334}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int \left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q dx$$

$$\Big\downarrow \text{ 333}$$

$$x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

input
```
Int[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -(
(d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

3.341.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.341.4 Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{q}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

3.341.5 Fricas [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q\,dx = \int (bx^2+a)^p (dx^2+c)^q\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.341.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q,x)
```

output
```
Timed out
```

3.341.7 Maxima [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.341.8 Giac [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q \, dx$$

input
```
int((a + b*x^2)^p*(c + d*x^2)^q,x)
```

output
```
int((a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.342 $\int \left(a+bx^2\right)^p \left(c+dx^2\right)^3 \, dx$

3.342.1 Optimal result

Integrand size = 19, antiderivative size = 296

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^3 \, dx = \frac{d(15a^2d^2-8abcd(6+p)+b^2c^2(57+28p+4p^2))\, x(a+bx^2)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)} - \frac{d(5ad-bc(11+2p))x(a+bx^2)^{1+p}\left(c+dx^2\right)}{b^2(5+2p)(7+2p)} + \frac{dx(a+bx^2)^{1+p}\left(c+dx^2\right)^2}{b(7+2p)} - \frac{(15a^3d^3-9a^2bcd^2(7+2p)+3ab^2c^2d(35+24p+4p^2)-b^3c^3(105+142p+60p^2+8p^3))\, x(a+bx^2)^p \left(1 \right.}{b^3(3+2p)(5+2p)(7+2p)}$$

output
```
d*(15*a^2*d^2-8*a*b*c*d*(6+p)+b^2*c^2*(4*p^2+28*p+57))*x*(b*x^2+a)^(p+1)/b
^3/(8*p^3+60*p^2+142*p+105)-d*(5*a*d-b*c*(11+2*p))*x*(b*x^2+a)^(p+1)*(d*x^
2+c)/b^2/(4*p^2+24*p+35)+d*x*(b*x^2+a)^(p+1)*(d*x^2+c)^2/b/(7+2*p)-(15*a^3
*d^3-9*a^2*b*c*d^2*(7+2*p)+3*a*b^2*c^2*d*(4*p^2+24*p+35)-b^3*c^3*(8*p^3+60
*p^2+142*p+105))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b^3/(8*
p^3+60*p^2+142*p+105)/((1+b*x^2/a)^p)
```

3.342.2 Mathematica [A] (verified)

Time = 7.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.46

$$\int (a+bx^2)^p (c+dx^2)^3 \, dx$$
$$= \frac{1}{35} x (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(35c^3 \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + dx^2\left(35c^2 \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right) + dx^2\left(21c \text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right) + 5dx^2 \text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)\right)\right)$$

input
```
Integrate[(a + b*x^2)^p*(c + d*x^2)^3,x]
```

output
```
(x*(a + b*x^2)^p*(35*c^3*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + d
*x^2*(35*c^2*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + d*x^2*(21*c*H
ypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)] + 5*d*x^2*Hypergeometric2F1[7
/2, -p, 9/2, -((b*x^2)/a)]))))/(35*(1 + (b*x^2)/a)^p)
```

3.342.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {318, 25, 403, 25, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^2)^3 (a+bx^2)^p \, dx$$

$$\Big\downarrow \; 318$$

$$\frac{\int -(bx^2+a)^p (dx^2+c) \left(d(5ad-bc(2p+11))x^2+c(ad-bc(2p+7))\right) dx}{b(2p+7)} + \frac{dx(c+dx^2)^2 (a+bx^2)^{p+1}}{b(2p+7)}$$

$$\Big\downarrow \; 25$$

$$\frac{dx\left(c+dx^2\right)^2\left(a+bx^2\right)^{p+1}}{b(2p+7)}-\frac{\int\left(bx^2+a\right)^p\left(dx^2+c\right)\left(d(5ad-bc(2p+11))x^2+c(ad-bc(2p+7))\right)dx}{b(2p+7)}$$

$\big\downarrow$ 403

$$\frac{dx\left(c+dx^2\right)^2\left(a+bx^2\right)^{p+1}}{b(2p+7)}-\frac{\frac{\int-\left(bx^2+a\right)^p\left(d\left(b^2\left(4p^2+28p+57\right)c^2-8abd(p+6)c+15a^2d^2\right)x^2+c\left(b^2\left(4p^2+24p+35\right)c^2-4abd(p+4)c+5a^2d^2\right)\right)dx}{b(2p+5)}+\frac{dx\left(c+dx^2\right)\left(a+bx^2\right)^{p+1}(5ad-}{b(2p+5)}}{b(2p+7)}$$

$\big\downarrow$ 25

$$\frac{dx\left(c+dx^2\right)^2\left(a+bx^2\right)^{p+1}}{b(2p+7)}-\frac{\frac{dx\left(c+dx^2\right)\left(a+bx^2\right)^{p+1}(5ad-bc(2p+11))}{b(2p+5)}-\frac{\int\left(bx^2+a\right)^p\left(d\left(b^2\left(4p^2+28p+57\right)c^2-8abd(p+6)c+15a^2d^2\right)x^2+c\left(b^2\left(4p^2+24p+35\right)c^2-4abd(p+4)c+5\right.\right.}{b(2p+5)}}{b(2p+7)}$$

$\big\downarrow$ 299

$$\frac{dx\left(c+dx^2\right)^2\left(a+bx^2\right)^{p+1}}{b(2p+7)}-\frac{\frac{dx\left(c+dx^2\right)\left(a+bx^2\right)^{p+1}(5ad-bc(2p+11))}{b(2p+5)}-\frac{\frac{dx\left(a+bx^2\right)^{p+1}\left(15a^2d^2-8abcd(p+6)+b^2c^2\left(4p^2+28p+57\right)\right)}{b(2p+3)}-\frac{\left(15a^3d^3-9a^2bcd^2(2p+7)+3ab^2c^2d\left(4p^2+24p+\right.\right.}{b(2}}{b(2p+5)}}{b(2p+7)}$$

$\big\downarrow$ 238

$$\frac{dx\left(c+dx^2\right)^2\left(a+bx^2\right)^{p+1}}{b(2p+7)}-\frac{\frac{dx\left(c+dx^2\right)\left(a+bx^2\right)^{p+1}(5ad-bc(2p+11))}{b(2p+5)}-\frac{\frac{dx\left(a+bx^2\right)^{p+1}\left(15a^2d^2-8abcd(p+6)+b^2c^2\left(4p^2+28p+57\right)\right)}{b(2p+3)}-\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(15a^3d^3-9a^2bcd^2(2p+\right.}{}}{b(2p+5)}}{b(2p+7)}$$

$\big\downarrow$ 237

$$\frac{dx\left(c+dx^2\right)^2\left(a+bx^2\right)^{p+1}}{b(2p+7)}-\frac{\frac{dx\left(c+dx^2\right)\left(a+bx^2\right)^{p+1}(5ad-bc(2p+11))}{b(2p+5)}-\frac{\frac{dx\left(a+bx^2\right)^{p+1}\left(15a^2d^2-8abcd(p+6)+b^2c^2\left(4p^2+28p+57\right)\right)}{b(2p+3)}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(15a^3d^3-9a^2bcd^2(2p-\right.}{}}{b(2p+5)}}{b(2p+7)}$$

input
```
Int[(a + b*x^2)^p*(c + d*x^2)^3,x]
```

3.342. $\int\left(a+bx^2\right)^p\left(c+dx^2\right)^3\,dx$

output
```
(d*x*(a + b*x^2)^(1 + p)*(c + d*x^2)^2)/(b*(7 + 2*p)) - ((d*(5*a*d - b*c*(
11 + 2*p))*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b*(5 + 2*p)) - ((d*(15*a^2*
d^2 - 8*a*b*c*d*(6 + p) + b^2*c^2*(57 + 28*p + 4*p^2))*x*(a + b*x^2)^(1 +
p))/(b*(3 + 2*p)) - ((15*a^3*d^3 - 9*a^2*b*c*d^2*(7 + 2*p) + 3*a*b^2*c^2*d
*(35 + 24*p + 4*p^2) - b^3*c^3*(105 + 142*p + 60*p^2 + 8*p^3))*x*(a + b*x^
2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(b*(3 + 2*p)*(1 + (b*x
^2)/a)^p))/(b*(5 + 2*p)))/(b*(7 + 2*p))
```

3.342.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.342.4 Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{3}d x$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^3,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^3,x)
```

3.342.5 Fricas [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^3 \,dx=\int \left(dx^2+c\right)^3\left(bx^2+a\right)^p\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*(b*x^2 + a)^p, x)
```

3.342.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.41

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^3 \,dx=a^p c^3 x {}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2}, -p\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)+a^p c^2 d x^3 {}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{2}, -p\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)+\frac{3a^p c d^2 x^5 {}_{2}F_{1}\left(\left.\begin{matrix}\frac{5}{2}, -p\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{5}+\frac{a^p d^3 x^7 {}_{2}F_{1}\left(\left.\begin{matrix}\frac{7}{2}, -p\\ \frac{9}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{7}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**3,x)
```

output
```
a**p*c**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*c**2
*d*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + 3*a**p*c*d**2
*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a**p*d**3*x**
7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

3.342.7 Maxima [F]

$$\int (a+bx^2)^p (c+dx^2)^3 \, dx = \int (dx^2+c)^3 (bx^2+a)^p \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^3*(b*x^2 + a)^p, x)
```

3.342.8 Giac [F]

$$\int (a+bx^2)^p (c+dx^2)^3 \, dx = \int (dx^2+c)^3 (bx^2+a)^p \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^3*(b*x^2 + a)^p, x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^p (c+dx^2)^3 \, dx = \int (b\,x^2+a)^p \, (d\,x^2+c)^3 \, dx$$

input
```
int((a + b*x^2)^p*(c + d*x^2)^3,x)
```

output
```
int((a + b*x^2)^p*(c + d*x^2)^3, x)
```

3.343 $\int (a+bx^2)^p (c+dx^2)^2 \, dx$

3.343.1 Optimal result

Integrand size = 19, antiderivative size = 176

$$\int (a+bx^2)^p (c+dx^2)^2 \, dx = -\frac{d(3ad-bc(7+2p))x(a+bx^2)^{1+p}}{b^2(3+2p)(5+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)}{b(5+2p)}$$
$$+ \frac{(3a^2d^2-2abcd(5+2p)+b^2c^2(15+16p+4p^2))\, x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2}\right.}{b^2(3+2p)(5+2p)}$$

output
```
-d*(3*a*d-b*c*(7+2*p))*x*(b*x^2+a)^(p+1)/b^2/(4*p^2+16*p+15)+d*x*(b*x^2+a)
^(p+1)*(d*x^2+c)/b/(5+2*p)+(3*a^2*d^2-2*a*b*c*d*(5+2*p)+b^2*c^2*(4*p^2+16*
p+15))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b^2/(4*p^2+16*p+1
5)/((1+b*x^2/a)^p)
```

3.343.2 Mathematica [A] (verified)

Time = 5.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

$$\int (a+bx^2)^p (c+dx^2)^2 \, dx = \frac{1}{15} x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(15c^2 \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + dx^2\left(10c \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right) + 3dx^2 \text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\right)\right)$$

input
```
Integrate[(a + b*x^2)^p*(c + d*x^2)^2,x]
```

output
```
(x*(a + b*x^2)^p*(15*c^2*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + d
*x^2*(10*c*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + 3*d*x^2*Hyperge
ometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])))/(15*(1 + (b*x^2)/a)^p)
```

3.343.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {318, 25, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^2)^2 (a+bx^2)^p\,dx$$

$$\Big\downarrow \text{318}$$

$$\frac{\int -(bx^2+a)^p \left(d(3ad-bc(2p+7))x^2+c(ad-bc(2p+5))\right)dx}{b(2p+5)}+\frac{dx(c+dx^2)(a+bx^2)^{p+1}}{b(2p+5)}$$

$$\Big\downarrow \text{25}$$

$$\frac{dx(c+dx^2)(a+bx^2)^{p+1}}{b(2p+5)}-\frac{\int (bx^2+a)^p \left(d(3ad-bc(2p+7))x^2+c(ad-bc(2p+5))\right)dx}{b(2p+5)}$$

$$\Big\downarrow \text{299}$$

$$\frac{dx(c+dx^2)(a+bx^2)^{p+1}}{b(2p+5)}-\frac{\frac{dx(a+bx^2)^{p+1}(3ad-bc(2p+7))}{b(2p+3)}-\frac{(3a^2d^2-2abcd(2p+5)+b^2c^2(4p^2+16p+15))\int (bx^2+a)^p dx}{b(2p+3)}}{b(2p+5)}$$

$$\Big\downarrow \text{238}$$

$$\frac{dx(c+dx^2)(a+bx^2)^{p+1}}{b(2p+5)}-\frac{\frac{dx(a+bx^2)^{p+1}(3ad-bc(2p+7))}{b(2p+3)}-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3a^2d^2-2abcd(2p+5)+b^2c^2(4p^2+16p+15))\int \left(\frac{bx^2}{a}+1\right)^p dx}{b(2p+3)}}{b(2p+5)}$$

$$\Big\downarrow \text{237}$$

$$\frac{dx(c+dx^2)(a+bx^2)^{p+1}}{b(2p+5)} - \frac{\frac{dx(a+bx^2)^{p+1}(3ad-bc(2p+7))}{b(2p+3)} - \frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3a^2d^2-2abcd(2p+5)+b^2c^2(4p^2+16p+15))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{b(2p+3)}}{b(2p+5)}$$

input
```
Int[(a + b*x^2)^p*(c + d*x^2)^2,x]
```

output
```
(d*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b*(5 + 2*p)) - ((d*(3*a*d - b*c*(7
+ 2*p))*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) - ((3*a^2*d^2 - 2*a*b*c*d*(5
+ 2*p) + b^2*c^2*(15 + 16*p + 4*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/
2, -p, 3/2, -((b*x^2)/a)])/(b*(3 + 2*p)*(1 + (b*x^2)/a)^p))/(b*(5 + 2*p))
```

3.343.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.343.4 Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{2}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^2,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^2,x)
```

3.343.5 Fricas [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2 \, dx = \int \left(dx^{2}+c\right)^{2}\left(bx^{2}+a\right)^{p}dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^4 + 2*c*d*x^2 + c^2)*(b*x^2 + a)^p, x)
```

3.343.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.50

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2\,dx = a^p c^2 x_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) + \frac{2a^p c d x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3} + \frac{a^p d^2 x^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**2,x)
```

output
```
a**p*c**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + 2*a**p*c*
d*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + a**p*d**2*x*
*5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

3.343.7 Maxima [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2\,dx = \int \left(dx^2+c\right)^2 \left(bx^2+a\right)^p\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2*(b*x^2 + a)^p, x)
```

3.343.8 Giac [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2\,dx = \int \left(dx^2+c\right)^2 \left(bx^2+a\right)^p\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^2*(b*x^2 + a)^p, x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2 \, dx = \int \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^2 \, dx$$

input
```
int((a + b*x^2)^p*(c + d*x^2)^2,x)
```

output
```
int((a + b*x^2)^p*(c + d*x^2)^2, x)
```

3.344 $\int (a+bx^2)^p (c+dx^2)\, dx$

3.344.1 Optimal result

Integrand size = 17, antiderivative size = 93

$$\int (a+bx^2)^p (c+dx^2)\, dx = \frac{dx(a+bx^2)^{1+p}}{b(3+2p)} - \frac{(ad-bc(3+2p))x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{b(3+2p)}$$

output
```
d*x*(b*x^2+a)^(p+1)/b/(3+2*p)-(a*d-b*c*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1
/2, -p],[3/2],-b*x^2/a)/b/(3+2*p)/((1+b*x^2/a)^p)
```

3.344.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int (a+bx^2)^p (c+dx^2)\, dx = \frac{x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(d(a+bx^2)\left(1+\frac{bx^2}{a}\right)^p + (-ad+bc(3+2p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\right.\right.}{b(3+2p)}$$

input
```
Integrate[(a + b*x^2)^p*(c + d*x^2),x]
```

output
```
(x*(a + b*x^2)^p*(d*(a + b*x^2)*(1 + (b*x^2)/a)^p + (-(a*d) + b*c*(3 + 2*p
))*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]))/(b*(3 + 2*p)*(1 + (b*x^
2)/a)^p)
```

3.344.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^2)(a+bx^2)^p\,dx$$

$$\Big\downarrow \text{299}$$

$$\left(c-\frac{ad}{2bp+3b}\right)\int (bx^2+a)^p\,dx+\frac{dx(a+bx^2)^{p+1}}{b(2p+3)}$$

$$\Big\downarrow \text{238}$$

$$(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c-\frac{ad}{2bp+3b}\right)\int \left(\frac{bx^2}{a}+1\right)^p dx+\frac{dx(a+bx^2)^{p+1}}{b(2p+3)}$$

$$\Big\downarrow \text{237}$$

$$x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c-\frac{ad}{2bp+3b}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+\frac{dx(a+bx^2)^{p+1}}{b(2p+3)}$$

input
```
Int[(a + b*x^2)^p*(c + d*x^2),x]
```

output
```
(d*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) + ((c - (a*d)/(3*b + 2*b*p))*x*(a
+ b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^
p
```

3.344.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.344.4 Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c),x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c),x)
```

3.344.5 Fricas [F]

$$\int (a+bx^2)^p\,(c+dx^2)\,dx=\int (dx^2+c)\,(bx^2+a)^p\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((d*x^2 + c)*(b*x^2 + a)^p, x)
```

3.344.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.76 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)\,dx = a^p c x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p d x^3\,{}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c),x)
```

output
```
a**p*c*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*d*x**3*
hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

3.344.7 Maxima [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)\,dx = \int \left(dx^2+c\right)\left(bx^2+a\right)^p\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(b*x^2 + a)^p, x)
```

3.344.8 Giac [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)\,dx = \int \left(dx^2+c\right)\left(bx^2+a\right)^p\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(b*x^2 + a)^p, x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right) \, dx = \int \left(b\,x^2+a\right)^p \left(d\,x^2+c\right) \, dx$$

input
```
int((a + b*x^2)^p*(c + d*x^2),x)
```

output
```
int((a + b*x^2)^p*(c + d*x^2), x)
```

3.345 $\int (a+bx^2)^p \, dx$

3.345.1 Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^2)^p \, dx = x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

output
```
x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

3.345.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^p \, dx = x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

input
```
Integrate[(a + b*x^2)^p,x]
```

output
```
(x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p
```

3.345.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^p \, dx$$

$$\Big\downarrow \; 238$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \; 237$$

$$x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[(a + b*x^2)^p,x]
```

output
```
(x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p
```

3.345.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

3.345.4 Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p}dx$$

input
```
int((b*x^2+a)^p,x)
```

output
```
int((b*x^2+a)^p,x)
```

3.345.5 Fricas [F]

$$\int \left(a+bx^2\right)^p\,dx=\int \left(bx^2+a\right)^p\,dx$$

input
```
integrate((b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p, x)
```

3.345.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \left(a+bx^2\right)^p\,dx=a^p x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**p,x)
```

output
```
a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

3.345.7 Maxima [F]

$$\int (a+bx^2)^p \, dx = \int (bx^2+a)^p \, dx$$

input
```
integrate((b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p, x)
```

3.345.8 Giac [F]

$$\int (a+bx^2)^p \, dx = \int (bx^2+a)^p \, dx$$

input
```
integrate((b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p, x)
```

3.345.9 Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int (a+bx^2)^p \, dx = \frac{x\,(b\,x^2+a)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^p}$$

input
```
int((a + b*x^2)^p,x)
```

output
```
(x*(a + b*x^2)^p*hypergeom([1/2, -p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

3.346 $\int \frac{(a+bx^2)^p}{c+dx^2}\,dx$

3.346.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^2)^p}{c+dx^2}\,dx = \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}$$

output
```
x*(b*x^2+a)^p*AppellF1(1/2,-p,1,3/2,-b*x^2/a,-d*x^2/c)/c/((1+b*x^2/a)^p)
```

3.346.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.19 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^2)^p}{c+dx^2}\,dx = -\frac{3acx(a+bx^2)^p\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)\left(-3ac\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+2x^2\left(-bcp\,\text{AppellF1}\left(\frac{3}{2},1-p,1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\right.\right.}$$

input
```
Integrate[(a + b*x^2)^p/(c + d*x^2),x]
```

output
```
(-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2)/
c)])/((c + d*x^2)*(-3*a*c*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2
)/c)] + 2*x^2*(-(b*c*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*x^2)/a), -((d*x^2
)/c)]) + a*d*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

3.346.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{c+dx^2} dx$$

$$\Big\downarrow \text{334}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \frac{\left(\frac{bx^2}{a}+1\right)^p}{dx^2+c} dx$$

$$\Big\downarrow \text{333}$$

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}$$

input
```
Int[(a + b*x^2)^p/(c + d*x^2),x]
```

output
```
(x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c
*(1 + (b*x^2)/a)^p)
```

3.346.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.346.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p}{d\,x^2+c}dx$$

input
```
int((b*x^2+a)^p/(d*x^2+c),x)
```

output
```
int((b*x^2+a)^p/(d*x^2+c),x)
```

3.346.5 Fricas [F]

$$\int \frac{(a+bx^2)^p}{c+dx^2}\,dx = \int \frac{(bx^2+a)^p}{dx^2+c}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(d*x^2 + c), x)
```

3.346.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p/(d*x**2+c),x)
```

output
```
Timed out
```

3.346.7 Maxima [F]

$$\int \frac{(a + bx^2)^p}{c + dx^2}\, dx = \int \frac{(bx^2 + a)^p}{dx^2 + c}\, dx$$

input
```
integrate((b*x^2+a)^p/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/(d*x^2 + c), x)
```

3.346.8 Giac [F]

$$\int \frac{(a + bx^2)^p}{c + dx^2}\, dx = \int \frac{(bx^2 + a)^p}{dx^2 + c}\, dx$$

input
```
integrate((b*x^2+a)^p/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/(d*x^2 + c), x)
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^2)^p}{c + dx^2}\, dx = \int \frac{(b\,x^2 + a)^p}{d\,x^2 + c}\, dx$$

input
```
int((a + b*x^2)^p/(c + d*x^2),x)
```

output
```
int((a + b*x^2)^p/(c + d*x^2), x)
```

3.347 $\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx$

3.347.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx=\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}$$

output
```
x*(b*x^2+a)^p*AppellF1(1/2,-p,2,3/2,-b*x^2/a,-d*x^2/c)/c^2/((1+b*x^2/a)^p)
```

3.347.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx=-\frac{3acx(a+bx^2)^p\operatorname{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)^2\left(-3ac\operatorname{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-2x^2\left(bcp\operatorname{AppellF1}\left(\frac{3}{2},1-p,2,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-\right.\right.}$$

input
```
Integrate[(a + b*x^2)^p/(c + d*x^2)^2,x]
```

output
```
(-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2)/
c)])/((c + d*x^2)^2*(-3*a*c*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x
^2)/c)] - 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, 2, 5/2, -((b*x^2)/a), -((d*x^2
)/c)] - 2*a*d*AppellF1[3/2, -p, 3, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

3.347.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx$$

$$\Big\downarrow \; 334$$

$$(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{(dx^2+c)^2}dx$$

$$\Big\downarrow \; 333$$

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}$$

input
```
Int[(a + b*x^2)^p/(c + d*x^2)^2,x]
```

output
```
(x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c
^2*(1 + (b*x^2)/a)^p)
```

3.347.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.347.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p}{(d\,x^2+c)^2}dx$$

input
```
int((b*x^2+a)^p/(d*x^2+c)^2,x)
```

output
```
int((b*x^2+a)^p/(d*x^2+c)^2,x)
```

3.347.5 Fricas [F]

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^p}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

3.347.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.347.7 Maxima [F]

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^p}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/(d*x^2 + c)^2, x)
```

3.347.8 Giac [F]

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^p}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/(d*x^2 + c)^2, x)
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^p}{(d\,x^2+c)^2}\,dx$$

input
```
int((a + b*x^2)^p/(c + d*x^2)^2,x)
```

output
```
int((a + b*x^2)^p/(c + d*x^2)^2, x)
```

3.348 $\int \frac{(a+bx^2)^p}{(c+dx^2)^3} dx$

3.348.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3} dx = \frac{x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}$$

output
```
x*(b*x^2+a)^p*AppellF1(1/2,-p,3,3/2,-b*x^2/a,-d*x^2/c)/c^3/((1+b*x^2/a)^p)
```

3.348.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3} dx =$$

$$-\frac{3acx(a+bx^2)^p \text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)^3 \left(-3ac\,\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-2x^2\left(bcp\,\text{AppellF1}\left(\frac{3}{2},1-p,3,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-\right.\right.}$$

input
```
Integrate[(a + b*x^2)^p/(c + d*x^2)^3,x]
```

output
```
(-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2)/
c)])/((c + d*x^2)^3*(-3*a*c*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x
^2)/c)] - 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, 3, 5/2, -((b*x^2)/a), -((d*x^2
)/c)] - 3*a*d*AppellF1[3/2, -p, 4, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

3.348.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3} \, dx$$

$$\Big\downarrow \text{ 334}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \frac{\left(\frac{bx^2}{a}+1\right)^p}{(dx^2+c)^3}dx$$

$$\Big\downarrow \text{ 333}$$

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}$$

input
```
Int[(a + b*x^2)^p/(c + d*x^2)^3,x]
```

output
```
(x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c
^3*(1 + (b*x^2)/a)^p)
```

3.348.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.348.4 Maple [F]

$$\int \frac{(b\,x^{2}+a)^{p}}{(d\,x^{2}+c)^{3}}dx$$

input
```
int((b*x^2+a)^p/(d*x^2+c)^3,x)
```

output
```
int((b*x^2+a)^p/(d*x^2+c)^3,x)
```

3.348.5 Fricas [F]

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^p}{(dx^2+c)^3}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

3.348.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.348.7 Maxima [F]

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^p}{(dx^2+c)^3}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/(d*x^2 + c)^3, x)
```

3.348.8 Giac [F]

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^p}{(dx^2+c)^3}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/(d*x^2 + c)^3, x)
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx = \int \frac{(b\,x^2+a)^p}{(d\,x^2+c)^3}\,dx$$

input
```
int((a + b*x^2)^p/(c + d*x^2)^3,x)
```

output
```
int((a + b*x^2)^p/(c + d*x^2)^3, x)
```

3.349 $\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}} dx$

3.349.1 Optimal result

Integrand size = 50, antiderivative size = 53

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}} dx = \frac{x\left(a+bx^2\right)^{-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

output
```
x*(d*x^2+c)^(a*d/(-2*a*d+2*b*c))/a/c/((b*x^2+a)^(b*c/(-2*a*d+2*b*c)))
```

3.349.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}} dx = \frac{x\left(a+bx^2\right)^{\frac{bc}{-2bc+2ad}} \left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

input
```
Integrate[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-1 + (a*d)
/(2*b*c - 2*a*d)),x]
```

output
```
(x*(a + b*x^2)^((b*c)/(-2*b*c + 2*a*d))*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d)
))/(a*c)
```

3.349.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {295}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{-\frac{bc}{2bc-2ad}-1} \left(c+dx^2\right)^{\frac{ad}{2bc-2ad}-1} \, dx$$

$$\Big\downarrow \ 295$$

$$\frac{x\left(a+bx^2\right)^{-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

input
```
Int[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-1 + (a*d)/(2*b*
c - 2*a*d)),x]
```

output
```
(x*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d)))/(a*c*(a + b*x^2)^((b*c)/(2*b*c - 2
*a*d)))
```

3.349.3.1 Defintions of rubi rules used

rule 295
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*c)), x] /; FreeQ[{a, b, c,
d, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && EqQ[a*d*(
p + 1) + b*c*(q + 1), 0]
```

3.349.4 Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

method	result	size
gosper	$\frac{x\left(b\,x^{2}+a\right)^{1-\frac{2ad-3bc}{2(ad-bc)}} \left(d\,x^{2}+c\right)^{1-\frac{3ad-2bc}{2(ad-bc)}}}{ac}$	71

input
```
int((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c)),x,
method=_RETURNVERBOSE)
```

output
```
x/a/c*(b*x^2+a)^(1-1/2*(2*a*d-3*b*c)/(a*d-b*c))*(d*x^2+c)^(1-1/2*(3*a*d-2*
b*c)/(a*d-b*c))
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.72

$$\int (a+bx^2)^{-1-\frac{bc}{2bc-2ad}} (c+dx^2)^{-1+\frac{ad}{2bc-2ad}}\,dx = \frac{bdx^5+(bc+ad)x^3+acx}{(bx^2+a)^{\frac{3\,bc-2\,ad}{2\,(bc-ad)}}(dx^2+c)^{\frac{2\,bc-3\,ad}{2\,(bc-ad)}}ac}$$

input
```
integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*
c)),x, algorithm="fricas")
```

output
```
(b*d*x^5 + (b*c + a*d)*x^3 + a*c*x)/((b*x^2 + a)^(1/2*(3*b*c - 2*a*d)/(b*c
 - a*d))*(d*x^2 + c)^(1/2*(2*b*c - 3*a*d)/(b*c - a*d))*a*c)
```

3.349.6 Sympy [F(-1)]

Timed out.

$$\int (a+bx^2)^{-1-\frac{bc}{2bc-2ad}} (c+dx^2)^{-1+\frac{ad}{2bc-2ad}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(-1-b*c/(-2*a*d+2*b*c))*(d*x**2+c)**(-1+a*d/(-2*a*d+
2*b*c)),x)
```

output
```
Timed out
```

3.349.7 Maxima [F]

$$\int (a+bx^2)^{-1-\frac{bc}{2bc-2ad}} (c+dx^2)^{-1+\frac{ad}{2bc-2ad}}\,dx = \int (bx^2+a)^{-\frac{bc}{2\,(bc-ad)}-1}(dx^2+c)^{\frac{ad}{2\,(bc-ad)}-1}\,dx$$

input
```
integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*
c)),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c
 - a*d) - 1), x)
```

3.349.8 Giac [F]

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}}\,dx=\int \left(bx^2+a\right)^{-\frac{bc}{2\,(bc-ad)}-1} \left(dx^2+c\right)^{\frac{ad}{2\,(bc-ad)}-1}\,dx$$

input
```
integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*
c)),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c
 - a*d) - 1), x)
```

3.349.9 Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.47

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}}\,dx$$
$$=\frac{x\left(b\,x^2+a\right)^{\frac{b\,c}{2\,a\,d-2\,b\,c}-1}+\frac{x^3\,\left(b\,x^2+a\right)^{\frac{b\,c}{2\,a\,d-2\,b\,c}-1}\,\left(a\,d+b\,c\right)}{a\,c}+\frac{b\,d\,x^5\,\left(b\,x^2+a\right)^{\frac{b\,c}{2\,a\,d-2\,b\,c}-1}}{a\,c}}{\left(d\,x^2+c\right)^{\frac{a\,d}{2\,a\,d-2\,b\,c}+1}}$$

input
```
int((a + b*x^2)^((b*c)/(2*a*d - 2*b*c) - 1)/(c + d*x^2)^((a*d)/(2*a*d - 2*
b*c) + 1),x)
```

output
```
(x*(a + b*x^2)^((b*c)/(2*a*d - 2*b*c) - 1) + (x^3*(a + b*x^2)^((b*c)/(2*a*
d - 2*b*c) - 1)*(a*d + b*c))/(a*c) + (b*d*x^5*(a + b*x^2)^((b*c)/(2*a*d -
2*b*c) - 1))/(a*c))/(c + d*x^2)^((a*d)/(2*a*d - 2*b*c) + 1)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```